Chiral lattice gauge theories from warped domain walls and Ginsparg-Wilson fermions

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+ work in progress with Joel Giedt (Minneapolis)

why lattice?

QFT tools of the trade...

- perturbation theory

- semiclassical expansions

- 1/N

... (very useful!) divergent expansions of something:

- the thing that is the **nonperturbative definition** of the theory

why do we need a nonperturbative definition
if various expansions work so well?

well, they do not always work (QCD)

by "nonperturbative definition,"
we mean something like:

arbitrary* Green's functions = an object that
a.) exists
b.) can, in principle, be exactly calculated

*not only a class, such as chiral, single-trace, etc...

known nonperturbative definitions:

constructive field theory

quite abstract:
Osterwalder-Schrader "reconstruction theorem:" Euclidean Weightman functions with right properties ("OS positivity") allow reconstruction of positive norm Hilbert space

string theory

- needs its own nonperturbative definition
- can be useful if enough symmetries around
- fairly helpless in non-supersymmetric situations

lattice field theory

very useful in vectorlike gauge theories (QCD)

the only one well-suited for generic QFTs

but:

breaks global symmetries:
- Poincare...
- chiral (*if naive!*)...
- supersymmetry...

at its best in Euclidean...
does not include gravity...

why formulate non-QCD like theories on the lattice?

(apart as a purely theoretical excersize)

- standard model is a chiral gauge theory

(weakly coupled, so no really strong incentive to bother...)

- extensions of the standard model?

if weakly coupled, also no strong reason

if strong dynamics is shown to be relevant, the issue of non-QCD like theories on the lattice will become more prominant strong dynamics can be relevant in many ways:

supersymmetric extensions: dynamical supersymmetry breaking... some progress in lattice supersymmetry in latter years, but limited to extended SUSY theories [vectorlike by nature]

reviewed at Santa Fe 2004 Workshop

strong electroweak breaking-renaissance as AdS/EWSB (a.k.a. RS)

- weak coupling duals of large-N vectorlike theories [no notion of large-N in chiral gauge theories]; fundamental dual 4d description strong

- other not-yet-imagined-not-large-N-not-QCD-like dynamics???

strong chiral gauge dynamics remains largely mysterious

- in non-SUSY case only tools are ‘t Hooft anomaly matching and MAC

- analytic methods, like large-N expansions, incl. recent “AdS/QCD dualities” do not apply to chiral case: eg SU(5) with 10 and 5*; SU(6) with 15 and 2x6* etc...

- further progress in understanding interesting supersymmetric theories on the lattice is tied to the chiral gauge theories problem

however:

numerical or analytic methods using the lattice face the difficulty of preserving chiral symmetries on the lattice

Nielsen-Ninomiya theorem

quickest argument:
if exact, gauge it, but where would anomalies come from?

there has been striking progress in understanding lattice (global) chiral symmetries in the last 10 years

we will make use of these developments:

Ginsparg and Wilson '82
D.B. Kaplan '92
Narayanan and Neuberger '94-5
Neuberger '97-8
Hasenfratz, Niedermayer '98
Luscher '99-'00

plan - present two new proposals:

1 domain wall and "waveguide" models
& their failure to obtain chiral spectrum

2 the use of warped domain walls Bhattacharya, Csaki, Martin, Shirman, Terning '05

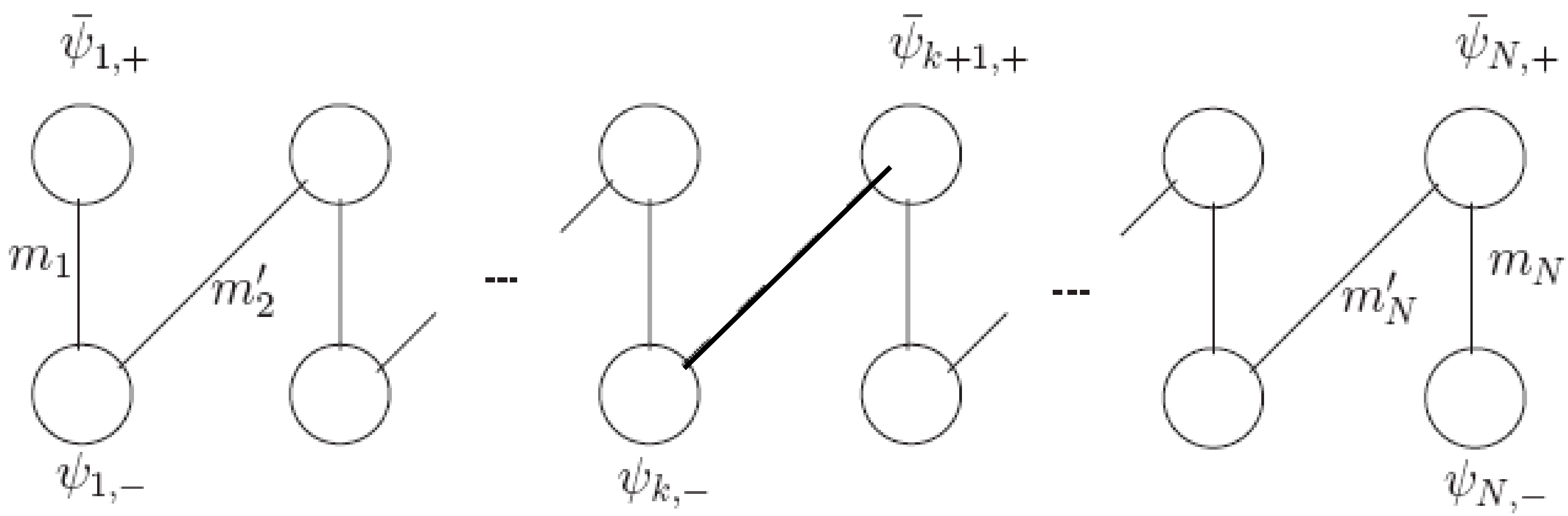
3 a proposal using Ginsparg-Wilson mechanism
to impose a modified exact lattice chiral symmetry
incl. recent analytical and numerical results supporting it!

