(1)

Phys 246B

Superpotential is a holomorphic function

eg: $W = \frac{m}{2}\phi^2 + \frac{\lambda}{3}\phi^3$

background fields

holomorphic in couplings

integrate out a mode above μ
(Wilsonian effective action)

W_{eff} must also holomorphic

ex $W \supset \lambda \mathcal{O}_{-1}$

assign charge +1 charge −1

calculate contribution $\mathcal{O}_{-10}$

$W_{eff} \supset \lambda^{10}\mathcal{O}_{-10} + \lambda^{11}\lambda^{\dagger}\mathcal{O}_{-10} + \lambda^{10}e^{-\frac{1}{\lambda\lambda^{\dagger}}}\mathcal{O}_{-10}, \ldots$

only this can appear

if weak coupling limit ($\lambda \to 0$) exist

no negative powers of λ

②

defined at some scale Λ

ex: $W = \frac{1}{2} m \phi^2 + \frac{1}{3} \lambda \phi^3$

$[R, Q_\alpha] = -Q_\alpha \qquad [R, Q^\dagger_{\dot\alpha}] = +Q^\dagger_{\dot\alpha}$

$\Phi = \phi - \theta\psi + \ldots \qquad R(\psi) = R(\phi) - 1$

$W = \phi^3$

$\mathcal{L} \supset \phi\psi\psi$

$R(\mathcal{L}) = 3R(\phi) - 2 = 0$

$R(\phi) = 2/3 \qquad R(W) = 2$

$R(\theta) = +1, \quad \mathcal{L}_{int} = \int d^2\theta\, W \qquad R(W) = 2$

$Q(W) = 0$

ex:

	$U(1)$	$U(1)_R$
ϕ	$+1$	$+1$
m	-2	0
λ	-3	-1

integrate out modes between m, and Λ

$$W_{eff} = f(\phi, m, \lambda) = m\phi^2 h\left(\frac{\lambda\phi}{m}\right)$$

$$= \sum_n a_n \lambda^n m^{1-n} \phi^{n+2}$$

$\lambda \to 0$ limit is free $\to \quad n \geq 0$

$m \to 0 \qquad\qquad n \leq 1$

$$W_{eff} = \frac{1}{2} m \phi^2 + \frac{\lambda}{3} \phi^3$$

superpotential couplings are not renormalized

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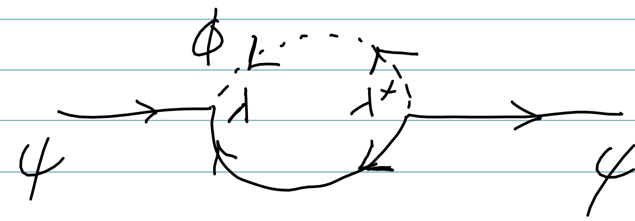
Wavefunction Renormalization

$$\mathcal{L} = Z[\partial_\mu \phi^* \partial_\mu \phi + i \psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi]$$

$$Z(m, \lambda, m^\dagger, \lambda^\dagger, \mu, \Lambda)$$



$\Lambda > \mu > m$ $\quad Z = 1 + c\lambda\lambda^\dagger \ln\left(\frac{\Lambda^2}{\mu^2}\right)$

$\Lambda > m > \mu$ $\quad Z = 1 + c\lambda\lambda^\dagger \ln\left(\frac{\Lambda}{m m^\dagger}\right)$

running couplings $\quad \frac{m}{Z}, \quad \frac{\lambda}{Z^{3/2}}$

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$$W = \frac{1}{2} m \phi_H^2 + \frac{\lambda}{2} \phi_H \phi^2$$

