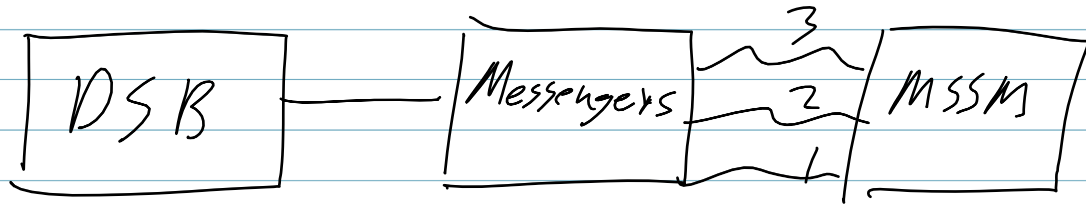
Lecture 19

Gauge Mediation



$$W = X \bar{\Phi}_i \phi_i \qquad i = 1, \ldots N_F$$

$$\langle X \rangle = M + \theta^2 F$$

Goldstino multiplet

to preserve gauge unification

$\bar{\Phi}_i, \phi_i$ should form complete GUT multiplets

eg $\bar{5} + 5$, $\overline{10} + 10$ of $SU(5)$

$$\delta \alpha^{-1}_{GUT} = -\frac{N}{2\pi} \ln\left(\frac{M_{GUT}}{M}\right)$$

$$N = \sum_{i=1}^{N_F} 2\, T(r_i)$$

perturbative unification $N \leq \dfrac{150}{\ln\left(\frac{M_{GUT}}{M}\right)}$

$M = 100$ TeV $\quad N \leq 5$

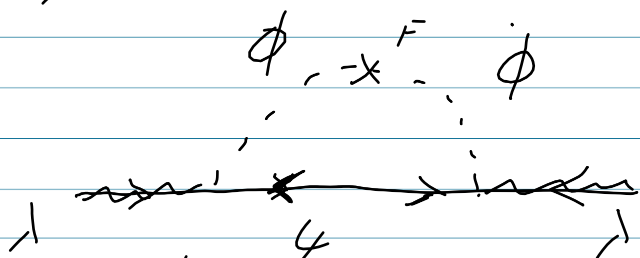
$M = 10^{10}$ GeV $\quad N \leq 10$

$\langle X \rangle \to$ fermions mass M

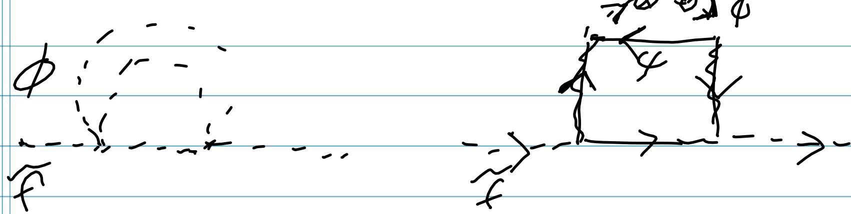
scalars: $\left(\phi^\dagger \;\; \bar{\phi}\right) \begin{pmatrix} M^\dagger M & F^\dagger \\ F & M M^\dagger \end{pmatrix} \begin{pmatrix} \phi \\ \bar{\phi}^\dagger \end{pmatrix}$

masses $M^2 \pm F$

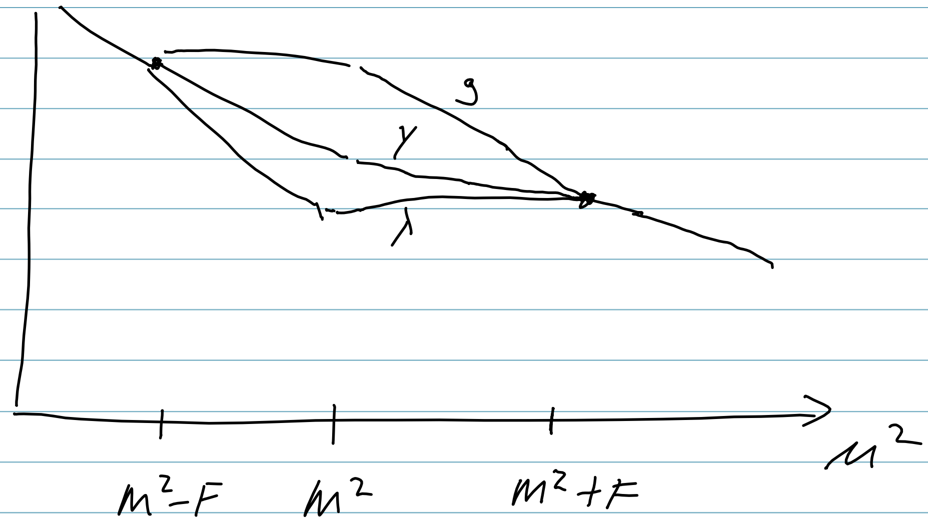
assume $F \ll M^2$ $\qquad$ $\text{TeV} \sim \frac{\alpha}{4\pi} \frac{F}{M}$



