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Instatons in Broken Gauge Theories

Turn on scalar VEV $\langle\phi\rangle \neq 0$
instantons are no longer solutions
of classical eq. of motion

approximate solutions

$D^\mu F_{\mu\nu} = 0$ ← neglect current

$A_\mu > i\,U(x)\,\partial_\mu U^\dagger(x)$ as $|x|\to\infty$

$D^\mu D_\mu \phi^j + \frac{\partial V}{\partial \phi^{*i}}(\phi) = 0$

$\phi^j \to U(x)\langle\phi^j\rangle$ as $|x|\to\infty$

for small istantons: $\rho \ll \frac{1}{gv}$

give Euclidean action

$S_{int} = \frac{8\pi^2}{g^2}$ $\quad S_\phi = 8\pi^2\rho^2 v^2$

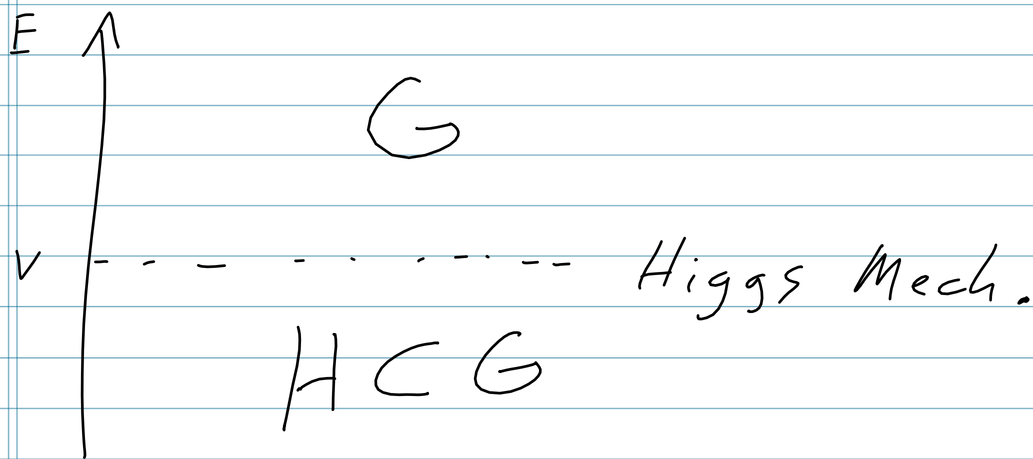
equiv. to minizing S_ϕ with fixed ϕ b.c.

$$\int d^4x_0 \int \frac{d\rho}{\rho^5}\, e^{-S_{int}-S_\phi} = \int d^4x_0 \int \frac{d\rho}{\rho^5}\,(\rho\Lambda)^b e^{-8\pi^2\rho^2 v^2}$$

dominated $\rho^2 \sim \frac{b}{16\pi^2 v^2}$

self consistent for $\frac{b\,g^2(\frac{1}{\rho})}{16\pi^2} < 1$

Instantons and Effective Theories



if inst. G/H can be gauge rotated into $SU(2) \subset H$ then all G instantons can be accounted for in the effective theory by H instantons

if not we must add new interactions in the effective theory to match the match physics properly

eg: $SU(N) \rightarrow U(1)$

$SU(N) \rightarrow$ breaks completely

$SU(N) \times SU(N) \rightarrow SU(N)_D$

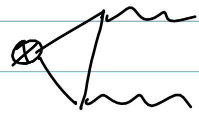
$SU(N) \rightarrow SO(N)$

non-trivial $\pi_3(G/H)$

rule-of-thumb: if there are different # of zero modes for G and H instantons new terms are needed

