Lecture 18

Goldstinos and Gravitinos

~~g~~ genera theory with W and gauge

$$\Psi = (\lambda^a, \psi_i)$$

$$M_{\text{fermion}} = \begin{pmatrix} 0 & \sqrt{2}\, g_a (\langle\phi^*\rangle T^a)^i \\ \sqrt{2}\, g_a (\langle\phi^*\rangle T^a)^j & \langle W^{ij}\rangle \end{pmatrix}$$

zero eigenvector $\propto \left(\frac{1}{\sqrt{2}}\langle D^a\rangle, \langle F_i\rangle\right)$

$$\tilde{\pi} = \frac{1}{\sqrt{N}} \begin{pmatrix} \frac{1}{\sqrt{2}}\langle D^a\rangle \lambda^a \\ \langle F_i\rangle \psi_i \end{pmatrix}$$

W gauge inv

$$(\langle\phi^*\rangle T^a)^i F_i = -(\langle\phi^*\rangle T^a)^i W_i^* = 0$$

$$V = \tfrac{1}{2} D^a D^a + F_i F^{*i}$$

$$\frac{\partial V}{\partial \phi_j} = -g_a (\phi^* T^a)^j D^a - W_i \frac{\partial W^i}{\partial \phi_j}$$

$$\left\langle \frac{\partial V}{\partial \phi_j} \right\rangle = -g_a (\langle\phi^*\rangle T^a)^j \langle D^a\rangle - \langle F_i\rangle\langle W^{ij}\rangle$$

$$= 0 \quad \text{stability}$$

$$J^\mu_\alpha = (\sigma^\nu \bar\sigma^\mu \psi_i)_\alpha D_\nu \phi^{*i} - i(\sigma^\mu \psi^{\dagger i})_\alpha W_i^*$$
$$- \frac{1}{2\sqrt{2}}(\sigma^\nu \bar\sigma^\rho \sigma^\mu \lambda^{\dagger a})_\alpha F^a_{\nu\rho} - \frac{i}{\sqrt{2}} g \phi^* T^a \phi (\sigma^\mu \lambda^{\dagger a})_\alpha$$

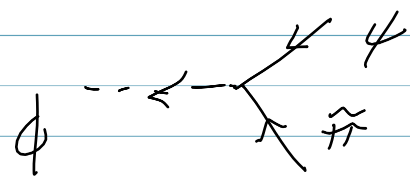
suppose $F_a \neq 0$ $\quad D = 0$ $\quad \langle F \rangle^2 = \sum_a |\langle F_a \rangle|^2$

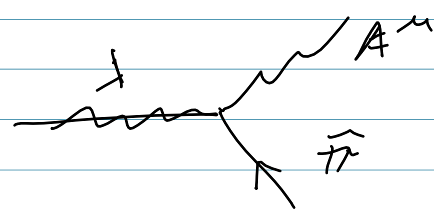
$$\partial_\mu J^\mu_\alpha = i \langle F_a \rangle (\sigma^\mu \partial_\mu \psi^{\dagger a})_\alpha + \partial_\mu j^\mu_\alpha$$
$$= i \langle F \rangle (\sigma^\mu \partial_\mu \tilde\pi^\dagger)_\alpha + \partial_\mu j^\mu_\alpha$$

$$\mathcal{L}_{\text{Goldstino}} = i \tilde\pi^\dagger \bar\sigma^\mu \partial_\mu \tilde\pi + \frac{1}{\langle F \rangle} \tilde\pi \partial_\mu j^\mu_\alpha$$

$$j^\mu_\alpha = (\sigma^\nu \bar\sigma^\mu \psi_i)_\alpha \partial_\nu \phi^{*i} - \frac{1}{2\sqrt{2}} \sigma^\nu \bar\sigma^\rho \sigma^\mu \lambda^{\dagger a} F^a_{\nu\rho}$$

