Quantum Moduli Space

heavy mass for flavor N through F
light masses for the rest

integrating out $\Lambda^{3N-F} \det m_H = \Lambda_{N,N-1}^{3N-N+1}$

$$\langle M_i^j \rangle = (m_L^{-1})_i^j \left(\det m_L \, \Lambda_{N,N-1}^{2N+1} \right)^{\frac{1}{N}}$$

$$= (m_L^{-1})_i^j \left(\underbrace{\det m_L \det m_H} \Lambda^{3N-F} \right)^{\frac{1}{N}}$$

$$= (m^{-1})_i^j \left(\det m \, \Lambda^{3N-F} \right)^{\frac{1}{N}}$$

$$\langle M_i^j \rangle \sim m^{\frac{F}{N}-1}$$

$F \geq N$ $\quad m_i^j \to 0$ with $\langle M \rangle \to$ finite or zero

quantum moduli space for $m = 0$

parameterized by $M, B, \bar{B}$
perturbative / $\langle M \rangle \langle B \rangle \langle \bar{B} \rangle$ are large
singular if $\langle M \rangle, \langle B \rangle, \langle \bar{B} \rangle \to 0$
gluons become light
what happen here?

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	$SU(N)$	$SU(F)$	$SU(F)$	$U(1)$	$U(1)_R$
Q	$\square$	$\square$	1	1	$\frac{F-N}{F}$
$\bar{Q}$	$\bar{\square}$	1	$\bar{\square}$	-1	$\frac{F-N}{F}$

$F \geq 3N$ loose AF

F just below $3N$

IR Fixed Point Banks-Zaks

$$\beta = -bg^3 + cg^5 + \ldots$$

large N, $b \ll N$

$$\beta \sim g\left(bg^2 + c(g^2N)^2 + \mathcal{O}(g^2N)^3\right)$$

$$g_*^2 N^2 = \frac{b}{c} \qquad \text{perturbative}$$

exact:

$$\beta = \frac{-g^3}{16\pi^2}\,\frac{3N - F + F\gamma}{1 - N\frac{g^2}{8\pi^2}}$$

$$\gamma = \frac{-g^2}{8\pi^2}\,\frac{N^2-1}{N} + \mathcal{O}(g^4) \qquad \text{pert.}$$

$$\beta = \frac{-g^3}{16\pi^2}\left(3N - F + F\gamma + (3N-F)\left(\frac{Ng^2}{8\pi^2}\right)\right.$$

$$= \frac{-g^3}{16\pi^2}\left(3N - F \quad \frac{-g^2}{8\pi^2}\left(-F\frac{(N^2-1)}{N} + 3N^2 - FN\right)\right.$$

$$+ \ldots$$

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$$16\pi^2 \beta = -g^3 (3N - F) - \frac{g^5}{8\pi^2}\left(3N^2 - FN - F\frac{(N^2-1)}{N}\right)$$

$$F = 3N - \epsilon N$$

$$16\pi^2 \beta = -g^3 \epsilon N - \frac{g^5}{8\pi^2}\left(3N^2 - 3N^2 + \epsilon N^2 - (3N - \epsilon N)\frac{(N^2-1)}{N}\right)$$

$$= -g^3 \epsilon N + \frac{g^5}{8\pi^2}\left(3(N^2-1) + \mathcal{O}(\epsilon)\right)$$

$$\beta(g_*) = 0 \qquad \epsilon N = \frac{g_*^2}{8\pi^2}\left(3(N^2-1) + \mathcal{O}(\epsilon)\right)$$

$$g_*^2 = \frac{8\pi^2}{3}\frac{N}{N^2-1}\epsilon$$

higher order terms

$$\beta \sim -g\left(\underset{\mathcal{O}(\epsilon^2)}{g^2(3N-F)} - \underset{\mathcal{O}(\epsilon^2)}{c(g^2N)^2} + \underset{\mathcal{O}(\epsilon^3)}{(g^2N)^3}\right)$$

