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$SO(N)$, $F = N-1$ flavors

dual:

	$SO(3)$	$SU(N-1)$	$U(1)_R$
q	$\square$	$\overline{\square}$	$\frac{N-2}{N-1}$
M	1	$\boxminus$	$\frac{2}{N-1}$

$$W = \frac{M_{ij}}{2\mu} \phi^i \phi^j - \frac{\det M}{64 \Lambda^{2N-5}}$$

integrate out one flavor

$$M = \begin{pmatrix} M' & P_j \\ P_i & X \end{pmatrix} \qquad \text{mass term } \tfrac{1}{2} m X$$

$$\frac{\partial W}{\partial P_j} = \frac{1}{2\mu} \phi^{N-1} \phi^j - \frac{\operatorname{cof}(P_j)}{64 \Lambda^{2N-5}} = 0$$

$$\frac{\partial W}{\partial X} = \frac{1}{2\mu} \phi^{N-1} \phi^{N-1} - \frac{\det M'}{64 \Lambda^{2N-5}} + \tfrac{1}{2} m = 0$$

$$\phi^{N-1} \phi^{N-1} = \frac{\mu \det M'}{32 \Lambda^{2N-5}} - \mu m$$

$$\frac{\partial W}{\partial \phi^{N-1}} = \frac{1}{2\mu} \left(P_j \phi^j + P_i \phi^i + X \phi^{N-1} \right) = 0$$

$$W_{eff} = \frac{1}{2\mu} f\left(\frac{\det M'}{\Lambda_{N,N-2}^{2N-4}} \right) M'_{ij} \phi^{+i} \phi^{-j}$$

normalize ϕ so that $f(0) = 1$

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$r = \text{rank}\, M'$

$(F-r)$ ϕ^+, ϕ^- are massless at $\det M' = 0$

consider a point M_0 where $\det M_0 = 0$

$$M' \to e^{2\pi i} M''$$

$$z = \det M' \to e^{2(F-r)2\pi i} z$$

$$\tilde{\tau} = -i \frac{1}{4\pi} \ln(\det M') + c$$

$$\tilde{\tau} \to \tilde{\tau} + 2(F-r)$$

in M, $M_0 = D_0^{-1} T^{2(F-r)} D_0$

in z, $M_0 = D_0^{-1} T^2 D_0$

$$[M_0, M_\infty] \neq 0$$

$$D \propto S^{2n+1}$$

$$\tilde{\tau} \to \infty \Rightarrow \tau \to 0$$

ϕ^+, ϕ^- are monopoles

SO(3) Diversion

Recall $SO(N),\ F=N-1 \xrightarrow{\text{dual}} SO(3);\ F=N-1$

$$W=0,\quad \tilde{W}=\frac{1}{2\mu}M\phi\phi-\frac{\det M}{64\Lambda_{N,N-1}^{2N-5}}$$

$N=3$ is a special case

$$SO(3),F \xrightarrow{\text{dual}} SO(F+1),F$$

QQ

$W=0$

$$\tilde{W}=\frac{1}{2\mu}M\phi\phi+\epsilon\alpha\det(\phi\phi)$$

$$\Lambda_{LF}^{6-F}\Lambda_{RF}^{6-F}=\Lambda_{3,F-1}^{6-2(F-1)}M_{FF}^{2}$$

$SO(3)$ has discrete Z_{4F} axial
$SO(F+1)$ has " Z_{2F} axial
$F>2$

$$Q\to e^{\frac{2\pi i}{4F}}Q \qquad Z_{4F}\begin{cases}\det\phi\phi\to e^{i\pi}\det\phi\phi\\ \theta\to\theta+\pi\end{cases}$$

$$\tilde{\tilde{W}}=\frac{1}{2\mu}MN+\frac{1}{2\tilde{\mu}}Ndd-\frac{\det M}{64\Lambda_{N,N-1}^{2N-5}}+\epsilon\alpha\det dd$$

