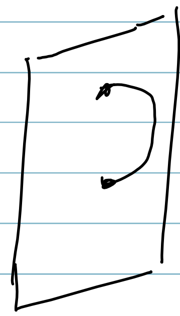
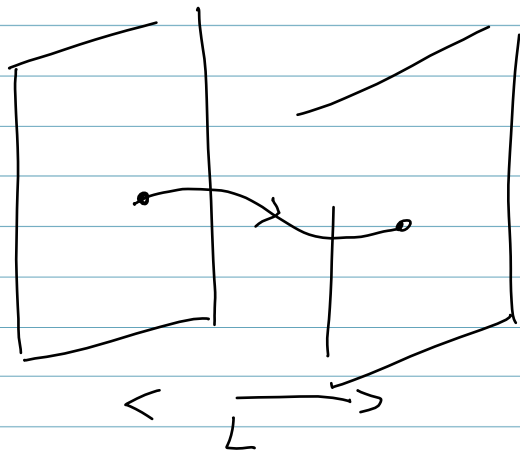
String 10 Dimensions IIA even p-branes
IIB odd p-branes

D branes



strings ending on
D branes
with Dirichlet B.C.



mass $= \frac{L}{\alpha}$

$L \to 0$, mass $\to 0$

massless gauge boson
+ superpartners

Type IIB $N \times D3$

low-energy limit
only massles modes

$g^2 = 4\pi g_s$

large N $U(N) \simeq SU(N)$

$\mathcal{N} = 4$

$SU(4)_R$

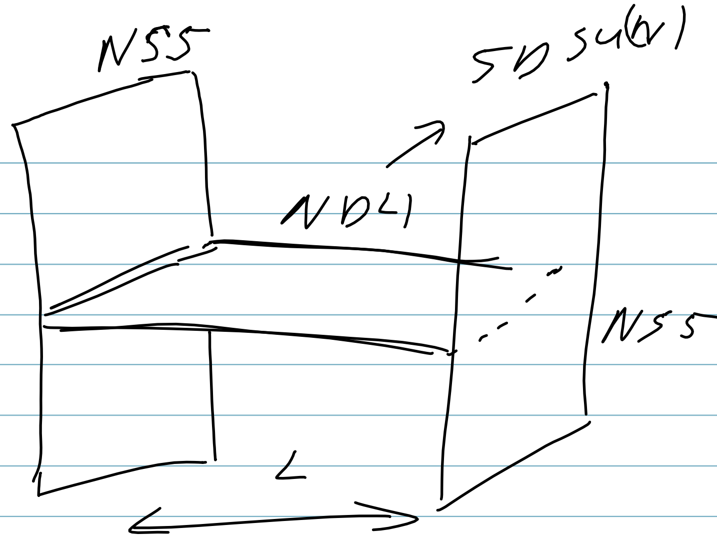
A^a_μ — 1

λ^a_α — $\square$

ϕ^{ab} — $\begin{array}{|c|}\hline \\ \hline \\ \hline\end{array}$

move branes apart in $6 \perp$ directions
$\langle \phi \rangle \to$ mass for gauge bosons
moduli space $\Longleftrightarrow$ geometry

Type IIA

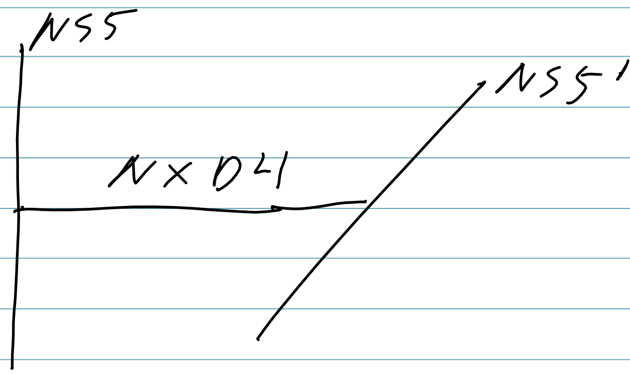


$\mathcal{N}=2$

	SU(2)
A^a_μ	1
λ^a_α	$\square$
ϕ^a	1

$$g_4^2 = \frac{g_5^2}{L}$$

move branes apart in 2 $\perp$ directions
2 real scalars
2 sets of // BPS states invariant under ½ SUSY