$\epsilon \to 0$ IR Fixed Point at weak coupling

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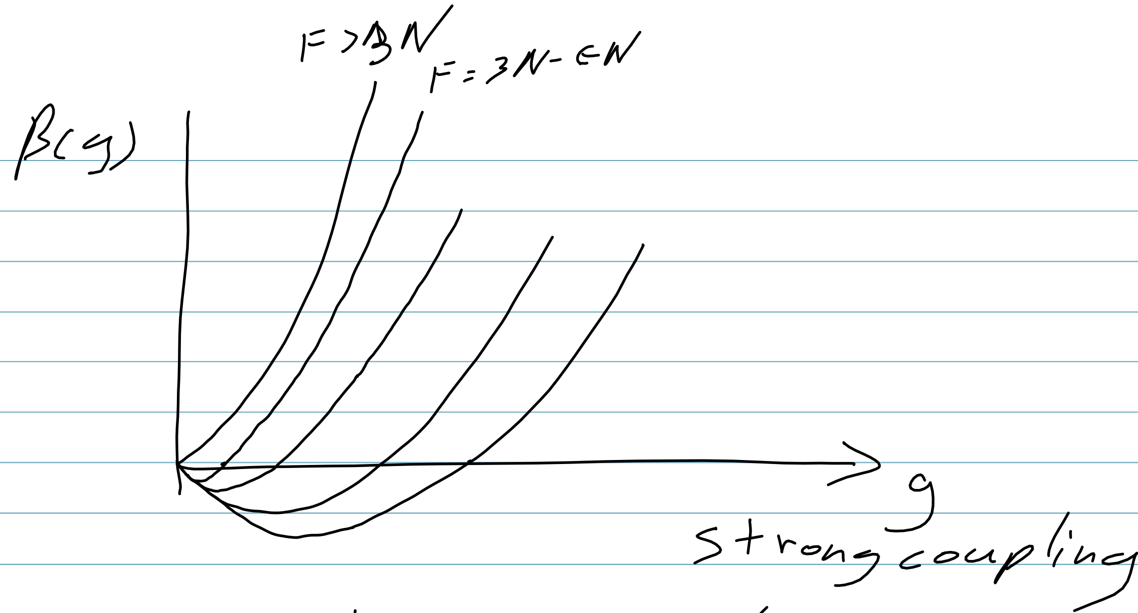

Scale invariant theory
spin ≤ 1

$\Rightarrow$ conformal theory
\+ SUSY

SUSY algebra $\rightarrow$ superconformal algebra

dimension $D \geq \frac{3}{2} |R_{sc}|$ superconformal R-charge

$D = \frac{3}{2} R_{sc}$ for a chiral superfield

$D = -\frac{3}{2} R_{sc}$ for anti-chiral "

for chiral

$$R_{sc}(\mathcal{O}_1 \mathcal{O}_2) = R_{sc}(\mathcal{O}_1) + R(\mathcal{O}_2)$$

$$\Rightarrow D(\mathcal{O}_1 \mathcal{O}_2) = D(\mathcal{O}_1) + D(\mathcal{O}_2)$$

OPE is just multiplication
"chiral ring"

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$$R_{sc} = R + \sum_i q_i$$

SUSYQCD: $R_{sc}(Q) = R_{sc}(\bar{Q}) \Rightarrow R_{sc} = R$

$$D(M) = D(\tilde{Q}Q) = 2 + \gamma_* = \frac{3}{2}\, 2\left(\frac{F-N}{F}\right)$$

$$= 3\left(\frac{F-N}{F}\right) = 3 - \frac{3N}{F}$$

$$\gamma_* = 1 - \frac{3N}{F}$$

$$\beta \propto 3N - F + F\gamma_* = 0$$

gauge inv, scalar $D \geq 1$ saturated for free scalar

$$3 - \frac{3N}{F} \geq 1$$

$$2 \geq \frac{3N}{F}$$

$$F \geq \frac{3}{2}N$$

IR is an interacting conformal theory

$$\frac{3}{2}N < F < 3N$$

conformal theory suggests no χSB

't Hooft anomaly matching should work

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	$SU(F)$	$SU(F)$
B	$\left.\begin{array}{\|c\|}\hline \\ \hline \\ \hline \vdots \\ \hline \\ \hline \end{array}\right\} N$	1
$\bar{B}$	1	$\left.\overline{\begin{array}{\|c\|}\hline \\ \hline \\ \hline \vdots \\ \hline \\ \hline \end{array}}\right\} N$
M	$\square$	$\bar{\square}$

$$SU(F)^3 : N \stackrel{?}{=} \mathcal{O}(F^{N-1}) + F$$

$$U(1)\, SU(F)^2 : \tfrac{1}{2}N \stackrel{?}{=} \mathcal{O}(F^{N-1})$$

$$U(1)_R\, SU(F)^2 : \frac{N}{2}\left(\frac{-N}{F}\right) \stackrel{?}{=} \mathcal{O}(F^{N-1})\left(\frac{N(F-N)}{F} - 1\right) + 2(F-N).$$

Seiberg Found

	$SU(F-N)$	$SU(F)$	$SU(F)$	$U(1)$	$U(1)_R$
q	$\square$	$\overline{\square}$	1	$\frac{N}{F-N}$	N/F
$\tilde{q}$	$\overline{\square}$	1	$\square$	$-N/F-N$	N/F
M	1	$\square$	$\overline{\square}$	0	$\frac{2}{F}(F-N)$

