# Dynamical GUT breaking and $\mu$-term driven SUSY breaking

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Ryuichiro Kitano (SLAC)

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# 1. Introduction

Standard Model

**matter**
$q, u^c, e^c, l, d^c$

**gauge**
$g, W^a, B$

**Higgs**
$h$

$$V = \frac{\lambda}{4}(|h|^2 - v^2)^2$$

`it seems that` $\lambda \ll 4\pi$

(Little) Hierarchy Problem????

supersymmetrize

MSSM

**matter**
$Q, U^c, E^c, L, D^c$

**gauge**
$g, W^a, B$

**Higgs**
$H_u, H_d$

+

~~SUSY~~ sector
?

$$V = (m_{H_u}^2 + \mu^2)|H_u|^2 + (m_{H_d}^2 + \mu^2)|H_d|^2$$
$$+B\mu H_u H_d + \text{h.c.} + D \text{ term}$$

$$m_{H_u}^2 \sim m_{H_d}^2 \sim \mu^2 \sim B\mu \sim M_W^2$$ μ-Problem????

Good News: Gauge coupling unification!!!

Grand Unification

## GUT

matter

$10+\bar{5}$

gauge

$A^{\mu}_{\mathrm{SU(5)}}$

Higgs

$H,\bar{H}\quad(5+\bar{5})$

\+

~~SUSY~~ sector

?

\+

GUT breaking sector

$\Sigma(24)$ ?

$$H=(H_C,H_u),\quad \bar{H}=(\bar{H}_C,H_d)$$

Doublet-Triplet Splitting Problem????

$$W\ni\frac{f_u f_d}{M_C}QQQL$$

Dim-5 Proton Decay Problem????

At the every stage, SM, MSSM, and GUT,

Something is wrong in the Higgs sector!!!

Let's be ambitious. Top-down approach!!!

We first solve the D-T splitting/proton decay problem.

➡ The solution may give us a hint for other problems in low energy. And hopefully, we can get some prediction in low energy physics.

My proposal today: **Composite Higgs model @GUT scale**

this solves everything beautifully...

particle content: $Q$, $\bar{Q}$ and $T \Rightarrow H \sim QT$, $\bar{H} \sim \bar{Q}T$

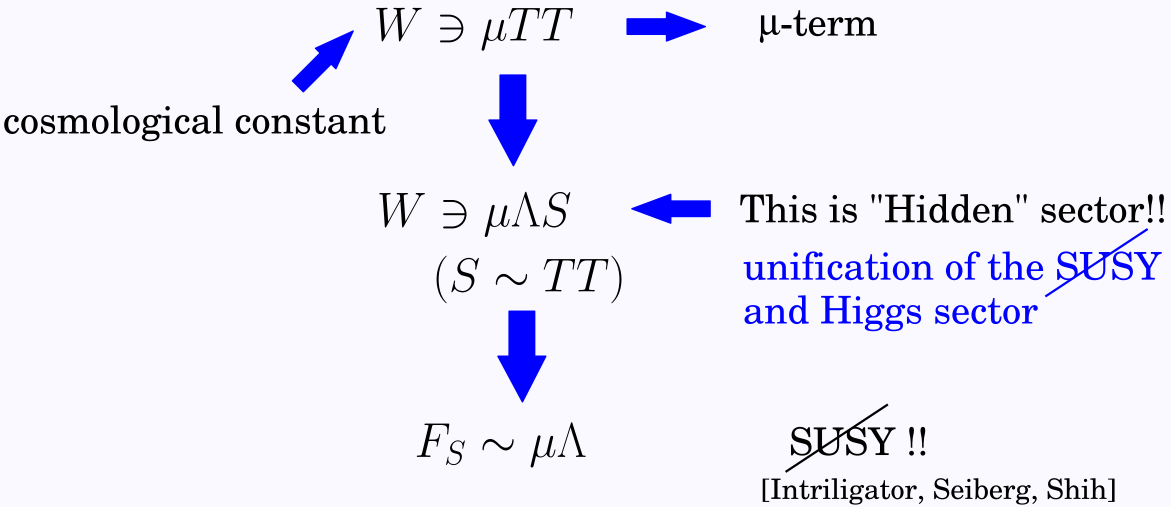
$$\langle Q\bar{Q}\rangle = \begin{pmatrix} 0 & & & & \\ & 0 & & & \\ & & 0 & & \\ & & & v^2 & \\ & & & & v^2 \end{pmatrix} \tag{1}$$

SU(5) → SU(3)×SU(2)×U(1) and D-T splitting/proton decay OK.

