# SUSY Reps : Supermultiplets

Massive Particles : $|m, s, s_3\rangle$

rest frame $P_\mu = (m, \vec{0})$

$$\{Q_\alpha, Q^\dagger_{\dot\beta}\} = 2m$$

"Clifford vacuum" $|\Omega_s\rangle = Q_1 Q_2 |m, s', s_3'\rangle$ degen. $2s+1$

$$Q_1 |\Omega_s\rangle = 0 \qquad Q_2 |\Omega_s\rangle = 0$$

massive supermultipet
$$\left\{ \begin{array}{c} |\Omega_s\rangle \\ Q_1^\dagger |\Omega_s\rangle, \quad Q_2^\dagger |\Omega_s\rangle \\ Q_1^\dagger Q_2^\dagger |\Omega_s\rangle \end{array} \right.$$

| state | $s_3$ | |
| --- | --- | --- |
| $\|\Omega_0\rangle$ | $0$ | massive chiral mutiplet |
| $Q_1^\dagger \|\Omega_0\rangle$, $Q_2^\dagger \|\Omega_0\rangle$ | $\pm 1/2$ | |
| $Q_1^\dagger Q_2^\dagger \|\Omega_0\rangle$ | $0$ | |
| | | |
| $\|\Omega_{1/2}\rangle$ | $\pm 1/2$ | massive vector multiplet |
| $Q_1^\dagger \|\Omega_{1/2}\rangle$, $Q_2^\dagger \|\Omega_{1/2}\rangle$ | $0, -1, 0, 1$ | |
| $Q_1^\dagger Q_2^\dagger \|\Omega_{1/2}\rangle$ | $\pm 1/2$ | |

massless particles: $|E, \lambda\rangle$

$$P_\mu = (E, 0, 0, E)$$

$$\{Q_1, Q_1^\dagger\} = 2P_0 + 2P_3 = 4E$$
$$\{Q_2, Q_2^\dagger\} = 2P_0 - 2P_3 = 0$$

$$|\Omega_\lambda\rangle = Q_1 Q_2 |E, \lambda'\rangle \qquad Q_1|\Omega_\lambda\rangle = 0$$
$$Q_2|\Omega_\lambda\rangle = 0$$

$$Q_2 Q_2^\dagger + Q_2^\dagger Q_2 = 0$$
$$\langle\Omega_\lambda| Q_2 Q_2^\dagger |\Omega_\lambda\rangle + \langle\Omega_\lambda| Q_2^\dagger Q_2 |\Omega_\lambda\rangle = 0$$
$$\langle\Omega_\lambda| Q_2 Q_2^\dagger |\Omega_\lambda\rangle = 0$$
$$Q_2^\dagger |\Omega_\lambda\rangle = 0$$

| State | helicity |
| --- | --- |
| $\vert\Omega_\lambda\rangle$ | $\lambda$ |
| $Q_1^\dagger\vert\Omega_\lambda\rangle$ | $\lambda + 1/2$ |
| CPT: $\vert\Omega_{-\lambda-1/2}\rangle$ | $-\lambda - 1/2$ |
| $Q_1^\dagger\vert\Omega_{-\lambda-1/2}\rangle$ | $-\lambda$ |

$$\left.\begin{array}{ll} |\Omega_0\rangle & 0 \\ Q_1^\dagger|\Omega_0\rangle & 1/2 \\ |\Omega_{-1/2}\rangle & -1/2 \\ Q_1^\dagger|\Omega_{-1/2}\rangle & 0 \end{array}\right\} \text{massless chiral multiplet}$$

| | State | helicity | |
|---|---|---|---|
| | $\|\Omega_{1/2}\rangle$ | $1/2$ | massless vector multiplet |
| | $Q_1^\dagger \|\Omega_{1/2}\rangle$ | $1$ | |
| CPT conjugate | $\|\Omega_{-1}\rangle$ | $-1$ | |
| | $Q_1^\dagger \|\Omega_{-1}\rangle$ | $-1/2$ | |

massive vector = massless vector + massless chiral

| superpartner | names |
|---|---|
| fermion | sfermion |
| quark | squark |
| gauge boson | gaugino |
| gluon | gluino |

Extended SUSY

$$\{ Q^a_\alpha , Q^{\dagger}_{\dot\beta b} \} = 2 \sigma^\mu_{\alpha\dot\beta} \delta^a_b P_\mu \qquad a, b = 1 \dots \mathcal{N}$$