Homework:

$$SU(F)^3: \quad N = -(F-N) + F = N$$

$$U(1)SU(F)^2: \ \tfrac{1}{2}N = \frac{N}{F-N}(F-N)\tfrac{1}{2}$$

$$U(1)_R SU(F)^2: \ -\frac{N^2}{2F} = \frac{N-F}{F}(F-N)\tfrac{1}{2} + \left(\frac{2(F-N)-F}{F}\right)F\tfrac{1}{2}$$

$$= \frac{-1}{2F}(F-N)^2 + F - N - \frac{F}{2}$$

$$= -\frac{1}{2F}(F^2 - 2FN + N^2) - N + F/2$$

$$= -\frac{F}{2} + N - \frac{N^2}{2F} - N + F/2$$

$$U(1)^3: \quad 0$$

$$U(1): \quad 0$$

$$U(1)_R^3: \ N^2 - 1 - \frac{2N^4}{F^2} = (F-N)^2 - 1 + 2(F-N)F\left(\frac{N-F}{F}\right)^3 + F^2\left(\frac{F-2N}{F}\right)$$

$$= F^2 - 2FN + N^2 - 1 - 2F\left(\frac{N^4 - 4N^3F + 6N^2F^2 - 4NF^3 + F^4}{F^3}\right)$$

$$+ \frac{F^2}{F^3}\left(F^3 - 6F^2N + 12FN^2 - 8N^3\right)$$

$$= N^2 - 1 - \frac{2N^4}{F^2} + 8\frac{N^3}{F} - 12N^2 + 12N^2 - 8\frac{N^3}{F}$$

$$U(1)_R: \ N^2 - 1 - 2N^2 = (F-N)^2 - 1 + 2(F-N)F\left(\frac{N-F}{F}\right) + F^2\left(\frac{F-2N}{F}\right)$$

$$= F^2 - 2FN + N^2 - 1 - 2(F^2 - 2FN + N^2) + (F^2 - 2FN)$$

$$= -N^2 - 1$$

$$U(1)^2 U(1)_R: \ 2(F-N)F\left(\frac{N}{F-N}\right)^2\left(\frac{N-F}{F}\right) = -2N^2$$

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duals admits a unique superpotential

$W = \lambda \tilde{M}^j_i \phi_j \bar{\phi}^i$ eq. o m. $\phi\bar{\phi} = 0$
color singlet

$M \longleftrightarrow \tilde{M}$

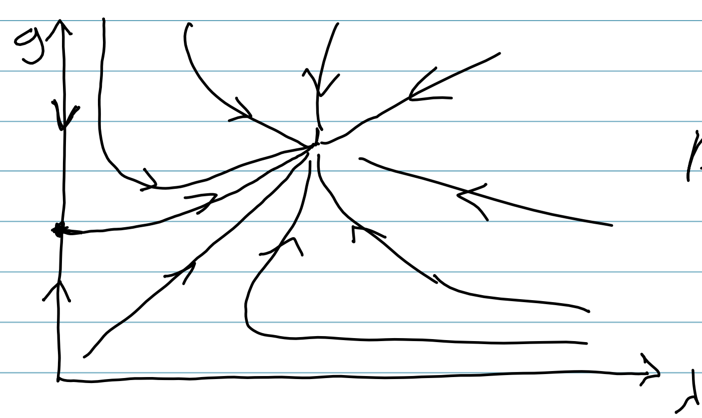
$B_{i_1 \dots i_N} \longleftrightarrow \epsilon_{i_1 \dots i_N j_1 \dots j_{F-N}} b^{j_1 \dots j_{F-N}}$

$\bar{B}^{i_1 \dots i_N} \longleftrightarrow \epsilon^{i_1 \dots i_N, j_1 \dots j_{F-N}} \bar{b}_{j_1 \dots j_{F-N}}$

$$\beta(\tilde{g}) = -\tilde{g}^3 (3\tilde{N} - F) = -\tilde{g}^3 (3(F-N) - F$$
$$= -\tilde{g}^3 (2F - 3N)$$

loose AF $F < 3/2 N$

$$F = \frac{3}{2} N + \epsilon N$$
$$\tilde{g}_*^2 = \frac{8\pi^2}{3} \frac{\tilde{N}}{\tilde{N}^2 - 1} \epsilon \left(1 + \frac{F}{\tilde{N}}\right)$$
$$\lambda_* = \frac{16\pi^2}{3\tilde{N}} \epsilon$$



