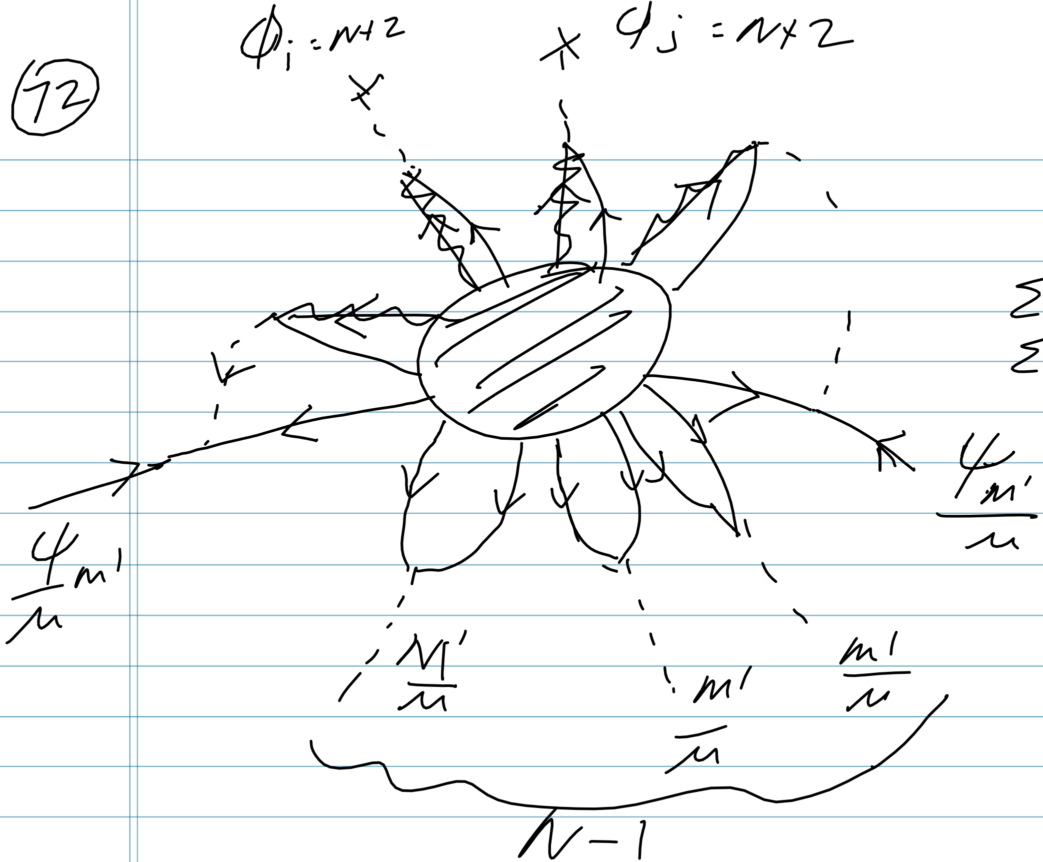
(72)



$2T(\lambda) = 4$
$\Sigma\, 2T(q) = F = N+2$
$\Sigma\, 2T(\bar{q}) = F = N+2$

$\langle \bar{\phi}^F \phi^F \rangle = -\mu m$

$$W_{int} = \tilde{\Lambda}^{b}_{N,N+2} \det\left(\frac{M'}{\mu}\right) \frac{1}{\bar{\phi}^F \phi_F}$$

$$= \frac{\tilde{\Lambda}^{4-N}}{(-\mu m)} \frac{\det M'}{\mu^{N+1}}$$

$$W = \frac{1}{\Lambda^{2N-1}_{N,N+1}} \left(\bar{B} M' B - \det M'\right)$$

$$c\bar{c} = \frac{\mu}{\Lambda^{2N-1}}$$

$$\frac{\tilde{\Lambda}^{4-N}_{N,N+2}}{\mu^{N+2} m} = \frac{1}{\Lambda^{2N-1}_{N,N+1}}$$

$$\frac{\Lambda^{2N-1}_{N,N+1}}{m} = \Lambda^{2N-2}_{N,N+2} = \mu^{N+2} \tilde{\Lambda}^{N-4}_{N,N+2}$$

special case of $\tilde{\Lambda}^{3\tilde{N}-F} \Lambda^{3N-F} = (-1)^{F-N} \mu^F$