4 outlook and remaining issues

- domain wall and "waveguide" models
& their failure to obtain chiral spectrum

lattice domain wall fermions D.B. Kaplan '92

Shamir's realization '93:



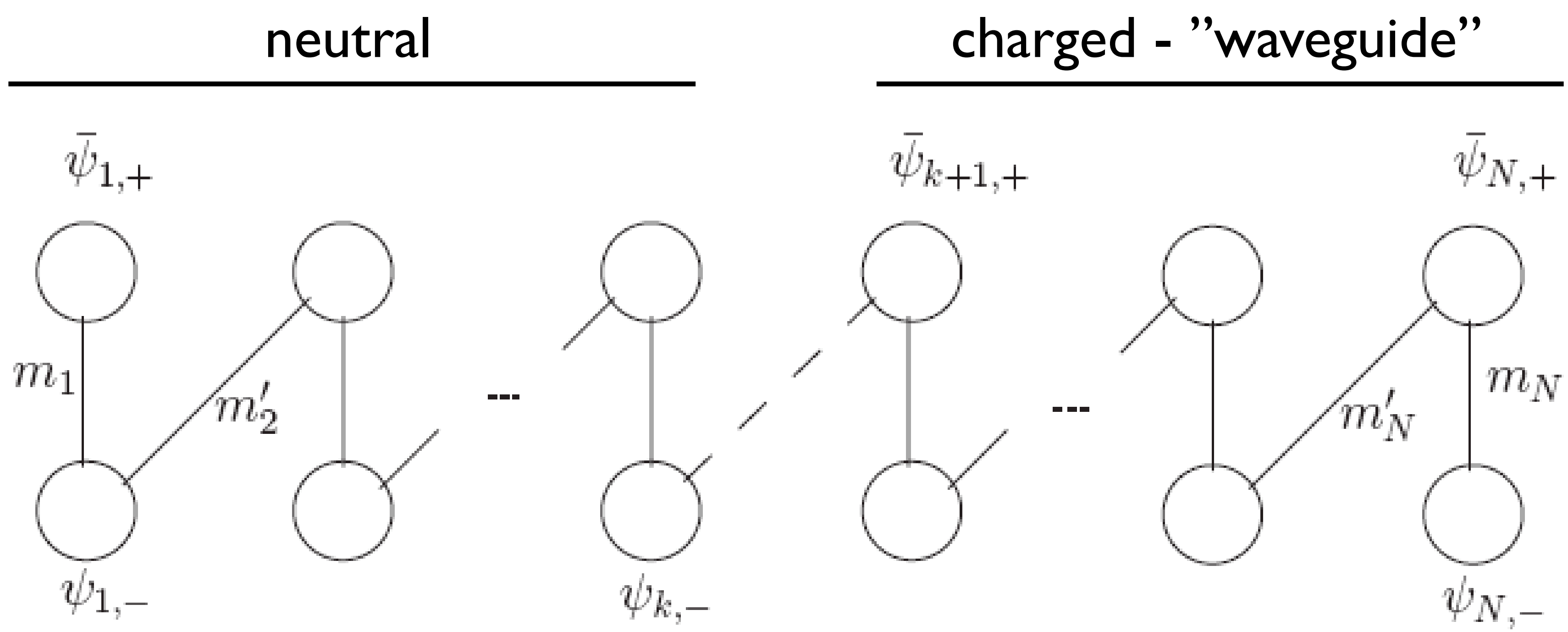
vectorlike gauge theory with exponentially light Dirac fermion;
becomes massless at infinite N, where chiral symmetry restored

