SUSY Moose Runs and Hops: An extra dimension from a deformed CFT

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Deconstructed Extra Dimensions

(Arkani-Hamed,Cohen,Georgi; Hill, Pokorski, Wang)

Motivations:

1. Extra Dimensional Phenomenology $\rightarrow$ 4D

2. Provides a definition of 5D gauge theories

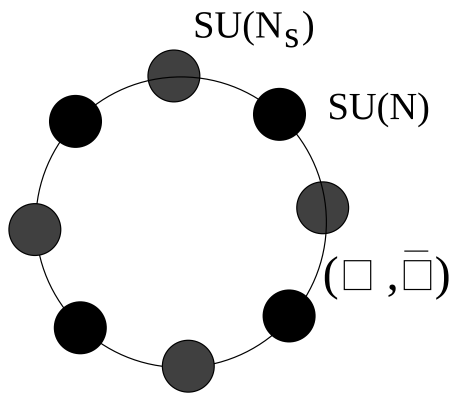
But...

Deconstruction is not always a magic bullet. Some scenarios may be preferred in the deconstructed approach.

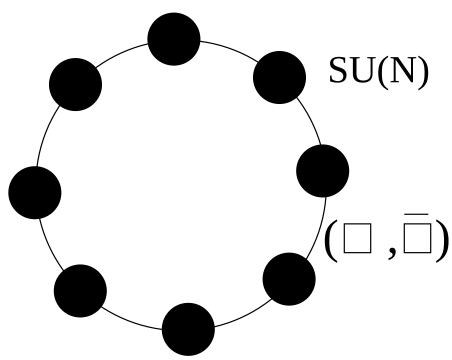
We will see that even the topology of an extra dimension may be predetermined by invoking deconstruction as a UV completion.

Two flavors of deconstruction:

1. Strongly coupled dynamics $\rightarrow$ nonlinear σ-model



2. Linear σ-model



Deconstructing SUSY

	$SU(N)_1$	$SU(N)_2$	$SU(N)_3$	$\cdots$	$SU(N)_k$	$U(1)_R$
Q_1	$\Box$	$\overline{\Box}$	1	$\cdots$	1	r_{Q_1}
Q_2	1	$\Box$	$\overline{\Box}$	$\cdots$	1	r_{Q_2}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
Q_k	$\overline{\Box}$	1	1	$\cdots$	$\Box$	r_{Q_k}
$N_f\ \ F_1$	$\Box$	1	1	$\cdots$	1	r_{F_1}
$N_f \overline{F}_1$	$\overline{\Box}$	1	1	$\cdots$	1	r_{F_1}
$N_f\ \ F_2$	1	$\Box$	1	$\cdots$	1	r_{F_2}
$N_f\ \ \overline{F}_2$	1	$\overline{\Box}$	1	$\cdots$	1	r_{F_2}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
$N_f\ \ F_k$	1	1	1	$\cdots$	$\Box$	r_{F_k}
$N_f\ \ \overline{F}_k$	1	1	1	$\cdots$	$\overline{\Box}$	r_{F_k}

Supersymmetry provides tools for studying nonperturbative properties of gauge theories

Can answer conceptual questions about SUSY gauge theories in extra dimensions

1. Nonperturbative equivalence of Deconstruction and extra dimensions

Nekrasov: 5D SU(2) SUSY YM on a circle has Seiberg-Witten description up to some minor ambiguities

Clarified in the deconstructed version of the theory (Csáki, Erlich, Khoze, Poppitz, Shadmi, Shirman)

2. Definition of higher dim'l CFT's

6D (0,2) theory on torus, Little String
(Arkani-Hamed,Cohen,Kaplan,Karch,Motl ;
Csáki, Erlich, Terning)

4D $\mathcal{N}=2$ SU(N)^k moose $\rightarrow$ 6D CFT

Seiberg-Witten description of the moose theory $\rightarrow$
interpretation as (0,2) theory on torus with angle

$\theta_{QCD} \rightarrow$ torus angle θ

3. New phenomenology

Secret SUSY with D-term VEVs.
(Carone,Erlich,Glover)

U(1)^k moose w/ Fayet-Iliopoulos terms.