$m_\lambda = \frac{\alpha}{4\pi} \frac{F}{M}$



$$m_s^2 \approx \left(\frac{\alpha}{4\pi}\right)^2 \left(\frac{F}{M}\right)^2$$


g
γ
λ
m^2-F
m^2
m^2+F
m^2

Integrate $\bar{\Phi}\,\phi$

$$\mathcal{L}_G = \frac{-i}{16\pi}\int d^2\theta\, \tau\, W^\alpha W_\alpha \qquad \tau_0 = \frac{\Theta_{YM}}{2\pi} + \frac{4\pi i}{g^2}$$

$\tau(X, \mu)$ holomorphic function of X

$$\langle X\rangle = M + \theta^2 F$$

$$\mathcal{L}_G = \frac{-i}{16\pi}\int d^2\theta \left(\tau\Big|_{\substack{F=0\\ \langle X\rangle = M}} + \frac{\partial\tau}{\partial X}\Big|_{\langle X\rangle = M} \theta^2 F \right) W^\alpha W_\alpha$$

$$M_{1/2} = \frac{i}{2}\frac{1}{\tau}\frac{\partial\tau}{\partial X}\Big|_{X=M} F = \frac{i}{2}\frac{X}{\tau}\frac{\partial\tau}{\partial X}\frac{F}{M}$$

$$= \frac{i}{2}\frac{\partial \ln\tau}{\partial \ln X}\frac{F}{M}$$

$$\tau(M,\mu) = \alpha^{-1}(M,\mu) = \alpha^{-1}(\mu_0) + \frac{b'}{2\pi}\ln\left(\frac{|M|}{\mu_0}\right) + \frac{b}{2\pi}\left(\frac{\mu}{|M|}\right.$$

$$b' = b - N$$

$$\tau(X,\mu) = \tau(\mu_0) + \frac{i b'}{2\pi} \ln\left(\frac{X}{\mu_0}\right) + \frac{i b}{2\pi} \ln\left(\frac{\mu}{X}\right)$$

$$\begin{aligned} \phi &\to e^{i\alpha}\phi \\ \bar{\phi} &\to e^{i\alpha}\bar{\phi} \\ X &\to e^{-2i\alpha} X \end{aligned} \qquad \Rightarrow \qquad \begin{aligned} \Theta_{YM} &\to \Theta_{YM} + 2\alpha(b'-b) \\ &\quad \Theta_{YM} - 2\alpha N \end{aligned}$$

$$M_\lambda = \frac{i}{2} \frac{\partial \ln \tau}{\partial \ln X}\Bigg|_{X=M} \frac{F}{M}$$

$$= -\frac{1}{2\tau(M,\mu)} \left(\frac{b'-b}{2\pi}\right) \frac{F}{M}$$

$$= -\frac{\alpha}{4\pi}(M,\mu)\,(-N)\,\frac{F}{M} = \frac{\alpha}{4\pi} N \frac{F}{M}$$

$$\frac{M_3}{\alpha_3} = \frac{M_2}{\alpha_2} = \frac{M_1}{\alpha_1} = \frac{N}{4\pi}\frac{F}{M}$$

same as SUGRA models with GUT

$$\mathcal{L} = \int d^4\theta \; Z \; Q^+ Q$$

$$Z = Z(X, X^+) \qquad \text{real}$$

$$\mathcal{L} = \int d^4\theta \left(Z + \frac{\partial Z}{\partial X} F\theta^2 + \frac{\partial Z}{\partial X^+} F^+\bar\theta^2 + \frac{\partial^2 Z}{\partial X \partial X^+} FF^+ \theta^2\bar\theta^2 \right)\Big|_{X=m} Q^+ Q$$

$$Q' = Z^{1/2}\left(1 + \frac{\partial \ln Z}{\partial X} F\theta^2 \right)\Big|_{X=M} Q$$

$$Q^+Q = Q'^+ Q' Z^{-1}\left(1 - \frac{\partial \ln Z}{\partial X} F\theta^2 - \frac{\partial \ln Z}{\partial X^+} F^+\bar\theta^2 + \frac{\partial \ln Z}{\partial X} F\theta^2 \frac{\partial \ln Z}{\partial X^+} F^+\bar\theta^2 \right)\Big|_{X=m}$$