- domain wall and "waveguide" models
  & their failure to obtain chiral spectrum

waveguide domain wall fermions

D.B. Kaplan '92

want: **A.) unbroken gauge theory**
B.) chiral light spectrum



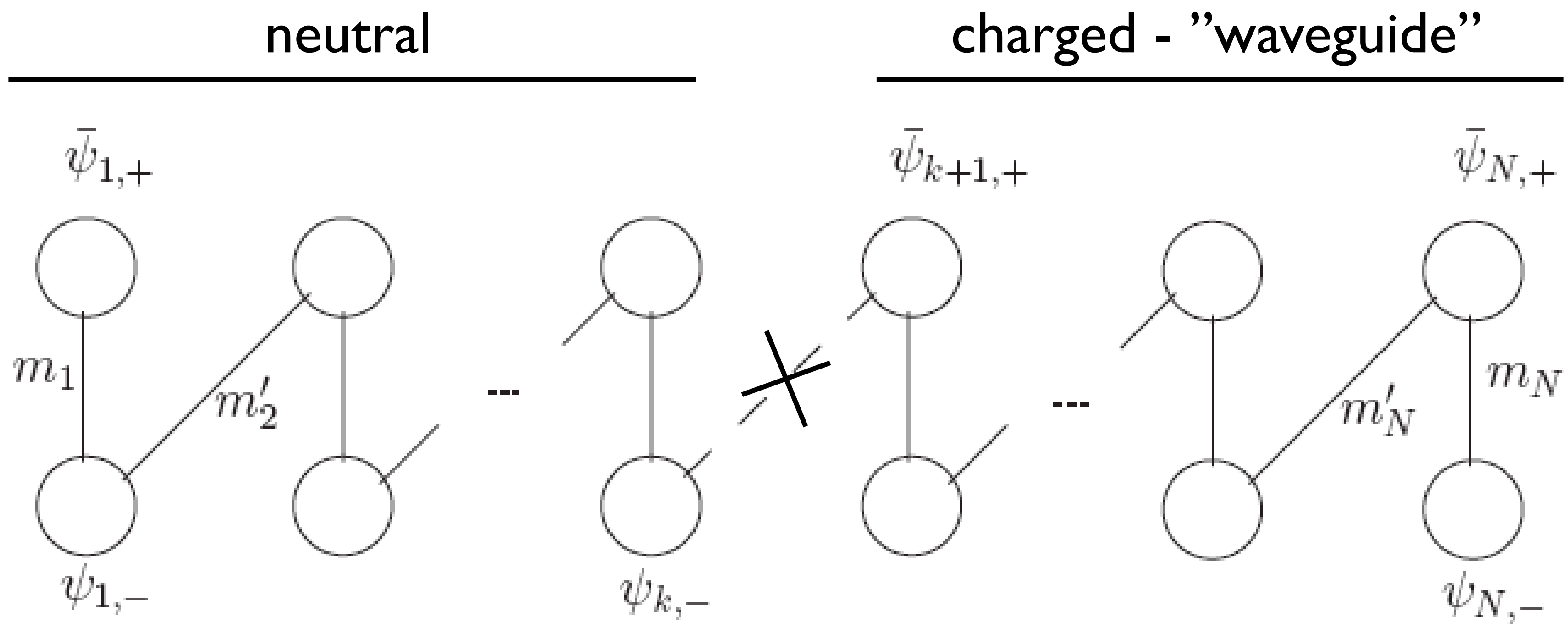
$$y\bar{\psi}_{k+1,+}\phi\psi_{k,-}$$

domain wall and "waveguide" models
& their failure to obtain chiral spectrum

waveguide at small Yukawa coupling

Golterman, Jansen, Petcher, Vink '93

- vectorlike fermion spectrum in the symmetric phase

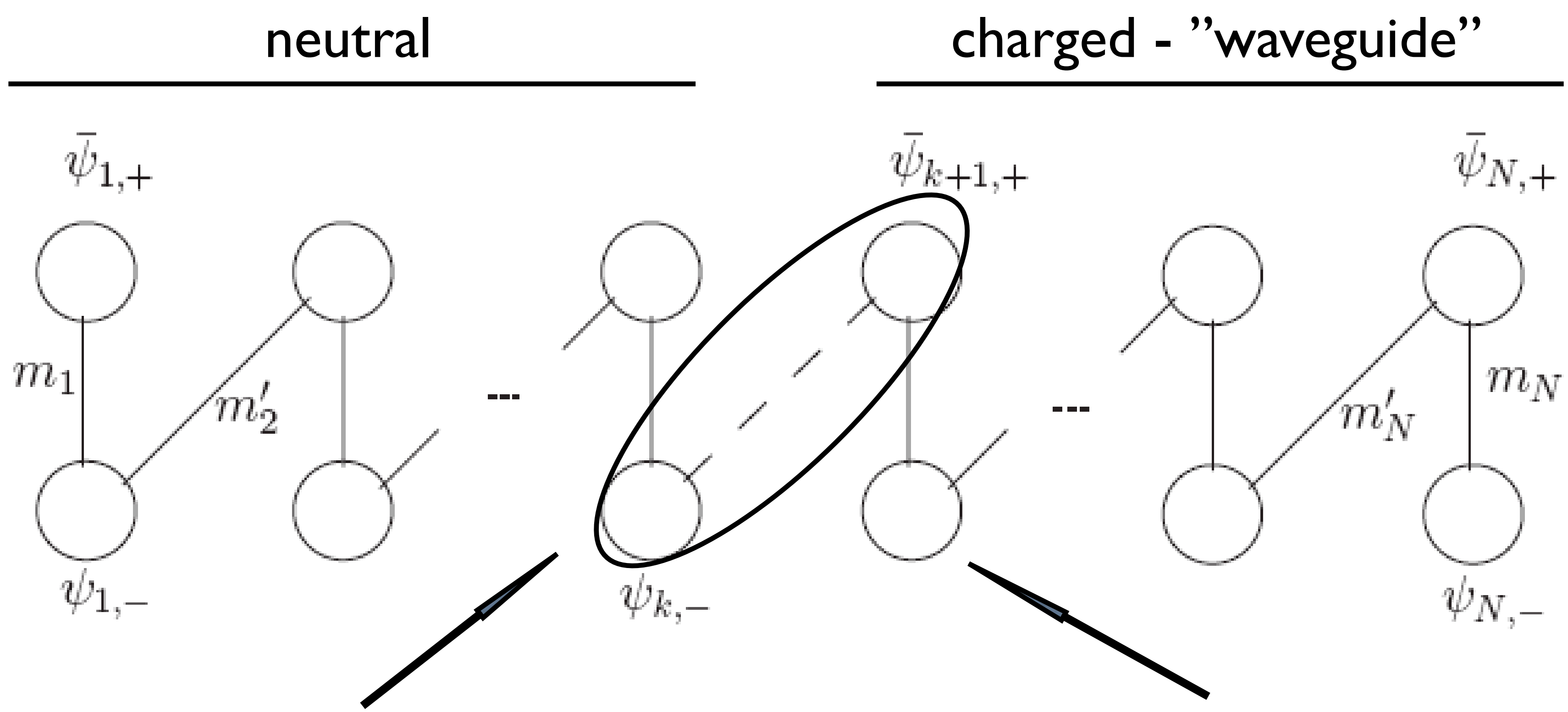


$$y\bar{\psi}_{k+1,+}\phi\psi_{k,-}$$

- domain wall and "waveguide" models
  & their failure to obtain chiral spectrum

waveguide at strong Yukawa - symmetric phase:

Golterman, Shamir '94



decouple at large y

$$y\bar\psi_{k+1,+}\phi\psi_{k,-}$$

charged massless doublers
due to lost Wilson term

vectorlike spectrum again!

1 domain wall and "waveguide" models
& their failure to obtain chiral spectrum

so far: waveguide doesn't work at both weak and strong Yukawa coupling

"mirror" fermion and gauge boson mass both determined by Higgs vev; in the symmetric phase "mirror" becomes massless:

weak Yukawa proposal:
2 the use of warped domain walls

Bhattacharya, Csaki, Martin, Shirman, Terning '05

extra "mirrors" appear near boundary because of loss of Wilson term of nearest neighbor in the bulk

strong Yukawa proposal

3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

so far, gauge field was purely 4d (not an extra dimension, rather 4d YM with N flavors)

introduce curvature and make gauge field 5d

in AdS fermion zero modes have similar localization properties
"mirror" fermion mass determined by Higgs, as in waveguide

