give masses to all flavors $\qquad (m^{-1})^j_i \det M$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad //$

$$W = W_{ADS} + m^i_j M^j_i$$

$$0 = \frac{\partial W}{\partial M^j_i} = (N-F)\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}}\left(\frac{-1}{N-F}\right)\frac{\mathrm{cof}(M^j_i)}{\det M} + m^i_j$$

$$(m^{-1})^j_i\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}} = M^j_i$$

$$\Rightarrow \mathrm{Tr}\, I\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}} = \mathrm{Tr}\, m M$$

$$\det M = \det(m^{-1})\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{F}{N-F}}$$

$$(\det M)^{N-F} = (\det(m^{-1}))^{N-F}\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{F}$$

$$(\det M)^{N} = \frac{1}{(\det m)^{N-F}}\left(\Lambda^{3N-F}\right)^{F}$$

$$(\det M)^{\frac{-1}{N-F}} = (\det m)^{\frac{1}{N}}\left(\Lambda^{3N-F}\right)^{\frac{-F}{N(N-F)}}$$

$$M^j_i = (m^{-1})^j_i\, \Lambda^{\frac{3N-F}{N-F}} (\det m)^{\frac{1}{N}}\left(\Lambda^{3N-F}\right)^{\frac{-F}{N(N-F)}}$$

$$= (m^{-1})^j_i\, (\det m)^{\frac{1}{N}}\left(\Lambda^{3N-F}\right)^{\frac{N-F}{N(N-F)}}$$

$$= (m^{-1})^j_i\left(\det m\, \Lambda^{3N-F}\right)^{\frac{1}{N}}$$

N different vacua

different phases

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matching: $$\left(\frac{\Lambda_{N,F}}{m_1}\right)^{b} = \left(\frac{\Lambda_{N,F-1}}{m_1}\right)^{b+1}$$

$$m_1 \Lambda_{N,F}^{b} = \Lambda_{N,F-1}^{b+1}$$

$$\left(\frac{\Lambda_{N,F-1}}{m_2}\right)^{b+1} = \left(\frac{\Lambda_{N,F-2}}{m_2}\right)^{b+2}$$

$$m_2 \Lambda_{N,F-1}^{b+1} = \Lambda_{N,F-2}^{b+2}$$

$$m_2\, m_1 \Lambda^{b} = \Lambda_{N,F-2}^{b+2}$$

$$\vdots$$

$$\det m\, \Lambda_{N,F}^{3N-F} = \Lambda_{N,0}^{3N}$$

$$W_{N,0} = \left(\frac{N-F}{F} + 1\right) m^{i}_{j} M^{j}_{i}$$

$$= \frac{N}{F} F \left(\det m\, \Lambda^{3N-F}\right)^{\frac{1}{N}}$$

$$= N \Lambda_{N,0}^{3}$$

agrees with the previous analysis where we assumed pure SUSY Yang-Mills had a mass gap

$$a = N$$

$$\langle \lambda^a \lambda^a \rangle = 32\pi^2 \Lambda_{N,0}^{3}$$

(37)

Summary: starting with $F = N-1$ we can consistently derive the correct effective superpotential for $0 \le F < N-1$

can we identify the physics that generates the superpotential?

recall $W \sim (\Lambda^b)^{\frac{1}{N-F}}$

instanton effects $\sim e^{-S_{inst}} \sim \Lambda^b$

for $F = N-1$ instantons can generate the superpotential (not for $F < N-1$)