73

check $\tilde{\Lambda}^{3\tilde{N}-F}\Lambda^{3N-F} = (-1)^{F-N}\mu^F$

generic $\langle M \rangle$ in dual

all dual quarks are massive

$$\tilde{\Lambda}_L^{3\tilde{N}} = \tilde{\Lambda}^{3\tilde{N}-F}\left(\det\frac{M}{\mu}\right)$$

gaugino condensation

$$W_L = \tilde{N}\tilde{\Lambda}_L^3 = (F-N)\left(\frac{\tilde{\Lambda}^{3N-F}\det M}{\mu^F}\right)^{\frac{1}{F-N}}$$

$$= (F-N)\left((-1)^{F-N}\frac{\det M}{\Lambda^{3N-F}}\right)^{\frac{1}{F-N}}$$

$$= (N-F)\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}}$$ for $F > N$

adding a mass term for quarks

$$W_{mass} = m^i_j M^j_i$$

$$\langle M^j_i \rangle = (m^{-1})^j_i \left(\det m \, \Lambda^{3N-F}\right)^{\frac{1}{N}}$$

74

dual of the dual

assume $\tilde{\tilde{\Lambda}} = \Lambda$

$$\tilde{\Lambda}^{3\tilde{N}-F}\Lambda^{3N-F} = (-1)^{F-N}\mu^F$$

$$\Lambda^{3N-F}\tilde{\Lambda}^{3\tilde{N}-F} = (-1)^{F-\tilde{N}}\hat{\mu}^F = (-1)^N\hat{\mu}^F$$