Bhattacharya, Csaki, Martin, Shirman, Terning '05

if gauge field in curved space (AdS) gauge mass independent of Higgs vev

$$m_{A_0}^2 = \frac{2}{R'^2}\frac{1}{\ln(R'/R)}\left(1+\mathcal{O}\left(\frac{1}{\ln(R'/R)}\right)\right)$$

while

$$m_{KK} \equiv \frac{\pi}{R'}$$

thus can decouple gauge boson and "mirror" fermion masses

take a limit where $m_{KK} \gg \Lambda_{\chi GT} \gg m_{A_0}$ while keeping mirror fermion massive

get massless chiral spectrum as

$$m_{KK} \to \infty$$
$$m_{A_0} \to 0$$

does it work?

deconstructed AdS_5 version was studied in detail
found strong goldstone mode/fermion coupling

Bhattacharya, Csaki, Martin, Shirman, Terning '05

- is analysis of spectrum valid, then?

then, clearly, 4-dim case requires a lattice simulation to settle

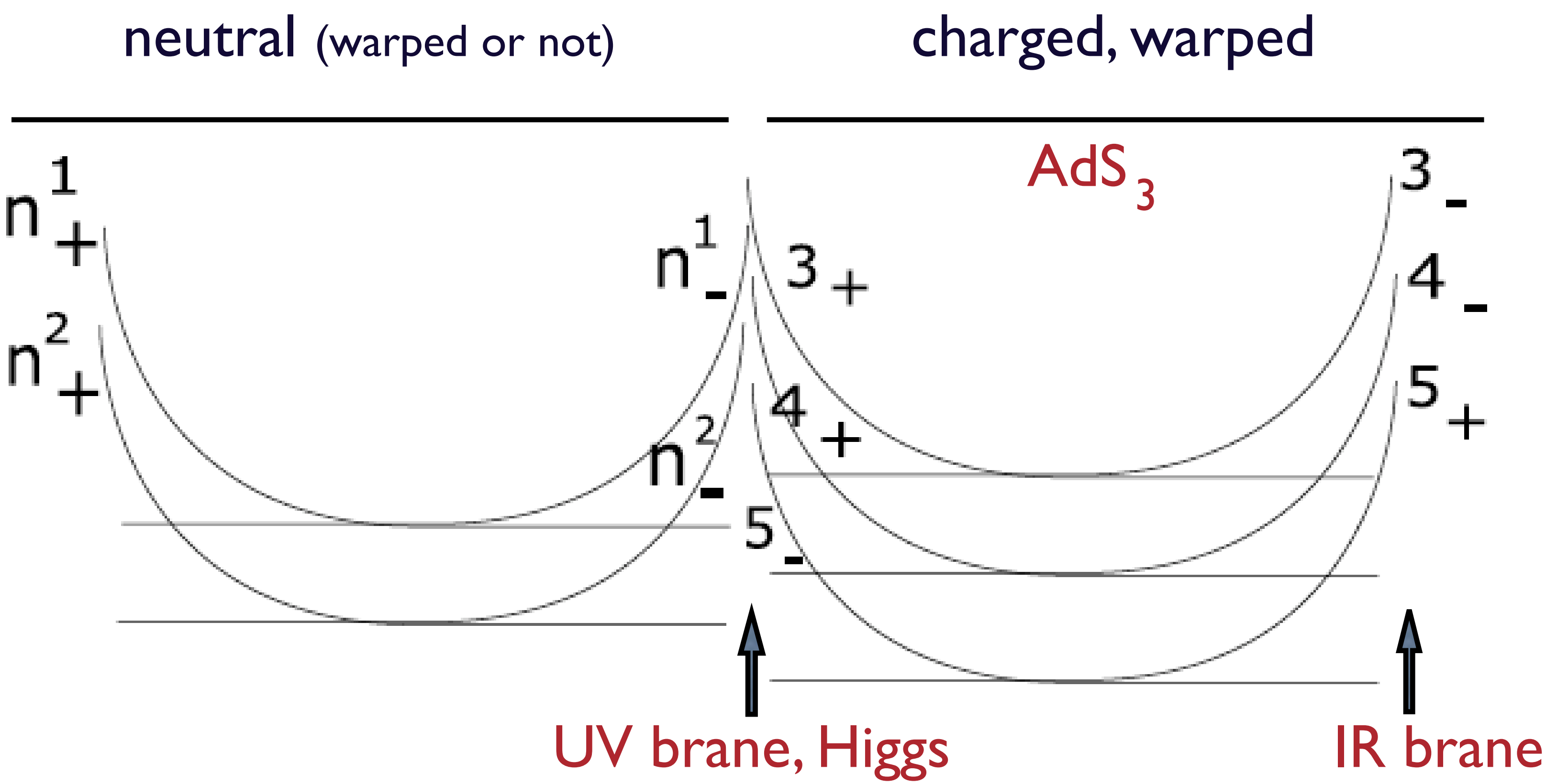
we took on a less ambitious (and easier) task: study the 2-dim case

- 2-dim chiral gauge theories are of interest on their own
- numerical simulation easier in 2 dimensions
- so it is of interest to have a formulation for AdS_3
- various 2d subtleties and differences... (will skip)

2-dim chiral theory: U(1) "345" theory $3_-, 4_-, 5_+$ chiral matter

133 global U(1) anomaly free

111 global U(1) anomalous, 't Hooft vertex $(3_-)^3\,\partial_+(4_-)^4\,(\bar{5}_+)^5$



(exponentially light modes shown; 345 symmetry; 133 broken - IR restoration?)

all couplings are weak, so perturbative analysis self-consistent to conclude the discussion of "warped domain walls" proposal:

- has better chances in 2d than 4d...
- 2d doesn't teach us if it will work in 4d...
  ... but would certainly make one more hopeful!