$$\frac{\partial W}{\partial N}=0=\frac{1}{2\mu}M+\frac{dd}{2\tilde{\mu}}$$

$$\tilde{\mu}=-\mu \qquad M=dd \qquad \epsilon=1$$

$$\alpha=\frac{1}{64\Lambda_{N,N-1}^{2N-5}}$$

$$\Downarrow$$

$$\tilde{\tilde{W}}=0$$

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$\epsilon = -1$

	$SO(N)$	$SU(F=N-1)$	$U(1)_R$
d	$\square$	$\square$	#

$$W = -\frac{\det(dd)}{32\,\Lambda_{N,N-1}^{2N-5}}$$ dyonic dual

integrate out a flavor

$$W = -\frac{\det(dd)}{32\,\Lambda_{N,N-1}^{2N-5}} + \tfrac{1}{2} m\, d_F d_F$$

$d = (d_i, d_F) \quad i = 1, \ldots N-2$

$$\frac{\partial W}{\partial M_{iF}} = -\frac{d_i d_j \,\mathrm{cof}(d_F d_i)}{32\,\Lambda_{N,N-1}^{2N-5}} + d_F d_i\left(m - \frac{2\det(d_i d_j)}{32\,\Lambda^{2N-5}}\right)$$

$d_i d_F = 0, \quad \det(d_i d_j) \neq 0 \quad SO(N) \to SO(2) \sim U(1)$

$$W = \tfrac{1}{2} m \left(1 - \frac{\det(d_i d_j)}{16\, m\, \Lambda^{2N-5}}\right) d_F^+ d_F^-$$

$$= \tfrac{1}{2} m \left(1 - \frac{\det(d_i d_j)}{16\,\Lambda_{N,N-2}^{2N-4}}\right) d_F^+ d_F^-$$

near $\det(d_i d_j) \simeq 16\,\Lambda_{N,N-2}^{2N-4}$

$d_F^+ d_F^-$ are light

duals of monopoles with $\theta \to \theta + \pi$
electric and magnetic charge.
dyons!

$\phi^{\pm i} Q_i \sim d_F^{\pm}$

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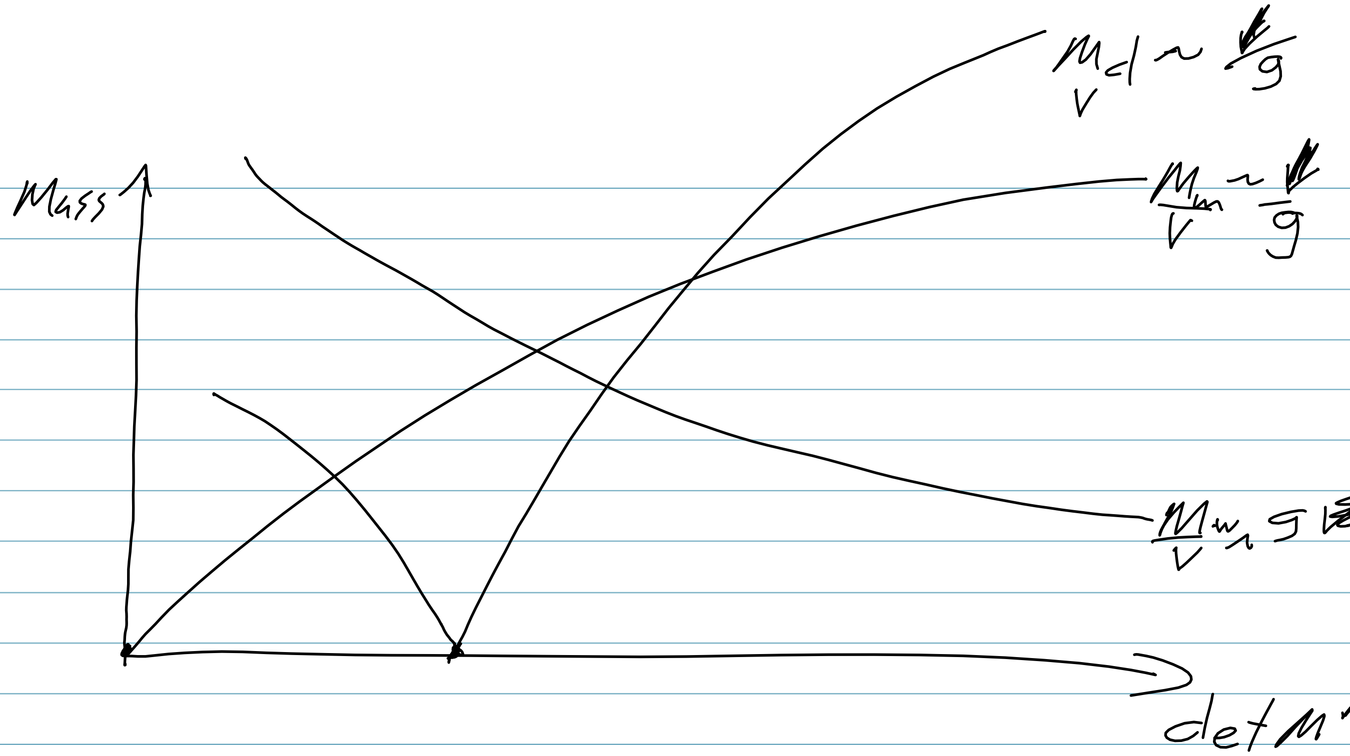

