LITTLE HIGGS GOES TO TASI

M. SCHMALTZ*

Physics Department, Boston University
590 Commonwealth Avenue, Boston, MA 02215, USA
Email: schmaltz@bu.edu

These TASI 2004 lecture notes provide a pedagogical introduction to Little Higgs models. The "Simplest Little Higgs" is used wherever explicit examples are given. Precision electroweak constraints and collider phenomenology as well as T-parity are briefly discussed.

1. Introduction

A few years before the start of the LHC program, electroweak symmetry breaking remains poorly understood. The detailed quantitative fit of Standard Model predictions to precision experiments at the weak scale strongly suggests that electroweak symmetry is broken by one or more weakly coupled Higgs doublets. However, fundamental scalar particles suffer from a radiative instability to their masses, leading us to expect additional structure (such as compositeness, supersymmetry, little Higgs, ...) near the weak scale.

Interestingly, we can turn this problem into a prediction for the LHC. The argument goes as follows: Let us assume that precision electroweak data are indeed telling us that there are no new particles beyond the Standard Model (with the exception of possible additional Higgs doublets) with masses at or below the weak scale. Then physics at the weak scale may be described by an "effective Standard Model" which has the particle content of the Standard Model and in which possible new physics is parametrized by higher dimensional operators suppressed by the new physics scale $\Lambda \gtrsim$ TeV. All renormalizable couplings are as in the Standard Model. If there are additional Higgs fields then more complicated Higgs self-couplings as

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well as Yukawa couplings are possible. Since no Higgs particles have been discovered so far, the effects of additional Higgs fields can be parametrized by effective operators for the Standard Model fields.

The higher dimensional operators can be categorized by the symmetries which they break. The relevant symmetries are baryon and lepton number (B and L), CP and flavor symmetries, and custodial $SU(2)$ symmetry. The wealth of indirect experimental data can then be translated into bounds on the scale suppressing the operators [1,2,3,4]. Examples of such operators and the resulting bounds are summarized in Table 1.

Table 1. Lower bounds on the scale which suppresses higher dimensional operators that violate approximate symmetries of the Standard Model.

broken symmetry	operators	scale Λ
B, L	$(QQQL)/\Lambda^2$	10^{13} TeV
flavor (1,2$^{\text{nd}}$ family), CP	$(\bar{d}s\bar{d}s)/\Lambda^2$	1000 TeV
flavor (2,3$^{\text{rd}}$ family)	$m_b(\bar{s}\sigma_{\mu\nu}F^{\mu\nu}b)/\Lambda^2$	50 TeV
custodial $SU(2)$	$(h^\dagger D_\mu h)^2/\Lambda^2$	5 TeV
none (S-parameter)	$(D^2h^\dagger D^2h)/\Lambda^2$	5 TeV

The bounds imply that physics at the TeV scale has to conserve B and L, flavor and CP to a very high accuracy, and that violations of custodial symmetry and contributions to the S-parameter should also be small.

The question then becomes if it is possible to add new physics at the TeV scale to the SM which stabilizes the Higgs mass but does not violate the above bounds. To understand the requirements on this new physics better we must look at the source of the Higgs mass instability. The three most dangerous radiative corrections to the Higgs mass in the Standard Model come from one-loop diagrams with top quarks, $SU(2)$ gauge bosons, and the Higgs itself running in the loop (Figure 1).

All other diagrams give smaller contributions because they involve small coupling constants. Assuming that the Standard Model remains valid up to a cut-off scale Λ the three diagrams give

$$\begin{aligned}
\text{top loop} \quad & -\tfrac{3}{8\pi^2}\lambda_t^2\Lambda_{top}^2 && \sim -(2\ \text{TeV})^2 \\
SU(2)\ \text{gauge boson loops} \quad & \tfrac{9}{64\pi^2}g^2\Lambda_{gauge}^2 && \sim (700\ \text{GeV})^2 \\
\text{Higgs loop} \quad & \tfrac{1}{16\pi^2}\lambda^2\Lambda_{Higgs}^2 && \sim (500\ \text{GeV})^2
\end{aligned}$$

The Higgs soft mass includes the sum of these contributions and a tree level

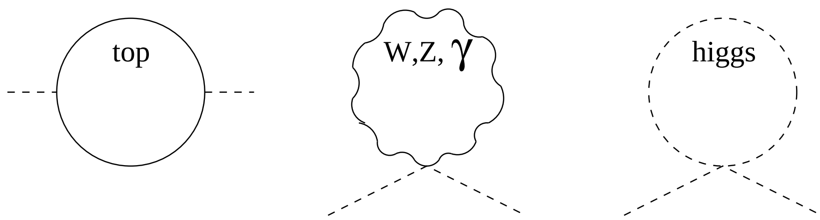

Figure 1. *The most significant quadratically divergent contributions to the Higgs mass in the Standard Model.*

mass-squared parameter.

In order for this to produce an expectation value for the Higgs of order the weak scale without fine tuning to more than 10%, the cut-off must satisfy

$$\Lambda_{top} \lesssim 2 \text{ TeV} \qquad \Lambda_{gauge} \lesssim 5 \text{ TeV} \qquad \Lambda_{Higgs} \lesssim 10 \text{ TeV} \tag{1}$$

We see that the Standard Model with a cut-off near the maximum attainable energy at the Tevatron (~ 1 TeV) is natural, and we should not be surprised that we have not observed any new physics. However, the Standard Model with a cutoff of order the LHC energy would be fine tuned and we expect to see new physics at the LHC.

More specifically, we expect new physics which cuts off the diverging top loop at or below 2 TeV. In a weakly coupled theory this implies that there are new particles with masses at or below 2 TeV. These particles must couple to the Higgs, and enable us to write a one-loop quadratically divergent diagram which cancels the contribution from the top loop. In order for this cancellation to be natural, the new particles must be related to the top quark by a symmetry which implies that the new particles have similar quantum numbers to top quarks. Thus naturalness arguments predict a new multiplet of particles with mass below 2 TeV which carry color and are therefore easily produced at the LHC. In supersymmetry these new particles are of course the top squarks.

The contributions from $SU(2)$ gauge loops must also be canceled by new particles which are related to the Standard Model $SU(2)$ gauge bosons by a symmetry. The masses of these particles must be at or below 5 TeV for the cancellation to be natural. Similarly, the Higgs loop requires new particles related to the Higgs at 10 TeV. We summarize the upper bounds on new particle masses which we obtain from naturalness in Table 2.

Given the center of mass energy of the LHC of 14 TeV these predictions are very exciting, and encourage us to explore different possibilities for what

Table 2. Predictions for maximum masses of "partner" particles.

Standard Model loop	maximum partner mass
top	2 TeV
weak bosons	5 TeV
Higgs	10 TeV

the new particles could be.

One example of new particles near the TeV scale which can appear in loops to cancel quadratic divergences are the superpartners predicted in the Minimal Supersymmetric Standard Model [5]. There the top quark loop is canceled by a corresponding loop with stop squarks. Supersymmetry also predicts the necessary relationship between top and stop coupling constants. Furthermore, the two diagrams are each proportional to Λ^2, the cut-off used to regulate the two divergences. In general, the cut-offs for the two diagrams need not be the same, and therefore the divergences from two diagrams do not cancel. However, if a supersymmetric cut-off is used, then the Λ's for the Standard Model particles and their superpartners are the same.

Another possibility is that the Higgs is a composite resonance at the TeV scale as in technicolor [6] or composite Higgs models [7,8,9]. Or extra dimensions might be lurking at the TeV scale with possible new mechanisms to stabilize the Higgs mass [10].

Here, we explore another possibility, that the Higgs is a pseudo-Nambu-Goldstone boson as suggested in [11,12]. This idea was recently revived by Arkani-Hamed, Cohen and Georgi who also constructed the first successful "little Higgs" model [13], and thereby started an industry of "little model building" [14–33].

2. Nambu-Goldstone Bosons

Nambu-Goldstone bosons (NGBs) arise whenever a continuous global symmetry is spontaneously broken. If the symmetry is exact, the NGBs are exactly massless and have only derivative couplings.