μ-Problem?

$$H \sim QT, \quad \bar{H} \sim \bar{Q}T$$

small mass term for quarks => meson mass



$$W \ni \mu TT$$

μ-term

cosmological constant

$$W \ni \mu\Lambda S$$

$$(S \sim TT)$$

This is "Hidden" sector!!

unification of the ~~SUSY~~ and Higgs sector

$$F_S \sim \mu\Lambda$$

~~SUSY~~ !!

[Intriligator, Seiberg, Shih]

$$m_{H_u}^2 \Leftarrow K \ni \frac{1}{\Lambda^2} S^\dagger S H_u^\dagger H_u \Rightarrow O(\mu^2) \quad !!$$

Direct mediation in the sense of the gravity mediation.
An opposite way of solving the mu-problem

## **Doublet-Triplet splitting and Proton decay** (In general)

$$H : \begin{pmatrix} H_C \\ H_u \end{pmatrix} \leftrightarrow \begin{pmatrix} \bar{H}_C \\ H_d \end{pmatrix} : \bar{H} \tag{2}$$

Let's give a mass term

$$W = M_C H_C \bar{H}_C \Rightarrow W = \frac{f_u f_d}{M_C} QQQL \tag{3}$$

dangerous dim-5 proton decay operators. This isn't good.

PQ symmetry,

$$\mathrm{PQ}(H) = \mathrm{PQ}(\bar{H}) = 1, \quad \mathrm{PQ}(10) = \mathrm{PQ}(\bar{5}) = -1/2,$$

can forbid the dim-5 proton decay.

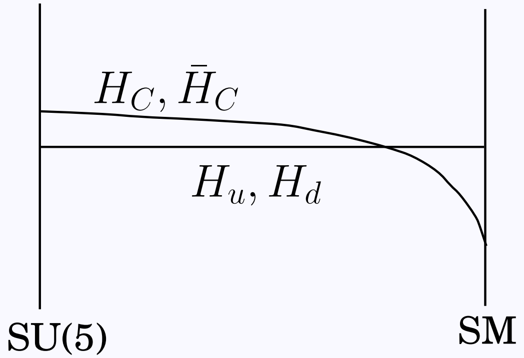
⇒ but also forbids the colored Higgs mass.....

⇒ **Explicit GUT breaking?**

## Examples of models without dim-5 proton decay

### Extra dimension models

[Kawamura, Hall, Nomura ….]



