Action is quad. in F

$$\mathcal{L}_F = F^{*a} F_a + W^a F_a + W_a^* F^{*a}$$

(exact!) $\quad F_a = -W_a^* \qquad F^{*a} = -W^a$

algebraic $\quad \int \mathcal{D}F \mathcal{D}\phi \, e^{-\int F^2 + WF + h.c.} = \int \mathcal{D}\phi \, e^{-\int |W|^2}$

$$\mathcal{L}_{WZ} = \partial^\mu \phi^{*a} \partial_\mu \phi^a + i \psi^{\dagger a} \bar{\sigma}^\mu \partial_\mu \psi_a - \frac{1}{2}\left(W^{ab} \psi_a \psi_b + W^*_{ab} \psi^{\dagger a} \psi^{\dagger b} \right) - W^a W_a^*$$

$$V(\phi, \phi^*) = W^a W_a^* = F^{*a} F_a$$

$F=0$ $\quad = M^*_{ac} M^{cb} \phi^{*a} \phi_b + \frac{1}{2} M^{ad} y^*_{bcd} \phi_a \phi^{*b} \phi^{*c} + \frac{1}{2} M^*_{ad} y^{bcd} \phi^{*a} \phi_b \phi_c + \frac{1}{4} y^{abe} y^*_{ced} \phi_a \phi_b \phi^{*c} \phi^{*d}$

$$V \geq 0$$

$$\mathcal{L}_{WZ} = \partial^\mu \phi^{*a} \partial_\mu \phi^a + i \psi^{\dagger} \bar{\sigma}^\mu \partial_\mu \psi_a - \frac{1}{2} M^{ab} \psi_a \psi_b - \frac{1}{2} M^*_{ab} \psi^{\dagger a} \psi^{\dagger b} - V(\phi, \phi^*) - \frac{1}{2} y^{abc} \phi_a \psi_b \psi_c - \frac{1}{2} y^*_{abc} \phi^{*a} \psi^{\dagger b} \psi^{\dagger c}$$

$$\partial_\mu \partial^\mu \phi_a = - M^*_{ab} M^{bc} \phi_c$$

$$\mathcal{L} = i\psi^\dagger_{\dot\alpha}\bar\sigma^{\mu\dot\alpha\alpha}\partial_\mu\psi_\alpha = -i(\partial_\mu\psi)\sigma^\mu\psi^\dagger$$

$$= i\psi^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\psi^\dagger$$

$\downarrow$ h.c.

$$= -i\,\partial_\mu\psi^\dagger\bar\sigma^\mu\psi$$

$$= i\psi^\dagger\bar\sigma^\mu\partial_\mu\psi$$

$$\partial_\mu\frac{\delta\mathcal{L}}{\delta(\partial_\mu\psi^\dagger)} = \frac{\delta\mathcal{L}}{\delta\psi^\dagger}$$

$$\cancel{\partial_\mu i\psi^\alpha\sigma^\mu_{\alpha\dot\alpha} = M\psi}$$

$$\mathcal{L} = +i\psi^\dagger\bar\sigma\partial_\mu\psi - \frac{1}{2}M\psi\psi - \frac{1}{2}M^*\psi^\dagger\psi^\dagger$$

$$\frac{\delta\mathcal{L}}{\delta\psi^\dagger} = i\bar\sigma^\mu\partial_\mu\psi - M^*\psi^\dagger = 0$$

$$i\bar\sigma^\mu\partial_\mu\psi_i = M^*_{ij}\psi^{\dagger j}$$

$$\frac{\delta\mathcal{L}}{\delta\psi} = i\sigma^\mu\partial_\mu\psi^\dagger - \cancel{M^*\psi}\, M\psi$$

$$i\sigma^\mu\partial_\mu\psi^{\dagger i} = M^{ij}\psi_j$$

$$i\sigma^\nu\partial_\nu(i\bar\sigma^\mu\partial_\mu\psi_i) = M^*_{ij}\,i\sigma^\nu\partial_\nu\psi^{\dagger j}$$