<u>$U(1)$ example:</u> Consider for example a theory with a single complex scalar field ϕ with potential $V = V(\phi^*\phi)$. The kinetic energy term $\partial_\mu \phi^* \partial^\mu \phi$ and the potential are invariant under the $U(1)$ symmetry transformation

$$\phi \to e^{i\alpha}\phi \tag{2}$$

If the minimum of the potential is not at the origin but at some distance f away as in the famous “wine bottle” or “Mexican hat” potential (Figure 2), then the $U(1)$ symmetry is spontaneously broken in the vacuum. We

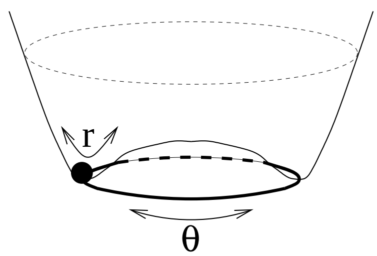

Figure 2. *The “Mexican hat” potential for Φ. The black dot represents the vacuum expectation value f, r is the radial mode and θ the Nambu-Goldstone boson.*

expand the field for small fluctuations around the vacuum expectation value (VEV)

$$\phi(x) = \frac{1}{\sqrt{2}}(f + r(x))\ e^{i\theta(x)/f} \tag{3}$$

where f is the VEV of r, $r(x)$ is the massive “radial mode” and $\theta(x)$ is the NGB. The factor of $1/\sqrt{2}$ ensures canonical kinetic terms for the real fields r and θ.

The radial field r is invariant under the $U(1)$ symmetry transformation Eq. (2) whereas the NGB field θ shifts

$$\theta \to \theta + \alpha \tag{4}$$

under $U(1)$ transformations. We say that the $U(1)$ symmetry is ”non-linearly” realized. We may now imagine integrating out the massive field r and writing the resulting effective Lagrangian for the NGB $\theta(x)$. θ cannot have a mass or any potential, because the shift symmetry forbids all non-derivative couplings of θ.

<u>Non-Abelian examples:</u> In the generalization to spontaneously broken non-Abelian symmetries we find one NGB for every broken symmetry generator. For example, we may break $SU(N) \to SU(N-1)$ with a VEV for a single fundamental ϕ of $SU(N)$. The number of broken generators is the total number of generators of $SU(N)$ minus the number of unbroken generators,

i.e.

$$[N^2-1]-[(N-1)^2-1]=2N-1 \tag{5}$$

The NGBs are conveniently parametrized by writing

$$\phi = exp\left\{ \frac{i}{f} \left(\begin{array}{c|c} & \pi_1 \\ & \vdots \\ & \pi_{N-1} \\ \hline \pi_1^* \cdots \pi_{N-1}^* & \pi_0/\sqrt{2} \end{array} \right) \right\} \begin{pmatrix} 0 \\ \vdots \\ 0 \\ f \end{pmatrix} \equiv e^{i\pi/f}\phi_0 \tag{6}$$

The field π_0 is real whereas the the fields $\pi_1 \cdots \pi_{N-1}$ are complex. The last equality defines a convenient short-hand notation which we will employ whenever the precise form of π and ϕ_0 is clear from the context.

Another example of symmetry breaking and NGBs which has found applications in little Higgs model building is

$$SU(N) \to SO(N)\ . \tag{7}$$

Here the number of NGBs is the number of fields in the adjoint of $SU(N)$ minus the number of fields in the adjoint of $SO(N)$ (antisymmetric tensor), i.e.

$$[N^2-1]-\frac{N(N-1)}{2}=\frac{N(N+1)}{2}-1\ . \tag{8}$$

For even N we also have

$$SU(N) \to SP(N) \tag{9}$$

and the number of NGBs is the number of fields in the adjoint of $SU(N)$ minus the number of fields in the adjoint of $SP(N)$ (symmetric tensor), i.e.

$$[N^2-1]-\frac{N(N+1)}{2}=\frac{N(N-1)}{2}-1\ . \tag{10}$$

Finally, for

$$SU(N)\times SU(N) \to SU(N) \tag{11}$$

the number of NGBs is

$$2[N^2-1]-[N^2-1]=N^2-1\ . \tag{12}$$

In this last case the symmetry breaking is achieved by a VEV which transforms as a bi-fundamental under the two $SU(N)$ symmetries. Denoting transformation matrices of the two $SU(N)$ as L and R respectively we have

$$\phi \to L\,\phi\,R^\dagger\ . \tag{13}$$

The symmetry breaking VEV is proportional to the unit matrix

$$< \phi > \equiv \phi_0 = f \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix} \tag{14}$$

This VEV is left invariant under "vector" transformations for which $L = R \equiv U$

$$\phi_0 \ \longrightarrow U \, \phi_0 \, U^\dagger = \phi_0 \ , \tag{15}$$

all other symmetry generators (the "axial" generators) are broken and correspond to NGBs which can be parametrized as

$$\phi = \phi_0 \, e^{i\pi/f} = f \, e^{i\pi/f} \tag{16}$$

where π is a Hermitian traceless matrix with $N^2 - 1$ independent components.

2.1. *How do NGBs transform ?*

We now show how NGBs transform under the broken and unbroken symmetries in the example of $SU(N) \to SU(N-1)$ which is often denoted in more mathematical notation as $SU(N)/SU(N-1)$. The NGBs are parametrized as $\phi \equiv e^{i\pi}\phi_0$ as in Eq. (6). Let's consider first transformations under the unbroken $SU(N-1)$. Then we have

$$\phi \to U_{N-1} \ \phi = (U_{N-1} \ e^{i\pi} \ U^\dagger_{N-1}) \, U_{N-1} \ \phi_0 = e^{i(U_{N-1} \, \pi \, U^\dagger_{N-1})} \, \phi_0 \tag{17}$$

where in the second equality we used the fact that the symmetry breaking ϕ_0 is invariant under the unbroken U_{N-1} transformations. Therefore the NGBs transform in the usual "linear" way under $SU(N-1)$ transformations $\pi \to U_{N-1} \, \pi \, U^\dagger_{N-1}$. Explicitly, in the case of $SU(N)/SU(N-1)$ the unbroken $SU(N-1)$ transformations are

$$U_{N-1} = \begin{pmatrix} \hat{U}_{N-1} & 0 \\ 0 & 1 \end{pmatrix} \ . \tag{18}$$

The single real NGB transforms as a singlet whereas the $N-1$ complex NGBs transform as

$$\left(\begin{array}{c|c} 0 & \vec{\pi} \\ \hline \vec{\pi}^\dagger & 0 \end{array} \right) \to U_{N-1} \left(\begin{array}{c|c} 0 & \vec{\pi} \\ \hline \vec{\pi}^\dagger & 0 \end{array} \right) U^\dagger_{N-1} = \left(\begin{array}{c|c} 0 & \hat{U}_{N-1}\vec{\pi} \\ \hline \vec{\pi}^\dagger \hat{U}^\dagger_{N-1} & 0 \end{array} \right) \tag{19}$$

where we used a vector notation $\vec{\pi}$ to represent the $N-1$ complex NGBs as a column vector. We see that $\vec{\pi} \to \hat{U}_{N-1}\vec{\pi}$, i.e. $\vec{\pi}$ transforms in the fundamental representation of $SU(N-1)$.

Under the broken symmetry transformations we have

$$\begin{aligned} \phi \to U\, e^{i\pi}\, \phi_0 &= exp\left\{ i \begin{pmatrix} 0 & \vec{\alpha} \\ \vec{\alpha}^\dagger & 0 \end{pmatrix} \right\} exp\left\{ i \begin{pmatrix} 0 & \vec{\pi} \\ \vec{\pi}^\dagger & 0 \end{pmatrix} \right\} \phi_0 \\ &\equiv exp\left\{ i \begin{pmatrix} 0 & \vec{\pi}' \\ \vec{\pi}'^\dagger & 0 \end{pmatrix} \right\} U_{N-1}(\vec{\alpha}, \vec{\pi})\ \phi_0 \\ &= exp\left\{ i \begin{pmatrix} 0 & \vec{\pi}' \\ \vec{\pi}'^\dagger & 0 \end{pmatrix} \right\} \phi_0 \end{aligned} \tag{20}$$

where in the second equality we used the fact that any $SU(N)$ transformation can be written as a product of a transformation in the coset $SU(N)/SU(N)$ times an $SU(N-1)$ transformation [34]. The $U_{N-1}(\vec{\alpha}, \vec{\pi})$ transformation which depends on $\vec{\alpha}$ and $\vec{\pi}$ leaves ϕ_0 invariant and can therefore be removed. Equation (20) defines the transformed field $\vec{\pi}'$ which – in general – is a complicated function of $\vec{\alpha}$ and $\vec{\pi}$. To linear order the transformation is simple

$$\vec{\pi} \to \vec{\pi}' = \vec{\pi} + \vec{\alpha} \tag{21}$$

which shows that the NGBs shift under the non-linearly realized symmetry transformations. As in the $U(1)$ case, the shift symmetry ensures that NGBs can only have derivative interactions.