$SU(\mathcal{N})$ R symmetry

massless multiplets $\qquad p_\mu = (E, 0, 0, E)$

$$\{ Q^a_1 , Q^{\dagger}_{1b} \} = 4E \delta^a_b$$
$$\{ Q^a_2 , Q^{\dagger}_{2b} \} = 0$$

| state | helicity | # |
|---|---|---|
| $\lvert \Omega_\lambda \rangle$ | $\lambda$ | $1$ |
| $Q^{\dagger a} \lvert \Omega_\lambda \rangle$ | $\lambda + 1/2$ | $\mathcal{N}$ |
| $Q^{\dagger a} Q^{\dagger b} \lvert \Omega_\lambda \rangle$ | $\lambda + 1$ | $\frac{\mathcal{N}(\mathcal{N}-1)}{2}$ |
| $\vdots$ | | |
| $Q^{1\dagger} Q^{2\dagger} \dots Q^{\mathcal{N}\dagger} \lvert \Omega_\lambda \rangle$ | $\lambda + \frac{\mathcal{N}}{2}$ | $1$ |

$$|\lambda| \le 1 \qquad \left| \lambda + \frac{\mathcal{N}}{2} \right| \le 1 \quad \Rightarrow \quad \mathcal{N} \le 4$$

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Group theory $\qquad SU(2) \qquad \lvert\uparrow\rangle, \ \lvert\downarrow\rangle$

$\lvert\uparrow\uparrow\rangle$, $\ \lvert\uparrow\downarrow\rangle + \lvert\downarrow\uparrow\rangle$, $\ \lvert\downarrow\downarrow\rangle$ — spin 1 $\qquad \lvert\uparrow\downarrow\rangle - \lvert\downarrow\uparrow\rangle$ — spin 0

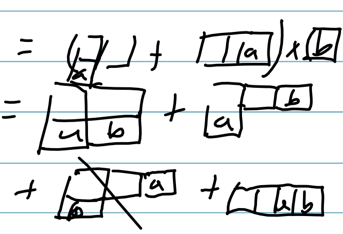
Young Tableaux $\qquad \square = \overline{\square} = 2 \qquad \begin{smallmatrix}\square\\\square\end{smallmatrix} = 1_A , \quad d = 2s+1$

$\square \times \square = 1 + \square\square$ $\qquad 2 \times 2 = 1_A + 3_S$

$\square\square \times \square = \square + \square\square\square$ $\qquad 3 \times 2 = 2 + 4$

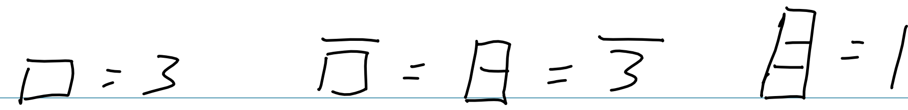
$\square\square\square \times \square = \square\square + \square\square\square\square$ $\qquad 4 \times 2 = 3 + 5$

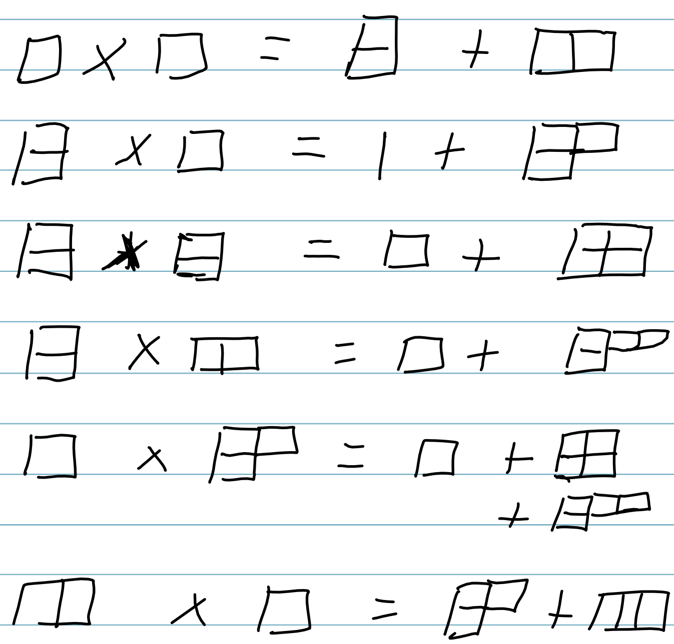
$\square\square \times \boxed{a}\boxed{b} =$



$3 \times 3 = 1 + 3 + 5$

SU(3)