Gauge Condensation
SUSY Yang-Mills

SU(N) $U(1)_R$ is anomalous

 $\sim (1) - T(Ad) = N$

$$\lambda^a \to e^{i\alpha}\lambda^a$$

$$\theta_{YM} \to \theta_{YM} - \underbrace{2N}_{\#}\alpha$$

\# zero modes of λ^a in one instanton solution

only a symmetry for $2N\alpha = k\,2\pi$

$$\alpha = \frac{k\pi}{N}$$

explicitly broken to
Z_{2N} subgroup $U(1)_R$

spurious R symmetry $\lambda \to e^{i\alpha}\lambda$
$\tau \to \tau - \frac{N\alpha}{\pi}$

expect that SUSY YM has no massless particles, just color singlet composites

$AA,\ \lambda\lambda,\ A\lambda,\ \ldots$

integrating out gives $W_{eff}(\tau)$

$$\left.\begin{array}{l} \text{R-charge } 2 \\ e^{2\pi i\tau} \to e^{i2N\alpha}\, e^{2\pi i\tau} \end{array}\right\} \Rightarrow W_{eff} = a\mu^3 e^{\frac{2\pi i\tau}{N}}$$

$$W_{eff} \to e^{2i\alpha} W_{eff}$$

(21)

$$\langle\lambda\lambda\rangle = 16\pi i \frac{\partial}{\partial F_\tau} \ln Z = 16\pi i \frac{\partial}{\partial F_\tau} \int d^2\theta\, W_{eff}(\tau, u)$$

$$= 16\pi i \frac{\partial}{\partial \tau} W_{eff} = 16\pi i \frac{2\pi i}{N} a \mu^3 e^{\frac{2\pi i \tau}{N}}$$

dropping non-pert^t coor, i.e. β

$$\langle\lambda\lambda\rangle = -\frac{32\pi^2}{N} a \Lambda^3$$

under Z_{2N} $\langle\lambda\lambda\rangle \to e^{2i\alpha}\langle\lambda\lambda\rangle$

$\alpha = \frac{k\pi}{N}$

invariant of $\alpha = 0, \pi$; $k = 0, N$

broken

Z_2 $\Rightarrow$ N degenerate vacua

$\theta_{YM} \to \theta_{YM} + 2\pi$ or $\tau \to \tau + 1$

gives N different values for $\langle\lambda\lambda\rangle$

still need to justify assumption of no massless states

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β function Mysteries

① the SUSY gauge coupling only runs at one loop

$$\beta = \frac{-g^3(3N-F)}{16\pi^2} = \frac{-g^3\left(3T(Ad) - \sum_i \ldots\right.}{16\pi^2}$$

② Novikov-Shifman-Vainstein-Zakharov derived the "exact" β-function

$$\beta(g) = \frac{-g^3}{16\pi^2}\left(\frac{3T(Ad) - \sum_j T(r_j)(1-\gamma_j)}{1 - T(Ad)\frac{g^2}{8\pi^2}}\right)$$

γ_j is the anomalous dimension of the matter field mass term

verified up to two loops

③ the one and two loop terms in the β function are scheme independent

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$$g' = g + ag^3 + \mathcal{O}(g^5)$$

$$\beta(g) = b_1 g^3 + b_2 g^5 + \mathcal{O}(g^7)$$

$$\beta(g') = \frac{dg'}{d\ln\mu} = \frac{\partial g'}{\partial g}\frac{\partial g}{\partial \ln\mu} = \beta(g)\frac{\partial g'}{\partial g}$$

$$= \beta(g)\frac{\partial}{\partial g}\left(g + ag^3 + \mathcal{O}(g^5)\right)$$

$$= \beta(g)\left(1 + 3ag^3 + \mathcal{O}(g^4)\right)$$

$$= \left(\begin{matrix} b_1\left(g' - ag'^3 + \mathcal{O}(g'^5)\right)^3 \\ + b_2 g'^5 + \mathcal{O}(g'^7) \end{matrix}\right)\left(\begin{matrix} 1 + 3ag'^2 \\ + \mathcal{O}(g'^4) \end{matrix}\right)$$

