Poincaré coordinates

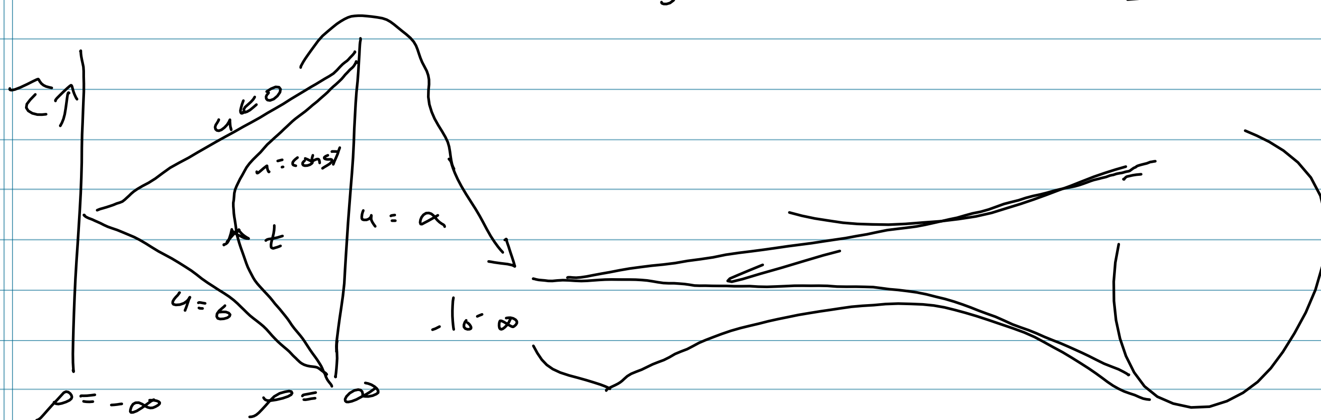
$X_0 = \frac{1}{2u}\left(1+u^2(R^2+\vec{x}^2-t^2)\right)$ $\qquad X_5 = Rut$

$X_i = Rux_i$

$X_4 = \frac{1}{2u}\left(1-u^2(R^2-\vec{x}^2+t^2)\right)$

$ds^2 = R^2\left(\frac{du}{u^2}+u^2\left(-dt^2+d\vec{x}^2\right)\right)$

covers half of global coordinates



wick rotation $\qquad \tau \to \tau_E = -i\tau$

$\qquad t \to t_E = -it$

$ds_E^2 = R^2\left(\cosh^2\rho\, d\tau_E^2 + d\rho^2 + \sinh^2\rho\, d\Omega_3^2\right)$

$\quad = R^2\left(\frac{du}{u^2} + u\left(dt_E^2 + d\vec{x}^2\right)\right)$

$u = 1/z \qquad x_4 = t_E$

$ds_E^2 = \frac{R^2}{z^2}\left(dz^2 + \sum_{i=1}^{4} dx_i^2\right)$

conformally flat metric

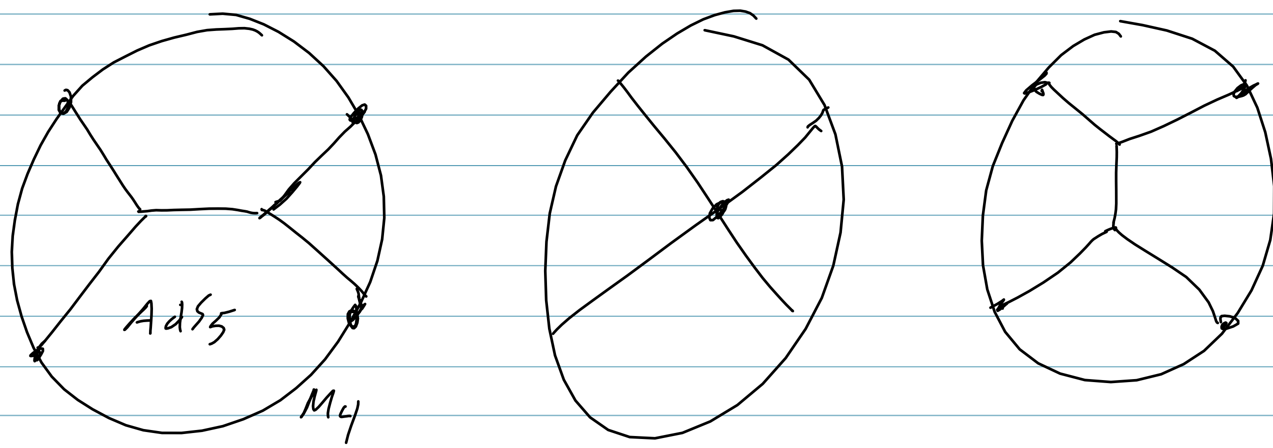
at a $z=0$ R^4 $\qquad$ point at $z=\infty$

