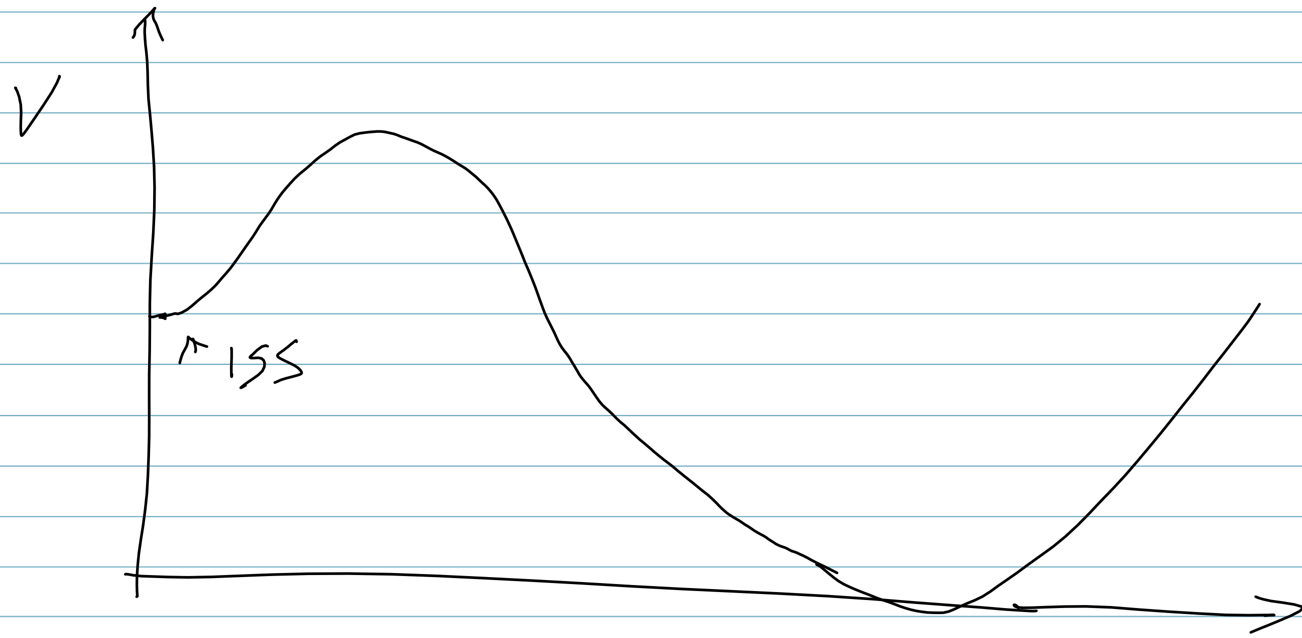
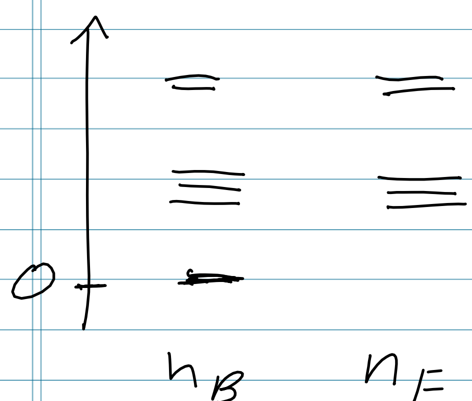
43



