A MINIMAL SET OF ELECTROWEAK PRECISION PARAMETERS

G. Cacciapaglia
C. Csàki
G. Marandella
A. Strumia

hep-ph/0604111

MOTIVATION

Notwithstanding its success, the SM has unsatisfactory aspects:

- based on $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge symmetry

the symmetry breaking is described via the Higgs mech.:

$$V(\phi) = \lambda \left(|\phi|^2 - v^2 \right)^2$$

- forces the Higgs boson in a vacuum where the gauge symmetry is broken
- gives mass to W, Z and fermions.

However:

- ☆ no explanation of EWSB!
- ☆ if the SM is an effective low energy theory (we know there is something more out there...) the potential is unstable:

$$\delta m_H^2 \sim \frac{g^2}{16\pi^2} \Lambda_{NP}^2$$

- ☆ the Higgs cannot be too heavy:
  - → W_L scattering unitarization: $m_H \lesssim 800$ GeV
  - → indirect measurements: $m_H \lesssim 300$ GeV

$$\Rightarrow \Lambda_{NP} \sim \frac{1\,\text{TeV}}{g}$$

★ the Higgs has not been discovered yet!

The need for a model with natural EWSB pushes us to add new Physics at a scale not far from 1 TeV.

The main challenge for such models is the boring success of the SM!

★ EWPT's (LEP1 and LEP2 mainly) generically constraints the new Physics into an unnatural corner of the parameter space. LEP Paradox

To be more concrete:

$$\mathcal{L}_{NP} = \sum_i \frac{1}{\Lambda_{NP}^2} \mathcal{O}_i$$

EWPT's : $\Lambda_{NP} \gtrsim 5 \div 10$ TeV

Barbieri, Strumia

Rules of the game:

- natural EWSB (NP @ ~ 1 TeV)
- evade EWPT's

Thus, it is crucial to have a clear understanding of what EWPT's are actually telling us.

$\boxed{\text{Our aim: find a minimal set of super-constrained parameters.}}$

List of measurements:

Inputs: $G_F = 1.16637 \cdot 10^{-5}$ GeV^{-2}

$M_Z = 91.1875$ GeV

$\alpha_{em}(M_Z) = 1/128.949$

Γ_Z	$= 2.4952 \pm 0.0023$ GeV	.1 %
σ_h	$= 41.540 \pm 0.037$ nb	.1 %
R_h	$= 20.767 \pm 0.025$	.1 %
R_b	$= 0.21644 \pm 0.00065$	.3 %
R_c	$= 0.1718 \pm 0.0031$	2 %
A^{τ}_{Pol}	$= 0.1465 \pm 0.0032$	2 %
A^{e}_{LR}	$= 0.1513 \pm 0.0021$	1.5 %
A^{b}_{LR}	$= 0.922 \pm 0.02$	2 %
A^{c}_{LR}	$= 0.670 \pm 0.026$	4 %
A^{ℓ}_{FB}	$= 0.01714 \pm 0.00095$	5 %
A^{b}_{FB}	$= 0.099 \pm 0.0017$	2 %
A^{c}_{FB}	$= 0.067 \pm 0.0026$	4 %
M_W	$= 80.426 \pm 0.034$ GeV	.4 %

$\sigma(e^+e^- \to f\bar{f})$ @ LEP 2

If we assume:

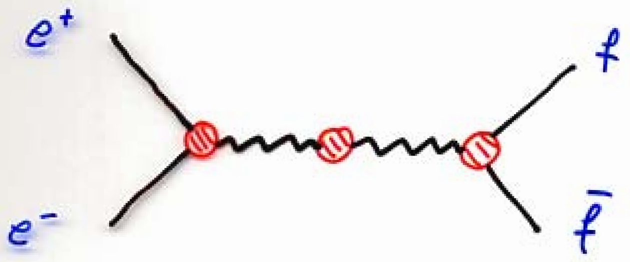
- CP conservation
- flavour cons.