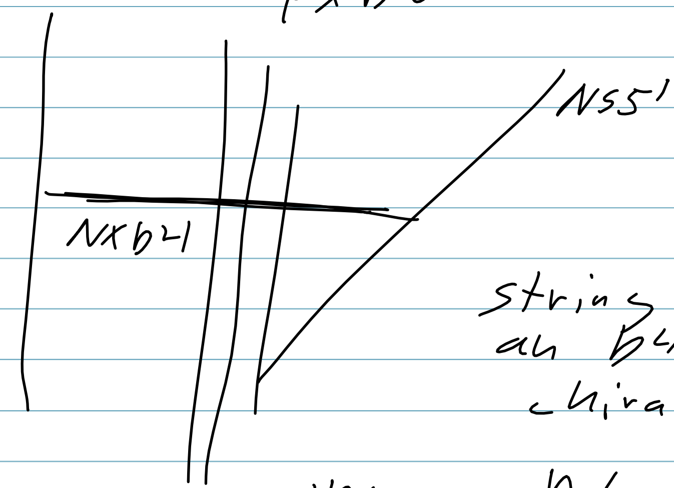
$\mathcal{N}=1$

A^a_μ

λ^a_α

branes are pinned, no scalars

F x D6 along 2D of NS5 ⊥ D4

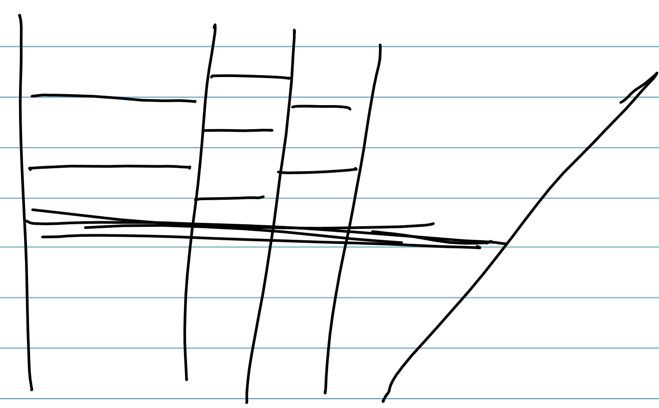


strings between D6 branes an D4

chiral multiplets $F(Q + \bar{Q})$

move D6 ⊥ ⇒ mass term $Q\bar{Q}$

$\langle Q \rangle$
$\langle \bar{Q} \rangle$

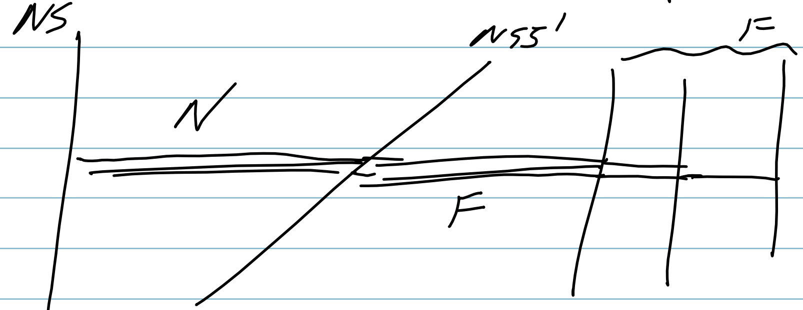


Higgs branch

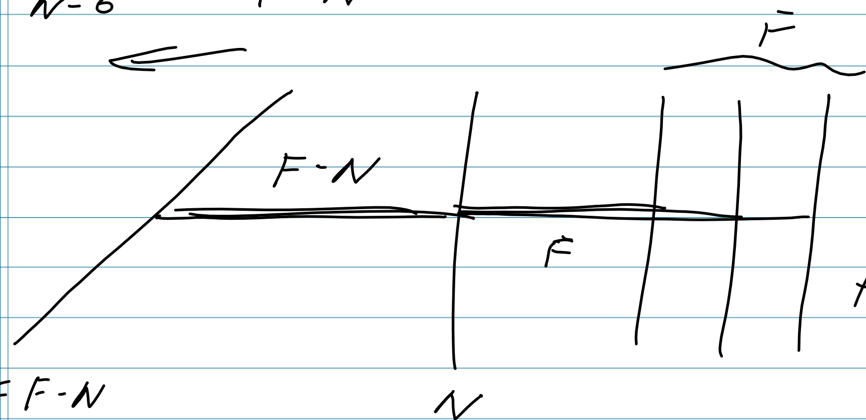
moduli space $= 2NF - N^2$

classical $U(N)$

move flavor branes



$R-L = N-0$ $\quad$ $F-N$



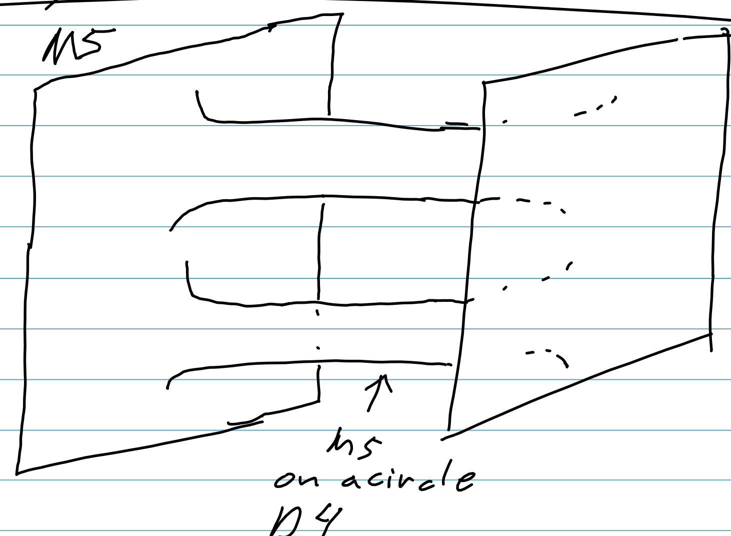
F^2 complex directions that don't Higgs dual quark mass

$R-L = F-N$

$SU(F-N)$ $\quad$ $q, \bar{q}, M$

$\mathcal{N}=1$ Seiberg duality $\quad$ $\langle M \rangle \Rightarrow$ mass for $q, \bar{q}$

M ~~T~~theory



IIA $\to$ M on a circle

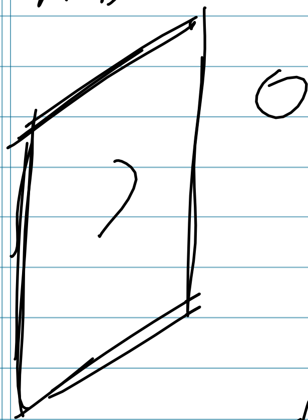
$g_s = (R_{10} M_{pl})^{3/2}$

$\mathcal{N}=2$ $SU(2)$

M theory curve 6D space $\to$ 4D spacetime ~~and~~ and 2D elliptic curve