massive colored Higgs fields
with PQ (or R) symmetry

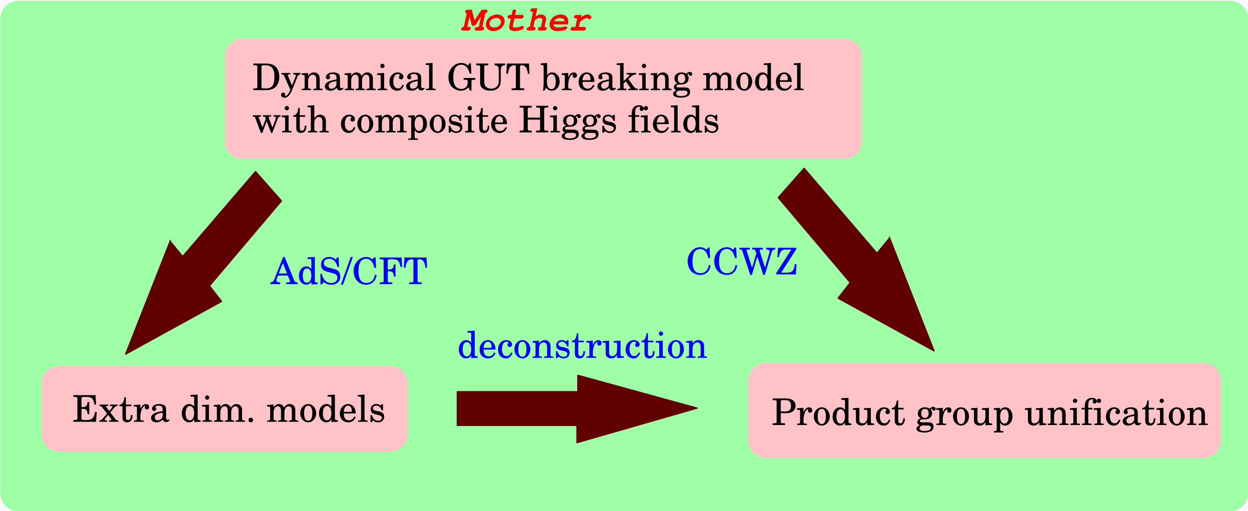
### Product group unification

[Hotta, Izawa, Yanagida, Weiner, ….]

| | $SU(5)\times$ | $(SU(3))\times$ | $SU(2)\times$ | $U(1)$ |
|---|---|---|---|---|
| $H_u, H_d$ | **1** | **1** | **2** | $\pm$**1/2** |

No colored Higgs.

# Dynamical GUT breaking model



***Mother***

Dynamical GUT breaking model
with composite Higgs fields

AdS/CFT

CCWZ

Extra dim. models

deconstruction

Product group unification

D-T splitting problem is similar to the situation of nucleons in QCD.

The success of Extra-dim models and Product group unification model suggests the presence of a really unified dynamical GUT breaking model.

By the explicit construction of the dynamical model, we can discuss the stability of the SM vacuum and also the SUSY breaking without knowledge of the quantum gravity.

With the knowledge of many non-perturbative effects in SUSY gauge theory [Seiberg ...], we can explicitly construct models.

# 2. Model

| | $SO(N_c)$ | $SU(5)_{\rm GUT}$ | PQ |
|---|---|---|---|
| $Q$ | $N_c$ | $5$ | $0$ |
| $\bar{Q}$ | $N_c$ | $\bar{5}$ | $0$ |
| $T$ | $N_c$ | $1$ | $1$ |

$$\boxed{W = mQ\bar{Q} - \frac{1}{M}(Q\bar{Q})^2 + \cdots}$$

with

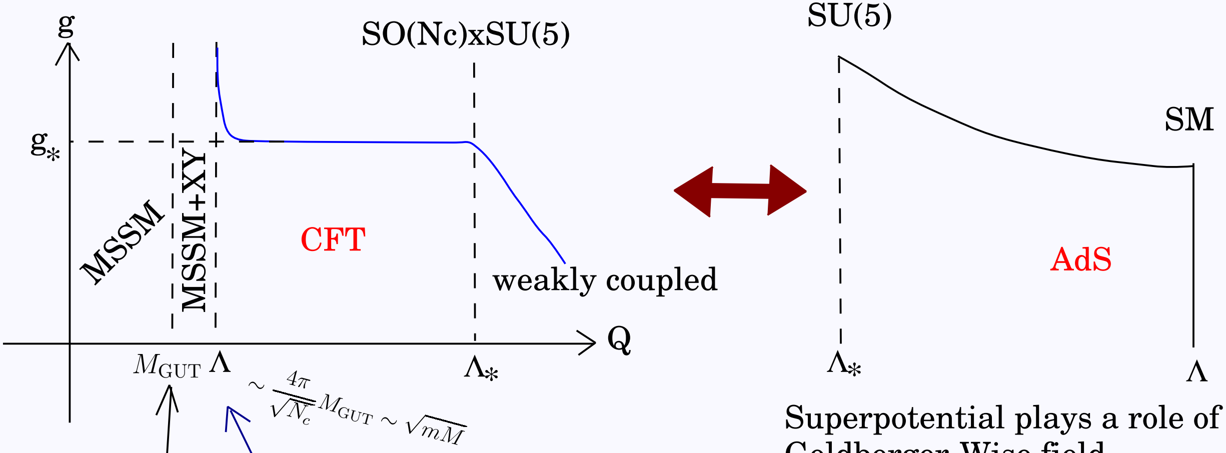
$$\langle M_{Q\bar{Q}} \rangle \sim \langle Q\bar{Q} \rangle = \begin{pmatrix} 0 & & & & \\ & 0 & & & \\ & & 0 & & \\ & & & v^2 & \\ & & & & v^2 \end{pmatrix}$$