$$-\partial_\mu\partial^\mu\psi_i = M^*_{ij}M^{jk}\psi_k$$

$$i\bar\sigma^\nu\partial_\nu(i\sigma^\mu\partial_\mu\psi^{\dagger i}) = M^{ij}\,i\bar\sigma^\nu\partial_\nu\psi_j$$

$$-\partial_\mu\partial^\mu\psi^{\dagger i} = M^{ij}M^*_{jk}\psi^{\dagger k}$$

$$= \psi^{\dagger k}M^*_{kj}M^{ji}$$

$\therefore$ fermions and scalars have same mass eigenvalues

diagonalizing gives set of massive chiral supermultiplets

SUSY Gauge Interactions

$$\delta A^a_\mu = -\partial_\mu \Lambda + g f^{abc} A^b_\mu \Lambda^c$$

$$\delta \lambda^a = g f^{abc} \lambda^b \Lambda^c$$

	off shell	On shell
A^a_μ	3 dof	2 dof
λ^a	4 dof	2 dof

bookkeeping: add a D^a (boson)

D^a	1 dof	0 dof.

real $D^{a*} = D^a$ (dumb name!)

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + i \lambda^{\dagger a} \bar{\sigma}^\mu D_\mu \lambda^a + \frac{1}{2} D^a D^a$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$$

$$D_\mu \lambda^a = \partial_\mu \lambda^a - g f^{abc} A^b_\mu \lambda^c$$

SUSY transformations satisfy

linear $\epsilon, \epsilon^\dagger$ dimension $(\text{mass})^{-1/2}$

δA^a_μ real $\rightarrow \lambda^a$

$\lambda^a \rightarrow \partial_\nu A^\mu_\mu \rightarrow$ but $F_{\mu\nu}$ covariant

δD^a real $\propto \lambda^a$ eqn. of motion

$$\delta A^a_\mu = \frac{-1}{\sqrt{2}} \left(\epsilon^\dagger \bar\sigma_\mu \lambda^a + \lambda^{\dagger a} \bar\sigma_\mu \epsilon \right)$$

$$\delta \lambda^a_\alpha = \frac{-i}{2\sqrt{2}} \left(\sigma^\mu \bar\sigma^\nu \epsilon \right)_\alpha F^a_{\mu\nu} + \frac{1}{\sqrt{2}} \epsilon_\alpha D^a$$

$$\delta \lambda^{\dagger a}_{\dot\alpha} = \frac{i}{2\sqrt{2}} \left(\epsilon^\dagger \bar\sigma^\nu \sigma^\mu \right)_{\dot\alpha} F^a_{\mu\nu} + \frac{1}{\sqrt{2}} \epsilon_{\dot\alpha} D^a$$

$$\delta D^a = \frac{-i}{\sqrt{2}} \left[\epsilon^\dagger \bar\sigma^\mu D_\mu \lambda^a - D_\mu \lambda^{\dagger a} \bar\sigma^\mu \epsilon \right]$$

$$(\delta_{\epsilon_2} \delta_{\epsilon_1} - \delta_{\epsilon_1} \delta_{\epsilon_2}) D^a$$

$$= \frac{i}{\sqrt{2}} \delta_{\epsilon_2} \Big(\epsilon_1^\dagger \bar\sigma^\mu (\partial_\mu \lambda^a - g f^{abc} A^b_\mu \lambda^c) - (\partial_\mu \lambda^{a\dagger} - g f^{abc} A^b_\mu \lambda^{\dagger c}) \bar\sigma^\mu \epsilon_1 \Big)$$

$$- 1 \leftrightarrow 2$$

$$= \frac{i}{\sqrt{2}} \Bigg(\epsilon_1^\dagger \bar\sigma^\mu \Big(\partial_\mu \Big(\frac{-i}{2\sqrt{2}} \sigma^\rho \bar\sigma^\tau \epsilon_2 F^a_{\rho\tau} + \frac{1}{\sqrt{2}} \epsilon_2 D^a \Big)$$