2.2. *Effective Lagrangian for NGBs*

Our goal for this section is to write the most general allowed effective Lagrangian for only the massless NGB fields, and respecting the full $SU(N)$ symmetry. This is where the utility of the exponentiated fields ϕ becomes obvious: while the full $SU(N)$ transformations on the π's are complicated, the ϕ's transform very simply. To get the low energy effective Lagrangian we expand in powers of ∂_μ/Λ and write the most general possible $SU(N)$ invariant function of $\phi = e^{i\pi/f}\phi_0$ at every order. With no derivatives we can form two basic gauge invariants objects $\phi^\dagger\phi = f^2$ and $\epsilon^{a_1\cdots a_N}\phi_{a_1}\phi_{a_2}\cdots\phi_{a_N} = 0$. Thus the most general invariant contribution to the potential is simply a constant. You can convince yourself that the most general term that can be written at quadratic order is a constant times $|\partial_\mu\phi|^2$ and therefore, we have

$$\mathcal{L} = const + f^2|\partial_\mu\phi|^2 + \mathcal{O}(\partial^4) \tag{22}$$

where we normalized the coefficient of the second order term such that the π fields have canonical kinetic terms. Note that the second order term expanded to higher order in the π fields contains interactions which determine the scattering of arbitrary numbers of π's at low energies in terms of the single parameter f.

3. Constructing a little Higgs model

Now that we know how to write a Lagrangian for NGBs we would like to use this knowledge to write a model where the Higgs is a NGB. The explicit model we are going to construct in the remainder of this section is the "simplest little Higgs" [19,20,29]. For example, consider the symmetry breaking pattern $SU(3)/SU(2)$ with NGBs

$$\pi = \left(\begin{array}{cc|c} -\eta/2 & 0 & \multirow{2}{*}{h} \\ 0 & -\eta/2 & \\ \hline \multicolumn{2}{c|}{h^\dagger} & \eta \end{array}\right) \tag{23}$$

Note that h is a doublet under the unbroken $SU(2)$ as required for the Standard Model and it is an NGB, it shifts under "broken" $SU(3)$ transformations. η is an $SU(2)$ singlet which we will ignore for simplicity in most of the following. To see what interactions we get for h from the Lagrangian we expand

$$\phi = exp\left\{\frac{i}{f}\begin{pmatrix} 0 & h \\ h^\dagger & 0 \end{pmatrix}\right\}\begin{pmatrix} 0 \\ f \end{pmatrix} = \begin{pmatrix} 0 \\ f \end{pmatrix} + i\begin{pmatrix} h \\ 0 \end{pmatrix} - \frac{1}{2f}\begin{pmatrix} 0 \\ h^\dagger h \end{pmatrix} + \cdots \tag{24}$$

and therefore

$$f^2|\partial_\mu\phi|^2 = \left|\begin{pmatrix} \partial_\mu h \\ 0 \end{pmatrix} + \cdots\right|^2 = |\partial_\mu h|^2 + \frac{|\partial_\mu h|^2 h^\dagger h}{f^2} + \cdots \tag{25}$$

which contains the Higgs kinetic term as well as interactions suppressed by the symmetry breaking scale f. Since the Lagrangian contains non-renormalizable interactions, it can only be an effective low-energy description of physics. To determine the cut-off Λ at which the theory becomes strongly coupled we can compute a loop and ask at which scale it becomes as as important as a corresponding tree level diagram. The simplest example is quadratically divergent one-loop contribution to the kinetic term which stems from contracting $h^\dagger h$ in the second term in Eq. (25) into a loop. Cutting the divergence off at Λ we find a renormalization of the kinetic term proportional to

$$\frac{1}{f^2}\frac{\Lambda^2}{16\pi^2} \tag{26}$$

and therefore $\Lambda \lesssim 4\pi f$.

Summarizing, we now have a theory which produces a "Higgs" doublet transforming under an exactly preserved (global) $SU(2)$. This "Higgs" is a NGB and therefore exactly massless. It has non-renormalizable interactions suppressed by the scale f which become strongly coupled at $\Lambda = 4\pi f$. Because of the shift symmetry no diagrams, divergent or not, can give rise to a mass for h. Anticipating that we are going to use this theory as a model for the Higgs in the Standard Model we summarize the relevant scales in Figure 3. On the other hand, this theory is still vary far from

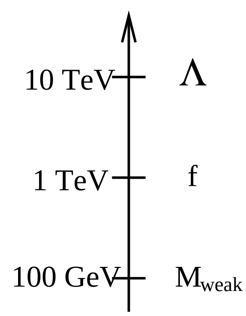

Figure 3. *Energy scales of a typical Little Higgs theory.*

what we want. An NGB can only have derivative interactions, i.e. no gauge interactions, no Yukawa couplings and no quartic potential. Any of these interactions explicitly break the shift symmetry $h \to h + const$. In the following subsections we discuss how to add these interactions without re-introducing quadratic divergences.

3.1. *Gauge interactions*

Let us try to introduce the $SU(2)$ gauge interactions for h (we ignore hypercharge for the moment, it will be easy to add later). To do so we simply follow our nose and see where it leads us. We will arrive at the right answer after a few unsuccessful attempts.

First attempt: Let's simply add couplings to $SU(2)$ gauge bosons in the usual way, i.e. in addition to the Lagrangian Eq. (25) we add the term

$$|gW_\mu h|^2 \tag{27}$$

and another term with one derivative and one $SU(2)$ gauge boson W_μ as required by gauge invariance. These terms allow us to write Feynman

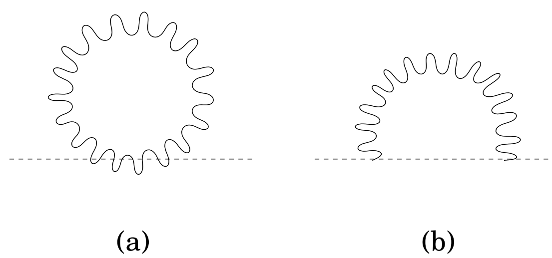

Figure 4. *Quadratically divergent gauge loop contributions to the Higgs mass.*

diagrams with quadratic divergences (Figure 4) which contribute to the "Higgs" mass

$$\frac{g^2}{16\pi^2}\Lambda^2 h^\dagger h \tag{28}$$

Note that these diagrams are exactly the quadratically divergent Standard Model gauge loops which we set out to cancel. We apparently gained nothing, we started with a theory in which the Higgs was protected by a non-linearly realized $SU(3)$ symmetry (under which h shifts) but then we added the term Eq. (27) which completely and explicitly breaks the symmetry. Of course, we necessarily have to break the shift symmetry in order to generate gauge interactions for h but we must break the symmetry in a subtler way to avoid quadratic divergences in the Higgs mass.

Second attempt: Let's write more $SU(3)$ symmetric looking expressions and add the coupling

$$|g \begin{pmatrix} W_\mu & \\ & 0 \end{pmatrix} \phi\,|^2 \tag{29}$$

where W_μ contains the three $SU(2)$ gauge bosons. (Really, we write $|D_\mu \phi|^2$ where the covariant derivative involves only $SU(2)$ gauge bosons. The two-gauge-boson-coupling is then Eq. (29)). This still allows a quadratically divergent contribution to the Higgs mass. The diagram is the same as before except with external ϕ fields and gives

$$\frac{g^2}{16\pi^2}\Lambda^2\ \phi^\dagger \begin{pmatrix} 1 & & \\ & 1 & \\ & & 0 \end{pmatrix} \phi = \frac{g^2}{16\pi^2}\Lambda^2\, h^\dagger h + \cdots \tag{30}$$

where the projection matrix $diag(1,1,0)$ arises from summing over the three $SU(2)$ gauge bosons running in the loop. Not surprisingly, we got the same answer as before because we added the same interactions, just using a fancier notation.

