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Adding matter fields

$$Q'_j = (Z_j(\mu, \mu'))^{-1/2} Q_j$$

$$\mathcal{D}(Z^{1/2} Q'_j) \neq \mathcal{D}(Q_j)$$

$$Z_j = e^{i\alpha} \quad \text{(axial anomaly again!)}$$

$$\mathcal{D}(e^{i\alpha/2} Q'_j)\, \mathcal{D}(e^{-i\alpha/2} Q'^{+}_j)$$

$$= \mathcal{D}(Q'_j)\, \mathcal{D}(Q'^{+}_j) \exp\left(\frac{i}{4} \int d^4x \int d^2\theta \, \frac{T(r_j)}{8\pi^2} \ln(e^{i\alpha})\, W^a W^a + h.c. \right)$$

$$\ln e^{i\alpha} \to \ln Z$$

$$\frac{1}{g_c^2} = R\left(\frac{1}{g_h^2}\right) - \frac{2T(Ad)}{8\pi^2} \ln g_c - \sum_j \frac{T(r_j)}{8\pi^2} \ln Z_j$$

$$\beta(g_c) = -\frac{g_c^3}{16\pi^2} \frac{\left(3T(Ad) - \sum_j T(r_j)(1-\gamma_j)\right)}{1 - \frac{T(Ad)\, g^2}{8\pi^2}}$$

where $\gamma_j \equiv -\mu \frac{d}{d\mu} \ln Z_j$

note $Z_j = f_j\left(Re\left(\frac{1}{g_h^2}\right)\right)$

$$\frac{1}{g_c^2} = Re\left(\frac{1}{g_h^2}\right) - a \ln g_c - \sum_j b_j \ln f_j\left(Re\left(\frac{1}{g_h^2}\right)\right)$$

no Taylor expansion around $g_c = 0$

$g_h = 0$

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for pure $SU(N)$

$$\beta(g_c) = -\frac{3Ng_c^3}{1-\frac{Ng_c^2}{8\pi^2}} \Leftarrow \text{pole}$$

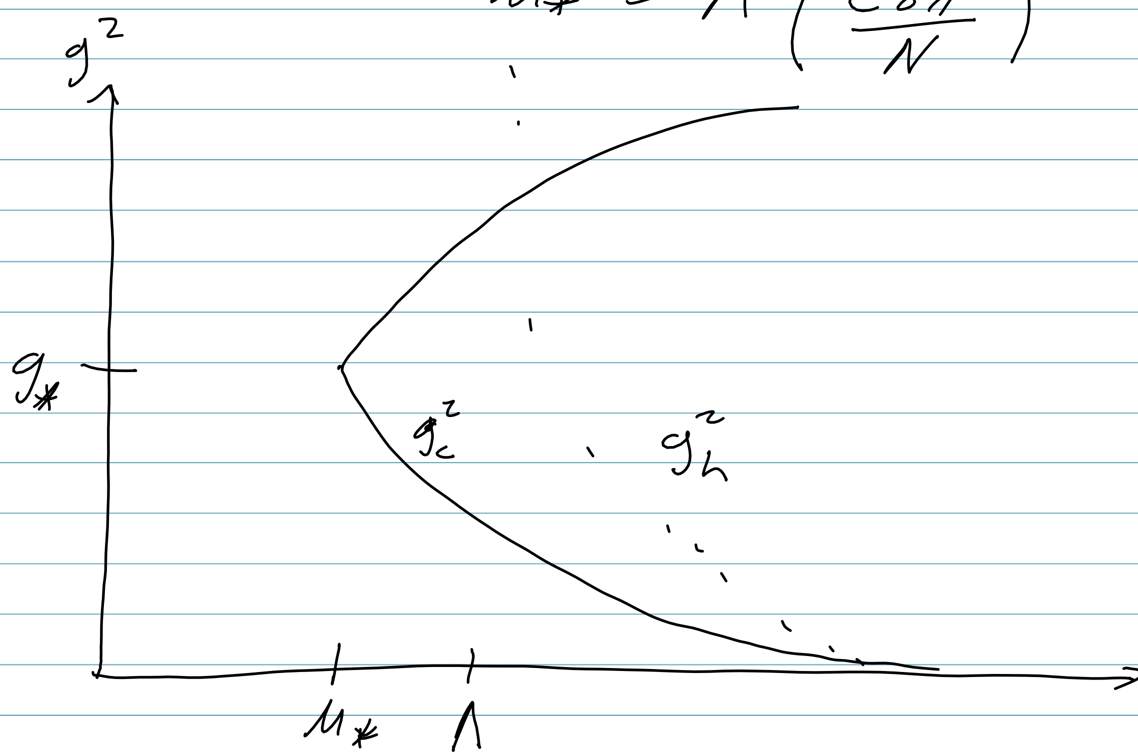
$$\frac{1}{g_c^2} + \frac{2N}{8\pi^2}\ln g_c = \frac{1}{g_h^2} = \frac{3N}{8\pi^2}\ln\left(\frac{\mu}{\Lambda}\right)$$