D-term VEVs$\neq 0$, but unbroken gauge group has no charged fields so D-term = Cosmological Constant

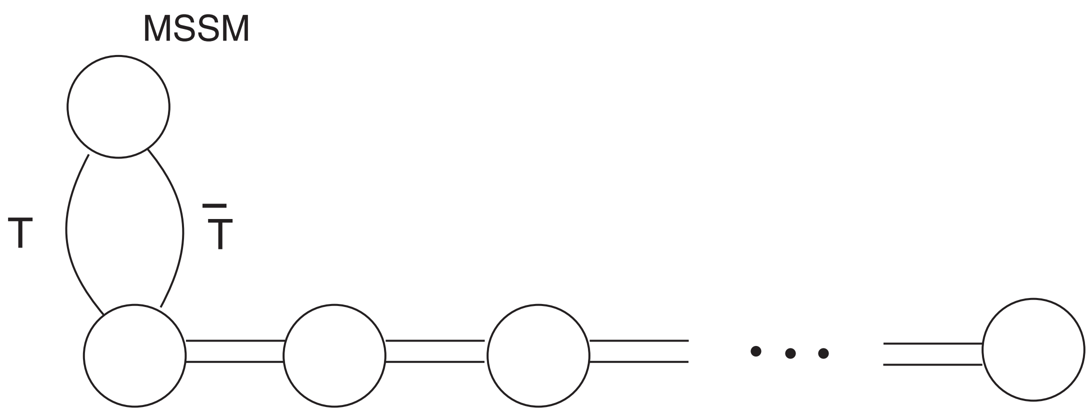
SUSY not broken globally: redefine SUSY action on gauginos,

$$\lambda_i \to \lambda_i + i(D_i - \langle D \rangle)\,\epsilon + \cdots$$

All D_j are equal, so non-homogeneous part of redefined SUSY transformation vanishes.

Consequences of Secret SUSY:

1) Couple one site to sector charged under diagonal U(1) $\rightarrow$ Supersoft SUSY breaking. (Fox,Nelson,Weiner)



2) Hierarchy between gravity mediated/gauge mediated SUSY breaking scale $\rightarrow$ gravitino can be heavy,

$$m_{3/2} \sim \sum_i D_i/M_{Pl} \gg D_i/M_{Pl}$$

4. Consequences of linear σ-model as a UV completion

Scales:

Lightest KK mode $m_{KK} \sim g_4 \, v/N$

SU$(N)^k \rightarrow$ SU(N) at scale $\Lambda_{KK} \sim g_4 v$

Cutoff of 4D theory $\Lambda_4 \gg \Lambda_{KK} \gg m_{KK}$

Strong coupling scale of each 4D gauge coupling, $\Lambda_{QCD} > \Lambda_{KK}$

Question: What can we say about flows from $\Lambda_4 \rightarrow \Lambda_{KK}$?

If operators in the action are irrelevant, then the deconstructed theory must be fine tuned.

Are there preferred values for the couplings at the scale Λ_{KK}?

Are they consistent with the extra dimensional interpretation?

A-maximization and operator dimensions (Intriligator,Wecht)

At a conformal fixed point R-charges $\to$ operator dimensions. R-charges maximize the Euler anomaly a.

$$\Delta_\phi = 3/2\, r_\phi$$

for any chiral superfield ϕ.

$$a = \frac{3}{32}\left(3\mathrm{Tr}\, R^3 - \mathrm{Tr} R\right)$$

$$\langle T \rangle = \frac{c}{16\pi^2}(W_{\mu\nu\lambda\sigma})^2 - \frac{a}{16\pi^2}(\tilde{R}_{\mu\nu\lambda\sigma})^2$$

Trace of Stress Tensor and $\partial_\mu J_R^\mu$ related by SUSY $\to$ can calculate c and a at fixed point from R-current anomalies.

