Local Super-Poicaré Invariance
aka Super-Gravity

onshell:

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{e^a e^b}{m_{pe}} \eta_{ab}$$

vierbein or tetrad

On-Shell		helicity	d.o.f
	e^a_μ	± 2	2
massless $Q_\alpha \Downarrow$			
	$\psi_{\nu\alpha}$	$\pm 3/2$	2

$$e = |\det e^a_\mu|$$

$$S = \frac{M_{pl}^2}{2} \int d^4x \, R + \frac{1}{2} \int d^4x \, \epsilon^{\mu\nu\rho\sigma} \bar{\Psi}_\mu \gamma_5 \gamma_\nu D_{\rho\sigma}$$

field strength

corresponds to Ψ_α

To go off-shell we first consider
Local superconformal Poincare Invariance

Recall Brans-Dicke scale invariant
Gravity theory

$$\mathcal{L}_{BD} = \frac{e}{2}\sigma^2 R + \frac{e}{12}\partial^\mu\sigma\partial_\mu\sigma$$

$\sigma = M_{pl}$ spurion field
gives back Einstein gravity

superconformal gravity
"gauge fields

							dof
	e_μ^a :	16	$-\ 4$ gen, coor	$-\ 6$ local Lorentz	$-\ 1$ dilations	=	5
	$\psi_{\mu\alpha}$:	16	$-\ 4$ susy	$-\ 4$ conformal susy		=	8
$U(1)_R$	A_μ :	4	$-\ 1$ gauge			=	3
	b_μ :	4	$-\ 4$ conformal boosts			=	0

no aux. fields needed

to get supergravity we need a spurion

$\Sigma = (\sigma, \chi, F_\Sigma)$ chiral supermultiplet

scale transformation

$$x^\mu \to \frac{x^\mu}{\rho} \qquad \theta \to \frac{\theta}{\rho^{1/2}}$$

$$\sigma \to \rho\sigma \quad , \chi\ \rho^{3/2}\chi \quad , \quad F_\Sigma \to \rho^2 F_\Sigma$$

$\Sigma \to \rho\,\Sigma$ conformal weight $\underline{1}$

$$\mathcal{L}_{sc} = \frac{e\sigma^2}{2}R + e\int d^4\theta\ \Sigma^\dagger\Sigma + \mathcal{L}_{gravitino}$$

conformal weight 4

covarianiant derivatives in e^a_μ, $\psi_{\mu\alpha}$, A_μ, b_μ

$\sigma = M_{pl}$ breaks local superconformal Poincare

$\chi = 0$

$b_\mu = 0$ local super Poincare

$$\mathcal{L} = \frac{M_{pl}^2}{2}R + F_\Sigma^\dagger F_\Sigma - \frac{M_{pl}^2}{8}A_\mu^2 + \mathcal{L}_{gravitino}$$

off shell

e_μ^a : $16 - \underset{\text{gen coord.}}{4} - \underset{\text{local Lorentz}}{6} = 6$

$\psi_{\mu\alpha}$: $16 - \underset{\text{susy}}{4} = 12$

Auxillary

A_μ : 4

F_Σ : 2

Given a globally SUSY inv. Theory

$$\mathcal{L} = \int d^4\theta\, K(Q^\dagger, e^V Q) + \int d^2\theta\, W(Q) + \frac{i\tau}{16\pi} W^\alpha W_\alpha + h.c.$$

we can make a local superconformal Poincare invariant theory

$Q \to \Sigma Q$ ← conformal weight 0
$m \to \Sigma m$

$$\mathcal{L} = e\left\{ \int d^4\theta\, f(Q^\dagger, e^V Q)\, \Sigma^\dagger \Sigma + \int d^2\theta\, \Sigma^3 W(Q) + \frac{i\tau}{16} W^\alpha W_\alpha + h.c. \right.$$
$$\left. - \frac{1}{6} f(q^\dagger, q)\, \sigma^2 R \quad \cancel{+} \right\}$$

$$+ \mathcal{L}_{aux} + \mathcal{L}_{gravitino}$$

to make contact with $M_{pl} \to \infty$ limit

$$f = -3\, e^{-K/3M_{pl}^2}$$

Weyl transformation

$$e_\mu^a \to e^{\frac{K}{12 M_{pl}^2}} e_\mu^a$$

integrate out aux. fields

$$\mathcal{L}_{Boson} = e \Big\{ \frac{M_{pl}^2}{2} K + K^i_j (q^+, q) D^\mu q^{i+} D_\mu q_j - V(q^+, q) + \frac{i\tau}{16\pi} (F^2 + i F_{\mu\nu} \tilde{F}^{\mu\nu}) + h.c.$$

$$K^i_j (q^+, q) = \frac{\partial^2 K}{\partial q^{j+} \partial q_i}$$

$$V = e^{K/M_{pl}^2} \left((K^{-1})^j_i \left(W^i + W \frac{K^i}{M_{pl}^2} \right) \left(W_j^* + \frac{W^* K_j}{M_{pl}^2} \right) - \frac{3|W|^2}{M_{pl}^2} \right) + g \left(K^i T^a q_i \right)^2$$

$$F_i^* = e^{-K/2M_{pl}^2} (K^{-1})^j_i \left(W_j^* + \frac{W^* K_j}{M_{pl}^2} \right)$$

tune $V = 0$

$$\bar{\sigma}^\mu D_\mu \supset \sigma^\mu \frac{\bar{\psi}_\mu}{M_{pl}} \tilde{q}$$

$$K_j^i \, \theta \sigma^\mu \frac{\bar{\psi}_\mu}{M_{pl}} \theta \tilde{q}_i \, \theta^{\dagger 2} F^{j*}$$

$$m_{3/2}^2 = \frac{K_j^i \, F_i F^{*j}}{3 M_{pl}^2}$$

taking $K = Z \, Q^{i\dagger} Q_i$

$$M_{pl} \to \infty$$

recover global SUSY results