	$U(1)_A$	$U(1)_B$	$U(1)_R$
ϕ_H	1	0	1
ϕ	0	1	$1/2$
m	-2	0	0
λ	-1	-2	0

$\mu < m$ integrate out ϕ_H

$$\phi^j m^k \lambda^p$$

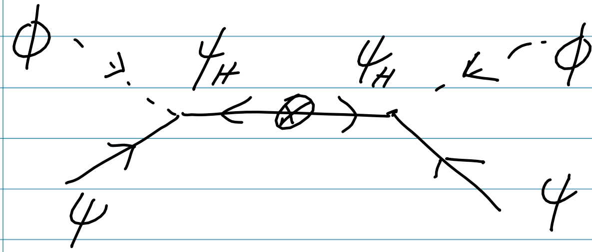
$U(1)_A$ $\quad -2k - p = 0$

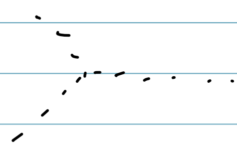
$U(1)_B$ $\quad j - 2p = 0$

$U(1)_R$ $\quad \frac{1}{2} j = 2$

$$j = 4$$
$$p = 2$$
$$k = -1$$

$$W_{eff} = \frac{-\lambda^2 \phi^4}{8m}$$





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$$\frac{\partial W}{\partial \phi_H} = m\phi_H + \frac{\lambda}{2}\phi^2 = 0$$

$$\phi_H = -\frac{\lambda\phi^2}{2m}$$

$$W_{eff} = \frac{1}{2}m\left(\frac{\lambda^2\phi^4}{4m^2}\right) + \frac{\lambda}{2}\left(\frac{-\lambda\phi^2}{2m}\right)\phi^2$$

$$= \frac{\lambda^2\phi^4}{8m} - \frac{\lambda^2\phi^4}{4m} = -\frac{\lambda^2\phi^4}{8m}$$

$$Ex = W = \frac{1}{2}m\phi_H^2 + \frac{\lambda}{2}\phi_H\phi^2 + \frac{y}{6}\phi_H^3$$

$$\frac{\partial W}{\partial \phi_H} = m\phi_H + \frac{\lambda}{2}\phi^2 + \frac{y}{2}\phi_H^2 = 0$$

$$\phi_H = \frac{-m}{y}\left(1 \pm \sqrt{1 - \frac{\lambda y\phi^2}{m^2}}\right)$$

$$W_{eff} = \frac{m}{6}\left(\frac{m^2}{y^2} \mp \frac{2m}{y}\sqrt{\frac{m^2}{y^2} - \frac{\lambda\phi^2}{y}} + \frac{m^2}{y^2} - \frac{\lambda\phi^2}{y}\right)$$
$$+ \frac{\lambda\phi^2}{3}\left(-\frac{m}{y} \pm \sqrt{\frac{m^2}{y^2} - \frac{\lambda\phi^2}{y}}\right)$$

$$W_{eff} = \frac{m^3}{3y^2}\left(1 - \frac{3}{2}\frac{\lambda y\phi^2}{m^2} \mp \left(1 - \frac{\lambda y\phi^2}{m^2}\right)\sqrt{1 - \frac{\lambda y\phi^2}{m^2}}\right)$$