Third attempt: Let us preserve $SU(3)$ by gauging the full $SU(3)$ symmetry, i.e. by adding $|D_\mu \phi|^2$ where now the covariant derivative contains the 8 gauge bosons of $SU(3)$. Again we can write the same quadratically divergent diagram and find

$$\frac{g^2}{16\pi^2}\Lambda^2\ \phi^\dagger \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \phi = \frac{g^2}{16\pi^2}\Lambda^2 f^2 \tag{31}$$

which has no dependence on the Higgs field. The quadratic divergence only contributes a constant term to the vacuum energy but no Higgs mass! Unfortunately, we have also lost the "Higgs"! The NGBs are "eaten" by the heavy $SU(3)$ gauge bosons corresponding to the broken generators, i.e. they become the longitudinal components of the gauge bosons.

We have now exhausted all possible ways of adding $SU(2)$ gauge interactions to our simple toy model for h. The lesson is that we can avoid the quadratically divergent contribution to the Higgs mass by writing $SU(3)$ invariant gauge interactions, the problem that remains is that our "Higgs" was eaten. But this is easy to fix.

Fourth attempt (successful): We use two copies of NGBs ϕ_1 and ϕ_2 and add $SU(3)$ invariant covariant derivatives for both. We expect no quadratic divergence for either of the NGBs but only one linear combination will be eaten. To see how this works in detail we parametrize

$$\phi_1 = e^{i\pi_1/f} \begin{pmatrix} \\ f \end{pmatrix} \qquad \phi_2 = e^{i\pi_2/f} \begin{pmatrix} \\ f \end{pmatrix} \tag{32}$$

where – for simplicity – we assumed identical symmetry breaking scales $f_1 = f_2 = f$, and we also assumed that the VEVs for ϕ_1 and ϕ_2 are aligned. We write the Lagrangian

$$\mathcal{L} = |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2 \tag{33}$$

The two interaction terms allow writing two quadratically divergent one-loop diagrams similar to the one's of the previous attempt (Figure 5.a) which give

$$\frac{g^2}{16\pi^2}\Lambda^2\ (\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2) = \frac{g^2}{16\pi^2}\Lambda^2\ (f^2 + f^2) \tag{34}$$

i.e. no potential or mass term for any of the NGBs. However only one linear combination of π_1 and π_2 is eaten as there is only one set of hungry massive

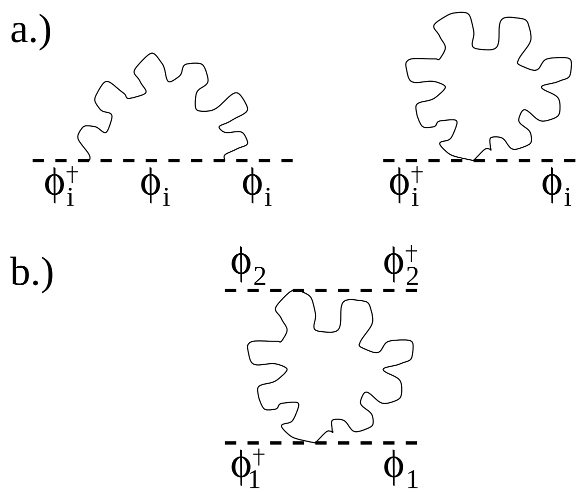

Figure 5. *a.) Quadratically divergent gauge loop contributions which do not contribute to the Higgs potential, b.) log-divergent contribution to the Higgs mass.*

$SU(3)$ gauge bosons. A simple way to understand this result is to notice that each of the two diagrams only involves one of the ϕ fields, therefore the diagrams are the same as in the theory with only one ϕ in which all NGBs are eaten, therefore none can get a potential. This reasoning changes once we consider diagrams involving both ϕ_1 and ϕ_2. For example, the diagram in Figure 5.b gives

$$\frac{g^4}{16\pi^2}\log\left(\frac{\Lambda^2}{\mu^2}\right)\ |\phi_1^\dagger\phi_2|^2 \tag{35}$$

which does depend on the "Higgs" field but is not quadratically divergent. To calculate the Higgs dependence we choose a convenient parametrization

$$\phi_1 = exp\left\{i\begin{pmatrix} & k \\ k^\dagger & \end{pmatrix}\right\} exp\left\{i\begin{pmatrix} & h \\ h^\dagger & \end{pmatrix}\right\}\begin{pmatrix} \\ f \end{pmatrix} \tag{36}$$

$$\phi_2 = exp\left\{i\begin{pmatrix} & k \\ k^\dagger & \end{pmatrix}\right\} exp\left\{-i\begin{pmatrix} & h \\ h^\dagger & \end{pmatrix}\right\}\begin{pmatrix} \\ f \end{pmatrix} \tag{37}$$

The field k can be removed by an $SU(3)$ gauge transformation, it corresponds to the "eaten" NGBs, h cannot simultaneously be removed from ϕ_1 and ϕ_2, it is physical. In the following we will work in the unitary gauge for $SU(3)$ where k has been rotated away. Then we have

$$\begin{aligned}\phi_1^\dagger \phi_2 &= \begin{pmatrix} 0 & f \end{pmatrix} exp\left\{ -\frac{2i}{f} \begin{pmatrix} & h \\ h^\dagger & \end{pmatrix} \right\} \begin{pmatrix} 0 \\ f \end{pmatrix} \\ &= \left[f^2 \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} - 2fi \begin{pmatrix} & h \\ h^\dagger & \end{pmatrix} - 2 \begin{pmatrix} h^\dagger h & \\ & h^\dagger h \end{pmatrix} + \cdots \right]_{33} \\ &= f^2 - 2h^\dagger h + \cdots \end{aligned} \tag{38}$$

and we see that Eq. (35) contains a mass for h $g^4/(16\pi^2) \log\left(\frac{\Lambda^2}{\mu^2}\right) f^2 \sim M_{weak}^2$ for g equal to the $SU(2)$ gauge coupling and $f \sim$ TeV. To summarize, the theory of two complex triplets which both break $SU(3) \to SU(2)$ automatically produces a "Higgs" doublet pseudo-NGB which does not receive quadratically divergent contributions to it's mass. There are log-divergent and finite contributions and from these the natural size for the "Higgs" mass is $f/4\pi \sim M_{weak}$.

3.2. *Symmetry argument, collective breaking*

Let us understand the absence of a quadratic divergence to the mass of h using symmetries. The lesson we learn is valuable as it generalizes to other couplings, it provides a general recipe for constructing little Higgs theories.

Without gauge interactions, our theory would consist of two non-linear sigma models corresponding to the spontaneous breaking of $SU(3)$ to $SU(2)$, the coset $[SU(3)/SU(2)]^2$. There are 10 spontaneously broken generators and therefore 10 NGBs. The gauge couplings explicitly break some of the global symmetries. For example, the two gauge boson - two scalar coupling

$$\mathcal{L} \sim |gA_\mu \phi_1|^2 + |gA_\mu \phi_2|^2 \tag{39}$$

breaks the two previously independent $SU(3)$ symmetries to the diagonal (gauged) $SU(3)$. Thus only one of the spontaneously broken symmetries is exact, and therefore only one set of exact NGBs arises, the eaten ones. The other linear combination, corresponding to the explicitly broken axial $SU(3)$, gets a potential from loops.

However, as we saw before, there is no quadratically divergent contribution to the potential. This is easy to understand by considering the symmetries left invariant by each of the terms in Eq. (39) separately. Imagine setting the gauge coupling of ϕ_2 to zero, then the Lagrangian has 2 independent $SU(3)$ symmetries, one acting on ϕ_1 (and A_μ) and the other acting on ϕ_2. Thus we now have two spontaneously broken $SU(3)$ symmetries and therefore 10 exact NGBs (5 of which are eaten). Similarly, if the gauge coupling of ϕ_1 is set to zero, there are again two spontaneously broken $SU(3)$'s. Only in the presence of gauge couplings for both ϕ_1 and ϕ_2 are the two $SU(3)$ symmetries explicitly broken to one $SU(3)$ and only then can the "Higgs" h develop a potential. Therefore any diagram which contributes to the Higgs mass must involve the gauge couplings for both ϕ_1 and ϕ_2. But there are no quadratically divergent one-loop diagrams involving both couplings.