at $\lambda = 0$

$D(\tilde{m}) = 1$

$D(\phi\bar{\phi}) = \frac{3}{2} \frac{(F - \tilde{N})}{F} = \frac{3N}{F}$

$< \frac{3N\,2}{3N}$

< 2

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$N+1 < F \leq 3N/2$

loose AF $\quad$ IR free

$g_* = 0 \qquad \lambda_* = 0$

$\Rightarrow \tilde{M}$ is free

accidental $U(1)$

$R_{sc} = R + Q_{accidental}$

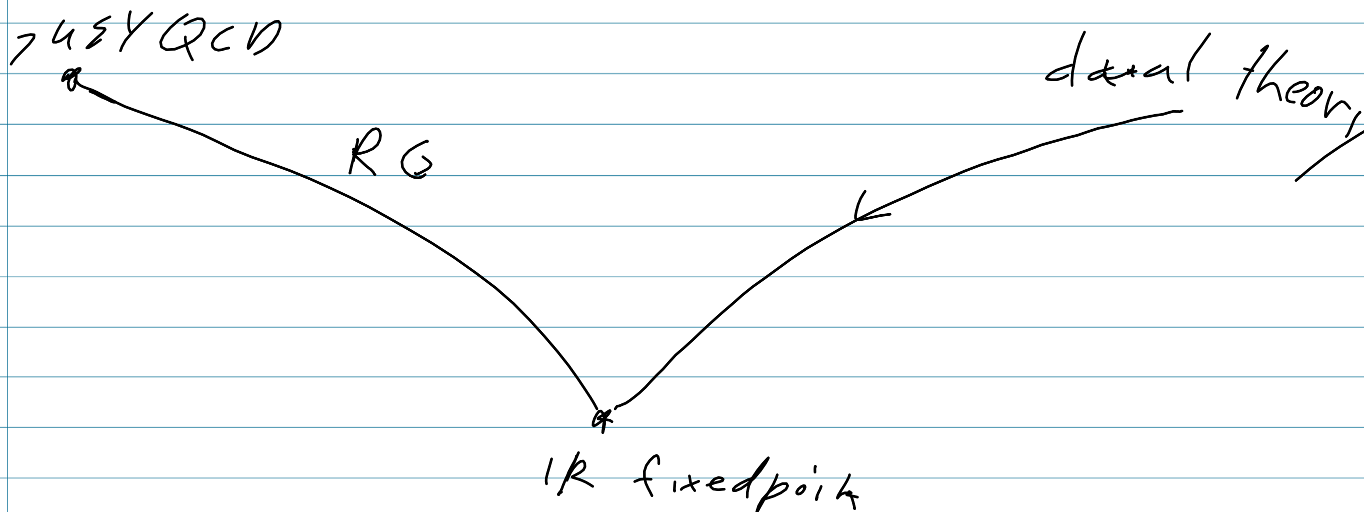
$R_{sc}(\tilde{M}) = 2/3$

$D(\tilde{M}) = 1$

IR Free $\quad$ massless composites

$\left.\begin{array}{l} \text{gauge bosons} \\ \text{quarks} \\ \text{mesons} \end{array}\right\}$ + superpartners

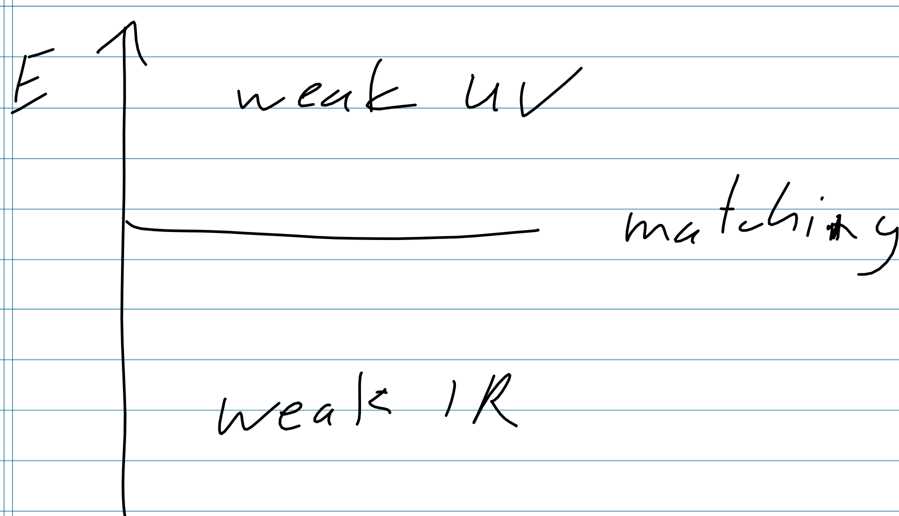
theory Space



cf QCD chiral lagrangian

same universality class

strong $\longleftrightarrow$ weakly



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$$\lambda \tilde{M} = \frac{M}{\mu}$$

Add a mass term for one flavor

$$W = m\, \bar{\Phi}^F \Phi_F$$

(dual

$$W_d = \frac{M^j{}_i}{\mu} \bar{\phi}^i \phi_j + m\, M^F{}_F$$

$$\frac{\partial W_d}{\partial M^F{}_F} = \frac{1}{\mu} \bar{\phi}^F \phi_F + m = 0$$

$$\phi = \left(\begin{array}{c|c} \phi' & H \\ \hline \phi'' & v+h \end{array}\right) \qquad \bar{\phi} = \left(\begin{array}{c|c} \bar{\phi}' & \bar{\phi}'' \\ \hline \bar{H} & v+\bar{h} \end{array}\right) \qquad M = \left(\begin{array}{c|c} M' & M^i{}_F \\ \hline M^F{}_i & M^F{}_F \end{array}\right)$$