$$\hat{\mu} = -\mu$$

$$M = \bar{\Phi}\Phi$$

$$N = \phi\bar{\phi} \quad \text{dual meson}$$

$$W_{dd} = \frac{N_i^j d_j \bar{d}^i}{\hat{\mu}} + \frac{1}{\mu} M_j^i N_i^j$$

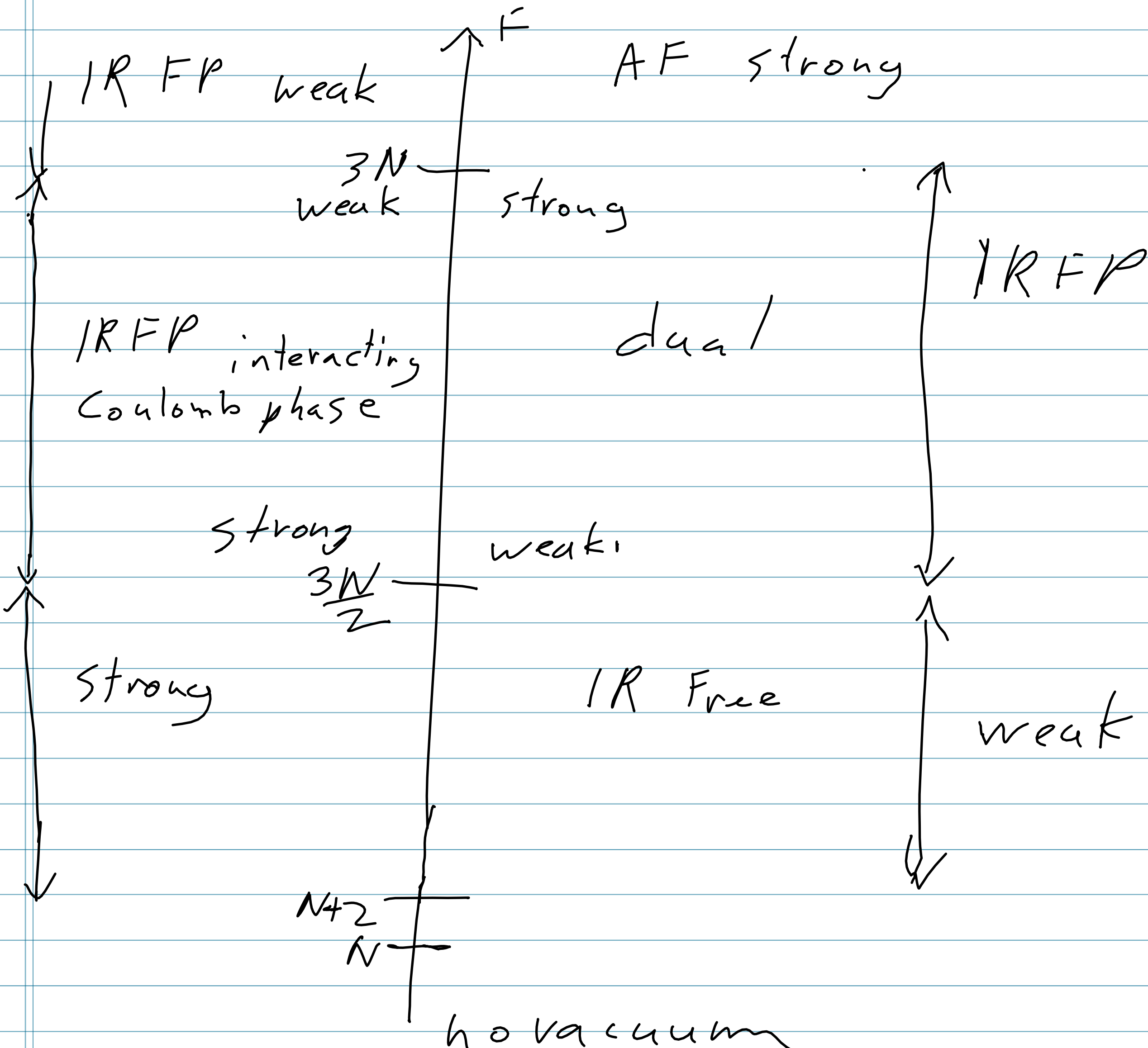
$$\frac{\partial W_{dd}}{\partial M_j^i} = \frac{1}{\mu} N_i^j = 0, \quad \frac{\partial W}{\partial N_i^j} = \frac{1}{\hat{\mu}} d_j \bar{d}^i + \frac{1}{\mu} M_j^i =$$

$$M_j^i = d_j \bar{d}^i$$

$$\bar{\Phi}_j = d_j$$

dual of dual is just is just SUSY QCD

Susy QCD



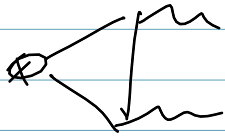
$F = N+1$ confinement without χSB

$F = N$ confinement with χSB

TG

$$SO(N)$$

	$SO(N)$	$SU(F)$	$U(1)_R$
Q	$\square$	$\square$	$\frac{F+2-N}{F}$



$$= 1\cdot T(Ad) + T(\square)\,F\,(R-1) = 0$$

$$= N-2 + F(R-1) = 0$$

$$R-1 = \frac{2-N}{F}$$

$$R = \frac{F+2-N}{F}$$

$$b = 3\,T(Ad) - F\,T(\square)$$

$$= 3(N-2) - F \qquad N>4$$

$N=4$ two couplings $\qquad SO(4) \simeq SU(2)\times SU(2)$

$N=3$ $\qquad SU(2) \simeq SO(3)$

$$b = 6-2F$$

Classical Moduli Space for $SO(N)$

$F < N$ $\quad \langle \phi \rangle = \begin{pmatrix} v_1 & & \\ & \ddots & \\ & & v_F \\ 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{pmatrix}$ $\quad N$ rows, F columns