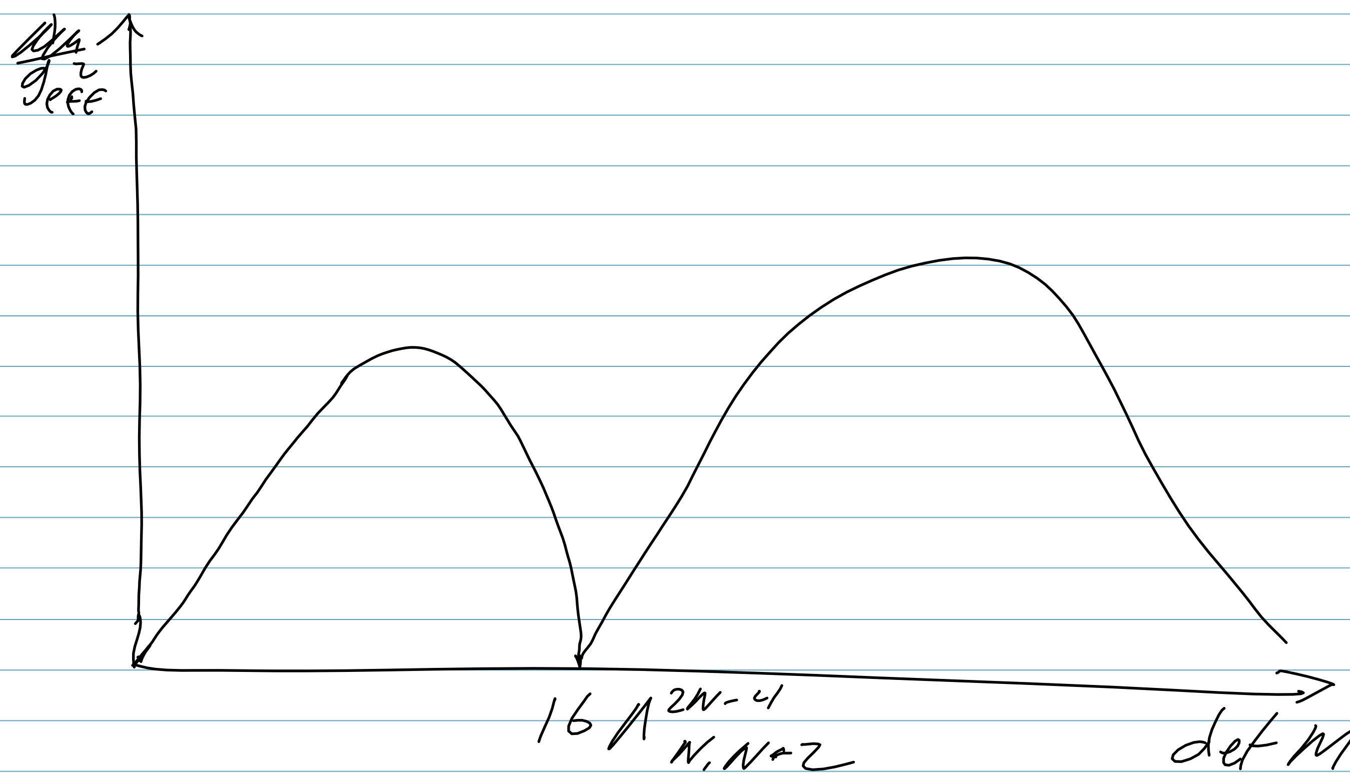

$$\Lambda^{2N-4}_{N,N-2} = m\,\Lambda^{2N-5}_{N,N-1}$$

$\to 0$ ~~as~~ $m \to 0$

recover SO(3) dual IRFP

Non-Abelian Coulomb phase

Web of dualities

electric

SO(N), N-1 □

magnetic

dyonic

SO(3), $\overline{N-1}$ □ + M

magnetic

magnetic

dyonic

SO(N) (N-1) □

$\Theta \to \Theta + \pi$

add a mass term $m \Phi_{N-1} \Phi_{N-1}$

SO(N) N-2 □

SO(2) $\overline{N-2}$ □

+ m

SO(N) (N-2) □

$\Theta \to \Theta + \pi$

add a mass $m_2 \Phi_{N-2} \Phi_{N-2}$

$SO(N)$ $(N-3)\,\square$

$\langle \Phi_{N-2} \Phi_{N-2} \rangle = -\mu m$

dual Meissner effect

confinement

remaining monopoles

identified $h^i = W_\alpha W^\alpha Q^{N-4}$