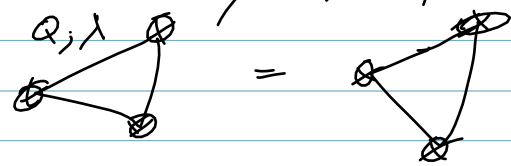
	$SU(F-N-1)$	$SU(F-1)$	$SU(F-1)$
ϕ'	$\square$	$\bar{\square}$	1
$\bar{\phi}'$	$\bar{\square}$	1	$\square$
M'	1	$\square$	$\bar{\square}$
ϕ''	1	$\bar{\square}$	1
$\bar{\phi}''$	1	1	$\square$
S	1	1	1
$M^F{}_j$	1	$\square$	1
$M^j{}_F$	1	1	$\bar{\square}$
$M^F{}_F$	1	1	1

$$W = \frac{1}{\mu} v \underbrace{\left(M^j{}_F \phi''_j + M^F{}_j \bar{\phi}'' + M^F{}_F S \right)}_{\text{integrate out}} + \frac{M'}{\mu} \phi' \bar{\phi}'$$

6)

1) Anomaly matching

Q, λ



$q, \tilde{\lambda}, M$

2) Moduli space

$M \Leftrightarrow M$

$B \Leftrightarrow b$

$\bar{B} \Leftrightarrow \bar{b}$

3)

$$\begin{tikzcd}
SU(N),\ F \arrow[r] \arrow[d, "W = m\,\bar{\Phi}_F \Phi^F"] & SU(F-N),\ F \arrow[d, "{W = M\phi\bar{\phi} + mM,\ \langle \phi \rangle \neq 0,\ \langle \bar{\phi} \rangle \neq 0}"] \\
SU(N),\ F-1 \arrow[r] & SU(F-N-1),\ F-1
\end{tikzcd}$$

classically rank $M \leq N$

dual $F -$ rank M light dual quarks

if $F -$ rank $M < \tilde{N} = F - N$

rank $M > N$

then ADS superpotential

no vacuum

rank $M \leq N$ enforced quantum effects in the dual

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Confinement

No gauge group in dual theory $F = N, N+1$

$F = N$

$B = \epsilon_{i_1 \cdots i_N} B^{i_1 \cdots i_N}$
$\bar{B} = \epsilon^{i_1 \cdots i_N} B_{i_1 \cdots i_N}$
$\Big\}$ flavor singlet

add in quark masses

$$\langle M^j_i \rangle = (m^{-1})^j_i \left(\det m \, \Lambda^{3N-F}\right)^{\frac{1}{N}}$$

$\det m \neq 0 \Rightarrow$ integrate out baryon # fields

set $\langle B \rangle = \langle \bar{B} \rangle = 0$

$$\det\langle M \rangle = \det(m^{-1})(\det m)\, \Lambda^{2N} = \Lambda^{2N}$$

classically $\det M = B\bar{B}$

	$U(1)$	$U(1)_A$	$U(1)_R$
$\det M$	0	$2N$	0
B	N	N	0
$\bar{B}$	$-N$	N	0
Λ^{2N}	0	$2N$	0

use holomorphy, $\Lambda \to 0$, $B, \bar{B} \to 0$

$$\det M - B\bar{B} = \Lambda^{2N}\left(1 + \sum_{a,b>0} C_{ab} \frac{(\Lambda^{2N})^a (\bar{B}B)^b}{(\det M)^{a+b}}\right)$$

$\langle \bar{B} B \rangle \gg \Lambda^{2N}$ perturbative

$$\det\langle M \rangle \sim \langle \bar{B} B \rangle^{\frac{b-1}{a+b}}$$

$\frac{b-1}{a+b} = 1$

$b - 1 = a + b$

$a = -1$

$$C_{a,b} = 0$$

$$\det M - \bar{B}B = \Lambda^{2N}$$

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Global symmetries are always broken

e.g. $M^{j}_{i} = \Lambda^2 \delta^{j}_{i}$ $\qquad B = \bar{B} = 0$

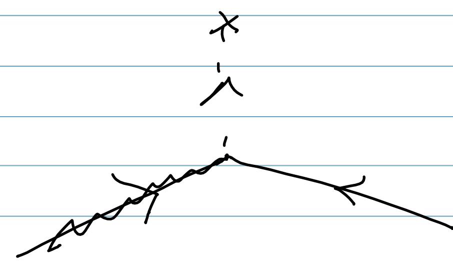
$$SU(F)\times SU(F)\times U(1)\times U(1)_R$$
$$\downarrow$$
$$SU(F)_d \times U(1)\times U(1)_R$$