$F \geq N$ $\quad \langle \phi \rangle = \begin{pmatrix} v_1 & & & 0 & \cdots & 0 \\ & \ddots & & \vdots & & \vdots \\ & & v_N & 0 & \cdots & 0 \end{pmatrix}$ $\quad N$ rows, F columns

$F < N$ $\qquad M_{ij} = \phi_i \phi_j \qquad \frac{1}{2} F(F+1)$

$F \geq N$ also $\quad B_{[i_1 \cdots i_N]} = Q_{[i_1} \cdots Q_{i_N]}$

classically $\quad \langle M \rangle = \begin{pmatrix} v_1^2 & & & & \\ & \ddots & & & \\ & & v_N^2 & & \\ & & & 0 & \\ & & & & \ddots \\ & & & & & 0 \end{pmatrix}$

$\langle B \rangle = v_1 \cdots v_N$

if rank $M = N \implies \langle B \rangle = \pm \sqrt{\det' M}$

78

	$U(1)_A$	$U(1)_R$
Q	1	$\frac{F+2-N}{F}$
W_α	0	1
Λ^b	$2F$	0
$\det QQ$	$2F$	$2(F+2-N)$

$$W_{eff} \propto \left(\frac{\Lambda^b}{\det QQ} \right)^{\frac{1}{N-2-F}}$$

$F < N-2$ superpotential can be generated

79

dual $F > N-2$

	$SO(F-N+4)$	$SU(F)$	$U(1)_R$
q	$\square$	$\bar{\square}$	$\frac{N-2}{F}$
M	1	$\square\!\square$	$\frac{2(F+2-N)}{F}$

$$W = \frac{M}{2\mu} \cancel{\phi} qq$$

	orig	dual
$SU(F)^3$	N	$-(F-N+4) + F+4$
$U(1)_R\, SU(F)^2$	$\frac{(2-N)}{F}\frac{N}{2}$	

$U(1)_R$

$$\frac{(2-N)}{F} F \cdot N + \frac{N(N-1)}{2}$$

$$= \frac{N}{2}(4-2N+N-1)$$

$$= \frac{N}{2}(3-N)$$

(80)

compare moduli spaces

$M^{\;i}_{\;i} = Q^{\xi}_i Q^{\xi}_j$

$B_{[i_1 \cdots i_{N}]} = Q_{i_1 \cdots i_N}$

$W_\alpha \sim \boxed{}$

$h_{[i_1 \cdots i_{N-4}]} = W_\alpha^2 Q_{[i_1 \cdots i_{N-4}]}$

$H_{\alpha [i_1 \cdots i_{N-2}]} = W_{\alpha [i_1 \cdots i_{N-2}]}$

$\Big\}$ hybrids

$$M \longleftrightarrow M$$
$$B \longleftrightarrow \epsilon_{i_1 \cdots i_F} \tilde{h}$$
$$h \longleftrightarrow \epsilon_{i_1 \cdots i_F} \tilde{B}$$
$$H_\alpha \longleftrightarrow \epsilon_{i_1 \cdots i_F} \tilde{H}_\alpha$$

$$\beta(\tilde{g}) = 3(\tilde{N}-2) - F = 3(F-N+4-2) - F$$
$$= 2F + 6 - 3N$$

AF $\quad F > \frac{3}{2}(N-2)$

IR Fixed Point : $N-2 < F\ 3(N-2)$ ~~Free~~

$N-2 \leq F < \frac{3}{2}(N-2)$ free dual

$$SO(N),\ F \longleftrightarrow SO(F-N+4),\ F$$

$\downarrow W = m Q^F Q^F \qquad \downarrow W = \frac{M}{2\mu} qq + m M$

$$SO(N),\ F-1 \longleftrightarrow SO(F-N+3),\ F-1$$

81

$F \leq N-5$

$SO(N) \xrightarrow[\text{break}]{} SO(N-F) \supset SO(5)$

no light flavors gaugino condensation

$$W_\alpha \langle \lambda\lambda \rangle \propto \left(\frac{16\Lambda^{3(N-2)-F}}{\det M} \right)^{\frac{1}{N-2-F}}$$