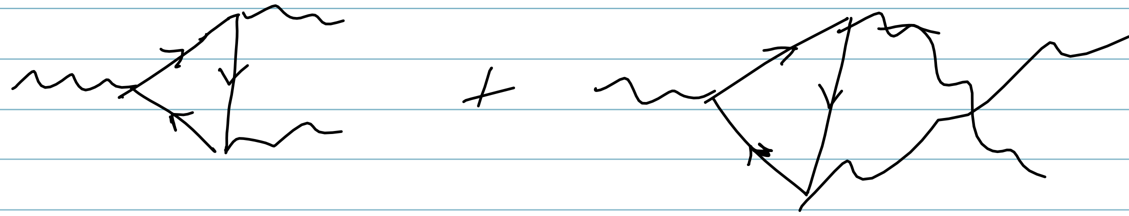
$$\mathrm{Tr}\, E(-1)^F = 0$$



Witten index $\quad \mathrm{Tr}(-1)^F$

N vacua

Gauge Anomalies



$$\propto \mathrm{Tr}\left(T^a\{T^b, T^c\}\right) = A^{abc}$$

potentially non-vanishing

$U(1)$, $SU(N)$ $N \geq 3$

Witten "global" anomaly $SU(2)$ $Sp(2N)$

$\sum_i Z\,T(r_i) = \text{even}$

$$A^{abc}(R_1 \oplus R_2) = A^{abc}(R_1) + A^{abc}(R_2) \Rightarrow T\begin{pmatrix} T_1 & \\ & T_2 \end{pmatrix}$$

$$A^{abc}(R_1 \otimes R_2) = \dim(R_1) A^{abc}(R_2) + \dim(R_2) A^{abc}(R_1)$$

$$T \sim \mathbb{1}_1 \otimes T_2 \oplus T_1 \otimes \mathbb{1}_2$$

for fundamental representation

$$d^{abc} \equiv \mathrm{Tr}\left(T_F^a\{T_F^b, T^c\}\right)$$

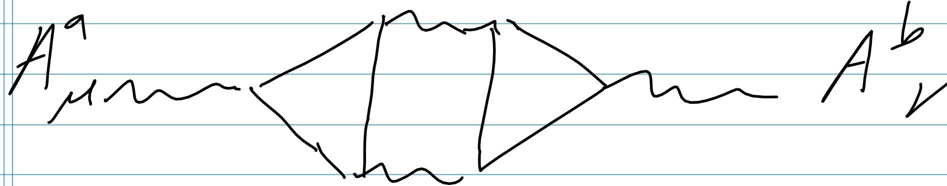
$$A^{abc}(R) = A(R)\, d^{abc}$$

$$A(\square) \equiv 1 \qquad A(\bar{\square}) = -1$$

$SU(3)$ $\quad \square \times \square = \square\square + \begin{smallmatrix}\square\\\square\end{smallmatrix} = \square\square + \bar{\square}$

$$A(\square \times \square) = 3 \cdot 1 + 1 \cdot 3 = A(6) - 1$$

$$A(6) = 7$$



generates a mass for A_μ

can only have anomalies in spontaneously broken gauge theories

eg $SU(N)$ $\quad F(\square + \bar{\square})$ $\qquad A$: $F - F = 0$

scalar vev $\square\square$

$$\lambda \phi \bar{\Psi} \bar{\Psi}$$

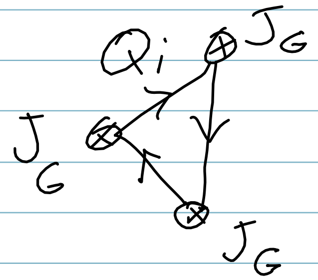
$\downarrow$ effective s

$F \square + (F-2)\bar{\square}$ $\qquad A$: 2

't Hooft

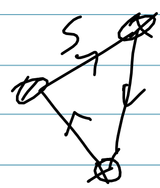
UV A.F. gauge theory global G

Q_i R_i r_i



$= A^{UV}$

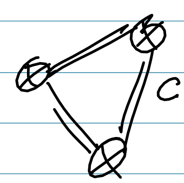
weakly gauge global symmetry
add spectators as needed

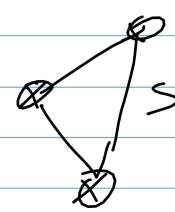


$= A^S = -A^{UV}$

$A_G = A^{UV} + A^S = 0$

integrate out down to $\mu < \Lambda$

IR $A_G^E =$ 

$+$ 

if G is unbroken then

$A_G = 0 = A^{IR} + + A^S$

$A^{IR} = A^{UV}$

G gauge coupling $\to 0$

Anomaly Matching

$SU(3)$	$SU(F)$	$SU(F)$	$U(1)_B$
$\square$	$\square$	1	$1/3$
$\overline{\square}$	1	$\overline{\square}$	$-1/3$