$$W = \frac{1}{2} m_1 \left(1 - \frac{\det M}{16\Lambda^{2N-4}}\right) d_F^+ \ldots$$

$$+ \frac{1}{2} m_2 M_{N-2\, N-2}$$

$$\frac{\partial W}{\partial M_{N-2, N-2}} = \frac{1}{2} m_2 + \frac{1}{2} m_1$$

$$\hookrightarrow \times \left(-\frac{\det M'}{16\Lambda^{2N}}\right) d_F^+ d_F \ldots$$

$$\langle d_F^+ d^- \rangle = \frac{m_2}{m_1} \frac{\left(16\Lambda_{N,N-2}^{2N-4}\right)}{\det M'}$$

oblique confinement

$$W_{eff} = \frac{m_2\, 8\, \Lambda_{N,N-2}^{2N-4}}{\det M'}$$

runaway vacuum

same physics $N=2$ Seiberg Witten

consistency checks between them

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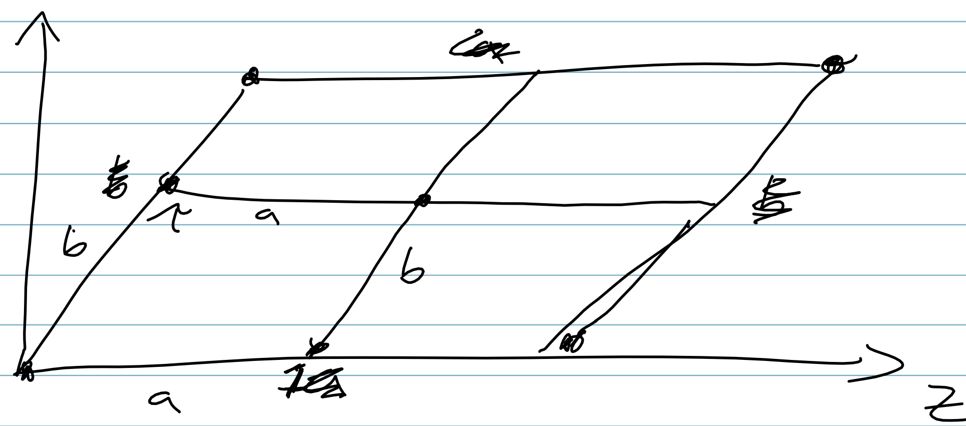
Torus