This is the general mechanism employed by "little Higgs" theories [13]: *The "little Higgs" is a pseudo-Nambu-Goldstone boson of a spontaneously broken symmetry. This symmetry is also explicitly broken but only "collectively", i.e. the symmetry is broken when two or more couplings in the Lagrangian are non-vanishing. Setting any one of these couplings to zero restores the symmetry and therefore the masslessness of the "little Higgs".*

We now know how to construct a theory with a naturally light scalar doublet coupling to $SU(2)$ gauge bosons. To turn this into an extension of the Standard Model we still need *i.* Yukawa couplings, *ii.* hypercharge and color, and *iii.* a Higgs potential with a quartic coupling.

3.3. *Top Yukawa coupling*

The numerically most significant quadratic divergence stems from top quark loops. Thus the cancellation of the quadratic divergence associated with the top Yukawa is the most important. Let us construct it explicitly. The crucial trick is to introduce $SU(3)$ symmetries into the Yukawa couplings which are only broken collectively. First, we enlarge the quark doublets into triplets $\Psi \equiv (t, b, T)$ transforming under the $SU(3)$ gauge symmetry. The quark singlets remain the same t^c and b^c except that we also need to add a Dirac partner T^c for T. Note that we are using a notation in which all quark fields are left-handed Weyl spinors, and the Standard Model Yukawa couplings are of the form $h^\dagger Q t^c$. Let us change notation slightly to reflect the fact that t^c and T^c mix and call them t^c_1 and t^c_2. We can now write two couplings which both look like they contribute to the top Yukawa coupling[a]

$$\mathcal{L}_{yuk} = \lambda_1 \phi_1^\dagger \Psi t_1^c + \lambda_2 \phi_2^\dagger \Psi t_2^c \tag{40}$$

[a]We do not write the couplings $\phi_1^\dagger \Psi t_2^c$ and $\phi_2^\dagger \Psi t_2^c$ as they would reintroduce quadratic divergences. They can be forbidden by global $U(1)$ symmetries and are therefore not generated by loops.

To see what couplings for the Higgs arise we substitute the parametrization Eq. (37) and expand in powers of h. For simplicity, let us also set $\lambda_1 \equiv \lambda_2 \equiv \lambda/\sqrt{2}$. This will reduce the number of terms we encounter because it preserves a parity $1 \leftrightarrow 2$, but the main points here are independent of this choice. We find

$$\begin{aligned}\mathcal{L} &\sim \frac{\lambda}{\sqrt{2}}\left[fT(t_2^c+t_1^c)+ih^\dagger Q(t_2^c-t_1^c)-\frac{1}{2f}h^\dagger hT(t_2^c+t_1^c)+\cdots\right]\\ &= \lambda f(1-\frac{1}{2f^2}h^\dagger h)TT^c+\lambda h^\dagger Qt^c+\cdots \end{aligned}\qquad (41)$$

where in the second line we have redefined fields $T^c = (t_2^c + t_1^c)/\sqrt{2}$ and $t^c = i(t_2^c - t_1^c)/\sqrt{2}$. We find a top Yukawa coupling and identify $\lambda = \lambda_t$. The Dirac fermion T, T^c has a mass $\lambda_t f$ and a coupling to two Higgs fields with coupling constant $\lambda_t/(2f)$. The couplings and masses are related by the underlying $SU(3)$ symmetries. To see how the new fermion and it's

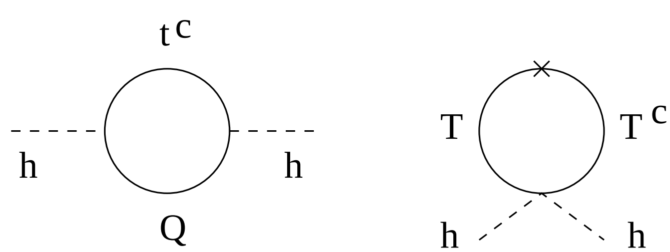

Figure 6. *The quadratically divergent contribution to the Higgs mass from the top loop is canceled by the T loop.*

couplings to the Higgs cancel the quadratic divergence from the top quark loop we compute the fermion loops including interactions to order λ^2. The two relevant diagrams (Figure 6) give

$$\frac{\lambda_t^2}{16\pi^2}\,\Lambda^2 h^\dagger h+\frac{\lambda_t^2 f^2}{16\pi^2}(1-\frac{h^\dagger h}{f^2})\,\Lambda^2+\mathcal{O}(h^4)=\text{const.}+\mathcal{O}(h^4) \qquad (42)$$

The quadratically divergent contribution to the Higgs mass from the top and T loops cancel![b]

While this computation allowed us to see explicitly that the quadratic divergence from t and T cancel, the absence of a quadratic divergence to the

[b]In order for the two cut-offs for the two loops to be identical, the new physics at the cut-off must respect the $SU(3)$ symmetries. This is analogous to the situation in SUSY where the boson-fermion cancellation also relies on a supersymmetric regulator/cutoff.

Higgs mass is much more naturally understood by analyzing the symmetries of the Lagrangian for the ϕ_i fields, Eq. (40). First note that the Yukawa coupling Lagrangian preserves one $SU(3)$ symmetry, the gauge symmetry. The term proportional to λ_1 forces symmetry transformations of ϕ_1 and Ψ to be aligned and the term proportional to λ_2 also forces ϕ_2 to transform like Ψ. Thus, in the presence of both terms the global symmetry breaking pattern is only $SU(3)/SU(2)$ with 5 NGBs which are all eaten by the heavy $SU(3)$ gauge bosons. However, if we set either of the λ_i to zero the symmetry of Eq. (40) is enhanced to $SU(3)^2$ because the ϕ_i can now rotate independently. Thus with either of the λ_i we expect two sets of NGBs. One linear combination is eaten but the other is the "little Higgs". To understand radiative stability of this result we observe that a contribution to the Higgs potential can only come from a diagram which involves both λ_i. The lowest order fermion diagram which involves both λ_i is the loop shown in Figure 7, it is proportional to $|\lambda_1\lambda_2|^2$. You can easily convince yourself

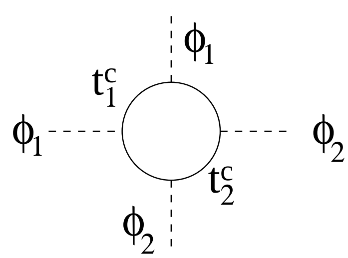

Figure 7. *A log divergent contribution to the Higgs mass from the top and T loops proportional to $|\lambda_1\lambda_2|^2$.*

that you cannot draw a diagram which contributes to the Higgs potential and is proportional to only a single power of $\lambda_1\lambda_2$. This also follows from an argument using "spurious" symmetries: assign t_1^c charge 1 under a $U(1)_1$ symmetry while all other fields are neutral. The symmetry is broken by the Yukawa coupling λ_1, but we can formally restore it by assigning the "spurion" λ_1 charge -1. Any effective operators which may be generated by loops must be invariant under this symmetry. In particular, operators which contribute to the Higgs potential and do not contain the fermion field t_1^c can depend on the spurion λ_1 only through $|\lambda_1|^2$. Of course, the same argument shows that the dependence on λ_2 is through $|\lambda_2|^2$ only. A contribution to the Higgs potential requires both couplings λ_1 and λ_2 to appear and therefore the potential is proportional to $|\lambda_1\lambda_2|^2$, i.e. at least four coupling constants. But a one-loop diagram with 4 coupling constants can at most be logarithmically divergent, and therefore does not destabilize the Higgs mass.

In the explicit formulae above, we assumed - for simplicity - that $f_1 = f_2 = f$ and $\lambda_1 = \lambda_2 = \lambda_t/\sqrt{2}$. In the general case we find

$$m_T = \sqrt{\lambda_1^2 f_1^2 + \lambda_2^2 f_2^2} \tag{43}$$

$$\lambda_t = \lambda_1 \lambda_2 \frac{\sqrt{f_1^2 + f_2^2}}{m_T} \tag{44}$$

Note that the top Yukawa coupling goes to zero as either of the λ_i is taken to zero as anticipated from the $SU(3)$ symmetry arguments. Furthermore note that the mass of the heavy T quark can be significantly lower than the larger of the two f_i if the corresponding λ_i is smaller than 1. This is a nice feature because it will allow us to take the heavy gauge boson's masses large ($\gtrsim$ few TeV as required by the precision electroweak constraints) while keeping the T mass near a TeV. Keeping the T mass as low as possible is desirable because the quadratic divergence of the top loop in the Standard Model is cut off at the scale of the mass of T.