M5 ~~theory~~ not 11, bending
move D4 branes, bending reproduces β function

N D3



$E << 1/l_s = \frac{1}{\sqrt{\alpha'}}$ massless modes

$S_{eff} = S_{bulk} + S_{brane} + S_{int}$

S_{bulk}: 10D S.G. + higher dim op

S_{brane}: $SU(N)$ $\mathcal{N}=4$ + higher dim op.

$$S_{bulk} = \frac{1}{2\kappa^2}\int \underbrace{\sqrt{g}}_{\text{dim 8}}\ \underbrace{R}_{\text{dim 2}} \sim \int (\partial h)^2 + \kappa(\partial \kappa)^2 h + \dots$$

$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$

$\to$ low energy $\int (\partial h)^2$

take low energy limit by $E =$ fixed g_s, N fixed
$\alpha' \to 0$

$S_{int} \to 0$
higher dim op $\to 0$

$S_{eff} \simeq \underbrace{S_{bulk}}_{\text{free}} + S_{SU(N)\ \mathcal{N}=4\ 4D}$

branes are a gravitational source

SG metric

$$ds^2 = f^{-1/2}\left(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right) + f^{1/2}\left(dr^2 + r^2 d\Omega_5\right)$$

$$f = 1 \cancel{+} \left(\frac{R}{r}\right)^4 , \quad R^4 = 4\pi g_s \alpha'^2 N$$