τ transforms under $SL(2,Z)$
τ is a section of an $SL(2,Z)$ bundle
↓
modular symm. grp. of a torus
τ <—> modular param of a torus



new basis vectors $\alpha\tau+\beta$, $\gamma\tau+\delta$
if $\alpha,\beta,\gamma,\delta \in Z$ and $\alpha\delta-\beta\gamma=1$
new lattice contained in old, invertable

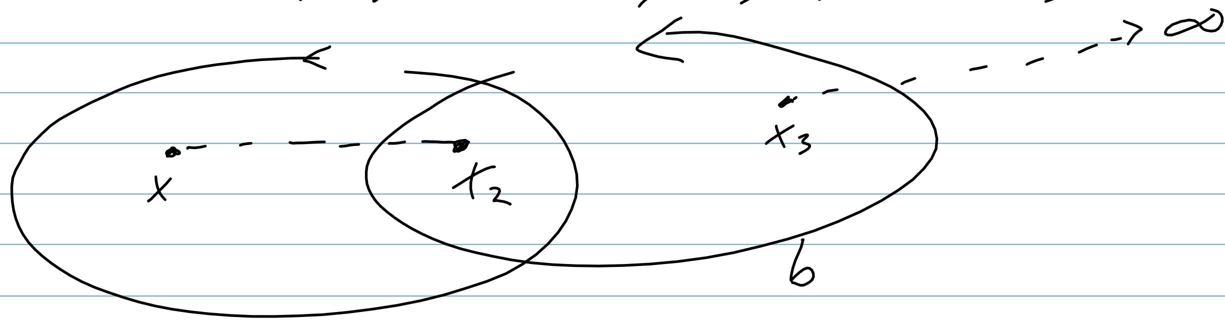
rescaling $\tau' = \dfrac{\alpha\tau+\beta}{\gamma\tau+\delta}$

$SL(2,Z)$ of torus <—> $SL(2,Z)$ of $U(1)$ gauge theory

Elliptic Curve

$y^2 = x^3 + Ax^2 + Bx + C = (x-x_1)(x-x_2)(x-x_3)$

two sheets meet along branch cuts



include $\overset{a}{\infty}$ cut planes ~ topologically a torus

~ two spheres connected by two tubes

~ torus, a, b are cycles

periods $\omega_1 = \int_a \frac{dx}{y}$ $\qquad \omega_2 = \int_b \frac{dx}{y}$

$\tau(A,B,C) = \frac{\omega_2}{\omega_1}$

singular when a cycle shrinks to zero

i.e. two roots meets

or one roots $\to \infty$

$\Rightarrow$ branch disappears

$\Delta = \prod_{i<j} (x_i - x_j)^2 = 4A^3C - B^2A^2 - 18ABC + 4B^3 + 27C^2 = 0$

A, B, C simple, single valued

use to calculate τ

$SO(N)$ $F = N-2$

singular $z = \det M$

$\cancel{z} = 0$ $z = 16\Lambda^{2N-4} = 16\Lambda^6$

at these points charged massless particles drives dual photon coupling to zero dual holomorphic coupling is singular