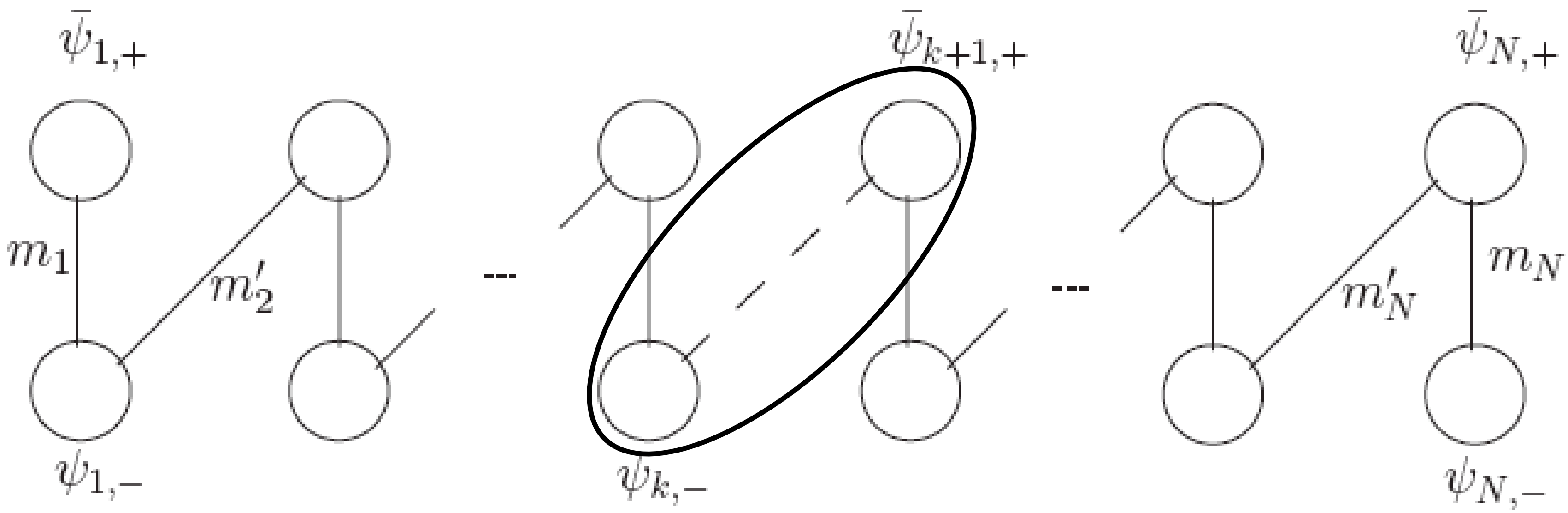
remains to be done:

- full lattice action
- restoration of all global symmetries? (133)
- simulation: 3d, many couplings...

3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

recall reason for failure to obtain vectorlike spectrum of strong coupling "waveguide"

neutral ——— charged - "waveguide"



$$y\bar\psi_{k+1,+}\phi\psi_{k,-}$$

at large y $\quad \psi_{k+1,+} \to \frac{1}{\sqrt{y}}\psi_{k+1,+}$

charged massless doublers due to lost Wilson term

$$\bar\psi_{k+1,-}\gamma\cdot D\psi_{k+1,-} + ra\bar\psi_{k+1,+}D^2\psi_{k+1,-} \to \bar\psi_{k+1,-}\gamma\cdot D\psi_{k+1,-} + \cancel{\frac{ra}{\sqrt{y}}}\bar\psi_{k+1,+}D^2\psi_{k+1,-}$$

hence, +/- mixing in Wilson term is source of problem!

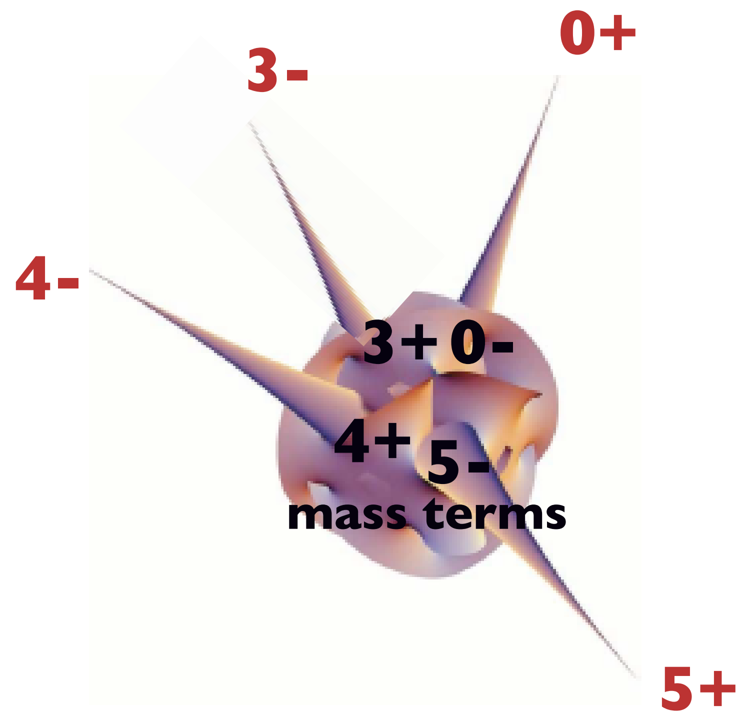
3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

what if we use fermions where +/- mixing does not happen?

Ginsparg-Wilson fermions obey $\bar{\psi} D^{GW} \psi = \bar{\psi}_+ D^{GW} \psi_+ + \bar{\psi}_- D^{GW} \psi_-$ while having no doublers

"345" theory fields: 3- 4- 5+ 0+

mirrors: 3+ 4+ 5- 0-



add Dirac and Majorana masses for mirrors break $U(1)^8$ exact lattice chiral symmetry to 345, 133, and 111 and $U(1)_0$

345, 133: anomaly free exact!
correct lattice anomalous WI

strong Yukawa symmetric phase

rescaling mirrors by $\frac{1}{\sqrt{y}}$ causes no appearance of doublers

3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

will discuss this one in more detail:

- it is a full *lattice* proposal (all spacetime is discrete)
- it is formulated in both 2d and 4d (2d simulations on the "fringe" possible)
- global symmetries, *including anomalous ones*, are realized exactly as expected in continuum theory

proposal is based on:

- Ginsparg-Wilson fermions and exact lattice chiral symmetry
- *"Foerster-Nielsen-Ninomiya" (FNN)* mechanism
- strong Yukawa symmetric phase
- *it is testable,* at a smaller cost...