AdS/CFT

$$\left\langle e^{\int d^4x\, \phi_0(x) \mathcal{O}(x)} \right\rangle_{CFT} = Z_{string}\left[z^{\Delta-4}\phi(x,z)\Big|_{z=0} = \phi_0(x) \right]$$

$g \ll 1$

$g^2 N \gg 1$ $\qquad Z_{string} \sim e^{-I_{suga}}$



consider a massive scalar in AdS_5

$$S = \int d^4x\, dz \frac{\sqrt{g}}{2}\left(g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi + m^2\phi^2 \right)$$

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu \qquad \partial_\mu = \frac{\partial}{\partial x^\mu}$$

$$g = \det g_{\mu\nu}$$

$$\phi(x, z) = e^{ipx} f(p, z)$$

$$ds^2 = \frac{R^2}{z^2}\left(z^2 + dx^i dx^i\right)$$

$$g_{ij} = \frac{R^2}{z^2} \qquad \sqrt{g} = \sqrt{\frac{R^{10}}{z^{10}}} = \frac{R^5}{z^5}$$

$$\frac{\partial}{\partial z}\left(\frac{R^5}{z^5}\frac{z^2}{R^2}\frac{\partial}{\partial z} e^{ipx} f(p, z)\right)$$

$$+ \frac{\partial}{\partial x^j}\left(\frac{R^5}{z^5}\frac{z^2}{R^2} i p^j e^{ipx} f(pz)\right) - m^2 \frac{R^5}{z^5} f = 0$$

$$z^5 \frac{\partial}{\partial z}\left(\frac{1}{z^3}\frac{\partial}{\partial z} f\right) - z^2 p^2 f - m^2 R^2 f = 0$$

$$y = pz$$

$$y^5 \frac{\partial}{\partial y}\left(\frac{1}{y^3}\frac{\partial}{\partial y} f\right) - y^2 f - m^2 R^2 f = 0$$

$$y^5\left(-\frac{3}{y^4} f' + \frac{1}{y^3} f''\right) - \left(m^2 R^2 + y^2\right) f = 0$$

$$y \sim 0 \qquad f \sim y^\alpha$$

$$y^5\left(-\frac{3\alpha}{y^4} y^{\alpha-1} + \alpha(\alpha-1)\frac{y^{\alpha-2}}{y^3}\right) - \left(m^2 R^2\right) y^\alpha = 0$$

$$(\alpha^2 - 4\alpha - m^2 R^2) y^\alpha = 0$$

$$\alpha^2 - 4\alpha - m^2R^2 = 0$$

$$\alpha = \frac{4 \pm \sqrt{16 + 4m^2R^2}}{2}$$

$$= 2 \pm \sqrt{4 + m^2R^2}$$

$$\Delta \equiv 2 + \sqrt{4 + m^2R^2}$$

modified Bessel function

$$f(y) = \begin{cases} y^2 I_{\Delta-2}(y) \xrightarrow[y \to 0]{} y^{\Delta} \\ y^2 K_{\Delta-2}(y) \xrightarrow[y \to 0]{} y^{4-\Delta} \end{cases}$$

$I_{\Delta-2}$ blows up as $y \to \infty$, infinite action

$$p \to \rho p \qquad x \to \frac{x}{\rho}$$

$y \sim 0$ $\quad \phi \to e^{ip \cdot x} \rho^{4-\Delta} f(\rho z)$, conformal weight $4-\Delta$

$$\lim_{z \to 0} \phi = \phi_0(x)$$

$$\int d^4x \, \phi_0 \, \mathcal{O}_{CFT}(x)$$

$\mathcal{O}_{CFT}$ must conformal weight Δ

$$\Delta = 2 + \sqrt{4 + m^2R^2}$$

$\delta(x)$ source $\rightarrow$ propagator

$$\phi(x,z) = c\int d^4x' \frac{z^{\Delta}}{(z^2+|x-x'|^2)^{\Delta}}\phi_0(x')$$

for small z

$\propto z^{4-\Delta}$

$z \sim 0$

$$\frac{\partial}{\partial z}\phi(x,\cancel{z}) = c\Delta\int d^4x' \frac{z^{\Delta-1}}{|x-x'|^{2\Delta}}\phi_0(x') + \mathcal{O}(z^{\Delta+1})$$