Instanton analysis is reliable

fermion mass

$$m \sim \frac{\partial^2 W}{\partial \phi_i \partial \bar{\phi}_i}$$

$$\approx \left(\frac{\Lambda^{3N-F}}{v^{2F}}\right)\frac{1}{v^2}$$

$$\approx \left(\frac{\Lambda^{3N-N+1}}{v^{2N-2}}\right)\frac{1}{v^2}$$

$$\approx \frac{\Lambda^{2N+1}}{v^{2N}}$$

Vaccuum Energy density $= |F_i|^2$

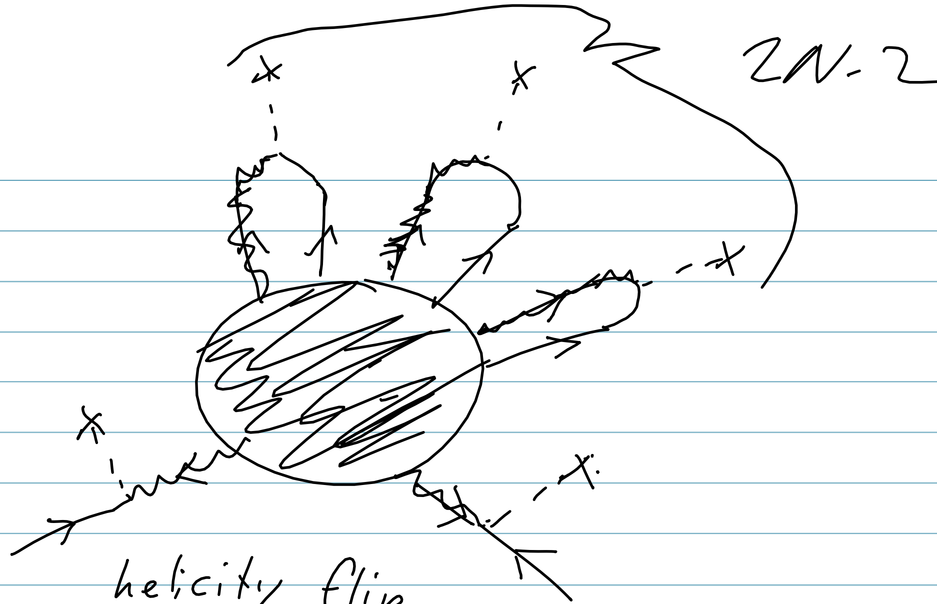
$$\simeq \left|\frac{\partial W}{\partial \phi_i}\right|^2$$

$$= \left|\left(\frac{\Lambda^{3N-F}}{v^{2F}}\right)^{\frac{1}{N-F}}\frac{1}{v}\right|^2$$

$$= \left|\frac{\Lambda^{3N-N+1}}{v^{2N-2}}\frac{1}{v}\right|$$

$$= \left|\frac{\Lambda^{2N+1}}{v^{2N-1}}\right|^2$$

$v \to \infty$



helicity flip

$$m \sim e^{-\frac{8\pi^2}{g^2(1/\rho)}}\, v^{2N} \rho^{2N-1}$$

$$\sim (\Lambda\rho)^{b}\, v^{2N} \rho^{2N-1}$$

$$\sim (\rho\Lambda)^{3N-(N-1)}\, v^{2N} \rho^{2N-1}$$

$$\sim \Lambda^{2N+1}\, v^{2N} \rho^{4N}$$

integration over ρ dominated by $\rho^2 = \frac{b}{16\pi^2 v^2}$

$$m \sim \Lambda^{2N+1}\, v^{2N} \left(\frac{b}{16\pi^2 v^2}\right)^{2N}$$

$$\sim \frac{\Lambda^{2N+1}}{v^{2N}} \left(\frac{b}{16\pi^2}\right)^{2N}$$

for $F < N-1$ instantons can't generate W_{ADS}

at generic point $\det M \neq 0$

$$SU(N) \to SU(N-F) \supset SU(2)$$

matching: $\Lambda^{3N-F} = \Lambda_{N-F,0}^{3(N-F)} \det M$

$\boxed{SU(N-F)}$ $\longleftarrow$ irrelevant ops $\longrightarrow$ $\boxed{F^2 \text{ gauge singlets } M}$

gaugino have R anomaly must be cancelled M

spontaneously broken

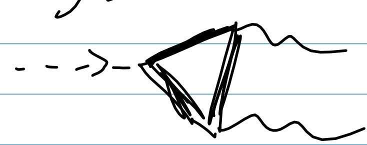
$$\tau = \frac{3(N-F)}{2\pi i} \ln\left(\frac{\Lambda_{N-F,0}}{\mu}\right)$$

$$\mathcal{L} \supset c \int d^2\theta \, \ln \det M \; W^a W^a + h.c.$$

$$\supset c\left(\mathrm{Tr}(F_m M^{-1}) \lambda^a \lambda^a + \mathrm{Arg}(\det M) F^{a\mu\nu} \tilde{F}^a_{\mu\nu} + \dots \right) h.c.$$