3.4. *Other Yukawa couplings*

The other up-type Yukawa couplings may be added in exactly the same way. We enlarge the $SU(2)$ quark doublets into triplets because of the gauged $SU(3)$. Then we add two sets of Yukawa couplings which couple the triplets to ϕ_1 and ϕ_2 and quark singlets q_1^c and q_2^c.

In the Standard Model, Yukawa couplings for down type quarks arise from a different operator where the $SU(2)$ indices of the Higgs doublet and the quark doublets are contracted using an epsilon tensor (or, equivalently, the conjugate Higgs field $h^c = i\sigma_2 h^\dagger$) is used. Before explicitly constructing this operator from the quark and ϕ_i fields note that even the bottom Yukawa coupling is too small to give a significant contribution to the Higgs mass. The quadratically divergent one loop diagram in the Standard Model yields

$$\frac{\lambda_b^2}{16\pi^2}\Lambda^2 \approx (30\ \text{GeV})^2\ . \tag{45}$$

Therefore, we need not pay attention to symmetries and collective breaking when constructing the down type Yukawa couplings. The Standard Model Yukawa is

$$\lambda_b \epsilon_{ij} h_i Q_j\, b^c \tag{46}$$

To obtain the epsilon contraction from an $SU(3)$ invariant operator we write

$$\frac{\lambda_b}{f}\,\epsilon_{ijk}\phi_1^i\phi_2^j\Psi_Q^k\,b^c \tag{47}$$

Note that the ϵ_{ijk} contraction breaks both $SU(3)$ symmetries (acting on the two scalar triplets ϕ_1 and ϕ_2) to the diagonal and therefore this operator does lead to a quadratic divergence. But the quadratic divergence is harmless because of the smallness of the bottom Yukawa coupling.

3.5. *Color and hypercharge*

Color is added by simply adding $SU(3)_{\text{color}}$ indices where we expect them from the Standard Model. $SU(3)_{\text{color}}$ commutes with all the symmetry arguments given above, therefore nothing significant changes.

hypercharge is slightly more complicated. The VEVs $\phi_i \propto (0,0,1)$ break the $SU(3)_{\text{weak}}$ gauge group to $SU(2)$, i.e. no $U(1)$ hypercharge candidate is left. Therefore, we gauge an additional $U(1)_X$. In order for the hypercharge of the Higgs to come out correctly, we assign the $SU(3)\times U(1)_X$ quantum numbers

$$\phi_i = 3_{-1/3} \tag{48}$$

The combination of generators which is unbroken by $\phi_i \sim (0,0,1)$ is

$$Y = \frac{-1}{\sqrt{3}}\,T^8 + X \quad \text{where} \quad T^8 = \frac{1}{2\sqrt{3}}\begin{pmatrix}1 & & \\ & 1 & \\ & & -2\end{pmatrix} \tag{49}$$

and X is the generator corresponding to $U(1)_X$. This uniquely fixes the $U(1)_X$ charges of all quarks and leptons once their $SU(3)$ transformation properties are chosen.

For example, the covariant derivative acting on ϕ_i is

$$D_\mu\phi = \partial_\mu\phi - \frac{1}{3}\,ig_X A_\mu^X\phi + igA_\mu^{SU(3)}\phi \tag{50}$$

Note that the $U(1)_X$ generator commutes with $SU(3)$, and the $U(1)_X$ gauge interactions do not change any of the symmetry arguments which we used to show that the Higgs does not receive quadratic divergences to its mass.

There are now three neutral gauge bosons corresponding to the generators T^3, T^8, X. These gauge bosons mix, the mass eigenstates are the photon, Z and a Z' which leads to interesting modifications to predictions for precision electroweak measurements.

3.6. *Quartic Higgs coupling*

To generate a quartic Higgs coupling we want to write a potential $V(\phi_1, \phi_2)$ that *i.* contains no mass at order f for the Higgs, *ii.* contains a quartic coupling, *iii.* preserves the "collective" symmetry breaking of the $SU(3)$'s: i.e. the quartic coupling is generated by at least two couplings in V and if one sets either one of them to zero the Higgs becomes an exact NGB. This last property is what guarantees radiative stability, no Λ^2 contributions to the Higgs mass.

Writing down a potential which satisfies these properties appears to be impossible for the pure $SU(3)$ model (if you can figure out how to do it please let me know, and write a paper about it). To see why it is not straightforward, note that $\phi_1^\dagger \phi_2$ is the only non-trivial gauge invariant which can be formed from ϕ_1 and ϕ_2. ($\phi_1^\dagger \phi_1 = \text{const} = \phi_2^\dagger \phi_2$ and $\epsilon^{ijk}\phi_i \phi_j \phi_k = 0$.) But the $\phi_1^\dagger \phi_2$ invariant is a bad starting point because it breaks the two $SU(3)$'s to the diagonal, and it is not surprising that generic functions of $\phi_1^\dagger \phi_2$ always contain a mass as well as a quartic. For example,

$$\phi_1^\dagger \phi_2 \sim f^2 - h^\dagger h + \frac{1}{f}(h^\dagger h)^2 + \cdots \tag{51}$$

so that

$$\frac{1}{f^{2n-4}}(\phi_1^\dagger \phi_2)^n \sim f^4 - f^2 h^\dagger h + (h^\dagger h)^2 + \cdots \tag{52}$$

By dialing the coefficient of this operator we can either get a small enough mass term or a large enough quartic coupling but not both. Of course, we could try to tune two terms with different powers n such that the mass terms cancel between them but that tuning is not radiatively stable.

There are two different solutions to the problem in the literature. Both require enlarging the model and symmetry structure. One solution, due to Kaplan and Schmaltz [20], involves enlarging the gauge symmetry to $SU(4)$ and introduce four ϕ fields which transform as a $\mathbf{4}$ of $SU(4)$. The four ϕ fields break $SU(4) \to SU(2)$, yielding 4 $SU(2)$ doublets. Two of them are eaten, the other two are "little Higgs" fields with a quartic potential similar to the quartic potential in SUSY.

The other solution, due to Skiba and Terning [22], keeps the $SU(3)$ gauge symmetry the same but enlarges the global $SU(3)^2$ symmetry to $SU(3)^3$ which is then embedded in a $SU(9)$. The larger symmetry also leads to two "little" Higgs doublets for which a quartic coupling can be written. Both of these solutions spoil some of the simplicity of the $SU(3)$ model but they allow a large quartic coupling for the Higgs fields with natural electroweak symmetry breaking. I refer you to the original papers for details on these models.

A third option [29] is to simply add a potential with a very small coefficient. The resulting quartic coupling is then also very small but radiative corrections from the top loop as in the MSSM give a contributions which can raise the Higgs mass above the experimental bound of 114 GeV. Explicitly,

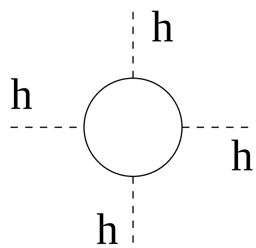

Figure 8. *The top loop contribution to the Higgs quartic coupling.*

below the T mass the cancellations in the top sector no longer occur and the diagram in Figure 8 gives a contribution to the quartic

$$\frac{3\lambda_t^4}{16\pi^2}\log(\frac{m_T^2}{m_t^2})\,(h^\dagger h)^2 \tag{53}$$

which is too small by itself but does give successful electroweak symmetry breaking when combined with a small tree level contribution. Since the tree level term also contributes to the soft mass for the Higgs a moderate amount of tuning (~10%) is required. While this is not completely satisfactory it is better than most other models of electroweak symmetry breaking and certainly better than the MSSM with gauge coupling unification which requires tuning at the few % level or worse.

3.7. *The simplest little Higgs*

This section summarizes the construction and salient features of the "simplest little Higgs" [29], the $SU(3)$ model in which the Higgs quartic coupling is predominantly generated from the top loop. In the phenomenology section, we will use the model as an example to discuss typical experimental signatures and constraints.