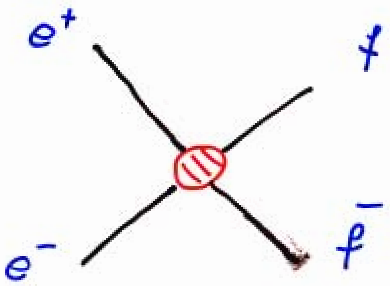
$\Big\}$ very tightly constrained by low energy exp.

there are 20 indep. operators of dimension 6 that contribute to the above observables

Skiba, Han

$$\mathcal{L}_{dim\,6} = \sum_{i=1}^{20} c_i \, \mathcal{O}_i \qquad c_i \sim \frac{1}{\Lambda_{NP}^2}$$





But, the observables are only sensitive to combinations of such ~~also~~ operators.

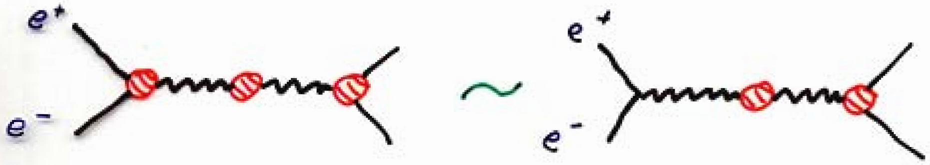
In other words, cancellations between operators can reduce the bounds on new physics!

We want to identify:

- a set of super-constrained combination
- no more cancellations allowed

A simple reparametrization:

can we remove the operators that affect the couplings of the electron and write them in terms of corrections to the propagators of gauge bosons?



$$\mathcal{L}_{BSM} = \sum_i c_i \mathcal{O}_i \longrightarrow \mathcal{L}_{GB} + \mathcal{L}_{\substack{\text{non} \\ \text{leptonic}}}$$

<u>constrained</u>

less constrained

Counting of the parameters

$$\mathcal{L}_{GB} = -W^+ \Pi_{ww}(p^2) W^- - \frac{1}{2} W^3 \Pi_{33}(p^2) W^3 - \frac{1}{2} B\, \Pi_{bb}(p^2) B$$

$$- W^3 \Pi_{3b}(p^2) B$$

is the most general GB lagrangian. Π's = corrected propagators

$$\Pi(p^2) = \underbrace{\Pi}_{\text{mass Term}} + \underbrace{\Pi'}_{\text{kinetic Term}} p^2 + \frac{1}{2} \underbrace{\Pi''}_{\text{higher order}} p^4 + \dots$$

$\hookrightarrow$ dim > 6 ops

Apparently $3 \times 4 = 12$ parameters. However:

- 3 can be absorbed in a redefinition of the SM parameters g g' v

$$\Pi'_{ww} = 1 \qquad \Pi'_{bb} = 1 \qquad \Pi_{ww} = -m_W^2$$

- $U(1)_{em}$ invariance imposes 2 relation among the mass Terms

$$\Pi_{33}\Pi_{bb} - \Pi_{3b}^2 = 0 \qquad \text{(massless photon)}$$

$$g^2 \Pi_{33} + g'^2 \Pi_{bb} + 2gg'\Pi_{3b} = 0 \qquad \text{(couplings of the photon)}$$

(authomatic if NP preserve electromagnetism)

We are left with 7 parameters:

Barbieri
Pomarol
Rattazzi
Strumia

$$\hat{S} = \frac{g}{g'}\, \Pi'_{3b} \qquad \hat{T} = \frac{1}{m_W^2}\left(\Pi_{33} - \Pi_{ww}\right)$$

$$W = \frac{m_W^2}{2}\, \Pi''_{33} \qquad Y = \frac{m_W^2}{2}\, \Pi''_{bb}$$

$$\hat{U} = -\left(\Pi'_{33} - \Pi'_{ww}\right) \qquad V = \frac{m_W^2}{2}\left(\Pi''_{33} - \Pi''_{ww}\right)$$

$$X = \frac{m_W^2}{2}\, \Pi''_{3b}$$

→ This are enough to encode all the corrections to purely leptonic observables (including neutrinos)

→ all of them are constrained by EWPT to be

$$\lesssim 10^{-3} \; !$$

→ no room for cancellations

List of operators:

→ Gauge bosons: 2 $\quad \mathcal{O}_{WB} \quad \mathcal{O}_H$

$$\mathcal{O}_{WB} = (H^\dagger \sigma^a H)\, W^a_{\mu\nu} B^{\mu\nu} \to p^2 W^3 B \qquad (\hat{S})$$

$$\mathcal{O}_H = |H^\dagger \mathcal{D}^\mu H|^2 \to \delta m^2 Z^2 \qquad (\hat{T})$$

$$\mathcal{O}_{WW} = (\mathcal{D}_\rho W^a_{\mu\nu})^2 \to p^4 W_a^2 \qquad (W)$$

$$\mathcal{O}_{BB} = (\mathcal{D}_\rho B_{\mu\nu})^2 \to p^4 B^2 \qquad (Y)$$

→ Vertex corrections: 3 $\quad \mathcal{O}_{HE} \quad \mathcal{O}_{HL} \quad \mathcal{O}'_{HL}$

$$\mathcal{O}_{HF} = (H^\dagger \mathcal{D}_\mu H)\, J^\mu_F \qquad J^\mu_F = \bar{f}\gamma^\mu f$$

$$\mathcal{O}'_{HF} = (H^\dagger \mathcal{D}_\mu \sigma^a H)\, J^{a\,\mu}_F \qquad J^{a\mu}_F = \bar{f}\gamma^\mu \sigma^a f \quad (\text{only doublets})$$

→ 4-fermion ops: 4 $\quad \mathcal{O}_{EE} \quad \mathcal{O}_{EL} \quad \mathcal{O}_{LL} \quad \mathcal{O}'_{LL}$

$$\mathcal{O}_{FF} = J_F\, J_{F'}$$

$$\mathcal{O}'_{FF} = J^a_F\, J^a_{F'}$$

How can I write these 11 ops in terms of the 7 parameters?

Equations of motion for the SM gauge bosons:

$$\partial^\nu B_{\nu\mu} + \frac{m_W^2}{g^2} g'(g'B_\mu - gW^3_\mu) + g' J_B = 0 + \ldots$$

$$\partial^\nu W^3_{\nu\mu} + \frac{m_W^2}{g^2} g(gW^3_\mu - g'B_\mu) + g J^3 = 0 + \ldots$$

$$\partial^\nu W^+_{\nu\mu} + m_W^2 W^+_\mu + \frac{g}{\sqrt{2}} J^+ = 0 + \ldots$$

$$J_B = \sum_f Y_f J_F \qquad J^a = \sum_{\text{doublets}} J_F^a$$

unconstrained ops
→ 3 GB's
→ multi-higgs
⋮

Solve for the 3 currents involving the electron:

$$J_E \quad J_{e_L} \quad J_L^+$$

This allows to write all the ops involving electron currents (vertex or 4-fermion) in terms of GB's!

$$\hat{S} = \frac{m_W^2}{g g'}\Big[4\,C_{WB} + 4t\left(c'_{HL} - c'_{LL}\right) + \frac{2}{t}\left(C_{HE} - C_{EE} - C_{EL}\right) +$$

$$+ 2t\left(C_{HE} + 2\,C_{HL} - C_{EE} - 2\,C_{LL} - 3\,C_{EL}\right)\Big]$$

$$\hat{T} = \frac{m_W^2}{g^2}\Big[-2\,C_H + 8\left(C_{HE} + C_{HL}\right) - 4\left(C_{EE} + C_{LL} + 2\,C_{EL}\right)\Big]$$

$$W = \frac{m_W^2}{g^2}\Big[2\,C_{WW}\,g^2 - 4\,c'_{LL} - \left(C_{EE} + 4\,C_{LL} + 4\,C_{EL}\right)\Big]$$

$$Y = \frac{m_W^2}{g'^2}\Big[2\,C_{BB}\,g'^2 - C_{EE}\Big]$$

$$\hat{U} = \frac{m_W^2}{g^2}\Big[4\left(C_{HE} + 2\,C_{HL}\right) - 4\left(C_{EE} + 2\,C_{LL} + 3\,C_{EL}\right)\Big]$$

$$V = -\frac{m_W^2}{g^2}\Big[C_{EE} + 4\,C_{LL} + 4\,C_{EL}\Big]$$

$$X = \frac{m_W^2}{g g'}\Big[C_{EE} + 2\,C_{EL}\Big]$$

$$\delta g_{\nu L} = -2\,\frac{m_W^2}{g^2}\left(C_{HE} + 2\,C_{HL}\right) = -\frac{1}{2}\,U + V - t\,X$$

UNIVERSAL THEORIES

This formalism was first proposed to analyse universal models.