E measured at $r = \infty$
redshifted at $r \neq \infty$

$$E = \sqrt{g_{tt}(r)}\, E_r = f^{-1/4} E_r$$

$$E \to 0 \iff r \to 0$$

low energy limit   states near $r = 0$
long wavelength bulk waves
$\sim$ S.G. in bulk

$\lambda \gg R$ decouple

$r \ll R$ $f \simeq \left(\frac{R}{r}\right)^4$

$$ds \simeq \frac{r^2}{R^2}\left(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right) + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2$$

$AdS_5 \times S^5$

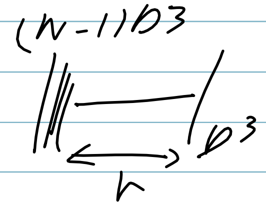
take limit carefully

$$E = f^{-1/4} E_r$$
$$\approx \frac{r}{R} E_r \approx \frac{r E_r}{(4\pi g_s \alpha'^2 N)^{1/4}}$$

keep E fixed

as $r \to 0$ keeping $u = \frac{r}{\alpha'}$ finite

$\alpha' \to 0$



$$m_W = \frac{r}{\alpha'} = u$$

$$\frac{ds^2}{\alpha'} = \left(\frac{u^2}{\sqrt{4\pi g_s N}} (-dt^2 + dx_i^2) + \sqrt{4\pi g_s N} \left(\frac{du^2}{u^2} + d\Omega_5^2 \right) \right)$$

near horizon limit $AdS_5 \times S^5$

S. gravity is a good approx. $g_s \ll 1$, $R \gg \sqrt{\alpha'}$

$$\frac{R^4}{\alpha'^2} = 4\pi g_s N \gg 1$$

$$g^2 N \gg 1$$
$$N \gg 1$$

gauge perturbation is good approx

$g^2 \ll 1$, $g^2 N \ll 1$

conjecture IIB string theory on $AdS_5 \times S^5$

$\equiv$ $\mathcal{N}=4$ $SU(N)$ on 3+1

S^5 : $R^2 = y_1^2 + y_2^2 + y_3^2 + y_4^2 + y_5^2 + y_6^2$

isometry $SO(6) \simeq SU(4)_R$

AdS_5 : neg. curvature, neg. cosmological const,

$R^2 : x_0^2 + x_5^2 - (x_1^2 + x_2^2 + x_3^2 + x_4^2)$

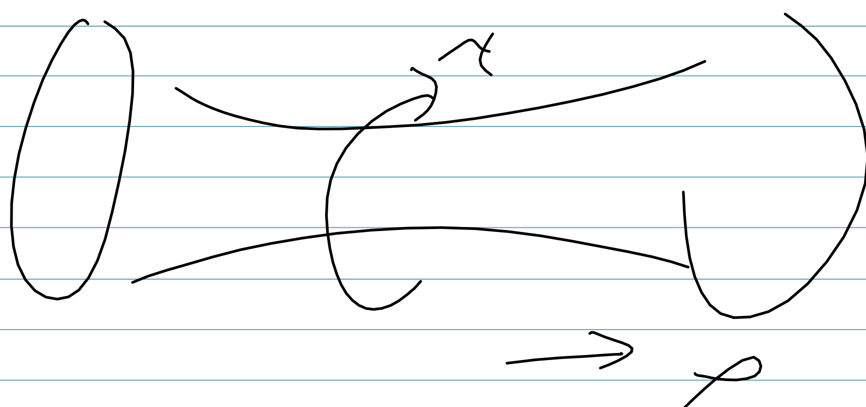
$$ds^2 = -dx_0^2 - dx_5^2 + \sum_i dx_i^2$$

$X_0 = R\cosh\rho\cos\tau \qquad X_5 = R\cosh\rho\sin\tau$

$X_i = R\sinh\rho\,\Omega_i \qquad i = 1 \ldots 4$

$\sum_i \Omega_i^2 = 1$

$$ds^2 = R^2(-\cosh^2\rho\, d\tau^2 + d\rho^2 + \sinh^2\rho\, d\Omega^2)$$



isometry $SO(4,2) \Longleftrightarrow$ conformal symmetry