(Anselmi,Freedman,Grisaru,Johanssen;
Anselmi,Erlich,Freedman,Johanssen)

Unitarity and Accidental Symmetry

Unitarity requires that the dimension of every gauge invariant operator is > 1, i.e. $R > 2/3$.

Sometimes the R-charges determined by A-max violate this.

The assumption is that in these cases there is an accidental symmetry in the IR under which only the unitarity violating operator transforms.

$\rightarrow$ Redo A-maximization including this accidental symmetry

Nontrivial fixed point theories
Example: SUSY QCD (Seiberg)

SU(N_c) gauge theory w/ N_f flavors.

$$\beta_e \propto (N_f - 3N)$$

Asymptotic Freedom: $N_f < 3N$.

Seiberg duality: $\rightarrow$ SU($N_f - N_c$) w/ N_f flavors.

$$\beta_m \propto N_f - 3(N_f - N_c).$$

Asymptotic Freedom: $N_f > 3N_c/2$.

In the range $3N_c/2 < N_f < 3N_c$ there is no weakly coupled description in the infrared

$\rightarrow$ Conformal window of SUSY QCD

Unitarity in SUSY QCD

$U(1)_R$ $SU(N_c)^2$ anomaly cancellation:

$$2N_c + 2N_f(r_F - 1) = 0$$

$$r_F = (N_f - N_c)/N_f$$

Unitarity: $r_{\overline{F}F} > 2/3$ if $N_f > 3N_c/2$

Deconstructed Theory: 5D theory on circle with a brane

	$SU(N)_1$	$SU(N)_2$	$SU(N)_3$	$\cdots$	$SU(N)_k$	$U(1)_R$
Q_1	$\square$	$\overline{\square}$	1	$\cdots$	1	r_{Q_1}
Q_2	1	$\square$	$\overline{\square}$	$\cdots$	1	r_{Q_2}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
Q_k	$\overline{\square}$	1	1	$\cdots$	$\square$	r_{Q_k}
$(N_f+N_b)\ F_1$	$\square$	1	1	$\cdots$	1	r_{F_1}
$(N_f+N_b)\overline{F}_1$	$\overline{\square}$	1	1	$\cdots$	1	r_{F_1}
$N_f\ \ F_2$	1	$\square$	1	$\cdots$	1	r_{F_2}
$N_f\ \ \overline{F}_2$	1	$\overline{\square}$	1	$\cdots$	1	r_{F_2}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
$N_f\ \ F_k$	1	1	1	$\cdots$	$\square$	r_{F_k}
$N_f\ \ \overline{F}_k$	1	1	1	$\cdots$	$\overline{\square}$	r_{F_k}

Naive Conformal Window: $N_c < N_f < 2N_c - N_b$

It can happen that fields which couple to more than one gauge group factor lead to modification of conformal window. (Intriligator,Wecht)

The Braneless Moose

A-Type fixed point:

Turn off all but one gauge coupling, say g_1:
Like SUSY QCD w/ $\tilde{N}_f = N_c + N_f$.

$$r_{F_1} = r_{Q_1} \equiv r = N_f/(N_f + N_c).$$

Perturb about fixed point by turning on neighboring gauge coupling: (cf. Poppitz,Shadmi,Trivedi)

	$SU(N)_1$	$SU(N)_2$	$U(1)_R$
Q_1	$\square$	$\bar{\square}$	r
$N_f\ F_1$	$\square$	1	r
$(N_f + N)\overline{F}_1$	$\bar{\square}$	1	r
$(N_f + N)\ F_2$	1	$\square$	2/3
$N_f\ \overline{F}_2$	1	$\bar{\square}$	2/3