Barbieri
Pomarol
Rattazzi
Strumia

UNIVERSAL: only the currents that couple to the B and W^a appear in $\mathcal{L}_{BSM}$.

$$J_B = \sum_F Y_F J_F \qquad J^a = \sum_{\text{doublets}} J^a_F$$

Barbieri et. al. showed that only 4 parameters are relevant: $\hat{S}$ $\hat{T}$ W Y

$$Y_L = -\frac{1}{2}$$
$$Y_E = 1$$

Universality $\rightarrow$

$$C_{HF} = Y_F C_v \qquad C'_{HF} = C'_v$$
$$C_{FF'} = Y_F Y_{F'} C_{4f} \qquad C'_{FF'} = C'_{4f}$$

$$\hat{S}_{univ} = \frac{m_W^2}{gg'}\left[4 C_{WB} + 4t\left(C'_v - C'_{4f}\right) + \frac{1}{t}\left(2C_v - C_{4f}\right)\right]$$

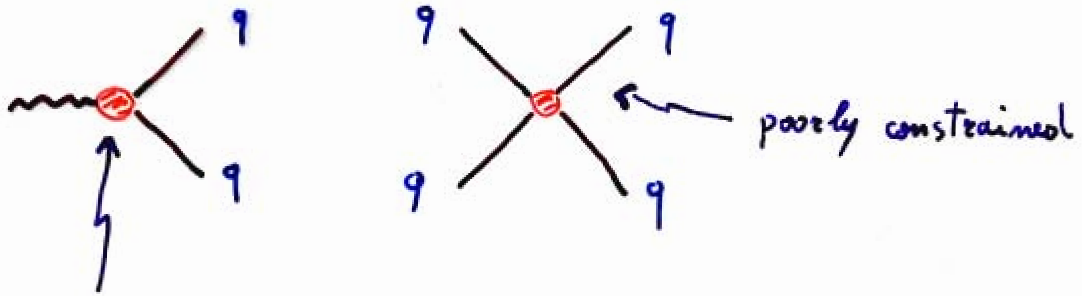
$$\hat{T}_{univ} = \frac{m_W^2}{g^2}\left[-2C_H + 4C_v - C_{4f}\right]$$

$$W_{univ} = \frac{m_W^2}{g^2}\left[2C_{WW}\, g^2 - C'_{4f}\right]$$

$$Y_{univ} = \frac{m_W^2}{g'^2}\left[2C_{BB}\, g'^2 - C_{4f}\right]$$

Including quarks: + 2 (3)

Besides the "oblique" parameters, we are left with corrections involving quark currents:



$$(\bar{q}\gamma^\mu q)\left[\sqrt{g^2+g'^2}\, Z_\mu \left(\delta g_q + \frac{C_q^Z}{m_Z^2}(p^2-m_Z^2)\right) + e\, A_\mu \frac{C_q^\gamma}{m_W^2}\, p^2\right]$$

δg's $\rightarrow$ vertex corrections

c's $\rightarrow$ 4-fermion operators

A throng of new parameters, but...

... only 2 of them are more constrained:

a simple guess

$$\Gamma(Z \to q\bar{q}) \sim g^{SM}_{q_L}\ \delta g_{q_L} + \cancel{\underbrace{g^{SM}_{q_R}}_{\text{smaller:}}\ \delta g_{q_R}}$$

$$\frac{g_R}{g_L} \sim \underset{(d)}{0.18} \quad \text{or} \quad \underset{(u)}{0.44}$$

$$\Gamma(Z \to \text{had}) \sim g^{SM}_{u_L}\ \delta g_{u_L} + g^{SM}_{d_L}\ \delta g_{d_L}$$

$$\sim \delta g_{u_L} - \delta g_{d_L} - \cancel{\underbrace{\frac{\tan\theta_W^2}{3}}_{\text{small}}\left(\delta g_{u_L} + \delta g_{d_L}\right)}$$