$SU(F)^3$: 3

$U(1)_B\, SU(F)^2$: $3 \cdot \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{2}$

$SU(F)$	$SU(F)$	$U(1)_B$
$\boxminus$	$\overline{\square}$	$1/3$
$\square$	$\boxminus$	$-1/3$
$\boxminus$	1	1
1	$\boxminus$	-1
$\vdots$	$\vdots$	

Solution for $F = 2$

$\square$	1	$1/3$
1	$\overline{\square}$	$-1/3$

only Witten global anomaly
and $U(1)\, SU(2)^2$

no solutions for $F \geq 3$
such that $F \to F-1$

Chiral

$SU(5)$	$U(1)$
$\overline{\square}$	3
$\boxminus$	-1

$U(1)^3 = 5 \cdot 3^3 + 10(-1)^3 = 135 - 10 = 125$

$B = \boxminus \times \overline{\square} \times \boxminus$

$U(1)^3 : (2 \cdot 3 - 1)^3 = 5^3 = 125$

Classical Moduli Space for $F \geq N$

	$SU(N)$	$SU(F)$	$SU(F)$	$U(1)$	$U(1)_R$
Q	$\square$	$\bar{\square}$	1	1	$\frac{F-N}{F}$
$\bar{Q}$	$\bar{\square}$	1	$\bar{\square}$	-1	$\frac{F-N}{F}$

$$D^a = \left(\phi^{*} T^a \phi - \bar{\phi}\, T^a \bar{\phi}^{*}\right) \qquad V = \frac{1}{2} D^a D^a$$

$$D^n_m = \phi^{*in}\, \phi_{mi}$$ ← gauge index

$$\bar{D}^n_m = \bar{\phi}^{in}\, \bar{\phi}^{*}_{mi}$$

$$D^a = T^{a\,m}_{\ \ n}\left(D^n_m - \bar{D}^n_m\right) = 0$$

$$D^n_m - \bar{D}^n_m = \alpha I$$

$$U^{+} D U$$

$$D = \begin{pmatrix} |v_1|^2 & & \\ & \ddots & \\ & & |v_N|^2 \end{pmatrix} \qquad \bar{D} = \begin{pmatrix} |\bar{v}_1|^2 & & \\ & \ddots & \\ & & |\bar{v}_N|^2 \end{pmatrix}$$

$$|v_i|^2 = |\bar{v}_i|^2 + \alpha$$

$$\phi = \underbrace{\begin{pmatrix} v_1 & & & 0 & \cdots & 0 \\ & \ddots & & \vdots & & \vdots \\ & & v_N & 0 & \cdots & 0 \end{pmatrix}}_{F} \Big\updownarrow N$$

$$\bar{\phi} = \begin{pmatrix} \bar{v}_1 & & \\ & \ddots & \\ & & \bar{v}_N \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{pmatrix} \Big\updownarrow F$$

generically $SU(N)$ is broken

$2NF - 2(N^2-1)$ massless chiral supermult,

$$M^j_i = \bar{\Phi}^{j\alpha}\phi_{\alpha i}$$

$$B_{i_1 \ldots i_N} = \phi_{\alpha_1 i_1} \ldots \phi_{\alpha_N i_N} \epsilon^{\alpha_1 \ldots \alpha_N}$$

$$\bar{B}^{i_1 \ldots i_N} = \bar{\Phi}^{\alpha_1 i_1} \ldots \bar{\Phi}^{\alpha_N i_N} \epsilon_{\alpha_1 \ldots \alpha_N}$$

$2\binom{F}{N} + F^2$ components

relations like

$$B_{i_1 \ldots i_N} \bar{B}^{j_1 \ldots j_N} \quad M^{j_1 \ldots j_N}_{[i_1 \ldots i_N]}$$