NSVZ beta function for neighboring gauge coupling:

$$\begin{aligned}\beta_2 &= -\frac{g_2^3}{16\pi^2}\frac{(3G-\sum_i \mu_i(1-\gamma_i))}{1-\frac{g_2^2}{8\pi^2}G} \\ &= -\frac{3g_2^3}{16\pi^2}\frac{(G-\sum_i \mu_i(1-r_i))}{1-\frac{g_2^2}{8\pi^2}G} \\ &= -\frac{3g_2^3}{32\pi^2}\frac{2N^2+3NN_f-2N_f^2}{\left(N+N_f\right)\left(1-\frac{g_2^2 N}{8\pi^2}\right)}\end{aligned}$$

The β function for bulk gauge groups near A-type fixed point are negative in the naive conformal window $\rightarrow$ fixed point is unstable

B1-type fixed pt: Two neighboring gauge couplings turned on.

	$SU(N)_1$	$SU(N)_2$	$U(1)_R$
Q_1	$\square$	$\overline{\square}$	r_Q
$N_f\ F_1$	$\square$	1	r_{F_1}
$(N_f+N)\overline{F}_1$	$\overline{\square}$	1	$r_{\overline{F}_1}$
$(N_f+N)\ F_2$	1	$\square$	r_{F_2}
$N_f\ \overline{F}_2$	1	$\overline{\square}$	$r_{\overline{F}_2}$

A-maximization$\longrightarrow$

$$r_{F_1} = r_{F_2} = r_{\overline{F}_1} = r_{\overline{F}_2} = \frac{9N^2 - 12N_f^2 - N\sqrt{(73N^2 - 4NN_f - 4N_f^2)}}{3(N^2 - 4NN_f - 4N_f^2)}$$

$$r_Q = \frac{-9N^2 - 12NN_f + (2N_f + N)\sqrt{(73N^2 - 4NN_f - 4N_f^2)} - 12N_f^2}{3(N^2 - 4NN_f - 4N_f^2)}$$

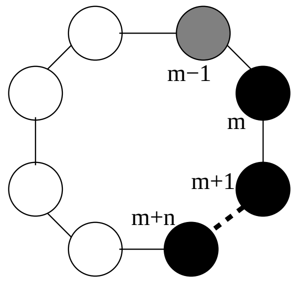
R-charges $\longrightarrow$ anomalous dimensions $\longrightarrow$ β-functions

Turn on neighboring gauge group $\longrightarrow$

$$\beta_3^{(B1)} = -\frac{g_3^3}{32\pi^2}\frac{11N^3 - 10N^2N_f + 8N_f^3 - 12NN_f^2 - N^2\sqrt{73N^2 - 4NN_f - 4N_f^2}}{(N^2 - 4NN_f - 4N_f^2)\left(1 - \frac{g_3^2 N}{8\pi^2}\right)}$$

$\beta_3 < 0$ for all N_c, N_f in the naive conformal window, so the B1-type fixed point is unstable.

General fixed point with a series of $n+1$ neighboring gauge groups turned on:



Turning on one more neighboring gauge coupling, find:

$$\beta_{m-1} = \beta_m + N_f \, \gamma_{F_m} + \frac{N}{2} \gamma_{Q_m}.$$

At the fixed point, $\beta_m = 0$.

If the fixed point is perturbative (Banks-Zaks), near $N_f = 2N_c$, can trust one-loop anomalous dimensions:

$$\begin{aligned} \gamma_{F_i} &= -\frac{(g_i^*)^2}{8\pi^2}\frac{(N^2-1)}{N} < 0 \\ \gamma_{Q_i} &= -\frac{\left((g_i^*)^2 + (g_{i+1}^*)^2\right)}{8\pi^2}\frac{(N^2-1)}{N} < 0 \end{aligned}$$

Hence, $\beta_{m-1} < 0$, and such fixed points are unstable.