$$S = \frac{1}{2}\int dz\, d^4x \frac{R^5}{z^5}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2\right)$$

$$= \frac{1}{2}\int dz\, d^4x\, \partial_{\cancel{z}}\left(\frac{R^3}{z^3}\phi\partial_z\phi\right) + S\Big|_{\text{eqm}} = c$$

$$= \frac{c\Delta}{2}\int d^4x\, d^4x' \frac{R^3}{z^3} z^{4-\Delta} \frac{\phi_0(x) z^{\Delta-1}\phi_0(x')}{|x-x'|^{2\Delta}}$$

$$\langle \mathcal{O}(x)\,\mathcal{O}(y)\rangle = \frac{\delta^2 S}{\delta\phi_0(x)\,\delta\phi_0(y)}$$

$$= \frac{c\Delta R^3}{|x-x'|^{2\Delta}}$$

Ops, large dimen $\longleftrightarrow$ bulk large mass

AdS_{d+1}

$d=4$
$m=0$

scalars $\Delta_{\pm} = \frac{1}{2}\left(d \pm \sqrt{d^2+4m^2R^2}\right)$ $\Delta_+ = 4$

vectors $\Delta_{\pm} = \frac{1}{2}\left(d \pm \sqrt{(d-2)^2+4m^2R^2}\right)$ $\Delta_+ = 3$

p form $\Delta_{\pm} = \frac{1}{2}\left(d \pm \sqrt{(d-2p)+4m^2R^2}\right)$

massless spin 2 $\Delta = d$ 4

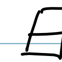
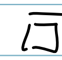
$\uparrow$ stress tensor

stringy states

$$m \sim \frac{1}{\sqrt{\alpha'}} = \frac{1}{l_s}$$

$$\Delta \sim (g^2 N)^{1/4}$$

large $N \Rightarrow$ large dim.

$\mathcal{N}=4$	h	$SU(4)$
A_μ^a	1	1
λ_I^a	$\frac{1}{2}$	□
ϕ_{IJ}	0	
λ_I	$-\frac{1}{2}$	
A_μ^a	-1	1

CFT Spectrum of Operators

~~chiral primary~~

primary: annihilated by superconformal raising op. S, K_μ

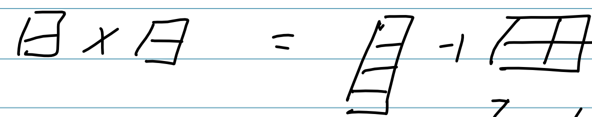
lowest dimension operator in superconformal multiplet

in $\mathcal{N}=4$ $SU(N)$

$$\mathcal{O}^{I_1 I_2 \cdots I_k} = \mathrm{Tr}\left(\phi^{I_1}\phi^{I_2}\cdots\phi^{I_k}\right)$$

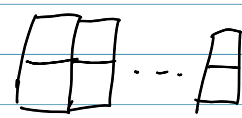
symmetric traceless

$$\mathrm{Tr}\,\phi^I\phi^J\big|_{symm} \sim 6\times 6\big|_{sym} = 1 + 20'$$



'' Traceless

1

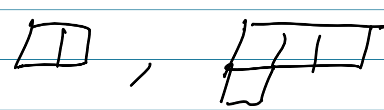
$\mathcal{O}^{I_1 I_2\cdots I_k} \sim$  $= (0,k,0)$ e.g. $20'$, 50, ...

k

R symmetry determines $\Delta = k$

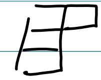
$\mathrm{Tr}\, W^\alpha W_\alpha \phi^{I_1}\cdots\phi^{I_k}$ $\quad \Delta = k+3$

$(0,k,2)$



10, 45, ...