$$g_c^2 = g_*^2 = \frac{8\pi^2}{N} \quad \text{at } \mu_*$$

$$\frac{N}{8\pi^2} + \frac{N}{8\pi^2}\ln\left(\frac{8\pi^2}{N}\right) = \frac{3N}{8\pi^2}\ln\left(\frac{\mu_*}{\Lambda}\right)$$

$$e\left(\frac{8\pi^2}{N}\right) = \left(\frac{\mu_*}{\Lambda}\right)^3$$

$$\mu_* = \Lambda\left(\frac{e\,8\pi^2}{N}\right)^{1/3}$$



large anomalous dimensions new relevant ops.

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SUSY QCD $F < N$

	$SU(N)$	$SU(F)$	$SU(F)$	$U(1)$	$U(1)_R$
ϕ, Q	$\square$	$\square$	1	1	$\frac{F-N}{F}$
$\bar{\phi}, \bar{Q}$	$\bar{\square}$	1	$\bar{\square}$	-1	$\frac{F-N}{F}$

Classically there is a D-flat moduli space

$$D^a = g\left(\phi^* T^a \phi - \bar{\phi} T^a \bar{\phi}^*\right) \qquad V = \frac{1}{2} D^a D^a$$

$$\langle \bar{\phi}^* \rangle = \langle \phi \rangle \quad \underbrace{\begin{pmatrix} v_1 & & \\ & \ddots & \\ & & v_F \\ & 0 & \end{pmatrix}}_{F} \Bigg\updownarrow N$$

generically $SU(N) \to SU(N-F)$

$N^2 - 1 - ((N-F)^2 - 1) = 2NF - F^2$ eaten chiral s.m.

$2NF - (2NF - F^2) = F^2$ massless chiral s.m.

We would like a gauge invariant description of the light degrees of freedom for an effective theory with scalars in the fundamental we have

Higgs $\xleftarrow{\hspace{1em}\text{complimentarity}\hspace{1em}}\!\!\longrightarrow$ confined

$\mathcal{O}^\alpha \qquad\qquad\qquad\qquad \phi_\alpha \mathcal{O}^\alpha$

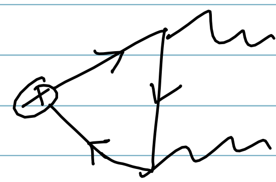
$\mathcal{O}^\alpha{}_\beta \qquad\qquad\qquad\quad \bar{\phi}^\beta \phi_\alpha \mathcal{O}^\alpha{}_\beta$

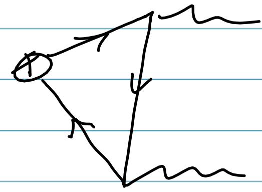
$\vdots \qquad\qquad\qquad\qquad\qquad \vdots$

$$M^j_i \equiv \phi_{i\alpha} \bar{\phi}^{\alpha j}$$

holomorphy $\Rightarrow$ multiplicative renormalization $Z_M = Z_\phi Z_{\bar{\phi}}$