3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

ingredient #1: *Ginsparg-Wilson relation [GW] 1982*

$$\{D_q, \gamma_5\} = D_q \gamma_5 D_q$$

but what is D? - Neuberger 1997

chiral projectors: $P_\pm = (1 \pm \gamma_5)/2$

define:

$$\hat{\gamma}_5 \equiv (1 - D)\gamma_5$$

$$\hat{\gamma}_5^2 = 1$$

(hence $\hat{P}_\pm = (1 \pm \hat{\gamma}_5)/2$ is a projector)

then, there is an exact chiral symmetry (GW, 1982; formulation of Luscher, 1999)

$$\Psi \to e^{i\alpha\gamma_5}\Psi \qquad \bar{\Psi} \to \bar{\Psi} e^{i\alpha\hat{\gamma}_5}$$

of lattice action

$$S_{kin} = \sum_{x,y} \bar{\Psi}_x D_{qxy} \Psi_y$$

since GW implies

$$\hat{\gamma}_5 D_q = -D_q \gamma_5$$

note that, really, we have $\bar{\Psi}_x \to \sum_{x'} \bar{\Psi}_{x'} \left(e^{i\alpha\hat{\gamma}_5}\right)_{x'x}$

3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

$$\Psi_q \longrightarrow e^{i\alpha_{q,\pm} P_\pm} \Psi_q$$
$$\bar{\Psi}_q \longrightarrow \bar{\Psi}_q e^{-i\alpha_{q,\pm} \hat{P}_\mp}$$

global L and R symmetries of action $U(1)_{q,-} \times U(1)_{q,+}$

field dependence of transformation leads to Jacobian (vanishes for vector)

$$\left[1 \pm i\alpha_{q,\pm} \mathrm{Tr}\left(\gamma_5 - \frac{1}{2} D_q \gamma_5\right)\right]$$

then properties of D are useful to (easily, really!) to show that:

$$\mathrm{Tr}(\gamma_5 - \tfrac{1}{2} D_q \gamma_5) = n_+^0 - n_-^0$$

moral:

exact lattice chiral symmetry (not usual one for all modes!), exact (anomalous) Ward identities, axial charge violation, etc... in vectorlike theories! **big success!**

3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

- tested extensively in Schwinger model (2d), works beautifully

- still expensive to run in 4d QCD because of non-sparseness of GW D

but recall our desire is not to study QCD:

start from vectorlike theory;
decouple mirrors;
get unbroken chiral theory -

- how can we do that?

3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

ingredient #2:

Fradkin, Shenker '79 *Phase diagrams of lattice field theories with Higgs fields*

Foerster, Nielsen, Ninomiya '80 *Creation of light from chaos: dynamical stability of local gauge symmetry*

(really, old work, '71, of F. Wegner on Z2 gauge theory...)

unitary Higgs field on the lattice

$$\frac{\kappa}{2}\sum_{x}\sum_{\hat{\mu}}[2-(\phi(x)^{*}U(x,\hat{\mu})\phi(x+\hat{\mu})+\mathrm{h.c.})]$$

symmetric phase $\kappa < \kappa_c$ = strong coupling (high-T expansion)
...disorder, small correlation length...
integrate out... irrelevant at large scales...

same for any compact gauge group: from Z_2 to U(N)...

moral:

local gauge-breaking terms on the lattice are harmless, provided they are small enough (exactly how small: must work out in each case!)

slide added after talk:

From the many questions I got, I want to make clear that the presence of a unitary higgs field \phi(x) does not mean that symmetry is broken.

We usually think of slowly varying unitary fields, when that is true (broken phase, \kappa > \kappa_c.

However, in the symmetric phase, <\phi(x)>=0 because of the uncorrelated fluctuations of \phi on neighboring sites. In other words, \phi is a random variable.

3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

finally, can explain proposal:

$$S_{kin} = \sum_{q=0,3,4,5} \sum_{x,y} \bar{\Psi}_q(x) D_q(x,y) \Psi_q(y)$$

"345" theory fields: 3- 4- 5+ 0+
mirrors: 3+ 4+ 5- 0-

8 global chiral U(1)s are symmetries of S_kin:

$$\prod_{q=0,3,4,5} U(1)_{q,-} \times U(1)_{q,+}$$

while target 3-4-5 theory has following exact classical ones

$$U(1)_{3,-} \times U(1)_{4,-} \times U(1)_{5,+} \times U(1)_{0,+}$$

introduce chiral components: $\Psi_\pm = P_\pm \Psi$ $\bar{\Psi}_\pm = \bar{\Psi} \hat{P}_\mp$

include Yukawa couplings involving mirrors that violate all unwanted U(1)s

e.g.: $\bar{\Psi}_{0,-} (\phi^*)^3 \Psi_{3,+}$ (Dirac) and $\Psi^T_{3,+} \gamma_2 (\phi^*)^8 \Psi_{5,-}$ (Majorana)

3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

from the remaining classical symmetries of the action

$$U(1)_{3,-} \times U(1)_{4,-} \times U(1)_{5,+} \times U(1)_{0,+}$$

three, 345, 133, U(1)_0, are exact and one is anomalous, 111, obeying an exact anomalous Ward identity:

$$\langle \delta_{\alpha_{111}} \mathcal{O} \rangle = i\, \frac{\alpha}{2}\, \langle \mathcal{O}\ \mathrm{Tr}\left[\gamma_5 (D_3 + D_4 - D_5)\right] \rangle$$

- this completes the definition of the model.

but does it behave as we want it to?
(symmetries vs. dynamics)

- heavy mirrors
- massless chiral matter
- unbroken gauge symmetry

we don't have proof...
... but only "evidence"...