**SU(5) -> SM**

$$\left.\begin{array}{ll} \text{Stability of this vacuum ==>} & 4 \le N_c \le 9 \\ \text{Asymptotic freedom of SO(Nc) ==>} & N_c \ge 6 \end{array}\right\} \quad 6 \le N_c \le 9$$

This coincides with the conformal window: $6 \le N_c \le 9$ of $N_F$ =11 SQCD

# CFT!!! (This is actually also true for SU(Nc) and Sp(Nc) models)



Decoupling of the heavy field --> CFT exit
(confinement)

SU(5) symmetry breaking at $\frac{\sqrt{N_c}\Lambda}{4\pi}$

Superpotential plays a role of the Goldberger-Wise field.

There is no coincidence problem between the parameters in the superpotential and the dynamical scale of SO(Nc).

Note: We cannot choose arbitrary breaking pattern once we fix Nc.

e.g., Nc=9

- $\mathrm{rank}(M_{Q\bar{Q}}) = 0$ SU(5) unbroken

  low energy: SO(9) 1 flavor theory --> No vacuum!!

  symmetry breaking must happen.

- $\mathrm{rank}(M_{Q\bar{Q}}) = 1$ SU(5) --> SU(4) x U(1)

  low energy: SO(7) 1 flavor theory --> No vacuum!!

- $\mathrm{rank}(M_{Q\bar{Q}}) = 2$ SU(5) --> SU(3) x SU(2) x U(1)

  low energy: SO(5) 1 flavor theory --> Stable vacuum exists.

  massless d.o.f: $H_u, H_d, S(\sim TT)$ (confining)

- $\mathrm{rank}(M_{Q\bar{Q}}) = 3$ SU(5) --> SU(3) x SU(2) x U(1)

  low energy: SO(3) 1 flavor theory --> Stable vacuum exists.

  massless d.o.f: $H_C, \bar{H}_C, S_0, S_+, S_-$ (Coulomb phase)

- $\mathrm{rank}(M_{Q\bar{Q}}) = 4$ SU(5) --> SU(4) x U(1)

  low energy: confining --> Stable vacuum exists.

- $\mathrm{rank}(M_{Q\bar{Q}}) = 5$ SU(5) unbroken

  low energy: SO(6) 1 flavor --> No vacuum!!

## Doublet-Triplet Splitting (SO(9) model)

- SO(9) 11 flavors

$$W = mQ\bar{Q} - \frac{1}{M}(Q\bar{Q})^2 + \cdots$$

Seiberg dual

- SO(6) 11 flavors $q, \bar{q}, t$ : dual quarks

$$\begin{aligned} W &= mM_{Q\bar{Q}} - \frac{1}{M}M_{Q\bar{Q}}^2 + \cdots \\ &\quad + \frac{1}{\hat{\Lambda}}\bar{q}M_{Q\bar{Q}}q + \cdots \\ &\quad + \frac{1}{\hat{\Lambda}}\bar{q}Ht + \frac{1}{\hat{\Lambda}}q\bar{H}t + \frac{1}{\hat{\Lambda}}Stt \end{aligned}$$

$$\langle M_{Q\bar{Q}} \rangle = \begin{pmatrix} 0 & & & & \\ & 0 & & & \\ & & 0 & & \\ & & & v^2 & \\ & & & & v^2 \end{pmatrix}$$