Recall

$U(1)_R$ R  $= 1\cdot N + \left(\frac{F\cdot N}{F} - 1\right) 2F\frac{1}{2} = 0$

$U(1)_A$  $= 1\cdot 2F\cdot\frac{1}{2} = F$

broken $U(1)_A \Rightarrow$ selection rule

$$e^{2\pi i\tau} = e^{-\frac{8\pi^2}{g^2(\mu)} + i\theta_{YM}} = \left(\frac{\Lambda}{\mu}\right)^b = \left(\frac{\Lambda}{\mu}\right)^{3N-F}$$

assign spurious axial charge to Λ

$Q \to e^{i\alpha}Q$, $\overline{Q} \to e^{i\alpha}\overline{Q} \iff \theta_{YM} \to \theta_{YM} - 2F\alpha$

Spurious symmetry $Q \to e^{i\alpha}Q$, $\overline{Q} \to e^{i\alpha}\overline{Q}$

$\theta_{YM} \to \theta_{YM} + 2F\alpha$

or $\tau \to \tau + \frac{F\alpha}{\pi}$

or $\Lambda^b \to e^{i2F\alpha}\Lambda^b$

construct most general superpotential for light fields

$\det M$ $\begin{cases} R \text{ charge } & 2F\left(\frac{F-N}{F}\right) = 2(F-N) \\ U(1)_A \text{ charge } & 2F \end{cases}$

$$W_{eff}^{N,F} = C_{N,F}\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}}$$

renormalization scheme dependent

non-perturbative

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Charges

	$U(1)_A$	$U(1)_R$	$U(1)_B$
$W^a W^a$	0	2	0
Λ^b	$2F$	0	0
$\det M$	$2F$	$2F\left(\frac{F-N}{F}\right) = 2(F-N)$	0

$$\delta W = \Lambda^{bn} (W^a W^a)^m (\det M^p)$$

$U(1)_A$: $\quad 0 = n2F + p2F$

$U(1)_R$: $\quad 2 = 2m + p2(F-N)$

$$p = -\frac{(1-m)}{N-F} = -n$$

locality: $m \geq 0$, integer

weak coupling $n \geq 0 \Rightarrow m \leq 1, \ p \leq 0$

$m = 1 \quad p = 0, \ n = 0$ ~~~~perturbative SYM

$\Rightarrow$ gauge coupling has no non-perturbative corrections

$m = 0$

$$W_{ADS} = C_{N,F} \left(\frac{\Lambda^b}{\det M} \right)^{\frac{1}{N-F}}$$

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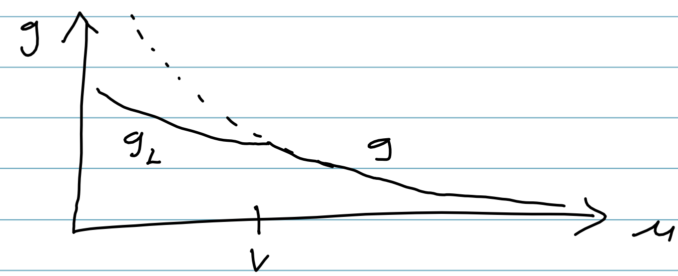
Consistency: Moduli Space

give a large VEV to one flavor
match on to effective theory at scale v
$SU(N-1)$, $F-1$ colored flavor + singlets



$$\frac{8\pi^2}{g^2(\mu)} = b \ln\left(\frac{\mu}{\Lambda}\right) \quad , \quad \frac{8\pi^2}{g_L^2(\mu)} = b_L \ln\left(\frac{\mu}{\Lambda_L}\right)$$

$$\frac{8\pi^2}{g^2(v)} = \frac{8\pi^2}{g_L^2(v)} + c$$

threshold correction vanishes in $\overline{DR}$

$$b \ln\left(\frac{v}{\Lambda}\right) = b_L \ln\left(\frac{v}{\Lambda_L}\right) + c$$

$$\left(\frac{\Lambda}{v}\right)^b = e^{-c}\left(\frac{\Lambda_L}{v}\right)^{b_L}$$

in $\overline{DR}$ scheme c vanishes, relation is exact for holomorphic coupling in $\overline{DR}$

$$\left(\frac{\Lambda}{v}\right)^{3N-F} = \left(\frac{\Lambda_L}{v}\right)^{3N-3-F+1} = \left(\frac{\Lambda_L}{v}\right)^{3N-F-2}$$

$$\frac{\Lambda^{3N-F}}{v^2} = \Lambda_L^{3N-F-2} \equiv \Lambda_{N-1,F-1}^{3N-F-2}$$