chiral symmetry is broken

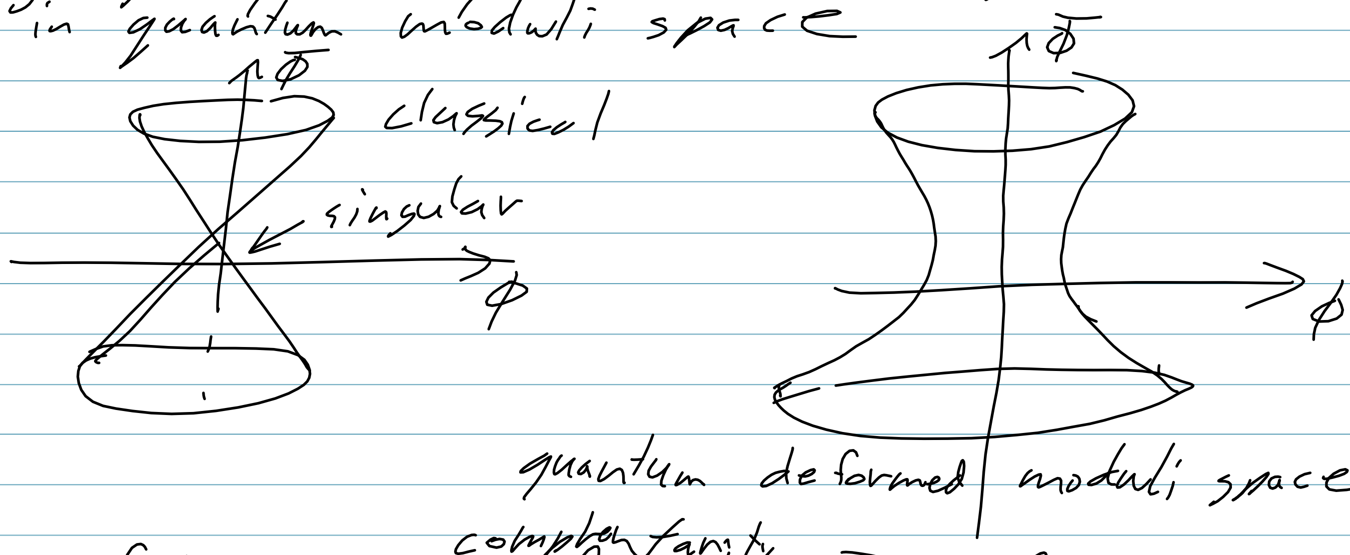
e.g. $M = 0 \qquad B\cdot\bar{B} = -\Lambda^{2N}$

$$SU(F)\times SU(F)\times U(1)_R$$

for large VEVs



gauge symmetry is broken everywhere in quantum moduli space



confining $\longleftarrow$ - complementarity / ~~confining~~ $\longrightarrow$ confining

$\downarrow$

strongly coupled color singlet degrees of freedom quantum constraint can be enforced by a Lagrange multiplier field

$$W = X\left(\det M - \bar{B}B - \Lambda^{2N}\right)$$

add a mass for Nth flavor

$$M = \left(\begin{array}{c|c} \tilde{M} & N^j \\ \hline P_i & Y \end{array}\right) \qquad \delta W_{mass} = mY$$

$$\frac{\partial W}{\partial B} = -X\bar{B} = 0 \qquad \frac{\partial W}{\partial \bar{B}} = -XB = 0$$

$$\frac{\partial W}{\partial N^j} = X\,\mathrm{cof}(N^j) = 0$$

$$\frac{\partial W}{\partial P_i} = X\,\mathrm{cof}(P_i) = 0$$

$$\frac{\partial W}{\partial Y} = X \det\tilde{M} + m = 0$$

$$X = -m \det\tilde{M}^{-1}$$

$$B = \bar{B} = N = P = 0$$

$$\frac{\partial W}{\partial X} = Y\det\tilde{M} - \Lambda^{2N} = 0 \Rightarrow Y = \Lambda^{2N}\det\tilde{M}^{-1}$$

$$W_{eff} = \frac{m\Lambda^{2N}}{\det\tilde{M}} = \frac{\Lambda_{N,N-1}^{2N+1}}{\det\tilde{M}}$$

$$= W_{ADS}(N, N-1)$$

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Anomaly Matching at maximal symmetry points