4 flavors decouple $q_D, \bar{q}_D$

- SO(6) 7 flavors

$$\begin{aligned} W &= mM_{Q\bar{Q}} - \frac{1}{M}M_{Q\bar{Q}}^2 + \cdots \\ &\quad + \frac{1}{\hat{\Lambda}}\bar{q}_C M_{Q\bar{Q}}^{(3\times3)} q_C + \cdots \\ &\quad + \frac{1}{\hat{\Lambda}}\bar{q}_C H_C t + \frac{1}{\hat{\Lambda}}q_C \bar{H}_C t + \frac{1}{\hat{\Lambda}}Stt - \frac{1}{v^2\hat{\Lambda}}H_u H_d tt \end{aligned}$$

Still interacting theory

Seiberg dual again

- SO(5) 7 flavors

It comes back to the original theory, but there is no doublet quarks anymore.

$$
\begin{aligned}
W = {} & m M_{Q\bar{Q}} - \frac{1}{M} M^2_{Q\bar{Q}} + \cdots \\
& + \frac{1}{\hat{\Lambda}} M^{(3\times3)}_{Q\bar{Q}} M^{(3\times3)}_{q\bar{q}} + \cdots \\
& + \frac{1}{\hat{\Lambda}} H_C \bar{H}'_C + \frac{1}{\hat{\Lambda}} \bar{H}_C H'_C \\
& - \frac{1}{v^2 \hat{\Lambda}} H_u H_d S' + \frac{1}{\hat{\Lambda}} S S' \\
& - \frac{1}{\hat{\Lambda}} \bar{Q}_C M^{(3\times3)}_{q\bar{q}} Q_C + \cdots \\
& - \frac{1}{\hat{\Lambda}} \bar{Q}_C H'_C T - \frac{1}{\hat{\Lambda}} Q_C \bar{H}'_C T - \frac{1}{\hat{\Lambda}} S' T T
\end{aligned}
$$

$$
\begin{cases}
F_{M^{(3\times3)}_{Q\bar{Q}}} = 0 \\
\Rightarrow \quad M^{(3\times3)}_{q\bar{q}} = \mathrm{diag.}(-m\hat{\Lambda}, -m\hat{\Lambda}, -m\hat{\Lambda})
\end{cases}
$$

mass terms for colored Higgs!!!

$F_S = 0 \quad \Rightarrow \quad S' = 0$

massless doublets!!!

6 flavors decouple $Q_C, \bar{Q}_C$

- SO(5) 1 flavor (massless T)

$W = 0$

confining

massless d.o.f: $H_u, H_d, S$

with no superpotential

Yukawa interactions

$$W = \frac{f_u}{M_Y}(10)(10)(QT) + \frac{f_d}{M_Y}(10)(\bar{5})(\bar{Q}T)$$

These operators look like irrelevant operators, but actually these are almost merginal operators by the large anomalous dimension of the CFT.

$$D(H) = D(\bar{H}) = \frac{3}{2}R(H) = \frac{12}{11} \simeq 1$$

Therefore, there is no problem with the O(1) top Yukawa couplings. It never hits the Landau pole in high energy.

In other words, the coefficients of the operators run almost linearly with scale. Planck suppressed operators can become GUT suppressed operators at the GUT scale.

In other words, the operators above have almost flat wave functions in the AdS picture.

# Colored Higgs mediated proton decay???

colored Higgs is massive by the superpotential term:

$$W = \frac{1}{\hat{\Lambda}} H_C \bar{H}'_C + \frac{1}{\hat{\Lambda}} \bar{H}_C H'_C$$

This is PQ symmetric mass term.

This preserves PQ symmetry if we assign $PQ(H'_C) = PQ(\bar{H}'_C) = -1$

⇒ No dangerous dim-5 proton decay.

