# Quantum Moduli Space

heavy mass for flavor $N$ through $F$
light masses for the rest

integrating out $\Lambda^{3N-F} \det m_H = \Lambda_{N,N-1}^{3N-N+1}$

$$\langle M_i^j \rangle = (m_L^{-1})_i^j \left( \det m_L \, \Lambda_{N,N-1}^{2N+1} \right)^{\frac{1}{N}}$$

$$= (m_L^{-1})_i^j \left( \underbrace{\det m_L \det m_H} \Lambda^{3N-F} \right)^{\frac{1}{N}}$$

$$= (m^{-1})_i^j \left( \det m \, \Lambda^{3N-F} \right)^{\frac{1}{N}}$$

$$\langle M_i^j \rangle \sim m^{\frac{F}{N}-1}$$

$F \geq N$ $\quad m_i^j \to 0$ with $\langle M \rangle \to$ finite or zero

quantum moduli space for $m = 0$

parameterized by $M, B, \overline{B}$
perturbative if $\langle M \rangle \langle B \rangle \langle \overline{B} \rangle$ are large

singular if $\langle M \rangle, \langle B \rangle, \langle \overline{B} \rangle \to 0$

gluons become light

what happen here?

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| | $SU(N)$ | $SU(F)$ | $SU(F)$ | $U(1)$ | $U(1)_R$ |
|---|---|---|---|---|---|
| $Q$ | $\square$ | $\square$ | $1$ | $1$ | $\frac{F-N}{F}$ |
| $\bar{Q}$ | $\bar{\square}$ | $1$ | $\bar{\square}$ | $-1$ | $\frac{F-N}{F}$ |

$F \geq 3N$ loose AF

$F$ just below $3N$

IR Fixed Point Banks-Zaks

$$\beta = -bg^3 + cg^5 + \dots$$

large $N$, $b \ll N$

$$\beta \sim -g\left(bg^2 + c(g^2N)^2 + O(g^2N)^3\right.$$

$$g_*^2 N^2 = \frac{b}{c} \qquad \text{perturbative}$$

exact:

$$\beta = \frac{-g^3}{16\pi^2}\,\frac{3N - F + F\gamma}{1 - N\frac{g^2}{8\pi^2}}$$

$$\gamma = \frac{-g^2}{8\pi^2}\,\frac{N^2-1}{N} + O(g^4) \qquad \text{pert.}$$

$$\beta = \frac{-g^3}{16\pi^2}\left(3N - F + F\gamma + (3N-F)\left(\frac{Ng^2}{8\pi^2}\right)\right.$$

$$= \frac{-g^3}{16\pi^2}\left(3N - F - \frac{g^2}{8\pi^2}\left(-F\frac{(N^2-1)}{N} + 3N^2 - FN\right)\right.$$

$$+ \dots$$

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$$16\pi^2 \beta = -g^3 (3N - F) - \frac{g^5}{8\pi^2} \left( 3N^2 - FN - F\frac{(N^2-1)}{N} \right)$$

$$F = 3N - \epsilon N$$

$$16\pi^2 \beta = -g^3 \epsilon N - \frac{g^5}{8\pi^2} \left( 3N^2 - 3N^2 + \epsilon N^2 - (3N - \epsilon N) \frac{(N^2-1)}{N} \right)$$

$$= -g^3 \epsilon N + \frac{g^5}{8\pi^2} \left( 3(N^2-1) + O(\epsilon) \right)$$

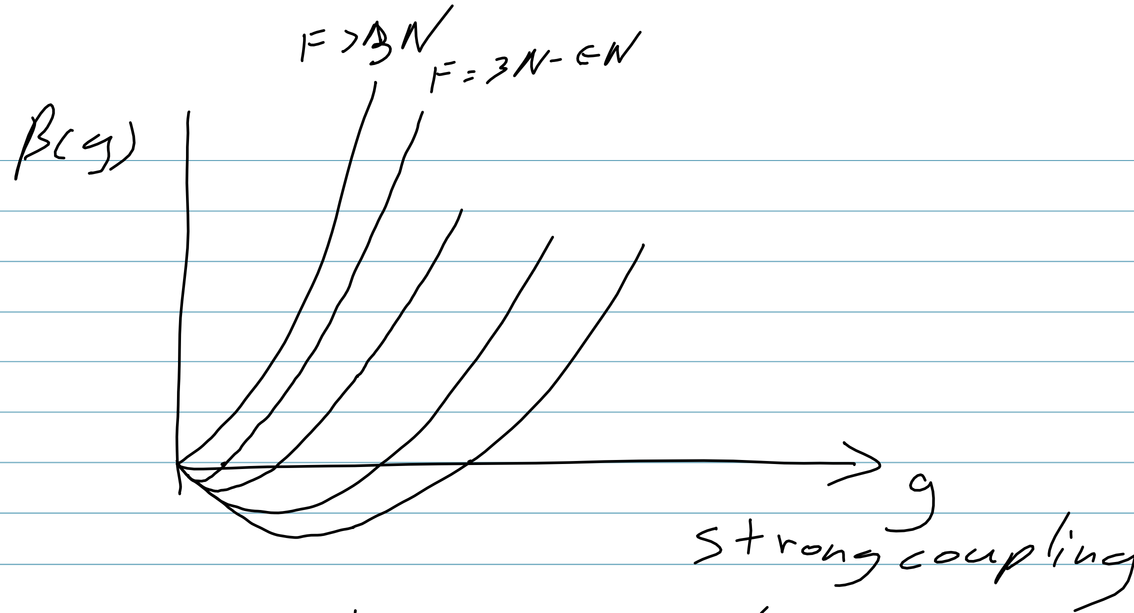
$$\beta(g_*) = 0 \qquad \epsilon N = \frac{g_*^2}{8\pi^2} \left( 3(N^2-1) + O(\epsilon) \right)$$

$$g_*^2 = \frac{8\pi^2}{3} \frac{N}{N^2-1} \epsilon$$

higher order terms

$$\beta \sim -g \left( \underset{O(\epsilon^2)}{g^2(3N-F)} - \underset{O(\epsilon^2)}{c(g^2N)^2} + \underset{O(\epsilon^3)}{(g^2N)^3} \right)$$