$$y^2 = x(x - 16\Lambda^6)(x - z)$$

weak coupling $\Lambda \to 0$ $y^2 = x^2(x-z)$

singular for all z, coupling always zero

A, B, C ~~holomorp~~ holomorphic in Λ, z

	$U(1)_R$	$U(1)_A$
$\det M$	0	$2F$
Λ^b	0	$2F$
x	0	$2F$
y	0	$3F$

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Consistency Check: Monodromy
near singularity z_0, $z = z_0 + \epsilon$
two roots approach $x_0 \pm a\epsilon^{n/2} \leftrightarrow \Delta \sim \epsilon^n$

$$y^2 = (x - x_1)(x - x_0 - a\epsilon^{n/2})(x - x_0 + a\epsilon^{n/2})$$

shift x by x_0 rescale x, y

$$y^2 = (x - \tilde{x})(x^2 - \epsilon^n)$$

$$\omega_1 = \int_{-\epsilon^{n/2}}^{\epsilon^{n/2}} \frac{dx}{y} \simeq \int_{-\epsilon^{n/2}}^{\epsilon^{n/2}} \frac{dx}{i\sqrt{\tilde{x}}\sqrt{x^2 - \epsilon^n}} = \frac{-\pi}{\sqrt{\tilde{x}}}$$

$$\omega_2 = \int_{\epsilon^{n/2}}^{\tilde{x}} \frac{dx}{y} \simeq \int_{\epsilon^{n/2}}^{\tilde{x}} \frac{dx}{\sqrt{(x - \tilde{x})(x^2 - \epsilon^n)}} = \frac{i}{\sqrt{\tilde{x}}} \ln \epsilon^{n/2}$$

$$\tau = \frac{\omega_2}{\omega_1} = \frac{1}{2\pi i} \ln \epsilon^n$$

monodromy at z_0 ~~is~~ T^n

$SO(N)$ $F = N - 2$ $z = \det M$
near zero $\Delta \sim z^2 \leftrightarrow M_0 \sim T^2$
up to duality $D^{-1} T^2 D$

monodromy on moduli space with rank $M = r$

$$M_0 \sim T^{2(F - r)}$$

(encircle singular point for each zero eigenvalue
$\ln \det = \operatorname{Tr} \ln$

near $z = z_d = \Lambda^b$; $\Delta \sim (z - z_d)^2$ $M_{z_d} \sim T^2$

for $z \approx \infty$ roots $\sim \left(0, \frac{16\Lambda^b}{z}, z \right)$

two sets of singular points approach each other simultaneously, rescale to get one set

$x \to x'(8\Lambda^b - z)$ $\quad y \to y'(8\Lambda^b - z)^{3/2}$

$$y'^2 = x'^3 + x'^2 + \frac{16\Lambda^{2b}}{(8\Lambda^b - z)^2} x'$$

near $z = \infty$; $\Delta \sim z^{-2}$ $\quad M_\infty \sim T^{-2}$

on moduli space $M_{\tilde{\infty}}$ T^{-2F}

in original x-y, change of varible gives a factor $\frac{1}{\sqrt{z}}$ in $\frac{dx}{y}$

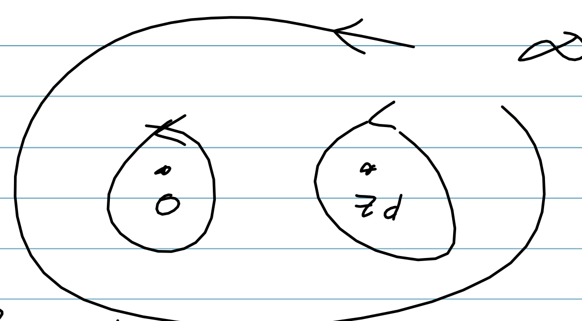
extra sign flip in τ

$$M_\infty = -T^{-2}$$

Assuming $M_0 = S^{-1} T^2 S$

simplest solution of

$$M_0 M_{zd} = M_\infty$$



is $M_{zd} = (ST^{-1})^{-1} T^2 ST^{-1}$