Explicit calculation of the effective superpotential gives

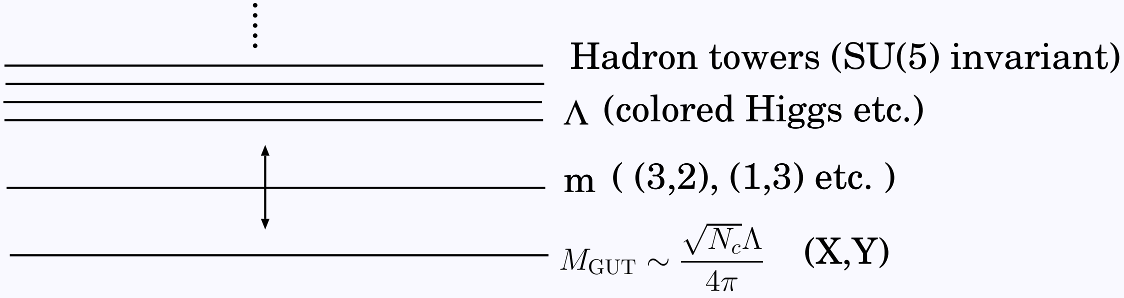
$$W = W_{\text{YUKAWA}} + \frac{y_u y_d}{m} \frac{S}{M_{\text{GUT}}} (\underbrace{QQQL + UUDE}_{\text{baryon number violating terms}} + QQUD + UEQL)$$

where S is flat direction.

As long as $\langle S \rangle < O(M_{\text{GUT}})$ , proton decay is suppressed.

(<S> depends on Kahler potential and SUSY breaking mechanism)

## Spectrum???



Hadron towers (SU(5) invariant)

$\Lambda$ (colored Higgs etc.)

m ( (3,2), (1,3) etc. )

$M_{\rm GUT} \sim \frac{\sqrt{N_c}\Lambda}{4\pi}$ **(X,Y)**

MSSM+S

NDA: $\Lambda^2 \sim mM$

For gauge coupling unification, we need

$$m \sim \underline{M \sim M_{\rm GUT}} \sim \Lambda$$

problem with $1/M_{\rm GUT}$ suppressed operators?

No. both of $Q\bar{Q}$ and $(Q\bar{Q})^2$ are relevant operators.
It does not become strong in high energy.

# μ-term driven ~~SUSY~~

We need μ-term. It is very easy to write down the μ-term.

$$W = \mu TT \quad \Leftarrow$$

quark mass of O(100GeV)
(small breaking of the PQ symmetry)

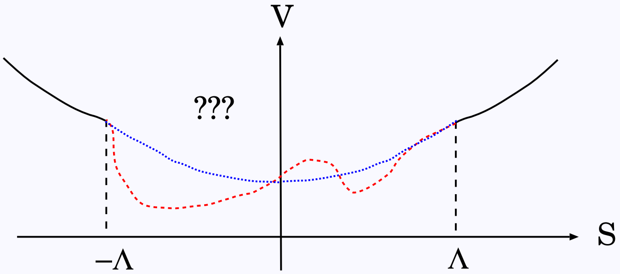
$$\Downarrow$$

$$W = \underbrace{\mu H_u H_d}_{\mu\text{-term}} + \underline{\mu M_{\rm GUT} S}$$

$F_S = 0$ cannot be satisfied ⇒ ~~SUSY~~ !!!

$$\boxed{F_S = \mu M_{\rm GUT} \quad \Rightarrow \quad m_{3/2} \sim F_S/M_{\rm Pl} \sim 1 \text{ GeV}}$$

meta stable? [Intriligator, Seiberg, Shih] **YES.**



We don't know the shape of the Kahler potential, but there must be minimum somewhere.

$$K = S^\dagger S + \frac{c}{M_{\rm GUT}^2}(S^\dagger S)^2 + \cdots$$

$$+\frac{c'}{M_{\rm GUT}}\left(\frac{m}{\Lambda}\right)^2 (S + S^\dagger)(S^\dagger S)$$

<-- R and PQ breaking term

$m^2_{H_u}, m^2_{H_d}$ ? really direct mediation (gravity mediation warped down)

$$K \ni \frac{1}{M^2_{\rm GUT}} S^\dagger S H_u^\dagger H_u + \frac{1}{M^2_{\rm GUT}} S^\dagger S H_d^\dagger H_d \quad \text{and} \quad F_S \sim \mu M_{\rm GUT}$$

$$\Rightarrow \boxed{m^2_{H_u} = m^2_{H_d} \sim \mu^2}$$

An opposite way of "solving" the mu-problem

mu-term is similar size as the SUSY breaking parameters
because mu-term breaks SUSY!!!

gaugino, sfermion masses?