$3 \times 3 = \bar{3}_A + 6_{\cancel{A}S}$

$\bar{3} \times 3 = 1 + 8$

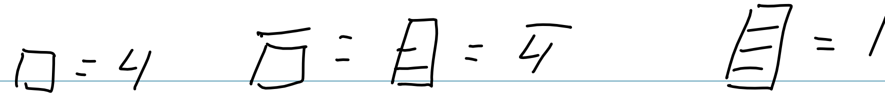
$\bar{3} \times \bar{3} = 3 + \bar{6}$

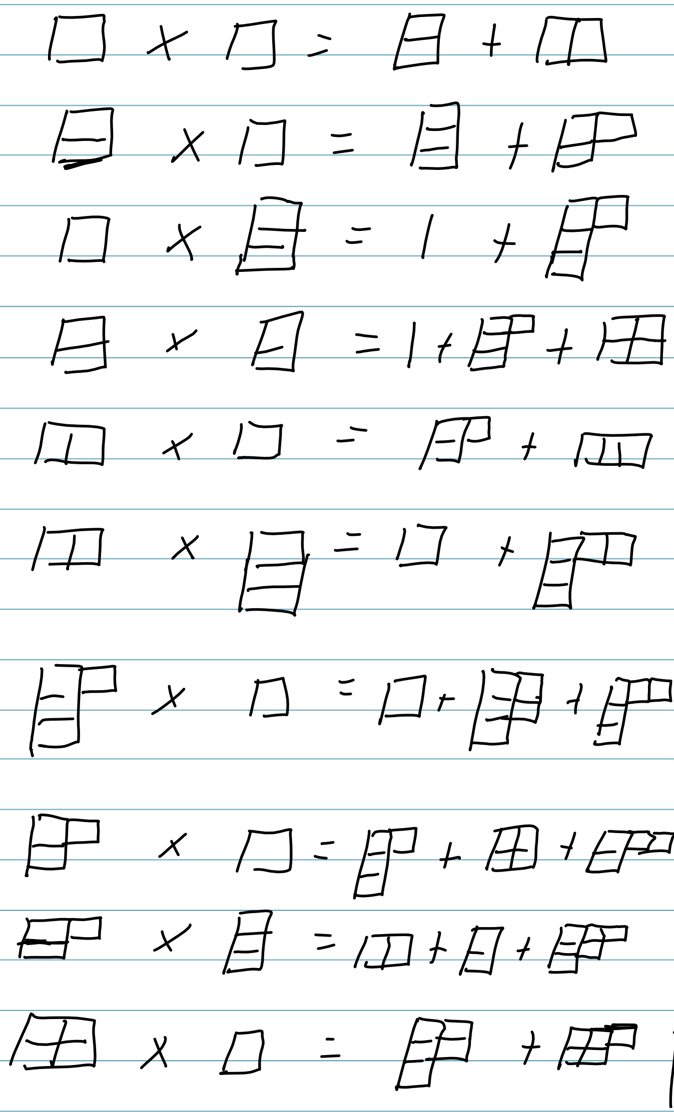
$\bar{3} \times 6 = 3 + 15$

$3 \times 8 = 3 + \bar{6} + 15$

$6 \times 3 = 8 + 10$

Su(4)



$4 \times 4 = 6_A + 10_S$

$6 \times 4 = \bar{4} + \overline{20}$

$4 \times \bar{4} = 1 + 15$

$6 \times 6 = 1 + 15 + 20'$

$10 \times 4 = \overline{20} + \overline{20''}$

$10 \times \bar{4} = 4 + 36$

$15 \times 4 = 4 + 20 + 36$

$\overline{20} \times 4 = 15 + 20' + \overline{45}$

$\overline{20} \times \bar{4} = 10 + 6 + 64$

$20' \times 4 = 20 + 60$

# Extended SUSY

$\mathcal{N}=2$

| | state | helicity | # | $SU(2)_R$ | |
|---|---|---|---|---|---|
| | $\lvert\Omega_0\rangle$ | $0$ | $1$ | $1$ | |
| | $Q^\dagger\lvert\Omega_0\rangle$ | $1/2$ | $2$ | $\square$ | |
| | $Q^\dagger Q^\dagger\lvert\Omega_0\rangle$ | $1$ | $1$ | $1$ | $\mathcal{N}=1$ vector |
| CPT conjugate | $\lvert\Omega_{-1}\rangle$ | $-1$ | $1$ | $1$ | |
| | $Q^\dagger\lvert\Omega_{-1}\rangle$ | $-1/2$ | $2$ | $\square$ | |
| | $Q^\dagger Q^\dagger\lvert\Omega_{-1}\rangle$ | $0$ | $1$ | $1$ | |

$\equiv$ $\mathcal{N}=1$ vector + chiral

| | | | | |
|---|---|---|---|---|
| $\lvert\Omega_{-1/2}\rangle$ | $-1/2$ | $1$ | $1$ | $\chi_L$ |
| $Q^\dagger\lvert\Omega_{-1/2}\rangle$ | $0$ | $2$ | $\square$ | $\phi$ |
| $Q^\dagger Q^\dagger\lvert\Omega_{-1/2}\rangle$ | $1/2$ | $1$ | $1$ | $\psi_R$ |

$\overline{\chi}_L \psi_R$ gauge invariant mass term

$\Rightarrow$ vector-like $\equiv$ non-chiral