$M^i_{\ j} = \Lambda^2 \delta^i_j \qquad B = \bar{B} = 0$

unbroken $SU(F)_d \times U(1) \times U(1)_R$

elem	Q	$1 + A_d$	1	0
	$\bar{Q}$	$1 + A_d$	-1	0
	λ	A_d	0	1
comp	$M - \mathrm{Tr}M$	A_d	0	0
	B	1	N	0
	$\bar{B}$	1	$-N$	0
constraint removes	$\mathrm{Tr}M$	1	0	0

non-vanishing anomalies

	elem	comp
$U(1)^2 U(1)_R$	$-2FN$	$-2N^2$
$U(1)_R$	$(-1)2FN + N^2 - 1$	$-(F^2-1) - 1 - 1$
$U(1)_R^3$	$(-1)^3 2FN + (1)^3 N^2 - 1$	$(-1)^3(F^2-1) - (-1)^3 - (-1)^3$
$U(1)_R SU(F)_d^2$	$(-1)2F + N$	$(-1)N$

$M=0 \qquad B\cdot\bar{B} = \Lambda^{2N}$

	$SU(N)$	$SU(F)$	$SU(F)$	$U(1)_R$
Q	$\square$	$\square$	1	0
$\bar{Q}$	$\bar{\square}$	1	$\bar{\square}$	0
λ	Ad	1	1	1
M	1	$\square$	$\bar{\square}$	0
B	1	1	1	0
$\bar{B}$	1	1	1	0

constraint eliminates 1 combination

Homework:

	elem	comp
$SU(F)^3$	N	F
$U(1)_R\, SU(F)^2$	$(-1)\, N\, \frac{1}{2}$	$(-1)\, F\, \frac{1}{2}$
$U(1)_R$	$(-1)\, 2NF + N^2 - 1$ $= -N^2 - 1$	$-N^2 - 1$
$U(1)_R^3$	$(-1)^3\, 2NF + N^2 - 1$ $= -N^2 - 1$	$(-1)^3 N^2 + (-1)^3$

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$F = N+1$: "confinement without χSB"

		$SU(F)$	$SU(F)$
$B^i = \epsilon^{i \ldots\ldots i_N, i} B_{i \ldots i_N}$		$\overline{\square}$	1
$\bar{B}_i = \epsilon_{i \ldots i_N, i} \bar{B}^{i \ldots i_N}$		1	$\square$

classical constraints are

$$(M^{-1})^i_j \det M = B^i \bar{B}_j$$

$$M^j_i B^i = M^j_i \bar{B}_j = 0$$

turn on masses

$$\langle M^j_i \rangle = (m^{-1})^j_i \left(\det m \, \Lambda^{2N-1}\right)^{\frac{1}{N}}$$

$$\langle B^i \rangle = 0 \quad \langle \bar{B}_j \rangle = 0$$

$$\det \langle M \rangle = (\det m)^{-1} \left(\det m \, \Lambda^{2N-1}\right)^{\frac{N+1}{N}}$$

$$= (\det m)^{\frac{1}{N}} \left(\Lambda^{2N-1}\right)^{\frac{N+1}{N}}$$

$$\langle M^{-1} \rangle^i_j \det \langle M \rangle = m^i_j \Lambda^{2N-1}$$

classical constraint is satisfied only as $m \to 0$

different $m^i_j \to 0$ limits

can cover the classical moduli space

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the most general superpotential is

$$W = \frac{1}{\Lambda^{2N-1}}\left(\alpha B^i M_i^j \bar{B}_j + \beta \det M + \det M \, f\left(\frac{\det M}{B^i M_i^j \bar{B}_j}\right)\right)$$

only $f=0$ reproduces the classical constraints

$$\frac{\partial W}{\partial M_i^j} = \frac{1}{\Lambda^{2N-1}}\left(\alpha B^i \bar{B}_j + \beta (M^{-1})_j^i \det M\right) = 0$$

$$\beta = -\alpha$$

$$\frac{\partial W}{\partial B^i} = \frac{1}{\Lambda^{2N-1}} \alpha M_i^j \bar{B}_j = 0$$

$$\frac{\partial W}{\partial \bar{B}} = \frac{1}{\Lambda^{2N-1}} \alpha B^i M_i^j = 0$$

add a mass for one flavo

$$W = \frac{\alpha}{\Lambda^{2N-1}}\left(B^i M_i^j \bar{B}_j - \det M\right) + m X$$

$$M = \underbrace{\begin{pmatrix} M' & Z \\ Y & X \end{pmatrix}}_{N} \updownarrow N \qquad B = (u, B') \qquad \bar{B} = \begin{pmatrix} \bar{u} \\ \bar{B}' \end{pmatrix}$$

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$$\frac{\partial W}{\partial Y} = \frac{\alpha}{\Lambda^{2N-1}}\left(B'\bar{u} - \mathrm{cof}(Y)\right) = 0$$

$$\frac{\partial W}{\partial Z} = \frac{\alpha}{\Lambda^{2N-1}}\left(u\bar{B}' - \mathrm{cof}(Z)\right) = 0$$

$$\frac{\partial W}{\partial u} = \frac{\alpha}{\Lambda^{2N-1}} Z\bar{B}' = 0$$

$$\frac{\partial W}{\partial \bar{u}} = \frac{\alpha}{\Lambda^{2N-1}} B'Y = 0$$

$$\frac{\partial W}{\partial X} = \frac{\alpha}{\Lambda^{2N-1}}\left(B'\bar{B}' - \det M'\right) + m = 0$$

$$\det M' - B'\bar{B}' = \frac{m}{\alpha}\Lambda^{2N-1} = \frac{\Lambda_{N,N}^{2N}}{\alpha}$$