3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

$$S = S_{Wilson} + S_{kin} + S_{mass} + \frac{\kappa}{2} \sum_x \sum_{\hat{\mu}} [2 - (\phi(x)^* U(x, \hat{\mu}) \phi(x + \hat{\mu}) + \text{h.c.})]$$

since for GW fermions $\bar{\Psi}_q D_q \Psi_q = \bar{\Psi}_{q,+} D_q \Psi_{q,+} + \bar{\Psi}_{q,-} D_q \Psi_{q,-}$

no coupling of mirror and light states via the strong Yukawas
=> cause of trouble (i.e., vectorlike spectrum of light states in the symmetric phase!) for all previous Higgs/Yukawa attempts, e.g. waveguide...

except: Hernandez/Sundrum two-cutoff proposal, '96, which has its own complications

as a result, since also

$d\Psi = d\Psi_+ d\Psi_-$ when g=0 partition function factorizes!

$$Z = Z_{light} \times Z_{mirror}$$

3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

$$Z_{light} = \int \prod_x d\Psi_{3,-} d\Psi_{4,-} d\Psi_{5,+} d\Psi_{0,+} e^{-S_{kin}(\Psi^{light})}$$

$$Z_{mirror} = \int \prod_x d\Psi_{3,+} d\Psi_{4,+} d\Psi_{5,-} d\Psi_{0,-} d\phi$$
$$\times \quad e^{-S_{kin}^{mirror}(\Psi^{mirror}) - S_{\kappa}(\phi) - S_{mass}(\Psi^{mirror})}$$

problem at hand:
study Z_mirror dynamics (enough, for g = 0)
is there a phase, where as $\kappa \to 0 \quad \lambda \to \infty$

one finds that scalar has small O(a) correlation length [=symmetric phase, no gauge boson mass] and mirrors are heavy?

3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

$$Y_- = \begin{pmatrix} \Psi_{5,-} \\ \bar{\Psi}^T_{5,-} \\ \Psi_{0,-} \\ \bar{\Psi}^T_{0,-} \end{pmatrix}$$

quick argument: $$S_{mass} = \lambda \sum_x X_+(x) M Y_-(x)$$

$$X_+ = (\Psi^T_{3,+}\ \bar{\Psi}_{3,+}\ \Psi^T_{4,+}\ \bar{\Psi}_{4,+})$$

M(x) contains powers of unitary Higgs field to make it gauge invariant

- at infinite Yukawa, drop kinetic term
- mirror determinant = product of dets at each x
- it is "gauge" symmetric and local (x), hence Higgs independent (since no local gauge invariant out of unitary Higgs)

=> fermions are:

a.) heavy and

b.) do not effect unitary Higgs dynamics and do not drive Higgs into ordered ("low-T") phase: **if they did, this would mean that they generated a large kinetic term for Higgs [= gauge boson mass term, once g is turned on] requiring fine-tuning of, possibly infinitely many, operators to obtain massless gauge bosons => drop the proposal!**

3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

too quick!

important quick claim was:
- mirror determinant = product of dets at each x

$$S_{mass} = \lambda \sum_x X_+(x) M Y_-(x)$$

elegant, but misleading... recall:

$$\bar{\Psi}_\pm = \bar{\Psi} \hat{P}_\mp$$

is actually somewhat smeared: $\bar{\Psi}_{\pm,x} = \sum_{x'} \bar{\Psi}_{x'} \left(\hat{P}_\mp\right)_{x'x}$

and so mirror determinant, even at infinite Yukawa, depends on Higgs

so, does it tend to order Higgs fluctuations?

[= induce large gauge breaking terms]

found no workable *analytic only* expansion ....combine with "experiment:"

3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

current work with Joel Giedt g=0 Higgs-GWfermion-Yukawa model:

analytic:

proper definition of measure (nontrivial because of smearing!)

$$\text{“}d\Psi = d\Psi_+ d\Psi_-\text{”}$$

and corresponding split of light and mirror action...

numerical:

simulation with backreaction of mirror fermions and study of the scalar correlation length

- fermion det turns out to be positive for Majorana > Dirac
- vanishes at Maj. = Dirac, does change sign for Maj. < Dirac

- remains small, O(a), in infinite Yukawa limit...

3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

$$S^{kin}_{mirror} = \sum_{\mathbf{k}} \lambda_{\mathbf{k}} \left(\bar{\alpha}_{\mathbf{k}-} \alpha_{\mathbf{k}-} + \bar{\beta}_{\mathbf{k}+} \beta_{\mathbf{k}+} \right) .$$

$$\frac{1}{\lambda} S^{Dirac}_{mirror} = \frac{1}{2} \sum_{\mathbf{k},\mathbf{p}} (2 - \lambda_{\mathbf{k}}) \left(\bar{\alpha}_{\mathbf{k}-} \beta_{\mathbf{p}+} \Phi^*_{\mathbf{k}-\mathbf{p}} + \bar{\beta}_{\mathbf{k}+} \alpha_{\mathbf{p}-} \Phi_{\mathbf{k}-\mathbf{p}} e^{i(\phi_{\mathbf{p}} - \phi_{\mathbf{k}})} \right) ,$$

$$\frac{1}{\lambda'} S^{Majorana}_{mirror} =$$

$$i \sum_{\mathbf{k},\mathbf{p}} \left[\alpha_{\mathbf{k}-} \beta_{\mathbf{p}+} \Phi_{-\mathbf{k}-\mathbf{p}} e^{i\phi_{\mathbf{k}}} - \bar{\beta}_{\mathbf{k}+} \bar{\alpha}_{\mathbf{p}-} \Phi^*_{\mathbf{k}+\mathbf{p}} \frac{(2-\lambda_{\mathbf{p}})(2-\lambda_{\mathbf{k}}) e^{-i\phi_{\mathbf{k}}} - \lambda_{\mathbf{p}} \lambda_{\mathbf{k}} e^{-i\phi_{\mathbf{p}}}}{4} \right]$$

$$\lambda_{\mathbf{k}} = a_{\mathbf{k}} \pm i \sqrt{b_{\mathbf{k}}^2 + c_{\mathbf{k}}^2} .$$

$$\Phi_{\mathbf{k}} \equiv \frac{1}{N^2} \sum_{\mathbf{x}} \omega_N^{-\mathbf{k}\cdot\mathbf{x}} \, e^{i\eta(x)}$$

$$e^{i\phi(\mathbf{k})} \equiv \begin{cases} \frac{ib_{\mathbf{k}} + c_{\mathbf{k}}}{\sqrt{b_{\mathbf{k}}^2 + c_{\mathbf{k}}^2}} & \text{if } \mathbf{k} \neq (N,N), (\frac{N}{2},N), (N,\frac{N}{2}), (\frac{N}{2},\frac{N}{2}) \\ 1 & \text{if } \mathbf{k} = (N,N), (\frac{N}{2},N), (N,\frac{N}{2}), (\frac{N}{2},\frac{N}{2}) \end{cases}$$