B2-type fixed point:
Next-nearest-neighbor interacting gauge groups

	$SU(N)_1$	$SU(N)_2$	$SU(N)_3$	$U(1)_R$
Q_1	$\square$	$\overline{\square}$	1	r
Q_2	1	$\square$	$\overline{\square}$	r
$N_f\ \ F_1$	$\square$	1	1	r
$N_f\ \ \overline{F}_1$	$\overline{\square}$	1	1	r
$N_f\ \ F_2$	1	$\square$	1	2/3
$N_f\ \ \overline{F}_2$	1	$\overline{\square}$	1	2/3
$N_f\ \ F_3$	1	1	$\square$	r
$N_f\ \ \overline{F}_3$	1	1	$\overline{\square}$	r

β-function for the weakly gauged middle group:

$$\beta_2^{(B2)} = -\frac{3g_2^3}{16\pi^2}\frac{\left(N - \frac{2N_f}{2}(1-2/3) - N(1-r)\right)}{1-\frac{g_2^2 N}{8\pi^2}}$$

$$= -\frac{3g_2^3}{16\pi^2}\frac{\left(2N - N_f\right)N_f}{(1-\frac{g_2^2 N}{8\pi^2})(N+N_f)}$$

$\beta_2^{(B2)} < 0$ in the entire naive conformal window.

Does the Braneless Moose Hop After it Runs?

The bulk flavors have a hopping superpotential of the form,

$$W_{\text{flavor}} = \sqrt{2} g \sum_i \text{tr}(\overline{F}_i Q_i F_{i+1}) + \sum_i m_i F_i \overline{F}_i$$

Near perturbative fixed point at scale $\Lambda_4 > \Lambda_{KK}$, we can calculate the R-charges perturbatively $\rightarrow W_{\text{flavor}}$ is relevant at Λ_{KK}.

We also have the nonperturbative R-charges from a-maximization $\rightarrow$

$$r_{\overline{F}_i Q_i F_{i+1}} = 2 - \frac{3N_f^2 - 6NN_f + (2N - N_f)\sqrt{20N^2 - N_f^2}}{6N^2 - 3N_f^2}$$

It's not immediately obvious, but this R-charge is < 2 in the naive conformal window $\rightarrow W_{\text{flavor}}$ is composed of relevant operators nonperturbatively, as well.

Summary of Results for Braneless Moose

1) There can be many physically inequivalent IR conformal fixed points.

2) Most of the fixed points are unstable to turning on additional gauge groups, as desired for the extra dimensional interpretation.

3) Hopping terms which give rise to derivatives in the extra dimension are relevant operators at the scale of the highest KK mode, also as desired.

The Moose with a Brane

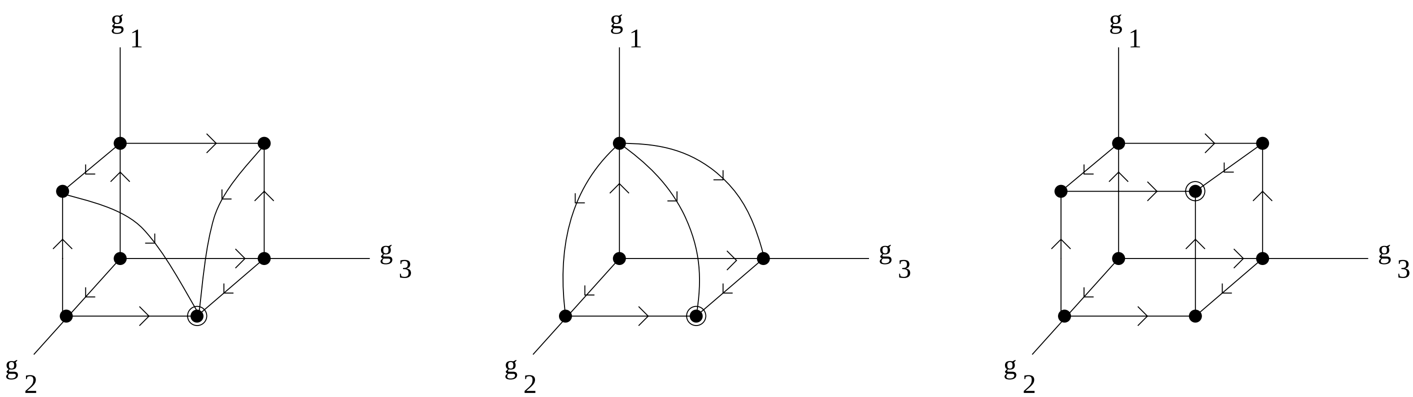
Add extra flavors at one lattice site.