$\alpha = 1 \Rightarrow$ correct quantum constraint for $F=N$

$$Y = Z = u = \bar{u} = 0$$

$$W_{eff} = \frac{X}{\Lambda^{2N-1}}\left(B'\bar{B}' - \det M' + m\Lambda^{2N-1}\right)$$

$$= \frac{X}{\Lambda^{2N-1}}\left(B'\bar{B}' - \det M' + \Lambda_{N,N}^{2N}\right)$$

hold $\Lambda_{N,N}$ fixed as $m \to \infty$

$\Lambda \to 0$ $\quad X$ becomes a Lagrange multiplier

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$F = N+1$ no chiral S.B.

Since $M=0$, $B=0$, $\bar{B}=0$ is on quantum moduli spaces what about the singular behavior?

naively massless gluons gluinos

Answer is that M, B, $\bar{B}$ are massless "confinement"

	$SU(F)$	$SU(F)$	$U(1)$	$U(1)_R$
M	$\square$	$\bar{\square}$	0	$\frac{2}{F}(F-N) = \frac{2}{N+1} = \frac{2}{F}$
B	$\bar{\square}$	1	N	$\frac{N}{F}(F-N) = \frac{N}{N+1} = N/F$
$\bar{B}$	1	$\square$	$-N$	$\frac{N}{F}(F-N) = N/F$

	elem	comp
$SU(F)^3$	N	$F-1=N$
$U(1)SU(F)^2$	$\frac{1}{2}N$	$\frac{1}{2}N$
$U(1)_R SU(F)^2$	$\left(\frac{-N}{N+1}\right)N\frac{1}{2}$	$\left(\frac{1-N}{N+1}\right)\frac{1}{2}F + \left(\frac{-1}{N+1}\right)\frac{1}{2} = \frac{1}{2}\left(\frac{-1-(N^2-1)}{N+1}\right)$
$U(1)_R$	$-\frac{N}{N+1}2N(N+1)+N^2-1$ $= -N^2 \cdot 1$	$\left(\frac{1-N}{N+1}\right)F^2 + \left(-\frac{1}{N+1}\right)2F$ $= -(N^2 \cdot 1) \quad -2$
$U(1)_R^3$	see below	see below

$U(1)_R^3$, elem:

$$\left(\frac{-N}{N+1}\right)^3 2N(N+1) + N^2 - 1$$
$$= -\frac{2N^4}{(N+1)^2} + N^2 - 1$$
$$= \frac{-2N^4 + (N^3 - N + N^2 - 1)(N+1)}{(N+1)^2}$$
$$= \frac{-2N^4 + N^4 + N^3 - N^2 - N + N^3 + N^2 - N - 1}{(N+1)^2}$$
$$= \frac{-N^4 + 2N^3 - 2N - 1}{(N+1)^2}$$

$U(1)_R^3$, comp:

$$\left(\frac{1-N}{N+1}\right)^3 (N+1)^2 + \left(-\frac{1}{N+1}\right)^3 2(N+1)$$
$$= \frac{-(N-1)^3}{N+1} - \frac{2}{(N+1)^2}$$
$$= -\frac{(N^3 - 3N^2 + 3N - 1)(N+1) - 2}{(N+1)^2}$$
$$= -\frac{\left(N^4 - 3N^3 + 3N^2 - N + N^3 - 3N^2 + 3N - 1\right) \cdot}{(N+1)^2}$$
$$= \frac{-N^4 + 2N^3 - 2N - 1}{(N+1)^2}$$

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go back to $F = N+2$

	dual $SU(2)$	$SU(N+2)$	$SU(N+2)$
q	$\square$	$\square$	1
$\bar{q}$	$\overline{\square}$	1	$\square$
M	1	$\square$	$\overline{\square}$

integrate out one flavor

dual squark VEVs break $SU(2)$ completely

	$SU(N+1)$	$SU(N+1)$
q'	$\overline{\square}$	1
$\bar{q}'$	1	$\overline{\overline{\square}}$
$\overline{M}$	$\square$	$\overline{\square}$

$W = \frac{M'}{\mu} q' \bar{q}'$

we can idenify $q'^{i} = c\, B^{i}$
$\bar{q}'_{j} = \bar{c}\, \bar{B}_{j}$

$W_{eff} = \frac{c\bar{c}}{\mu} B^{i} M_{i}^{\ j} \bar{B}_{j}$

broken $SU(2)$ instanton