$+ \ldots$





$\partial_\mu j^\mu_\alpha$ has two derivatives

$\to \dfrac{\Delta m^2}{\langle F \rangle} \to$ const as $\langle F \rangle \to 0$

for reasonable $\Delta m^2 \sim (100\ \text{GeV})^2$
$\langle F \rangle$ small enough
$\dfrac{\Delta m^2}{\langle F \rangle}$ interactions important

If include gravity $\Rightarrow$ SUSY gravity

SUSY transformations must be local

$\epsilon_\alpha \to \epsilon_\alpha(x) \Rightarrow$ supergravity

Spin 2 graviton $\Longleftrightarrow$ Spin 3/2 gravitino Ω_μ^α

inhomomegeous SUSY transformation

$$\delta \Omega_\mu^\alpha = -\partial_\mu \epsilon^\alpha + \ldots$$

analogous to gauge particle of susy

when SUSY is broken

gravitino eats the goldstino

(other SuperHiggs mechanism)

$2(3/2)+1 = 4$ helicity states

$$m_{3/2} = \frac{\langle F \rangle}{M_{pl}} \quad \text{cf} \quad m_W \propto g\langle H \rangle$$

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gravity mediated: $m_{3/2} \sim m_{soft}$

$\frac{\Delta m_{soft}^2}{F} = \frac{m_{soft}}{F}\frac{F}{M_{pl}} = \frac{m_{soft}}{M_{pl}}$

gravity strength interactions $\Rightarrow$ not important for colliders

can overclose universe

gauge mediation $m_{3/2} = \frac{F}{M_{pl}}$ $M_{soft} = \frac{\alpha}{4\pi} \frac{F}{M_{mess}}$

if $M_{mess} << M_{pl}$ $m_{3/2} << m_{soft}$

gravitino is the LSP

goldstinos have non-gravitational interaction

gravitino componen are irrellevant at colliders

$$\Gamma(\tilde{\chi} \to \chi \tilde{\pi}) = \frac{m_{\chi}^5}{16\pi \langle F \rangle^2} \left(1 - \frac{m_{\chi}^2}{m_{\tilde{\chi}}^2}\right)^4$$

$m_{\tilde{\chi}} \simeq 100$ GeV $>> m_{\chi}$

if $\sqrt{\langle F \rangle} < 10^3$ TeV

$\Gamma > 2 \times 10^{-16}$ GeV

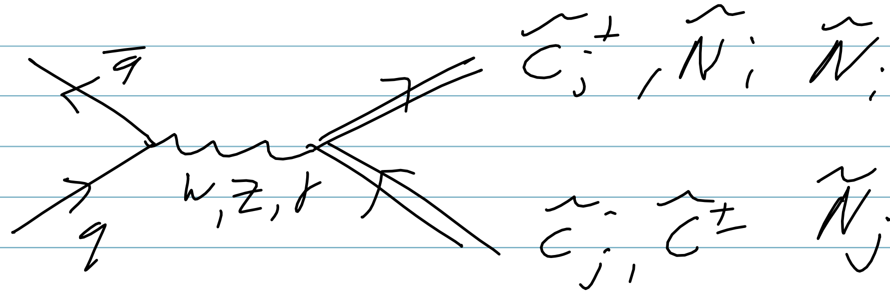
$$c\tau = \frac{c}{\Gamma} < 5 \times 10^{15} \text{ GeV}^{-1} \frac{1\text{ s} \quad 3 \times 10^8 \text{ m/s}}{1.5 \times 10^{24} \text{ GeV}^{-1}}$$