$F = N-4$: $SO(N) \to SO(4) \sim SU(2)_L \times SU(2)_R$

$$W = 2\langle\lambda\lambda\rangle_L + 2\langle\lambda\lambda\rangle_R = \frac{1}{2}(\epsilon_L + \epsilon_R)\left(\frac{16\Lambda^{2(N-1)}}{\det M}\right)^{1/2}$$

$\epsilon_L, \epsilon_R = \pm 1$

$\epsilon_L + \epsilon_R = \begin{cases} \pm 2 & \leftarrow \text{two branches runaway} \\ 0 & \leftarrow \text{no superpotential quantum moduli space} \end{cases}$

what happens at $\langle M \rangle = 0$?

confinement without χSB

	M matches elem		comp
$SU(F)^3$	N	$=$	$F+4$
$U(1)_R$	$\frac{2-N}{F} F \cdot N + \frac{N}{2}(N-1)$		$\frac{F+4-2N}{F} \frac{F}{2}(F+1)$
	$= \frac{1}{2} N(N-1+4-2N)$		$= \frac{1}{2}(F+4-2N)(F+1)$
	$= -\frac{1}{2}(N-3)N$	$=$	

(82)

theory one less flavor had only branch

adding a mass term { on runaway branch $\to$ get runaway $F = N-5$

second branch

$W = m M_{FF}$ $\qquad \frac{\partial W}{\partial M_{FF}} = m$

if there had d.o.f. in superpotential then we would find extra vacua

$F = N-3$ $\qquad$ had $SO(4) \sim SU(2)_L \times SU(2)_R$

$\qquad \qquad \downarrow$

$\qquad SU(2)_d \sim SO(3)$

instanton effect

$\Pi_3(G/H) = \Pi_3(SU(2)) = Z$

gausino condensation

$$W = 4(1+\epsilon)\frac{\Lambda^{2N-3}}{\det M}$$

instanton

two branches $\quad \epsilon = +1$ runaway

$\epsilon = -1$ quantum moduli space

83

integrating out one flavor
need to find two branches

$\Rightarrow$ $W \neq 0$ even for $\epsilon = -1$

M cannot match anomalies at $\langle M \rangle = 0$
(only works for $F = N-4$)

$F = N-3$

	$SU(F)$	$U(1)_R$
q	$\overline{\square}$	$\frac{N-2}{F}$
M	$\square\square$	$\frac{2(F+2-N)}{F}$

$$W = \frac{1}{2\mu} Mqq\, f\left(\frac{\det M \; Mqq}{\Lambda^{2N-2}}\right)$$

adding a mass $\Rightarrow$ $q_F = \pm i v$

correct # of ground states

$q \longleftrightarrow h = Q^{N-4} W_\alpha W^\alpha$ hybrid

$N = 4$ $\Rightarrow$ glueball

confinement without χSB

84

$F = N$ dual

	$SO(4)$	$SU(N)$	$U(1)_R$
q	$\square$	$\overline{\square}$	$\frac{N-2}{N}$
M	1	$\square\!\square$	$\frac{4}{N}$

mass term $\quad \Delta W = m M_{NN}$

$SO(4) \sim SU(2) \times SU(2) \rightarrow SU(2)_d \sim SO(3)$

$\pi_3(G/H) = \mathbb{Z}$ instanton

	~~SU~~ $SO(3)$	$SU(N-1)$
q'	$\square$	$\overline{\square}$
M'	1	$\square\!\square$

$$W_{eff} = \frac{1}{2\mu} M' q' q' - \frac{1}{64 \Lambda_{N,N-1}^{2N-5}} \det M'$$