$$- g f^{abc} \Big(\Big(\frac{-1}{\sqrt{2}} (\epsilon_2^\dagger \bar\sigma_\mu \lambda^b + \lambda^{\dagger b} \bar\sigma_\mu \epsilon_2 \Big) \lambda^c$$

$$A^b_\mu \Big(\frac{-i}{2\sqrt{2}} (\sigma^\rho \bar\sigma^\tau \epsilon_2 F^c_{\rho\tau} + \frac{1}{\sqrt{2}} \epsilon_2 D^c \Big) \Big) \Big)$$

$$- \Big(\partial_\mu \Big(\frac{i}{2\sqrt{2}} \epsilon_2^\dagger \bar\sigma^\tau \sigma^\rho F^a_{\rho\tau} + \frac{1}{\sqrt{2}} \epsilon_2^\dagger D^a \Big)$$

$$- g f^{abc} \Big(\frac{-1}{\sqrt{2}} \big(\epsilon_2^\dagger \bar\sigma_\mu \lambda^b + \lambda^{\dagger b} \bar\sigma_\mu \epsilon_2 \big) \lambda^{\dagger c}$$

$$+ A^b_\mu \Big(\frac{i}{2\sqrt{2}} \epsilon_2^\dagger \bar\sigma^\tau \sigma^\rho F^c_{\rho\tau} + \frac{1}{\sqrt{2}} \epsilon_2^\dagger D^c \Big) \Big) \Big) \Bigg)$$

$$- 1 \leftrightarrow 2$$

$$\sigma^\mu \sigma^\nu \bar\sigma^\rho = -\eta^{\mu\rho}\bar\sigma^\nu + \eta^{\nu\rho}\bar\sigma^\mu + \eta^{\mu\nu}\bar\sigma^\rho - i\epsilon^{\mu\nu\rho\kappa}\bar\sigma_\kappa$$

$$= i\Big(\epsilon_1^\dagger \bar\sigma^\mu \epsilon_2 \partial_\mu D^a - \epsilon_2^\dagger \bar\sigma^\mu \epsilon_1 \partial_\mu D^a - g f^{abc} A^b_\mu \epsilon_1^\dagger \bar\sigma^\mu \epsilon_2 D^c + g f^{abc} A^b_\mu \epsilon_2^\dagger \bar\sigma^\mu \epsilon_1 D^c \Big)$$

$$+\frac{1}{4}\Big(-\epsilon_1^\dagger \bar\sigma^\mu \sigma^\rho \bar\sigma^\tau \epsilon_2 \partial_\mu F^a_{\rho\tau} - \epsilon_2^\dagger \bar\sigma^\tau \sigma^\rho \bar\sigma^\mu \partial_\mu F^a_{\rho\tau}$$
$$+ \epsilon_2^\dagger \bar\sigma^\mu \sigma^\rho \bar\sigma^\tau \epsilon_1 \partial_\mu F^a_{\rho\tau} + \epsilon_1^\dagger \bar\sigma^\tau \sigma^\rho \bar\sigma^\mu \partial_\mu F^a_{\rho\tau}$$
$$- g f^{abc} A^b_\mu \epsilon_1^\dagger \bar\sigma^\mu \sigma^\rho \bar\sigma^\tau \epsilon_2 F^c_{\rho\tau}$$
$$+ g f^{abc} A^b_\mu \epsilon_2^\dagger \bar\sigma^\mu \sigma^\rho \bar\sigma^\tau \epsilon_1 F^c_{\rho\tau}$$
$$+ g f^{abc} A^b_\mu \epsilon_2^\dagger \bar\sigma^\rho \sigma^\tau \bar\sigma^\mu \epsilon_1 F^c_{\rho\tau}$$
$$- g f^{abc} A^b_\mu \epsilon_1^\dagger \bar\sigma^\rho \sigma^\tau \bar\sigma^\mu \epsilon_2 F^c_{\rho\tau}\Big)$$