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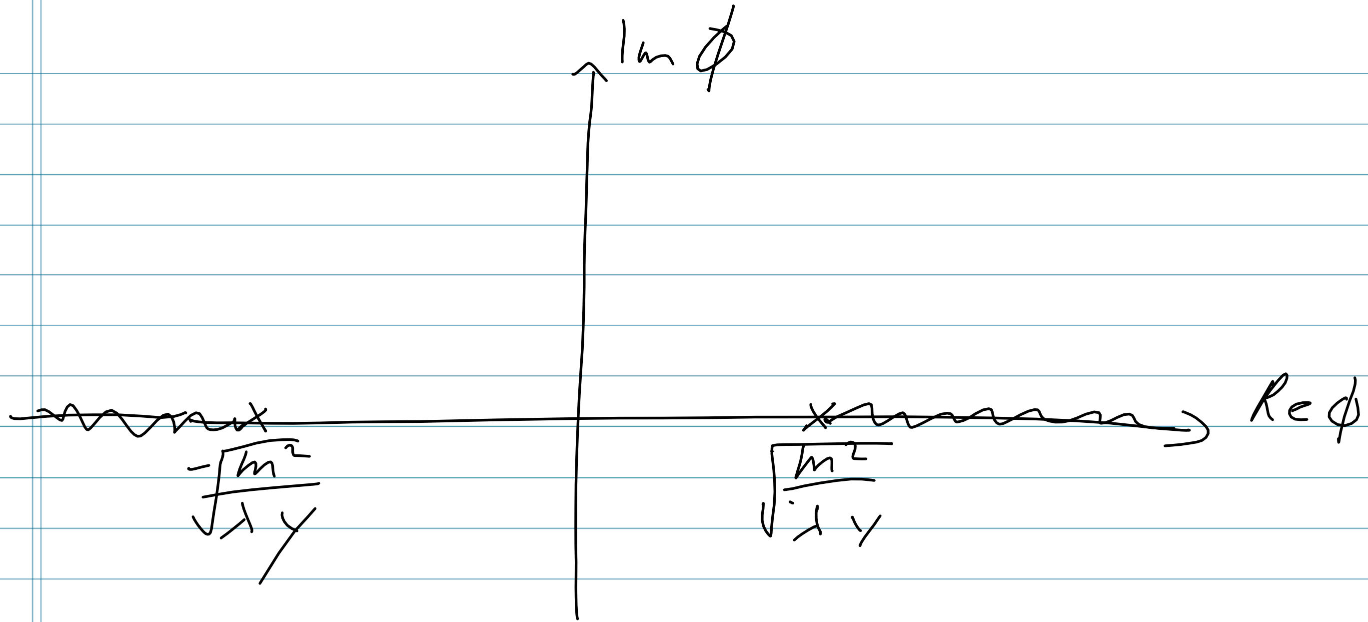

$$\frac{\partial^2 W}{\partial \phi_H^2} = m + y\phi_H$$

mass vanishes at $\phi_H = -\frac{m}{y}$

$$\frac{\partial W}{\partial \phi_H} = m\phi_H + \frac{\lambda}{2}\phi^2 + \frac{1}{2}y\phi_H^2 = 0$$

$$\left.\frac{\partial W}{\partial \phi_H}\right|_{\phi_H = -\frac{m}{y}} \left(= -\frac{m^2}{y} + \frac{\lambda}{2}\phi^2 + \frac{1}{2}y\frac{m^2}{y^2}\right) = 0$$

$$\frac{\lambda}{2}\phi^2 = \frac{m^2}{2y}$$

$$\phi^2 = \frac{m^2}{\lambda y}$$

$$\phi = \pm\frac{m}{\sqrt{\lambda y}}$$

singularity in effective superpotential indicate points where integrated out ~~points~~ fields become massless

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Holomorphic Gauge Coupling

$$y^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}$$

$$W^a_\alpha = -i\lambda^a_\alpha(y) + \theta_\alpha D^a(y) - (\sigma^{\mu\nu}\theta)_\alpha F^a_{\mu\nu}(y) - (\theta\theta)(\sigma^\mu D_\mu \lambda^{a\dagger}(y))_\alpha$$

$$\tau = \frac{\theta_{YM}}{2\pi} + \frac{4\pi i}{g^2} \qquad \sigma^{\mu\nu} = \frac{i}{4}(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu)$$

$$R(\lambda^a) = 1 \quad R(D^a) = 0 \quad R(F^a_{\mu\nu}) = 0$$

$$\mathcal{L} = \frac{-i}{16\pi}\int d^4x \int d^2\theta \, \tau W^{a\alpha} W^a_\alpha + h.c.$$

$$= \frac{-1}{4g^2} F^{a\mu\nu}F^a_{\mu\nu} - \frac{\theta_{YM}}{32\pi^2} F^{a\mu\nu}\tilde{F}^a_{\mu\nu} + \frac{1}{2g^2} D^a D^a + \frac{i}{g^2}\lambda^{a\dagger}\bar{\sigma}^\mu D_\mu \lambda^a$$

where $\tilde{F}^{a\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F^a_{\alpha\beta}$

gauge multiplet not canonically normalized

$$(A^a_\mu, \lambda^a_\alpha, D^a) \to g\,(A^a_\mu, \lambda^a_\alpha, D^a)$$

here g only appears in τ

holomorphic parameter

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3N - F

$$\mu \frac{dg}{d\mu} = \frac{-bg^3}{16\pi^2} \qquad \frac{dg}{g^3} = \frac{-b\,d\mu}{16\pi^2 \mu}$$