$\mathrm{Tr}\, F^2\phi^k \quad (0,k,0), \quad \Delta = k+4$

$J_R^\mu \quad \Delta = 3$  $= 15$

$T^{\mu\nu} \quad \Delta = 4 \quad 1$

Type IIB String Theory on $AdS_5 \times S^5$

KK spherical harmonics on S^5

irreps of $SU(4) \sim SO6$

low mass: spin $R^2 m^2 = k(k+4)$ $\quad k \geq 0$

$\sim (0, k, 0)$

$1, 6, 20', $

graviton $\longrightarrow T_{\mu\nu}$

spin 1 $\quad R^2 m^2 = (k-1)(k+1)$ $\quad k \geq 1$

$\sim (1, k-1, 1)$

$\sim 15, 64, \ldots$

SU(4) gauge boson $\longleftrightarrow J^\mu_k$

3 scalar families $\quad R^2 m^2 = k(k-4)$ $\quad k \geq 2$

$(0, k, 0) \sim 20', 50, \ldots$

$\longrightarrow \Delta = k$

$\mathrm{Tr}\, \phi^{I_1} \ldots \phi^{I_k}$

complex scalars $\quad R^2 m^2 = (k-1)(k+3), \; k \geq 0$

$(0,k,2) \sim 10, \; 45,$

$\ydiagram{2}, \; \ydiagram{3,1}$

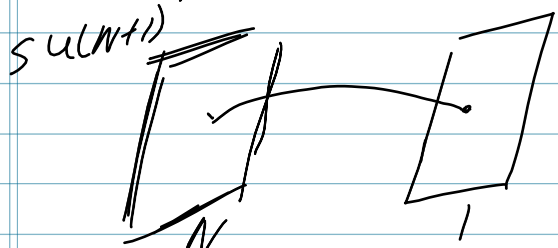
$\rightarrow \Delta = k+3 \quad \mathrm{Tr}\,(W^\alpha W_\alpha \phi^{I_1} \cdots \phi^{I_k})$

complex scalars $\quad R^2 m^2 = k(k+4) \quad k \geq 0$

$(0,k,0) \quad 1, 6, 20'$

$\rightarrow \Delta = k+4 \quad \mathrm{Tr}(\phi^k F^2)$

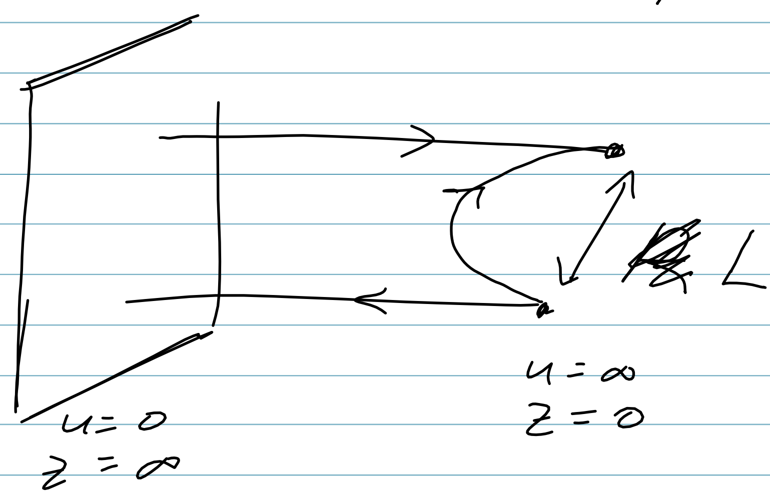
Applications of AdS/CFT



$m_W = \dfrac{u}{\alpha'}$

Fundamental of SU(N)