$$+\frac{i}{2}\Big(g f^{abc} \epsilon_1^\dagger \bar\sigma^\mu \lambda^c \big((\epsilon_2^\dagger \bar\sigma_\mu \lambda^b) + (\lambda^{\dagger b}\bar\sigma_\mu \epsilon_2)\big)$$
$$- g f^{abc} \epsilon_2^\dagger \bar\sigma^\mu \lambda^c \big((\epsilon_1^\dagger \bar\sigma_\mu \lambda^b) + (\lambda^{\dagger b}\bar\sigma_\mu \epsilon_1)\big)$$
$$- g f^{abc} \big(\epsilon_2^\dagger \bar\sigma_\mu \lambda^b \lambda^{\dagger c} \bar\sigma^\mu \epsilon_1 + \lambda^{\dagger b}\bar\sigma_\mu \epsilon_2 \lambda^{\dagger c}\bar\sigma^\mu \epsilon_1$$
$$+ g f^{abc} \big(\epsilon_1^\dagger \bar\sigma_\mu \lambda^b \lambda^{\dagger c} \bar\sigma^\mu \epsilon_2 + \lambda^{\dagger b}\bar\sigma_\mu \epsilon_1 \lambda^{\dagger} \bar\sigma^\mu \Big)$$

$$= i\big(\epsilon_1^\dagger \bar\sigma^\mu \epsilon_2 - \epsilon_2^\dagger \bar\sigma^\mu \epsilon_1 \big) D_\mu D^a$$

$$+\frac{1}{4}\Big(\epsilon_1^\dagger \big(\eta^{\mu\tau}\bar\sigma^\rho - \eta^{\rho\tau}\bar\sigma^\mu - \eta^{\mu\rho}\bar\sigma^\tau + i\epsilon^{\mu\rho\tau\kappa}\bar\sigma_\kappa$$
$$- \eta^{\tau\mu}\bar\sigma^\rho + \eta^{\rho\mu}\bar\sigma^\tau + \eta^{\rho\tau}\bar\sigma^\mu - i\epsilon^{\tau\rho\mu\kappa}\bar\sigma_\kappa \big)\epsilon_2$$
$$\times \big(\partial_\mu F^a_{\rho\tau} - g f^{abc} A^b F^c_{\rho\tau} \big)$$
$$- \epsilon_2^\dagger \big(-\eta^{\tau\mu}\bar\sigma^\rho + \eta^{\rho\mu}\bar\sigma^\tau + \eta^{\rho\tau}\bar\sigma^\mu - i\epsilon^{\tau\rho\mu\kappa}\bar\sigma_\kappa$$
$$+ \eta^{\mu\tau}\bar\sigma^\rho - \eta^{\rho\tau}\bar\sigma^\mu - \eta^{\mu\rho}\bar\sigma^\tau + i\epsilon^{\mu\rho\tau\kappa}\bar\sigma_\kappa \big)\epsilon_1$$
$$\times \big(\partial_\mu F^a_{\rho\tau} - g f^{abc} A^b_\mu F^c_{\rho\tau} \big)\Big)$$

$$+ g f^{abc}\Big((\epsilon_1^\dagger \epsilon_2^\dagger)(\lambda^b \lambda^c) + (\epsilon_1^\dagger \lambda^{\dagger b})(\epsilon_2 \lambda^c)$$
$$- \epsilon_2^\dagger \epsilon_1^\dagger \lambda^b \lambda^c - \epsilon_2^\dagger \lambda^{\dagger b} \epsilon_1 \lambda^c$$
$$- \epsilon_2^\dagger \lambda^{\dagger c} \epsilon_1 \lambda^b - \lambda^{\dagger c}\lambda^{\dagger b} \epsilon_1 \epsilon_2$$
$$+ \epsilon_1^\dagger \lambda^{\dagger c} \epsilon_2 \lambda^b + \lambda^{b\dagger}\lambda^{c\dagger} \epsilon_1 \epsilon_2$$