< 1 m

$\tilde{\chi} \to \chi + \tilde{\pi}$ could be observed

Finding SUSY

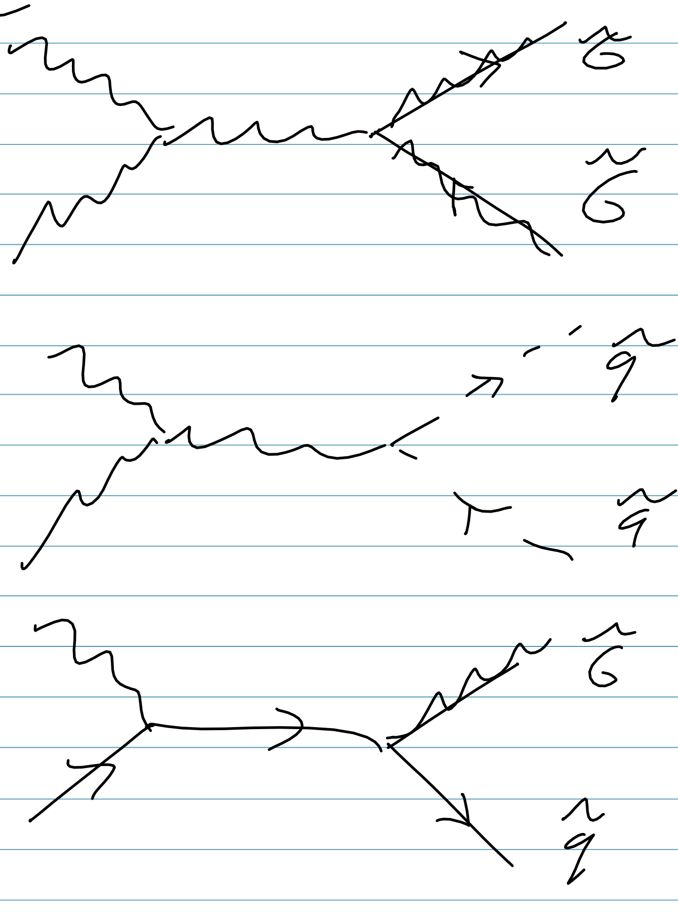
~~LHC~~



Tevatron

$p\bar{p}$

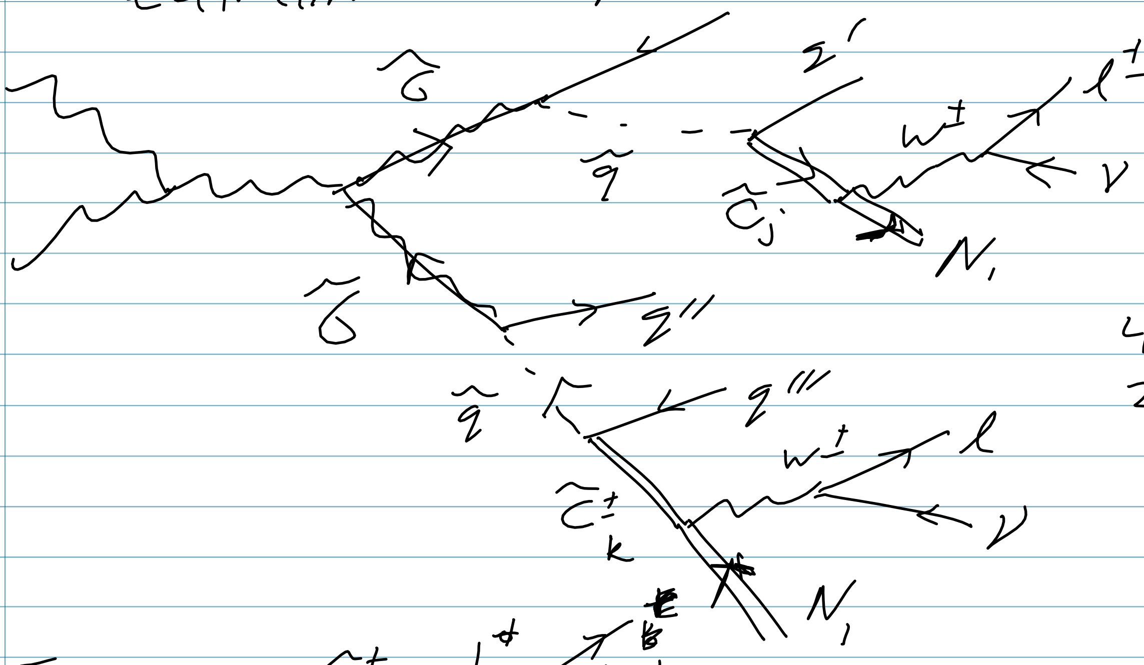
~~LHC~~



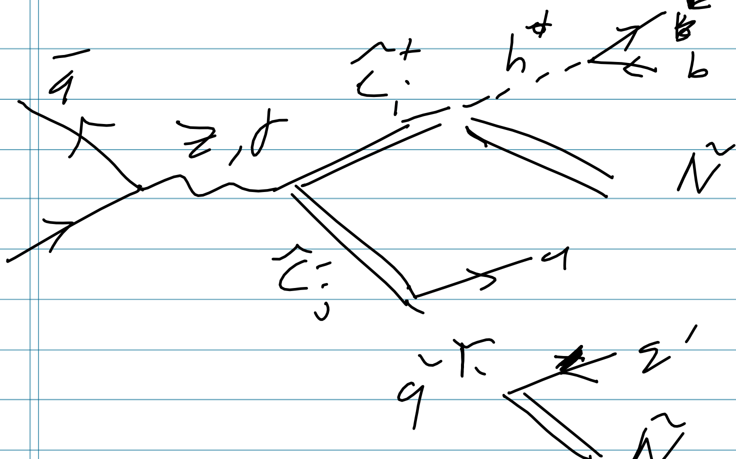
LHC

pp

Neutralino LSP



4 jets
2 leptons
$\not{E}$ ⇓
same
sign
50%



4jets
no leptons
$\not{E}$

goldstino LSP, NLSP $U(1)_Y \to \tilde N, \tilde l_R$

decay length submicron to multi km $\sqrt{F} \geq 10^3$ TeV

NLSP escape

$\tilde N$ NLSP superpair $\to X + \tilde N + \tilde N$
$\qquad \hookrightarrow \gamma \tilde\pi \quad \hookrightarrow \gamma \tilde\pi$

$\to X + 2\gamma + \not{E}$ short decay length

$\tilde l_R$ NLS superpair $\to X + 2\gamma + \not{E}$

Goldstino Theorem

$$Q^{\alpha}|0\rangle \neq 0 \iff \langle 0|H|0\rangle \neq 0$$

$$\{Q_\alpha, Q^{+}_{\dot\alpha}\} = 2\sigma^{\mu}_{\alpha\dot\alpha} P_\mu$$