$$\mathcal{L} = \int d^4\theta \frac{Q'^+ Q'}{Z}\left(1 - \frac{\partial \ln Z}{\partial X}\frac{\partial \ln Z}{\partial X^+} FF^+\theta^2\bar\theta^2 + \frac{1}{Z}\frac{\partial^2 Z}{\partial X \partial X^+} FF^+ \theta^2\bar\theta^2 \right)\Big|_{X=m}$$

$$\frac{\partial^2 \ln Z}{\partial X \partial X^+} = \frac{\partial}{\partial X}\left(\frac{1}{Z}\frac{\partial Z}{\partial X^+} \right)$$

$$= \frac{-1}{Z^2}\frac{\partial Z}{\partial X}\frac{\partial Z}{\partial X^+} + \frac{1}{Z}\frac{\partial^2 Z}{\partial X \partial X^+}$$

$$M_Q^2 = -\frac{\partial^2 \ln Z}{\partial \ln X \, \partial \ln X^+}\Big|_{X=M} \frac{FF^+}{MM^+}$$

$$\int d^2\theta\, W(Q) \to \int d^2\theta\, W\left(Z^{-1/2}\left(1 - \frac{\partial \ln Z}{\partial X}\Big|_{X=M} F\theta^2\right) Q'\right)$$

$$= Z^{-1/2}\left(-\frac{\partial \ln Z}{\partial X} F\right)\Bigg|_{X=M} \frac{Q'\,\partial W(Z^{-1/2}Q')}{\partial(Z^{-1/2}Q')} + \int d^2\theta\, W(Z^{-1/2}Q')$$

Z is invariant under $X \to e^{i\alpha}X$

$$Z(XX^\dagger, \mu) = Z(M \to \sqrt{X\cancel{X}X^\dagger}, \mu)$$

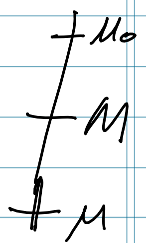
$$L_0 = \ln\left(\frac{\mu^2}{M_0}\right) \qquad L_X = \ln\left(\frac{\mu^2}{XX^\dagger}\right)$$

RG: ℓ loops

$$\ln Z = \alpha^{\ell-1}(\mu_0)\, f\left(\alpha(\mu_0) L_X, \alpha(\mu_0) L_0\right)$$

$$m^2 \propto \frac{\partial^2 \ln Z}{\partial \ln X\, \partial \ln X^\dagger} = \alpha^{\ell+1}(\mu)\, h\left(\alpha(\mu) L_X\right)$$

2 loop masses determined by 1 loop R.G. eq.



$$\frac{\partial \ln Z}{\partial \ln \mu} = \frac{C_2(r)}{\pi}\,\alpha(\mu)\;;\quad \frac{d\alpha^{-1}}{d\ln\mu} = \frac{+b}{2\pi}$$

$$Z = Z_0\left(\frac{\alpha(M_0)}{\alpha(X)}\right)^{\frac{2C_2(r)}{b'}}\left(\frac{\alpha(X)}{\alpha(\mu)}\right)^{\frac{2C_2(r)}{b}}$$

$$\alpha^{-1}(X) = \alpha^{-1}(M_0) + \frac{b'}{2\pi}\ln\left(\frac{XX^+}{\mu^2}\right)$$

$$\alpha^{-1}(\mu) = \alpha^{-1}(X) + \frac{b}{2\pi}\ln\left(\frac{\mu^2}{XX^+}\right)$$

$$m_Q^2 = 2C_2(r)\,\frac{\alpha^2(\mu)}{16\pi^2}\,N\left(\xi^2 + \frac{N}{b}\left(1-\xi^2\right)\right)\left(\frac{F}{M}\right)^2$$

$$\xi = \frac{\alpha(M)}{\alpha(\mu)} = \frac{1}{1 + \frac{b}{2\pi}\alpha(\mu)\ln\frac{M}{\mu}}$$

$$m_Q \sim \frac{M}{\sqrt{N}}$$

μ - problem

$W = \mu H_u H_d$ $\qquad\qquad$ $V = b H_u H_d$

$\mu \sim \frac{1}{16\pi^2} \frac{F}{M}$ $\qquad\qquad$ $b \sim \mu^2$

SUGRA : $\int d^4\theta \, H_u H_d \left(\frac{X}{M_{pl}} + \frac{X X^+}{M_{pl}^2} \right)$