$m_{3/2} \sim 1$ GeV $\Rightarrow$ We need gauge mediation.
Gravity mediation effect is too small, and moreover, there may be conformal sequestering effects.

$$W \ni \frac{1}{M_X} T^2 \Phi \bar{\Phi}$$

$\Phi, \ \bar{\Phi}$ : messenger field

(almost merginal operator)

$$\Rightarrow \lambda S \Phi \bar{\Phi}$$

$$\Rightarrow m_{1/2} = \frac{\alpha}{4\pi} \frac{F_S}{\langle S \rangle}$$

We need $\langle S \rangle \sim 10^{14}$ GeV for $m_{1/2} \sim O(\mu)$

This is quite natural if c<0 and $m < M_{\rm GUT}$

$$\langle S \rangle \sim \frac{M^2_{\rm GUT}}{M_{\rm Pl}} \sim 10^{14} \text{ GeV}$$

$$\begin{aligned} K = S^\dagger S &+ \frac{c}{M^2_{\rm GUT}} (S^\dagger S)^2 + \cdots \\ &+ \frac{c'}{M_{\rm GUT}} \left(\frac{m}{\Lambda}\right)^2 (S + S^\dagger)(S^\dagger S) \end{aligned}$$

This is also consistent with the supression of the proton decay.

Anyway, the general prediction of this scenario is that the pattern of the SUSY breaking parameters is the gauge mediation type with modification in the Higgs sector.

## Cosmological Constant driven SUSY breaking

What's the origin of the mu-term? Can it be the cosmological constant?

In any SUSY models

$$\Lambda_{CC}^4 = \underset{\uparrow}{|F|^2} \underbrace{- 3|W|^2}_{\text{Negative cosmological constant}} \simeq 0$$

Positive contribution from SUSY breaking

We already have a dimensionful parameter with the same size of F!!!

$\longrightarrow$ This can be origin of the mu-term.

**Giudice-Masiero Mechanism**

$$\left\{\begin{aligned} K &= T^\dagger T + \kappa(TT + \text{h.c.}) \\ W &= c \qquad (c = O(m_{3/2})) \\ &\qquad\quad\; (\kappa = O(1)) \end{aligned}\right. \xRightarrow{\text{Kahler transformation}} \left\{\begin{aligned} K &= T^\dagger T \\ W &= ce^{\kappa TT} \simeq c + c\kappa TT + O(1/M_{\text{Pl}}) \end{aligned}\right.$$

Large anomalous dimension

$$a \sim \frac{\Lambda_*}{M_{\text{GUT}}} \gg 1$$

$$\left\{\begin{aligned} K &= T^\dagger T \\ W &= ce^{\kappa TT} \simeq c + ca\kappa TT + O(1/M_{\text{Pl}}) \end{aligned}\right.$$

we can obtain O(100GeV) mu-term for $a\kappa = O(100)$

## 3. Summary

- Composite Higgs model@GUT scale easily solves the Doublet-Triplet splitting and proton decay dilemma.
- The explicit construction of the dynamical model allows us to discuss the stability of the standard model vacuum even with the supersymmetry breaking.
- Once we assume the compositeness of the Higgs boson, we can easily unify the hidden sector and the Higgs sector by using the meta-stable SUSY breaking vacua discussed by Intriligator, Seiberg and Shih. The origin of the SUSY breaking can actually be the $\mu$-term. This provides an interesting solution to the $\mu$-problem.
- The low energy prediction of this scenario is the unique SUSY spectrum. It is predicted to be the gauge mediation type with modification in the Higgs sector.
- Gravitino is the lightest LSP with $m_{3/2} \sim 1$ GeV.
- Negative cosmological constant can be the trigger of the SUSY breaking.
- I love this scenario.