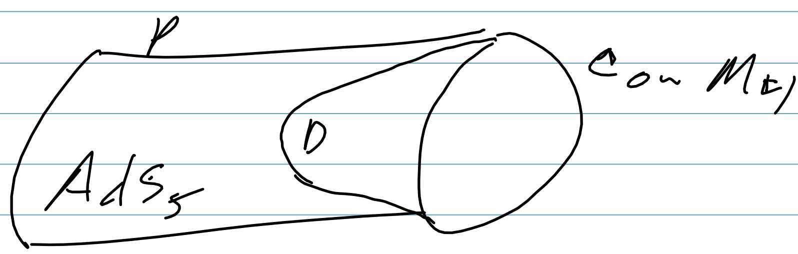
$u \to \infty$ static quark



$W(C) = \mathrm{Tr}\, P \exp \int_C A$

$C = T \times R$
for square

$\langle W(C) \rangle = e^{-\alpha(D)} = e^{-TV(R)}$



$\alpha(D) = A(D) - AP$

D mimizes area, action

$$ds^2 = \frac{R^2}{z^2}\left(dz^2 + dx^i dx^i\right)$$

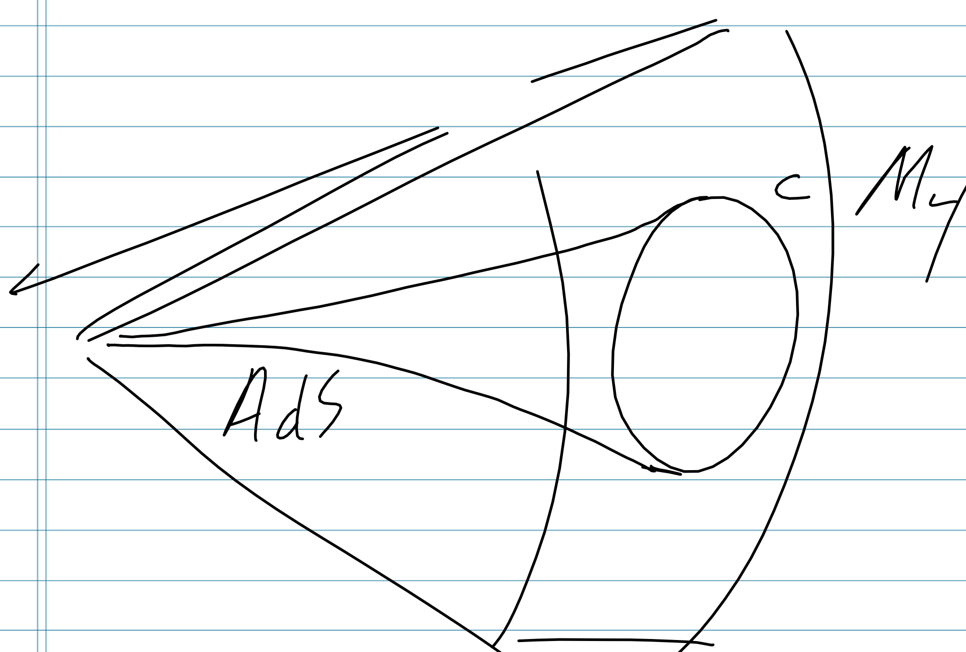
scale $C: \; x^i \to \rho x^i$

$D: \; x^i \to \rho x^i \qquad z \to \rho z$

$A(C) \sim \rho^2$

$\alpha(D) \sim \rho^0$

$$V(R) \sim -\frac{\sqrt{g^2 N}}{R}$$ ← conformal symmetry



Finite Temp
Periodic Euclidean Time

$t \to i t_E$ $\qquad t_E \sim t_E + \beta$

$e^{itE} \to e^{-\beta E} \to$ 3D finite Temp

b.c. periodic for bosons
anti-periodic for fermions
no zero ~~energy~~ mass ~~bosons~~ scalars

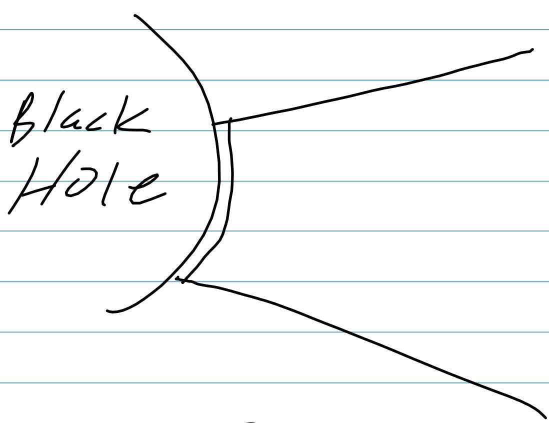
SUSY broken

gauge mediation $\longrightarrow$ massive gluino
Non-supersymmetric 3D SU(N)

& for large T AdS Hawking Page
Black Hole

$$\frac{ds^2}{R^2} = \frac{du^2}{\left(u^2 - \frac{b^4}{u^2}\right)} + \left(u^2 - \frac{b^4}{u^2}\right) d\tau^2 + u^2 dx_1 dx_2$$

horizon at $u = b$



$\alpha(b) = R^2 b^2 \,\text{Area}(C)$

Area Law confinement

$V = R^2 b^2 L$

$\sigma = R^2 b^2 \sim \sqrt{g^2 N}\,\alpha'$