$\mu = \frac{F}{M_{pl}}$ $\qquad$ $b = \left(\frac{F}{M_{pl}} \right)^2$

$W = \lambda X H_u H_d$

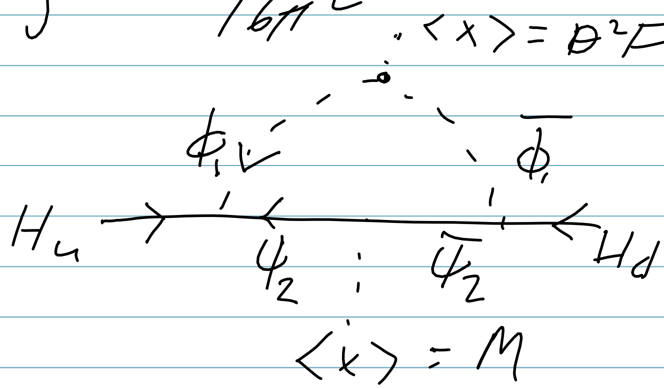
$\mu = \lambda M$ $\qquad\qquad$ $b = \lambda F = \mu \frac{F}{M}$

$= 16\pi^2 \mu$

$W = \lambda H_u \phi_1 \phi_2 + \bar{\lambda} H_d \bar{\Phi}_1 \bar{\Phi}_2$

$+ X (\lambda_1 \Phi_1 \bar{\Phi}_1 + \lambda_2 \Phi_2 \bar{\Phi}_2)$

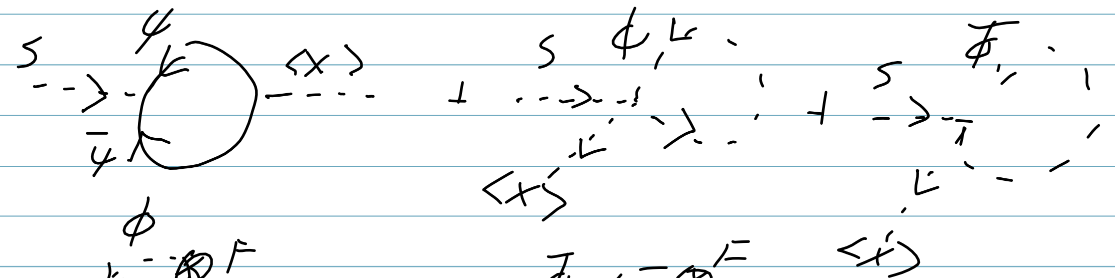
$\int d^4\theta \, \frac{\lambda \bar{\lambda}}{16\pi^2} f(\lambda_1, \lambda_2) H_u H_d \frac{X}{X^+}$

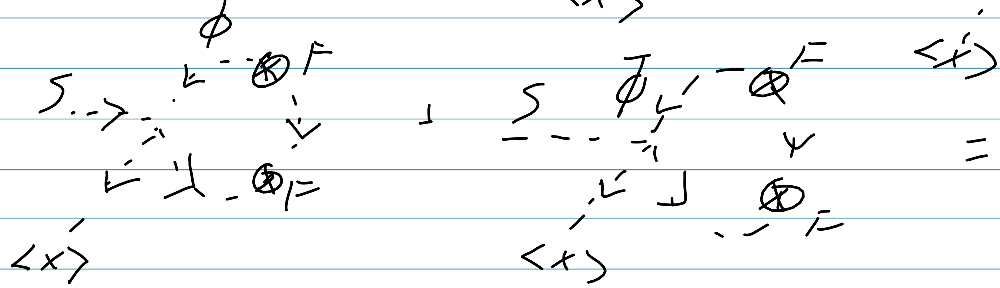


$B = \mu \frac{F}{M}$

$$W = S(\lambda_1 H_u H_d + \lambda_2 N^2 + \lambda \phi \bar{\phi} - M_N^2)$$
$$+ X \phi \bar{\phi} \qquad \langle N \rangle = \frac{M_N}{\sqrt{\lambda_2}}$$

$$\mu = \lambda_1 \langle S \rangle$$
$$b = \lambda_1 \langle F_S \rangle$$

CW  $= 0$

 $= \dfrac{F_X^2}{16\pi^2 M}$

$$V = \frac{S}{16\pi^2} \frac{F_X^2}{M} + \lambda_2^2 |SN|^2 + \dots$$

$$\frac{\partial V}{\partial S} = \frac{F_X^2}{16\pi^2 M} + \lambda_2^2 S^+ \langle N \rangle^2 = 0$$

$$\langle S^+ \rangle = \frac{F_X^2}{16\pi^2 M \lambda_2^2 \langle N^2 \rangle}$$
$$= \frac{F_X^2}{16\pi^2 M M_N}$$

one loop $F_S = 0$

two loops $F_S = \dfrac{1}{(16\pi^2)^2} \dfrac{F_X^2}{M^2} \approx \dfrac{1}{16\pi^2} \mu \dfrac{M_N^2}{M}$

$\sim \mu^2$ if $M_N^2 \sim F_X$