Analogously for the C's. Let's define:

$$\delta \varepsilon_q = \delta g_{u_L} - \delta g_{d_L} = \frac{4 m_W^2}{g^2}\left(c'_{HQ} - c'_{HL}\right) + V - \frac{1}{2}\hat{U} - \tan\theta_W\, X$$

$$\delta C_q = C^Z_{u_L} - C^Z_{d_L} = \frac{4 m_W^2}{g^2 + g'^2}\left(c'_{LQ} - c'_{LL}\right) + \cos\theta_W^2\, V - \cos\theta_W \sin\theta_W\, X$$

↓

Only involve c' (triplet current ops)
+ oblique corrections

A probe of the guess

20 indep. ops $\rightarrow$ 7 "oblique" + 2 quark pars

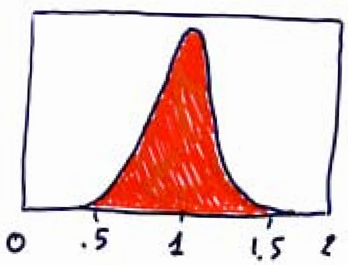
We generated a random set of "models", i.e. set of the operators

$$c_i = \frac{r_i}{\Lambda^2} \qquad -1 < r_i < 1$$

and computed the exact and approximate bound on Λ in 3 cases:

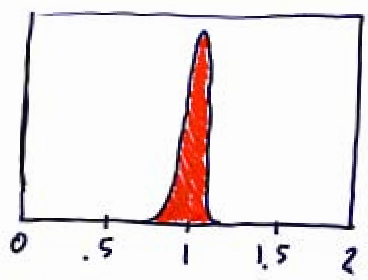
a) Oblique

$$\frac{\Lambda_{appr.}}{\Lambda_{exact}} = 0.95 \pm 0.16$$



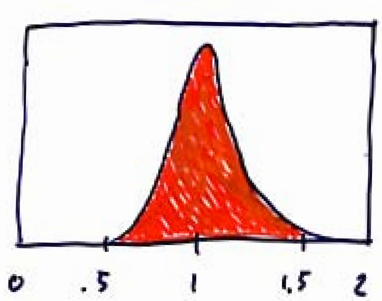
b) Oblique + $\delta\epsilon_q$ + δC_q

$$\frac{\Lambda_{appr.}}{\Lambda_{exact}} = 0.98 \pm 0.06$$



c) All but $\delta\epsilon_q$, δC_q

$$\frac{\Lambda_{appr.}}{\Lambda_{exact}} = 0.98 \pm 0.15$$



Example: Z's

	Universal?	H	L	E	Q	U	D	Exact	Approx.	Oblique
H	✓	1	0	0	0	0	0	6.7	6.7	6.7
B'	✓	$1/2$	$-1/2$	1	$1/6$	$-2/3$	$1/3$	6.7	6.7	6.7
B'_F	✓	0	$-1/2$	1	$1/6$	$-2/3$	$1/3$	4.8	4.8	4.8
$B-L$	no	0	-1	1	$1/3$	$-1/3$	$-1/3$	6.2	6.5	6.5
L	no	0	1	-1	0	0	0	5.8	6.5	6.5
10	no	0	0	1	1	1	0	2.8	3.3	4.3
5	no	0	1	0	0	0	1	4.3	3.7	5.6
13	no	$2/3$	1	$-1/3$	$-1/3$	$-1/3$	1	4.6	4.8	5.7
16	no	0	1	1	1	1	1	4.4	4.7	8.4
Simplest LH	no	—	—	—	—	—	—	2.8	2.6	2.9
SU(6) Super little H.	no	—	—	—	—	—	—	3.1	3.3	3.3

99% CL bounds on $\frac{M_{Z'}}{g_{Z'}}$ in TeV

CONCLUSIONS

We proposed a minimal and simple parametrization of EWPT's → few super-constrained parameters:

$$\begin{array}{rl} 7 & \text{"oblique" pars.} \to \text{gauge + leptons} \\ +2 & \text{quarks} \\ +1 & \text{bottom} \\ \hline 10 & \end{array}$$

→ Can be applied to ANY model beyond the SM

→ Makes the calculation easier and well-defined

- can be directly computed at tree level

RS models, Higgsless, Gauge-Higgs unification, Little Higgs, SM + Heavy particles ...

If the heavy states are integrated out properly, no need to do a global fit with ~20 ops!

→ Gives hints to model-builders