The model has an $SU(3)_{\rm color} \times SU(3)_{\rm weak} \times U(1)_X$ gauge group with each of the three generations of quark and lepton fields transforming as

$$\begin{aligned} \Psi_Q &= (3,3)_{\frac{1}{3}} & \Psi_L &= (1,3)_{-\frac{1}{3}} \\ d^c &= (\bar{3},1)_{\frac{1}{3}} & e^c &= (1,1)_1 \\ 2\times u^c &= (\bar{3},1)_{-\frac{2}{3}} & n^c &= (1,1)_0 \end{aligned} \tag{54}$$

The triplets Ψ_Q and Ψ_L contain the quark and lepton doublets, the (charge-conjugated) singlets are the fields u^c, d^c, e^c, n^c.[c]

The $SU(3)_w \times U(1)_X$ symmetry breaking stems from expectation values for $\phi_1 = \phi_2 = (1,3)_{-1/3}$.

The Lagrangian of the model contains the usual kinetic terms

$$\mathcal{L}_{kin} \sim \Psi_Q^\dagger \not{D} \Psi_Q + \cdots + |D_\mu \phi_1|^2 + \cdots \tag{55}$$

Yukawa couplings

$$\begin{aligned} \mathcal{L}_{yuk} \sim \lambda_1^u \phi_1^\dagger \Psi_Q u_1^c + \lambda_2^u \phi_2^\dagger \Psi_Q u_2^c + \frac{\lambda^d}{f} \phi_1 \phi_2 \Psi_Q \, d^c \\ + \lambda^n \phi_1^\dagger \Psi_L n^c + \frac{\lambda^e}{f} \phi_1 \phi_2 \Psi_L \, e^c + \text{h.c.} \end{aligned} \tag{56}$$

and the tree level Higgs potential arises from

$$\mathcal{L}_{pot} \sim \mu^2 \phi_1^\dagger \phi_2 + \text{h.c.} \tag{57}$$

We substitute the parametrization for the NGBs

$$\phi_1 = e^{i\Theta \frac{f_2}{f_1}} \begin{pmatrix} 0 \\ 0 \\ f_1 \end{pmatrix}, \quad \phi_2 = e^{-i\Theta \frac{f_1}{f_2}} \begin{pmatrix} 0 \\ 0 \\ f_2 \end{pmatrix} \tag{58}$$

where

$$\Theta = \frac{\eta}{\sqrt{2} f} + \frac{1}{f} \begin{pmatrix} 0 & 0 & \\ 0 & 0 & h \\ & h^\dagger & 0 \end{pmatrix} \quad \text{and} \quad f^2 = f_1^2 + f_2^2 \; . \tag{59}$$

and solve for the spectrum of massive particles. The numerical values provided below correspond to the example point $f_1 = 0.5$ TeV and $f_2 = 2$

---

[c]This fermion content is anomalous under the extended electroweak gauge group. The anomaly must be canceled by additional fermions which can be as heavy as Λ. It is also possible to change the charge assignments such that anomalies cancel among the fields $\Psi_Q, \Psi_L, u^c, d^c, e^c, n^c$ alone [29,35,36].

TeV ($f \sim 2$ TeV). In addition to the Standard Model particles we have

$$\begin{array}{rcl} \text{heavy gauge bosons} & \left\{ \begin{array}{c} \begin{pmatrix} W^{+\prime} \\ W^{0\prime} \end{pmatrix} \\ Z' \end{array} \right. & \begin{array}{l} \sim \sqrt{1/2}\, gf \sim 0.9\,\text{TeV} \\ \sim \sqrt{2/3}\, gf \sim 1.1\,\text{TeV} \end{array} \\ \text{top partner} & T & = \sqrt{(\lambda_1 f_1)^2 + (\lambda_2 f_2)^2} \sim 1\,\text{TeV} \\ \text{up, charm partners} & U, C & \sim \lambda_2 f_2 \sim 0.7\,\text{TeV} \\ \text{real scalar} & \eta & \sim 300\,\text{GeV} \end{array} \tag{60}$$

4. Phenomenology

4.1. *Direct production of little partners*

Precision electroweak constraints (which we will discuss in the next section) force the masses of the new states to be at or above 1 TeV, and therefore probably out of reach of the Tevatron.

LHC ("Little Higgs Collider") little Higgs phenomenology is very exciting [37–44]. All new states may be within reach and give interesting signatures.

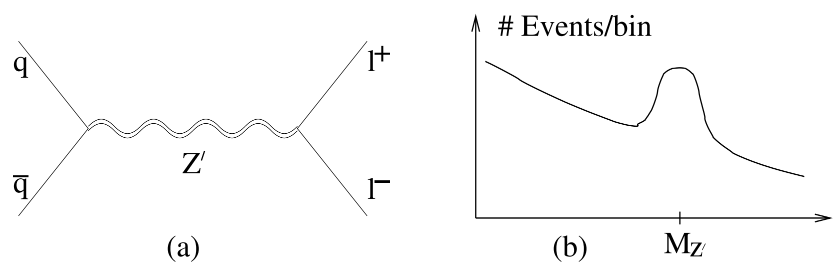

Figure 9. *a.) Lepton pairs from s-channel Z' production, b.) bump in the invariant mass distribution of lepton pairs.*

little Z (Z'): Heavy neutral gauge bosons would be produced in the s-channel in quark-antiquark collisions with large rates. An easy signature comes from the decay of the Z' to pairs of highly energetic leptons (Figure 9.a). A plot of the invariant mass of the lepton pairs should show a clear bump at the mass of the Z' (Figure 9.b). The reach in this channel is about 5-7 TeV [37].

little W (W'): The heavy $SU(2)$ doublet gauge bosons $W' = (W^{0\prime}, W^{+\prime})^T$ are less straightforward to see. The reason is that to lowest order in an expansion in m_W/f they couple an $SU(2)_{\rm weak}$ doublet fermion of the Standard Model Q to a heavy partner fermion as can be seen explicitly from the gauge couplings of the triplet fermions

$$g\left(Q^\dagger\; U^\dagger\right)\left(\begin{array}{c|c} & W' \\ \hline W'^\dagger & \end{array}\right)\begin{pmatrix} Q \\ U \end{pmatrix} \tag{61}$$

Thus W' gauge bosons would always have to be produced in association with heavy "little quarks" (U). This picture is not quite correct because we ignored mixing effects. In fact, little quarks mix with the light quarks

$$U_{\rm heavy} = U - \frac{v}{f}\, u \tag{62}$$

which results in v/f suppressed couplings of pairs of light quarks to W'. For example, a charged $W^{+'}$ may be produced (Figure 10.a) and then decay into a light quark and a little quark (Figure 10.b) which then decays further as in Figure 10.d. However the rates are expected to be small because the W' is heavy and because of the v/f in the coupling.

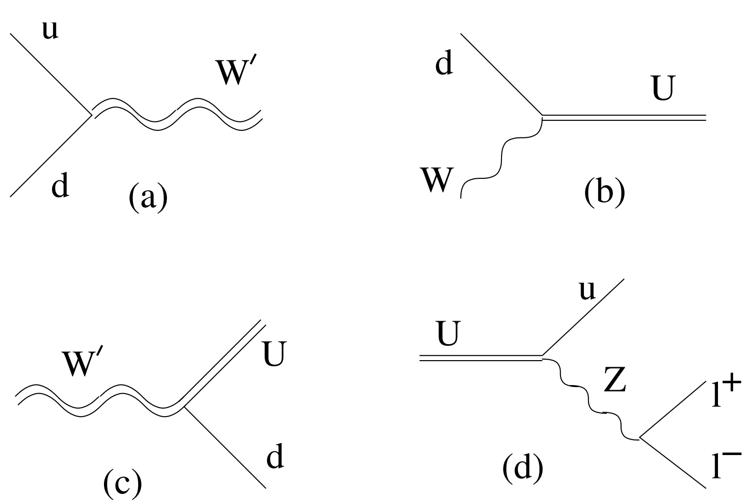

Figure 10. *a.) W' production, b.) U production, c.) W' decay d.) U decay.*

The signatures quoted here for little W's are unique to "simple little Higgs" models. Models based on product gauge groups such as $[SU(2)\times U(1)]^2$ have little W's which transform as triplets under $SU(2)_{weak}$ which can couple directly to pairs of light quarks. They are produced with larger cross sections and also contribute more strongly to precision electroweak observables.

little quarks (U): Heavy U quarks may be pair produced directly via their coupling to gluons. However, because of mixing there is also the possibility of single U production as shown in Figure 10.c. The produced U's would subsequently decay into a light quark and a Standard Model gauge boson ($U \to u + Z$, $U \to d + W$). The rate is suppressed by v^2/f^2 from the coupling in the production cross section and the reach is estimated to be of the order 2-3 TeV.