$$\frac{-1}{2g^2} = \frac{-b}{16\pi^2}\ln(\mu) + c$$

$$\frac{1}{g^2} = \frac{-b}{8\pi^2}\ln\left(\frac{\mu}{\Lambda}\right.$$

$$\frac{1}{g_0^2} = \frac{1}{g^2(\mu_0)} = \frac{b}{8\pi^2}\ln\left(\frac{|\Lambda|}{\mu_0}\right)$$

$$|\Lambda| = \mu_0 e^{-\frac{8\pi^2}{bg_0^2}}$$

$$\tau_0 = \frac{\theta_{ym}}{2\pi} + \frac{4\pi i}{g_0^2} = \frac{1}{2\pi i}\ln\left(\left(\frac{|\Lambda|}{\mu_0}\right)^b e^{i\theta_{ym}}\right)$$

$$\Lambda = |\Lambda| e^{\frac{i\theta_{ym}}{b}}$$

$$= \mu\, e^{\frac{2\pi i \tau}{b}}$$

$$\tau = \frac{b}{2\pi i}\ln\left(\frac{\Lambda}{\mu}\right)$$

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integrate down to μ

$$W_{eff} = \frac{\tau}{16\pi i}(\Lambda;\mu)\, W^{a\alpha} W^a_\alpha$$

holomorphic in Λ

periodicity: $\theta_{ym} \to \theta_{ym} + 2\pi$, $\tau \to \tau + 1$

or $\Lambda \to e^{\frac{2\pi i}{b}}\Lambda$

$$\tau = \frac{b}{2\pi i}\ln\left(\frac{\Lambda}{\mu}\right) + f(\Lambda^b;\mu)$$

arbitrary holomorphic function

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$\Lambda \to 0$ is weak coupling

$$\tau = \frac{b}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right) + \sum_{n=1}^{\infty} \left(\frac{\Lambda^b}{\mu^b}\right)^n a_n$$

one-loop

non-perturbative
n-instanton

not-much renormalization theorem

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Instantons

$$\frac{\theta_{ym}}{32\pi^2} F^{a\mu\nu} \tilde{F}^a_{\mu\nu} = \frac{\theta_{ym}}{8\pi^2} \epsilon^{\mu\nu\rho\sigma} \partial_\mu \mathrm{Tr}\left(A_\nu \partial_\rho A_\sigma + \frac{2}{3} A_\nu A_\rho A_\sigma \right)$$

total derivative

no effect in perturbation theory

semiclassical configurations: $A^a_\mu = \dfrac{h^a_{\mu\nu}(x-x_0)^\nu}{(x-x_0)^2+\rho^2}$

$$\frac{\theta_{ym}}{32\pi^2}\int d^4x\, F^{a\mu\nu}\tilde{F}^a_{\mu\nu} = n\,\theta_{ym}$$

n ↑ winding #

$$|\theta_{ym}\rangle = \sum_n e^{i\theta_{ym} n}|n\rangle$$

$$PI = \int \mathcal{D}A\,\mathcal{D}\lambda\,\mathcal{D}D\, e^{iS}$$

$\theta_{ym} \to \theta_{ym} + 2\pi$ is a symmetry

$$0 \le \int d^4x\, \mathrm{Tr}(F \pm \tilde{F})^2 = \int d^4x\, 2\mathrm{Tr}F^2 \pm 2\mathrm{Tr}F\tilde{F}$$

$$\int d^4x\, \mathrm{Tr}F^2 \ge \left|\int d^4x\, \mathrm{Tr}\, F\tilde{F}\right| = 16\pi^2|n|$$

$$e^{-S_{inst}} = e^{-\frac{8\pi^2}{g^2}}$$

$$\int d^4x_0 \int \frac{d\rho}{\rho^5}\, e^{-\frac{8\pi^2}{g(1/\rho)}} = \int d^4x_0 \int_0^\infty \frac{d\rho}{\rho^5}(\rho\Lambda)^{3N}$$

$N \ge 2$ IR divergent