

$M=0$ Soft SUSY Breaking

SUSY cancels quadr. divergences

for scalar masses
put in supermultiplets with fermions

fermions have chiral symmetries

fermion masses are at most log. div.

fermion mass vanishes as bare mass $\rightarrow 0$

$$m_f = m_0 + \frac{\alpha}{16\pi^2} m_0 \ln\left(\frac{\Lambda}{m_0}\right) + \dots$$

SUSY must be broken
in the real world

10⁻⁵ of ways to do this

$$e.g. \quad W = F^a \Phi_a$$

$$V = W_a^* W^a = F_a^* F_a \neq 0$$

dimensionless coupling need to
be related to cancel divergences

$$\delta m^2 \propto (\lambda - |\gamma|^2) \Lambda^2$$

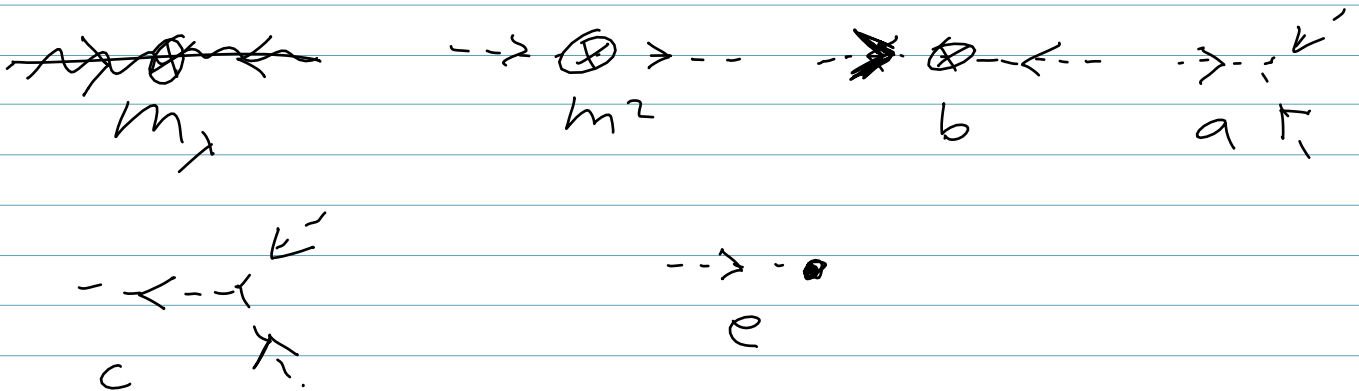
want effective theory spontaneously broken SUSY

with only soft breaking ($d_{im} < 4$) terms

$$\begin{aligned}
 \mathcal{L}_{\text{soft}} = & -\frac{1}{2} (M_{\lambda} \lambda^a \lambda^a + \text{h.c.}) - (m^2)_j^i \phi^* \phi_j \\
 & - \left(\frac{1}{2} b^{ij} \phi_i \phi_j + \frac{1}{6} a^{ijk} \phi_i \phi_j \phi_k + \text{h.c.} \right) \\
 & - \frac{1}{2} c_i^{ijk} \phi^* \phi_j \phi_k + \text{h.c.} \\
 & - e^i \phi_i + \text{h.c.}
 \end{aligned}$$

\uparrow
 only exist if ϕ_i is a gauge singlet

c_i^{jk} can introduce Λ^2 divergence if \exists a gauge singlet field



spontaneous ~~SUSY~~ $\xRightarrow{\text{feed down}}$ superpartners
 consistent SUSY theory

parameter \rightarrow $\langle \text{spurion field} \rangle$

$$\int d^4\theta \mathcal{Z} \Phi^\dagger \Phi$$

$\mathcal{Z} = 1 + \beta \theta^2 + \beta^* \bar{\theta}^2 + \gamma \theta^2 \bar{\theta}^2$
 only scalar component have VEVs
 if we preserve Lorentz inv.

$$= \mathcal{L}_{\text{free}} \psi + \beta F^* \phi + \beta^* \phi^* F + \gamma \phi^* \phi \quad \text{integrate out}$$

$$- \partial^\mu \phi^\dagger \partial_\mu \phi + i \psi^\dagger \not{\partial} \psi + (\gamma - |\beta|^2) \phi^* \phi$$

soft breaking $m^2 = |\beta|^2 - \gamma$

can do the same for superpot. couplings

Yukawa ϕ^3	mass ϕ^2	$\simeq W_\alpha W_\alpha$
\downarrow	\downarrow	\downarrow
a	b	m_\pm

$$\int d^4\theta C_i^{jk} \Phi_i^\dagger \Phi_j \Phi_k$$

$\hookrightarrow \theta^2 \bar{\theta}^2$ component nonvanishing

Susy QCD
 chiral superfields $Q_i = (\phi_i, \psi_i, F_i)$, $Q_i^\dagger = (\phi_i^\dagger, \psi_i^\dagger, F_i^\dagger)$
 $\bar{Q}_i = (\bar{\phi}_i, \bar{\psi}_i, \bar{F}_i)$

	$SU(N)$	$SU(F)$	$SU(F)$	$U(1)$	$U(1)_R$
Q	\square	\square	1	1	$\frac{F-N}{F}$
Q^\dagger	$\bar{\square}$	1	$\bar{\square}$	-1	$\frac{F-N}{F}$