$$= \left(\begin{matrix} b_1\left(g'^3 - 3g'^2 ag'^3 + \mathcal{O}(g'^5)\right) \\ + b_2 g'^5 + \mathcal{O}(g'^7) \end{matrix}\right)\left(1 + 3ag'^2 + \mathcal{O}(g'^4)\right)$$

$$= b_1 g'^3 + b_2 g'^5 + \mathcal{O}(g'^7)$$

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$$\frac{1}{g_h^2} = \frac{1}{g^2} - i\frac{\theta_{ym}}{8\pi^2} = \frac{\tau}{4\pi i}$$

$$= \frac{\cancel{b}}{8\pi^2} \ln\left(\frac{\mu}{\Lambda}\right) + \text{inst.}$$

$$\frac{1}{g_h^2(\mu')} = \frac{1}{g_h^2(\mu)} + \frac{b}{8\pi^2} \ln\left(\frac{\mu'}{\mu}\right)$$

$$\mathcal{L}_h = \frac{1}{4} \int d^2\theta \, \frac{1}{g_h^2} W^a(V_h) W^a(V_h) + \text{h.c.}$$

$$\mathcal{L}_c = \frac{1}{4} \int d^2\theta \left(\frac{1}{g_c^2} - i\frac{\theta_{ym}}{8\pi^2}\right) W^a(g_c V_c) W^a(g_c V_c)$$

change variables in P.I. $V_h = g_c V_c$

$$\mathcal{D}(g_c V_c) \neq \mathcal{D}(V_c)$$

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$$\lambda \to e^{\ln g_c}\lambda = e^{i\alpha}\lambda \qquad \alpha = \frac{1}{i}\ln g_c = -i\ln g_c$$

$$\mathcal{D}(g_c V_c) = \mathcal{D}V_c \exp\left(\frac{-i}{4}\int d^4x \int d^2\theta \, \frac{2T(Ad)}{8\pi^2}\ln(g_c)\, W^a(g_c V_c) W_a(g_c V_c) + h.c\right)$$

$$Z = \int \mathcal{D}V_h \exp\left(\frac{i}{4}\int d^4y \int d^2\theta \frac{1}{g_h^2} W^a(V_h) W_a(V_h) + h.c.\right)$$

$$= \int \mathcal{D}(g_c V_c) \exp\left(\frac{i}{4}\int d^4y \int d^2\theta \frac{1}{g_h^2} W^a(g_c V_c) W_a(g_c V_c) + h.c.\right)$$

$$= \int \mathcal{D}V_c \exp\left(\frac{i}{4}\int d^4y \int d^2\theta \left(\frac{1}{g_h^2} - \frac{2T(Ad)}{8\pi^2}\ln g_c\right) W^a(g_c V_c) W_a(g_c V_c) + h.c.\right)$$

$$\frac{1}{g_c^2} = Re\left(\frac{1}{g_h^2}\right) - \frac{2T(Ad)}{8\pi^2}\ln g_c$$

$$\frac{1}{g_c^2(\mu')} + \frac{2T(Ad)}{8\pi^2}\ln g_c(\mu') = \frac{1}{g_c^2(\mu)} + \frac{2T(Ad)}{8\pi^2}\ln g_c(\mu) + \frac{b}{8\pi^2}\ln\left(\frac{\mu'}{\mu}\right)$$

$$\mu'\frac{d}{d\mu'} \Rightarrow \frac{-2}{g_c^3(\mu')}\beta(g_c) + \frac{2T(Ad)}{8\pi^2}\frac{1}{g_c(\mu')}\beta(g_c) = \frac{b}{8\pi^2}$$

$$\beta(g_c)\left(1 - \frac{T(Ad)}{8\pi^2}g_c^2\right) = -\frac{b g_c^3}{16\pi^2}$$

$$\beta(g_c) = \frac{-b\, g_c^3}{1 - \frac{T(Ad)}{8\pi^2}g_c^2} = \frac{-3T(Ad)\, g_c^3}{1 - \frac{T(Ad)}{8\pi^2}g_c^2}$$