$$Q^{+}_{\alpha} = \sqrt{2}\int d^3x\, J^{+0}_{\alpha}(\vec{x})$$

$$\int d^3x \langle 0|\{Q_\alpha, \sqrt{2}\, J^{+0}_{\alpha}(\vec{x})\}|0\rangle = 2\sigma^{\mu}_{\alpha\dot\alpha}\int d^3x \langle 0|T^{0}_{\mu}(\vec{x})|0\rangle$$

$$\frac{1}{\sqrt{2}}\langle 0|\{Q_\alpha, J^{+\nu}_{\alpha}(0)\}|0\rangle = 2\sigma^{\mu}_{\alpha\dot\alpha}\langle 0|T^{\nu}_{\mu}(0)|0\rangle$$

$$\qquad\qquad\qquad\qquad\qquad\qquad \overset{\shortparallel}{E\,\delta^{\nu}_{\mu}}$$

$$0 \neq \langle 0|\{\int d^3x\, J^{0}_{\alpha}(\vec{x}), J^{+\nu}_{\dot\alpha}(0)\}|0\rangle$$

$$= \int d^3x \langle 0| J^{0}_{\alpha}(\vec{x}) J^{+\nu}_{\dot\alpha}(0) + J^{+\nu}_{\dot\alpha}(0) J^{0}_{\alpha}(\vec{x})|0\rangle$$

$$\neq 0 \implies \text{SUSY broken}$$

$$\int d^3x \, \langle 0 | J^0_\alpha(\vec{x}) J^{+\nu}_{\dot{\alpha}}(0) + J^{+\nu}_{\dot{\alpha}}(0) J^0_\alpha(\vec{x}) | 0 \rangle$$