$$= i\big(\epsilon_1^\dagger \bar\sigma^\mu \epsilon_2 - \epsilon_2^\dagger \bar\sigma^\mu \epsilon_1 \big) D_\mu D^a$$
$$+\frac{i}{2}\big(\epsilon_1^\dagger \bar\sigma_\kappa \epsilon_2 \epsilon^{\mu\rho\tau\kappa} - \epsilon_2^\dagger \bar\sigma_\kappa \epsilon_1 \epsilon^{\mu\rho\tau\kappa} \big) D_\mu F^a_{\rho\tau}$$
$$+ i g f^{abc}\big(\epsilon_1^\dagger \lambda^{\dagger b} \epsilon_2 \lambda^c - \epsilon_2^\dagger \lambda^{\dagger b} \epsilon_1 \lambda^c - \epsilon_2^\dagger \lambda^{\dagger c} \epsilon_1 \lambda^b + \epsilon_1^\dagger \lambda^{\dagger c} \epsilon_2 \lambda^b \big)$$

$$= 0$$

Bianchi $\epsilon^{\nu\rho\tau\kappa} D_\nu F^a_{\rho\tau} = 0$

$$(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})X^a = -i(\epsilon_1\sigma^\mu\epsilon_2^\dagger - \epsilon_2\sigma^\mu\epsilon_1^\dagger)D_\mu X^a$$

$$X^a = F^a_{\mu\nu},\ \lambda^a,\ \lambda^{\dagger a},\ D^a$$

Add Chiral Mutiplets

$$\delta_{gauge} X_j = i g \Lambda^a (T^a X)_j$$

$$X_j = \phi_j,\ \psi_j,\ F_j$$

$$D_\mu \phi_j = \partial_\mu \phi_j + i g A^a_\mu (T^a \phi)_j$$

$$D_\mu \phi^{*j} = \partial_\mu \phi^{*j} - i g A^a_\mu (\phi^* T^a)^j$$

$$D_\mu \psi_j = \partial_\mu \psi_j + i g A^a_\mu (T^a \psi)_j$$

New gauge invariant (renormalizable) interactions are allowed

$$(\phi^* T^a \psi)\lambda^a \qquad \lambda^{\dagger a}(\psi^\dagger T^a \phi)$$

$$(\phi^* T^a \phi) D^a$$

required by SUSY!

$$\mathcal{L} = \mathcal{L}_{sym} + \mathcal{L}_{WZ}$$
$$-\sqrt{2} g \left[(\phi^* T^a \psi) \lambda^a + \lambda^{\dagger a} (\psi^\dagger T^a \phi) \right]$$
$$+ g (\phi^* T^a \phi) D^a$$

SUSY Transformations

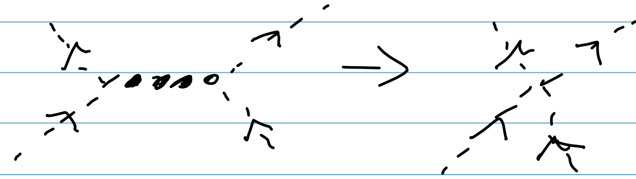
$$\delta \phi_j = \epsilon \psi_j$$
$$\delta (\psi_j)_\alpha = -i (\sigma^\mu \epsilon^\dagger)_\alpha D_\mu \phi_j + \epsilon_\alpha F$$
$$\delta F_j = -i \epsilon^\dagger \bar{\sigma}^\mu D_\mu \psi_j + \sqrt{2} g (T^a \phi)_j \epsilon^\dagger \lambda^{\dagger a}$$

new term

W must be gauge invariant

$$\delta_{gauge} W \propto \frac{\delta W}{\delta \phi_i} (T^a \phi)_i = 0$$

Integrate out D^a



$$D^a = -g (\phi^\dagger T^a \phi)$$
$$V(\phi, \phi^*) = F^{*i} F_i + \frac{1}{2} D^a D^a$$
$$= W_i W^{*i} + \frac{g^2}{2} (\phi^* T^a \phi)^2$$

$\delta \mathcal{L}_1 = -\delta \mathcal{L}_2 \qquad \delta^2 \mathcal{L}_1 = \mathcal{L}_2$