$\epsilon \to 0$ IR Fixed Point at weak coupling




Scale invariant theory
spin $\leq 1$

$\Rightarrow$ conformal Theory
+ SUSY

SUSY algebra $\rightarrow$ superconformal algebra

dimension $D \geq \frac{3}{2} |R_{sc}|$ superconformal R-charge

$D = \frac{3}{2} R_{sc}$ for a chiral superfield

$D = -\frac{3}{2} R_{sc}$ for anti-chiral "

for chiral

$$R_{sc}(\mathcal{O}_1 \mathcal{O}_2) = R_{sc}(\mathcal{O}_1) + R(\mathcal{O}_2)$$

$$\Rightarrow D(\mathcal{O}_1 \mathcal{O}_2) = D(\mathcal{O}_1) + D(\mathcal{O}_2)$$

OPE is just multiplication
"chiral ring"

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$$R_{sc} = R + \sum_i q_i$$

SUSYQCD: $R_{sc}(Q) = R_{sc}(\bar{Q}) \Rightarrow R_{sc} = R$

$$D(M) = D(\tilde{Q}Q) = 2 + \gamma_* = \frac{3}{2}\, 2\left(\frac{F-N}{F}\right)$$

$$= 3\left(\frac{F-N}{F}\right) = 3 - \frac{3N}{F}$$

$$\gamma_* = 1 - \frac{3N}{F}$$

$$\beta \propto 3N - F + F\gamma_* = 0$$

gauge inv, scalar $D \geq 1$ saturated for free scalar

$$3 - \frac{3N}{F} \geq 1$$

$$2 \geq \frac{3N}{F}$$

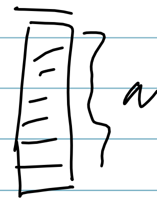
$$F \geq \frac{3}{2} N$$

IR is an interacting conformal theory

$$\frac{3}{2} N < F < 3N$$

conformal theory suggests no $\chi SB$

't Hooft anomaly matching should work

| | $SU(F)$ | $SU(F)$ |
|---|---|---|
| $B$ |  | $1$ |
| $\bar{B}$ | $1$ |  |
| $m$ | $\square$ | $\bar{\square}$ |

$$SU(F)^3 : N \stackrel{?}{=} \mathcal{O}(F^{N-1}) + F$$

$$U(1)\ SU(F)^2 : \frac{1}{2}N \stackrel{?}{=} \mathcal{O}(F^{N-1})$$

$$U(1)_R\ SU(F)^2 : \frac{N}{2}\left(\frac{-N}{F}\right) \stackrel{?}{=} \mathcal{O}(F^{N-1})\left(\frac{N(F-N)}{F} - 1\right) + 2(F-N.$$

Seiberg Found

| | $SU(F-N)$ | $SU(F)$ | $SU(F)$ | $U(1)$ | $U(1)_R$ |
|---|---|---|---|---|---|
| $q$ | $\square$ | $\bar{\square}$ | $1$ | $\frac{N}{F-N}$ | $N/F$ |
| $\tilde{q}$ | $\bar{\square}$ | $1$ | $\square$ | $-N/F-N$ | $N/F$ |
| $M$ | $1$ | $\square$ | $\bar{\square}$ | $0$ | $\frac{2}{F}(F-N)$ |

Homework:

$SU(F)^3$: $N = -(F-N) + F = N$

$U(1)\,SU(F)^2$: $\frac{1}{2}N = \frac{N}{F-N}(F-N)\frac{1}{2}$

$U(1)_R\,SU(F)^2$: $-\frac{N^2}{2F} = \frac{N-F}{F}(F-N)\frac{1}{2} + \left(\frac{2(F-N)-F}{F}\right)F\frac{1}{2}$

$$= \frac{-1}{2F}(F-N)^2 + F - N - \frac{F}{2}$$
$$= -\frac{1}{2F}(F^2-2FN+N^2) - N + F/2$$
$$= -\frac{F}{2} + N - \frac{N^2}{2F} - N + F/2$$

$U(1)^3$: $0$

$U(1)$: $0$

$U(1)_R^3$: $N^2-1-\frac{2N^4}{F^2} = (F-N)^2-1+2(F-N)F\left(\frac{N-F}{F}\right)^3 + F^2\left(\frac{F-2N}{F}\right)$

$$= F^2-2FN+N^2-1-2F\left(\frac{N^4-4N^3F+6N^2F^2-4NF^3+F^4}{F^3}\right)$$
$$+\frac{F^2}{F^3}\left(F^3-6F^2N+12FN^2-8N^3\right)$$
$$= N^2-1-\frac{2N^4}{F^2}+\frac{8N^3}{F}-12N^2+12N^2-\frac{8N^3}{F}$$

$U(1)_R$: $N^2-1-2N^2 = (F-N)^2-1+2(F-N)F\left(\frac{N-F}{F}\right)+F^2\left(\frac{F-2N}{F}\right)$

$$= F^2-2FN+N^2-1-2(F^2-2FN+N^2)+(F^2-2FN)$$
$$= -N^2-1$$

$U(1)^2U(1)_R$: $2(F-N)F\left(\frac{N}{F-N}\right)^2\left(\frac{N-F}{F}\right) = -2N^2$