$W=0$

$$[R, Q_\alpha] = -Q_\alpha \Rightarrow R_{Q_\alpha} = R_\phi - 1$$

$$R_\theta = +\theta$$

$$R_{\lambda^a} = +\lambda^a$$

$$R_W = +2W$$

Group Theory

$$\left(T_R^a \right)_l^m \left(T_R^a \right)_n^l = C_2(R) \delta_n^m$$

$$\left(T_R^a \right)_n^m \left(T_R^a \right)_m^l = T(R) \delta_n^l$$

Bird Track Notation

$$\left(T_R^a \right)_n^m = \begin{array}{c} a \\ \text{wavy line} \\ \leftarrow \quad \leftarrow \\ n \quad m \end{array}$$

$$C_2(R) \int_h^m = \text{diagram: a horizontal line with arrows pointing left from h to m, with a wavy line above it}$$

$$\int^{ab} T(R) = a \text{ --- } \text{diagram: a circle with arrows pointing clockwise} \text{ --- } b$$

$$\text{diagram: a circle with a wavy line on top and arrows pointing clockwise} = \text{diagram: a circle with a wavy line on the bottom and arrows pointing clockwise}$$

$$d(R) C_2(R) = d(Ad) T(R)$$

$$T(\square) = \frac{1}{2} \quad T(Ad) = N$$

$$C_2(\square) = \frac{N^2 - 1}{2N} \quad C_2(Ad) = N$$

for defining N dim. rep.

$$(T^a)_p^l (T^a)_h^m = \frac{1}{2} (\int_h^l \int_p^m - \frac{1}{N} \int_p^l \int_h^m)$$

$$\text{diagram: a vertical line with a wavy line on the left and arrows pointing left} = \frac{1}{2} \left(\text{diagram: a square with arrows pointing clockwise} - \frac{1}{N} \text{diagram: a horizontal line with arrows pointing right} \right)$$

$$\begin{aligned} C_2(\square) \int_h^m &= \text{diagram: a horizontal line with arrows pointing left from h to m, with a wavy line above it} = \frac{1}{2} \left(\text{diagram: a circle with a wavy line on top and arrows pointing clockwise} - \frac{1}{N} \text{diagram: a horizontal line with arrows pointing left} \right) \\ &= \frac{1}{2} \left(N - \frac{1}{N} \right) \int_h^m \\ &= \frac{N^2 - 1}{2N} \int_h^m \end{aligned}$$

For the R charge to be a quantum symmetry the anomaly must vanish

$$0 = \text{triangle diagram} + \text{triangle diagram with } Q, \bar{Q}$$

$$0 = 1 \cdot T(\text{Ad}) + (R-1) T(\square) 2F$$

$$0 = N + (R-1) \frac{2}{3} 2F$$

$$(R-1) = \frac{-N}{F} \Rightarrow R = \frac{F-N}{F}$$

$$\text{gluon vertex} = \gamma = \sqrt{2} g = \sqrt{2} \left\{ \text{gluon vertex} \right\}$$

$$\begin{aligned} \beta_g &= \mu \frac{dg}{d\mu} = \frac{-g^3}{16\pi^2} \left(\frac{11}{3} T(\text{Ad}) - \frac{2}{3} T(F) - \frac{1}{3} T(\square) \right) \\ &= \frac{-g^3}{16\pi^2} \left(\frac{11}{3} T(\text{Ad}) - \frac{2}{3} T(\square) 2F - \frac{2}{3} T(\text{Ad}) \right) \\ &= \frac{-g^3}{16\pi^2} \left(\frac{9}{3} N - \frac{3}{3} F \right) \\ &= \frac{-g^3}{16\pi^2} (3N - F) \end{aligned}$$

check $V = \sqrt{2} g$ independent of u
 Mahachek & Vaughlin

$$(4\pi)^2 \beta_Y = \frac{1}{2} (Y_2^+(F) Y^k + Y^k Y_2(F)) + 2 Y^p Y^{+k} Y^p$$

$$+ Y^p \text{Tr} Y^{+p} Y^k - 3g^2 \{ C_2(F), Y^k \}$$

$$Y^k_{ij} = \begin{array}{c} i \quad j \\ \diagdown \quad \diagup \\ \quad \quad k \end{array} \qquad Y^{k\dagger} \begin{array}{c} i \\ \diagdown \quad \diagup \\ \quad \quad \end{array}$$

$$Y_2(F) = Y^{+k} Y^k \sim \begin{array}{c} \quad \quad k \\ \text{---} \quad \quad \text{---} \\ \diagup \quad \diagdown \end{array}$$

$$Y_2^+(F) Y^k + Y^k Y_2(F) \sim \begin{array}{c} \quad \quad k \\ \text{---} \quad \quad \text{---} \\ \diagdown \quad \diagup \end{array} \qquad \begin{array}{c} \quad \quad k \\ \text{---} \quad \quad \text{---} \\ \diagup \quad \diagdown \end{array}$$

$$Y^p Y^{+k} Y^p \sim \begin{array}{c} \quad \quad k \\ \text{---} \quad \quad \text{---} \\ \diagdown \quad \diagup \end{array}$$

$$Y^p \text{Tr} Y^{+p} Y^k \sim \begin{array}{c} \quad \quad k \\ \text{---} \quad \quad \text{---} \\ \diagdown \quad \diagup \\ \text{---} \\ \text{---} \\ \diagup \quad \diagdown \\ \quad \quad k \end{array}$$

$$\{ C_2(F), Y^k \} \sim \begin{array}{c} \quad \quad k \\ \text{---} \quad \quad \text{---} \\ \diagdown \quad \diagup \\ \quad \quad k \end{array} + \begin{array}{c} \quad \quad k \\ \text{---} \quad \quad \text{---} \\ \diagup \quad \diagdown \\ \quad \quad k \end{array}$$