$\det M = v^2 \det \tilde{M} + \dots$

$$\left(\frac{\Lambda^{3N-F}}{v^2}\right)^{\frac{1}{N-F}} = \left(\Lambda_{N-1,F-1}^{3N-F-2}\right)^{\frac{1}{N-1-(F-1)}}$$

reproduces W_{ADS} if $C_{N,F} = C_{N-F}(N-F)$

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Giving equal VEVs to all flavors

$SU(N) \to SU(N-F)$ + singlets

matching gauge couplings at v:

$$\frac{\Lambda^{3N-F}}{v^{2F}} = \Lambda_{N-F,0}^{3(N-F)}$$

$$W_{eff} = C_{N-F}\left(\Lambda_{N-F,0}^{3(N-F)}\right)^{\frac{1}{N-F}} = C_{N-F}\,\Lambda_{N-F,0}^{3}$$

consistent with holomorphic argument for pure SUSY Yang-Mills and gaugino condensation

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Consistency: Mass Terms

give a large mass, m, to one flavor

$F \to F-1$

matching at scale m

$$\left(\frac{\Lambda}{m}\right)^{b} = \left(\frac{\Lambda_L}{m}\right)^{b_L}$$

$$\left(\frac{\Lambda}{m}\right)^{3N-F} = \left(\frac{\Lambda_{N,F-1}}{m}\right)^{3N-F+1}$$

$$m\,\Lambda^{3N-F} = \Lambda_{N,F-1}^{3N-F+1}$$

using holomorphy

$$W_{exact} = \left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}} f(t)$$

$$t = m\, M^F_F \left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{-1}{N-F}}$$

weak coupling: $\Lambda \to 0$, $m \to 0$

t is arbitrary

$f(t)$ must reproduce perturbation theory and W_{ADS}

$$\cancel{W_{exact}} = f(t) = C_{N-F} + t$$

$$W_{exact} = C_{N-F}\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}} + m\, M^F_F$$

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$$0 = \frac{\partial W}{\partial M^F_F} = C_{N-F}\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}}\left(\frac{-1}{N-F}\right)\frac{\mathrm{cof}(M^F_F)}{\det M} + m$$

$$0 = \frac{\partial W}{\partial M^i_F} = C_{N-F}\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}}\left(\frac{-1}{N-F}\right)\frac{\mathrm{cof}(M^i_F)}{\det M}$$

$$\mathrm{cof}(M^i_F) = 0 \quad \mathrm{cof}(M^F_i) = 0 \Rightarrow M = \begin{pmatrix} \tilde{M} & 0 \\ 0 & M^F_F \end{pmatrix}$$

$$\Rightarrow \mathrm{cof}(M^F_F) = \frac{\det M}{M_{FF}}$$

$$\frac{1}{N-F} C_{N-F}\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}} = m M^F_F$$

$$\therefore \frac{1}{N-F} C_{N-F}\left(\frac{\Lambda^{3N-F}}{\det \tilde{M}}\right)^{\frac{1}{N-F}} = m\left(M^F_F\right)^{1+\frac{1}{N-F}} = m\left(M^F_F\right)^{\frac{N-F+1}{N-F}}$$

$$\left(\frac{C_{N-F}}{(N-F)}\right)^{\frac{N-F}{N-F+1}}\left(\frac{m\Lambda^{3N-F}}{\det\tilde{M}}\right)^{\frac{1}{N-F+1}} = m M^F_F$$

plug into W_{exact}

$$W_{exact} = (N-F+1)\, m M^F_F$$ matching

$$= (N-F+1)\left(\frac{C_{N-F}}{N-F}\right)^{\frac{N-F}{N-F+1}}\left(\frac{\Lambda_{N,F-1}^{3N-F+1}}{\det\tilde{M}}\right)^{\frac{1}{N-F+1}}$$

for consistency $C_{N-(F-1)} = (N-F+1)\left(\frac{C_{N-F}}{N-F}\right)^{\frac{N-F}{N-F+1}}$

for $F = N-1$ instanton calculation gives $C_{N-(N-1)} = 1$

recursion gives $C_{N-F} = N-F$