$$\lambda \to e^{i\alpha} \lambda$$

$$\mathrm{Arg}\det M \to \mathrm{Arg}\det M + 2F\alpha$$



cancels R anomaly

gaugino condensation

$$F_M = M^{-1}\langle \lambda^a \lambda^a \rangle \propto M^{-1} \Lambda^3_{N-F,0}$$

$$\propto M^{-1}\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}}$$

agrees with ADS vacuum energy ✓

gaugino condensation

$\Rightarrow$ non-zero superpotential

W_{ADS} : only superpotential consistent with symmetries

$F < N-1$

gaugino condensation generates W_{ADS}

#11

Instanton $\Longrightarrow W_{ADS}$

$SU(N), F=N-1 \xrightarrow{VEV} SU(N-1), F=N-2 \xrightarrow{VEV} \cdots \longrightarrow SU(2)\ F=1$

$\downarrow$ mass

$SU(N), F=N-2$

$\downarrow$

$SU(N)\ F_{eff}$

$\downarrow$

$\vdots$

$\downarrow$

$SU(N)\ F=0$

$\langle \lambda\lambda \rangle = 32\pi^2 \Lambda^3_{N,0}$

$\uparrow$ derived from instantaton calc

$SU(N-F)\ F_{eff}=0$

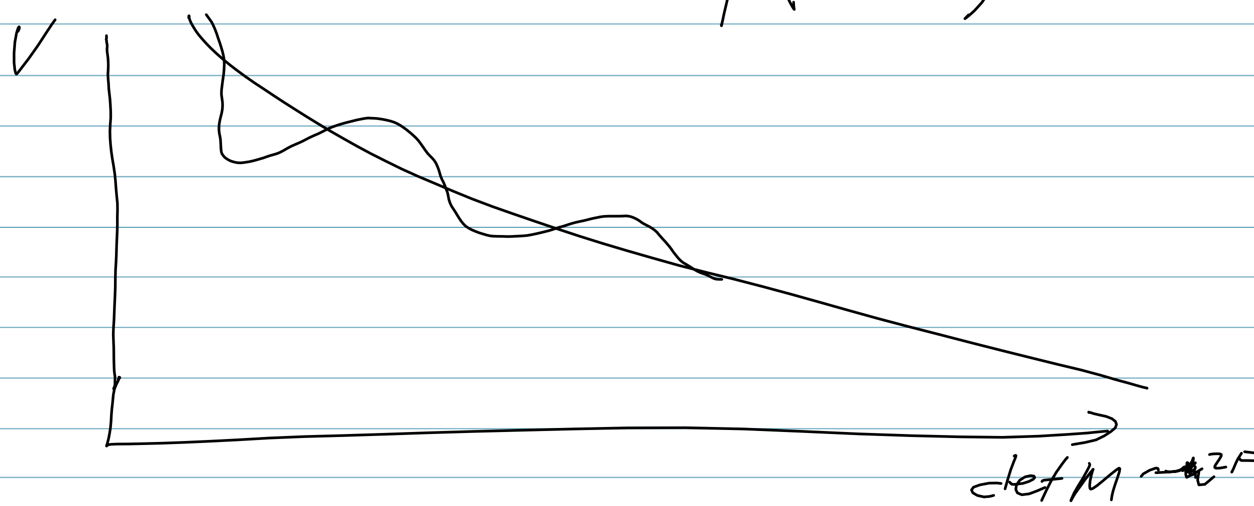
$\langle \lambda\lambda \rangle \neq 0$

$SU(2)\ F_{eff}=0$

$\langle \lambda\lambda \rangle \neq 0$

Now that we belive W_{ADS} what does it tell us about the vacuum structure:

recall $V = |F_i|^2 \sim \left| \left(\frac{\Lambda^{3N-F}}{v^{2F}} \right)^{\frac{1}{N-F}} \frac{1}{v} \right|^2$



no SUSY vacuum
"runaway vacuum

consequence of $\Lambda \to 0 \Rightarrow$ positive powers of Λ

when added masses for each flavor we could solve for $F_m = 0$