4.2. *Precision electroweak constraints*

As we already showed in the Introduction, indirect constrains from precision electroweak (PEW) measurements on any new Physics in the TeV energy regime are very severe. Little Higgs models are no exception and we must check that there are significant regions of parameter space in which the model is not ruled out by PEW, and where the little Higgs mechanism solves the Higgs naturalness problem.

In practice, the most significant effects arise from the exchange of the new neutral gauge boson, the Z'. The $SU(3)$ model has three neutral gauge bosons which mix (in the $SU(3)$ charge eigenstate basis: A_3, A_8, A_X). The mass eigenstates are the photon, Z, and a Z'. To understand all possible effects from the Z' we must work out it couplings.

From the gauge couplings of the fermions $\psi^\dagger i/D\psi$ and scalars $|D_\mu\phi|^2$ we find the mass and couplings. Neglecting order one group theory factors, these couplings are

$$
\begin{aligned}
|D_\mu\phi|^2 &\to \begin{cases} g^2 f^2 Z'_\mu Z^{\mu\prime} \\ Z'_\mu g h^\dagger D^\mu h \to g^2 v^2 Z'_\mu Z^\mu \end{cases} \\
\psi^\dagger i\not{D}\psi &\to g Z'_\mu \psi^\dagger i\gamma^\mu\psi
\end{aligned} \tag{63}
$$

The Feynman diagrams corresponding to these three Z' interactions are shown in Figure 11. Integrating out the Z' at tree level by solving it's classical equations of motion we find the following effective Lagrangian

$$
\mathcal{L}_{\rm eff} \sim \frac{(\psi^\dagger i\gamma^\mu\psi)^2 + 2(\psi^\dagger i\gamma^\mu\psi)(h^\dagger D_\mu h) + (h^\dagger D_\mu h)^2}{f^2} \tag{64}
$$

These three operators correspond to new four fermion operators, modifications of the Z couplings to fermions, and modifications of the Z mass, respectively. In terms of Feynman diagrams they arise as shown in Figure 12. The four fermion operators are most strongly constrained by limits from

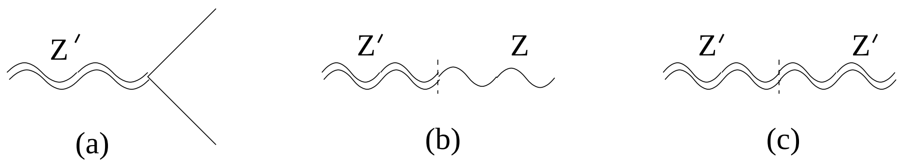

Figure 11. *a.) Z' coupling to fermions, b.) Z-Z' mixing c.) Z' mass.*

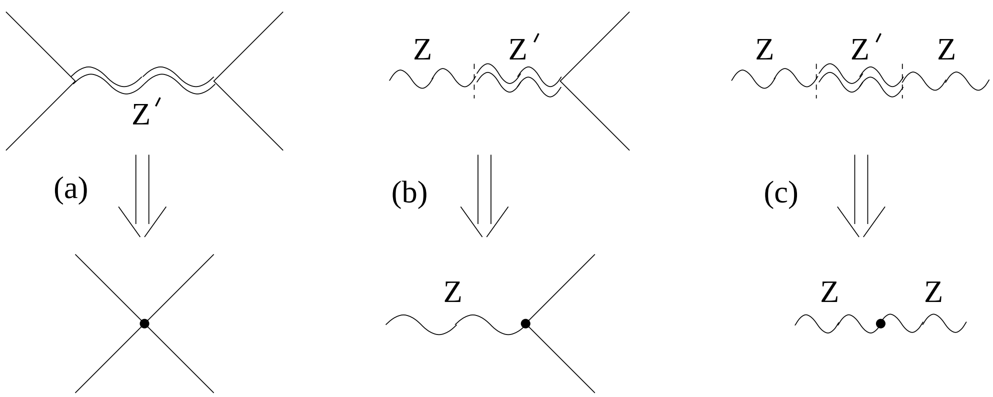

Figure 12. *Integrating out Z' to obtain a.) four fermion operators, b.) shift in Z couplings c.) shift in Z mass.*

LEP II and from atomic parity violation [2,45]. Modifications of the Z couplings are constrained by Z-pole data such as R_b and A_b^{FB}. Finally, shifts in the Z mass change the mass splitting between the Z and the W from the Standard Model prediction, deviations from the Standard Model value are usually parametrized with the ρ or T parameters. These constraints generally imply $f \geq 2-4$ TeV. A global fit of various little Higgs theories using a general operator analysis and the Han-Skiba precision electroweak matrix is currently being performed [4,46]. For more details regarding precision electroweak constraints on little Higgs models see [47–53].

4.3. *Precision electroweak, fine-tuning, and T-parity*

The result of PEW is that the scale of the new particles in most little Higgs models is somewhat higher than 1 TeV. This implies some fine tuning as quantum corrections to the Higgs $(\text{mass})^2$ parameter are directly proportional to the new particle masses. In particular, we would really like the mass of the top partner to be at or below 1 TeV, not 2-4 TeV. Since the naturalness problem we set out to solve involved fine tuning of order 1%, it is somewhat disappointing to find that fine-tuning is still on the order of 10%. Can we do better?

The answer is "yes". To understand the problem better, note that the problematic contributions to PEW all stem from tree level exchange of little partners (as in Figure 12). However, naturalness only requires the new particles to appear in loops in order to cancel quadratic divergences. Thus a solution would be to forbid all tree level couplings of little partners to Standard Model particles while keeping the loops. But how can this be enforced in a natural way?

To see how it might work, note that the MSSM does not appear to have this problem. In the MSSM superpartner masses can be well below 1 TeV without causing problems with PEW. Why is that? The answer is R-parity. R-parity has a reputation of being the ugliest part of the MSSM but with regards to PEW R-parity is really the MSSM's best part. Recall that R parity is defined such that all Standard Model particles are R-parity even and all superpartners are odd. If R-parity is exact then all interactions must involve an even number of superpartners. Interactions of single superpartners with Standard Model fields are forbidden. Therefore contributions from superpartners to processes in which all external states are Standard Model particles are loop suppressed as shown in Figure 13. Thus contributions to PEW from superpartners suppressed by $1/(16\pi^2)\, m_W^2/M_{SUSY}^2$ which is sufficiently small even for $M_{SUSY} \leq 1$ TeV.

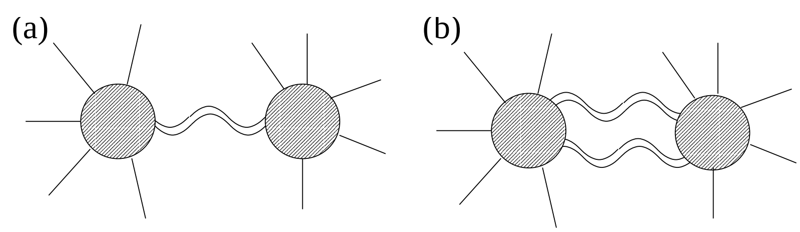

Figure 13. *a.) forbidden by R-parity, b.) superpartner loop, allowed.*

The equivalent of R-parity in the case of the little Higgs models has been dubbed T-parity by Cheng and Low [24,27,30]. Under T-parity all Standard Model fields are even whereas little partners are odd. Just like in the case of supersymmetry, this forbids the dangerous contributions to PEW while allowing the cancellation of quadratic divergences. The difficult part, of course, is to construct a model in which T-parity can be consistently imposed. It appears that this is impossible to do with any of the "simple group" models, such as our favorite $SU(3)$ model. However, it can be done with all the other little Higgs models. In models with T-parity the masses of little partners can be smaller than 1 TeV. The phenomenology of such a model was recently discussed in [54].

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