	$SU(N)_1$	$SU(N)_2$	$SU(N)_3$	$\cdots$	$SU(N)_k$	$U(1)_R$
Q_1	$\square$	$\overline{\square}$	1	$\cdots$	1	r_{Q_1}
Q_2	1	$\square$	$\overline{\square}$	$\cdots$	1	r_{Q_2}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
Q_k	$\overline{\square}$	1	1	$\cdots$	$\square$	r_{Q_k}
$(N_f + N_b)\ F_1$	$\square$	1	1	$\cdots$	1	r_{F_1}
$(N_f + N_b)\overline{F}_1$	$\overline{\square}$	1	1	$\cdots$	1	r_{F_1}
$N_f\ F_2$	1	$\square$	1	$\cdots$	1	r_{F_2}
$N_f\ \overline{F}_2$	1	$\overline{\square}$	1	$\cdots$	1	r_{F_2}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
$N_f\ F_k$	1	1	1	$\cdots$	$\square$	r_{F_k}
$N_f\ \overline{F}_k$	1	1	1	$\cdots$	$\overline{\square}$	r_{F_k}

The analysis of these theories is similar to the brane-less case, as are the conclusions...

Except, now there are generally several classes of RG flows:

The brane coupling is either relevant or irrelevant, depending on N_f, N_c, N_b.



(Brane gauge coupling is g_1.)

Boundary conditions for fields in the bulk depend on numbers of colors and flavors on and off the brane.

If the brane gauge coupling is irrelevant, then the extra dimensional circle transitions to an interval.

We learn that the boundary conditions and topology of the extra dimension depend on the numbers of colors and flavors.

The A-theorem

In 2D Zamolodchikov proved that the trace of the stress tensor in a gravitational background yields a C-function:

1) monotonically decreasing with energy scale

2) always positive

3) stationary at a conformal fixed point

$$T = \frac{c}{24\pi} R$$

The central charge c counts the (weighted) number of degrees of freedom, which decreases with energy as degrees of freedom are integrated out.

In 4D the trace of the stress tensor can have two different anomalies in a supergravity background: The Weyl anomaly c and the Euler anomaly a.

Cardy's Conjecture (the A-theorem):
The Euler anomaly is a C-function in 4D.

What is known: Any other linear combination of Weyl and Euler anomalies is NOT a C-function.

We can test the A-theorem by considering RG flows between fixed points.

With k lattice sites there are 2^k fixed points $\rightarrow$ a lot of RG flows!

We found for $k = 1, 2, 3$ and $N_c = 2, 3, 4, 5$ in the entire naive conformal window the A-theorem is satisfied.

An analytic proof seems possible, at least with small number of lattice sites.

Conclusions

1) Supersymmetric moose models can have an intricate structure of RG flows between conformal fixed points

2) Most of the fixed points are unstable, and prefer to turn on all of the bulk gauge couplings. The effective 5D gauge coupling is generally small compared to the size of the extra dimension.

3) Bulk field boundary conditions are determined dynamically, depend on numbers of colors and flavors.

4) The gauge couplings tend to be too small at the fixed point to correspond to an extra dimension $\rightarrow$ requires a new hierarchy between Λ_4 and Λ_{KK}.

5) Possible phenomenological applications:
Boundary conditions by fixed point dynamics
Higgsless models
GUT breaking by boundary conditions
Flavor dependence of boundary conditions