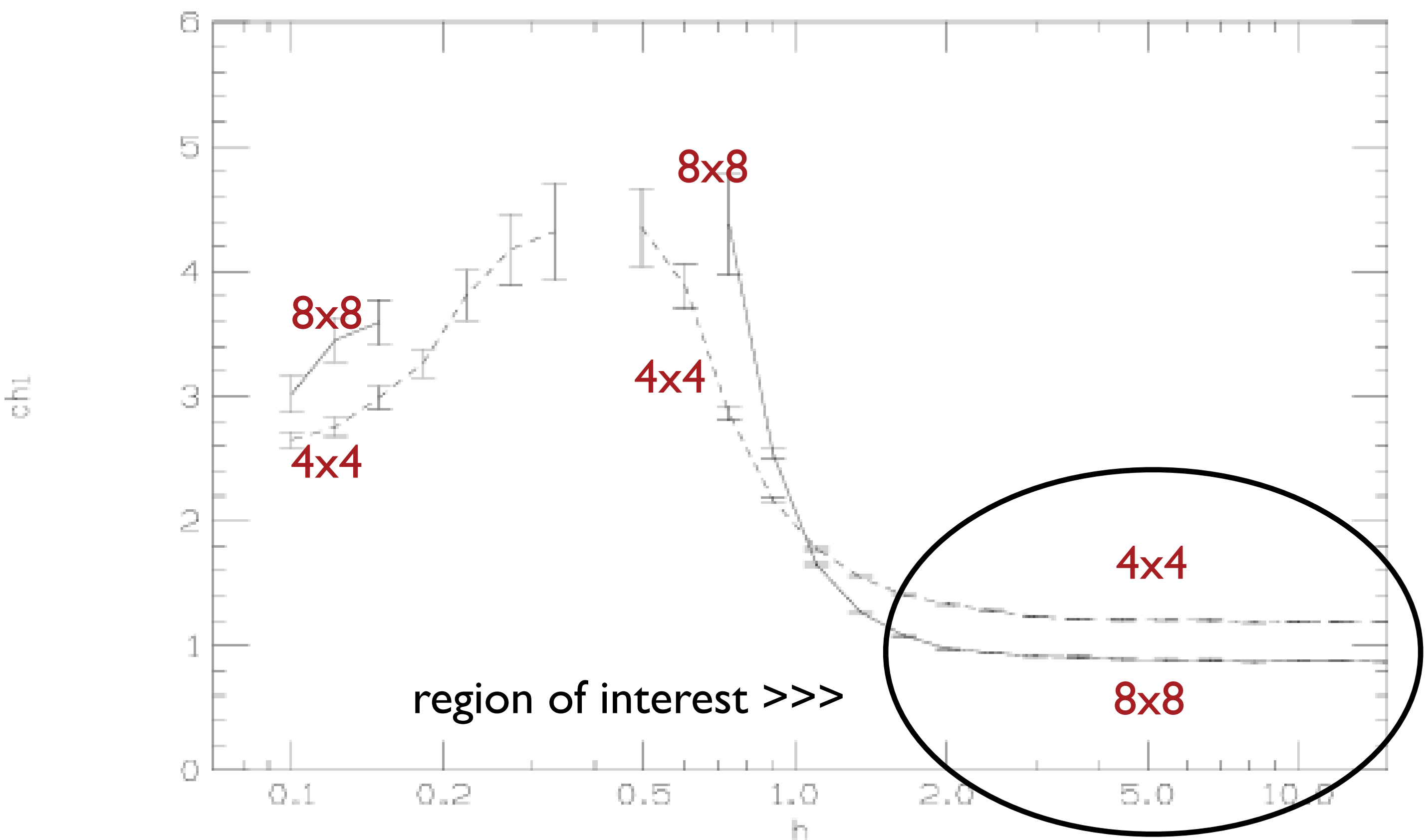
$$\begin{aligned} a_{\mathbf{k}} &\equiv 1 - \frac{1 - 2s_1^2 - 2s_2^2}{\sqrt{1 + 8s_1^2 s_2^2}} \\ b_{\mathbf{k}} &\equiv \frac{2s_2 c_2}{\sqrt{1 + 8s_1^2 s_2^2}} \\ c_{\mathbf{k}} &\equiv \frac{2s_1 c_1}{\sqrt{1 + 8s_1^2 s_2^2}} , \end{aligned}$$

in a basis where measure is simple:

$$\prod_x d\bar{\Psi}(x) d\Psi(x) = c \prod_{Im\lambda > 0} d\alpha_{\lambda+} d\alpha_{\lambda-} d\bar{\alpha}_{\lambda+} d\bar{\alpha}_{\lambda-} (1 - \lambda^*) .$$

3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

"Higgs susceptibility," ~ square of scalar correlation length, with dynamical fermions, in infinite Yukawa limit, as function of ratio of Majorana to Dirac mass:



moral:

- scalar correlation length is O(a) at large Yukawa ~as per quick argument!
- good! gauge breaking terms small, "FNN" applies, we can continue study...

now, will the "entire thing" work?

g=0 works better than any Higgs/Yukawa model so far!
... ours is the only unquenched study, btw... so I am optimistic...

but many issues need to be worked out and understood:

- stability of next order of strong coupling, g=0, expansion
- order g corrections (what if anomalous light content? verify Jackiw/Rajaraman!)
- fermion measure split into +/- chirality in nontrivial gauge backgrounds
- behavior in nontrivial topology backgrounds
- definition of Luscher's proposal's fermion measure?
- if it all holds up, is there a sign problem with gauge fields?

4 remaining issues and outlook

weak Yukawa proposal:

2 the use of warped domain walls

2-dim case seems to work better than AdS_5 case

strong Yukawa proposal

3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

preliminary indications promising!

not an intrinsically 2-dim proposal!