$$= \sum_n \int d^3x \langle 0 | e^{i\vec{P}\cdot\vec{x}} J^0_\alpha(0) e^{-i\vec{P}\cdot\vec{x}} | n \rangle \langle n | J^{+\nu}_{\dot{\alpha}}(0) | 0 \rangle$$
$$+ \langle 0 | J^{+\nu}_{\dot{\alpha}}(0) | n \rangle \langle n | e^{i\vec{P}\cdot\vec{x}} J^0_\alpha(0) e^{-i\vec{P}\cdot\vec{x}} | 0 \rangle$$

$$= \sum_n \int d^3x \langle 0 | J^0_\alpha(0) e^{-i\vec{p}_n\cdot\vec{x}} | n \rangle \langle n | J^{+\nu}_{\dot{\alpha}}(0) | 0 \rangle$$
$$+ \langle 0 | J^{+\nu}_{\dot{\alpha}}(0) | n \rangle \langle n | e^{+i\vec{p}_n\cdot\vec{x}} J^0_\alpha(0) | 0 \rangle$$

$$= \sum_n (2\pi)^3 \delta^{(3)}(\vec{p}_n) \Big(\langle 0 | J^0_\alpha(0) | n \rangle \langle n | J^{+\nu}_{\dot{\alpha}}(0) | 0 \rangle$$
$$+ \langle 0 | J^{+\nu}_{\alpha}(0) | n \rangle \langle n | J^0_\alpha(0) | 0 \rangle \Big)$$

$$= \sum_n (2\pi)^3 \delta^{(3)}(\vec{p}_n) \, f(E_n) \qquad ①$$

$$\int d^4x \, \langle 0 | J^0_\alpha(x) J^{+\nu}_{\dot{\alpha}}(0) + J^{+\nu}_{\dot{\alpha}}(0) J^0_\alpha(x) | 0 \rangle \delta(t)$$

$$= \int d^4x \, \partial_\rho \Big(\langle 0 | J^\rho_\alpha(x) J^{+\nu}_{\dot{\alpha}}(0) | 0 \rangle \theta(t) - \langle 0 | J^{+\nu}_{\dot{\alpha}}(0) J^\rho_\alpha(x) | 0 \rangle \theta(-t) \Big)$$

$$= \sum_n \int d^4x \, \partial_\rho \Big(\langle 0 | J^\rho_\alpha(0) e^{-ip_n\cdot x} | n \rangle \langle n | J^{+\nu}_{\dot{\alpha}}(0) | 0 \rangle \theta(t)$$
$$- \langle 0 | J^{+\nu}_{\dot{\alpha}}(0) | n \rangle \langle n | e^{ip_n\cdot x} J^\rho_\alpha(0) | 0 \rangle \theta(-t) \Big)$$

$$= \sum_n \int d^4x \Big[(-ip_{n\rho}) \Big(e^{-ip_n\cdot x} \langle 0 | J^\rho_\alpha(0) | n \rangle \langle n | J^{+\nu}_{\dot{\alpha}}(0) | 0 \rangle \theta(t)$$
$$+ e^{ip_n\cdot x} \langle 0 | J^{+\nu}_{\alpha}(0) | n \rangle \langle n | J^\rho_{\alpha}(0) | 0 \rangle \theta(-t) \Big)$$
$$\delta(t) \Big(e^{-ip_n\cdot x} \langle 0 | J^0_\alpha(0) | n \rangle \langle n | J^{+\nu}_{\alpha}(0) | 0 \rangle$$
$$+ e^{ip_n\cdot x} \langle 0 | J^{+\nu}_{\dot{\alpha}}(0) | n \rangle \langle n | J^\rho_\alpha(0) | 0 \rangle \Big)$$

$$= \sum_n (2\pi)^3 \delta^{(3)}(\vec{p}_n) \left(\int dt \, (-iE_n) \left(e^{-iE_n t} \langle 0 | J^0_\alpha(0) | n \rangle \langle n | J^{+\nu}_{\dot\alpha}(0) | 0 \rangle \theta(t) + e^{iE_n t} \langle 0 | J^{+\nu}_{\dot\alpha}(0) | n \rangle \langle n | J^0_\alpha(0) | 0 \rangle \theta(-t) \right) \right.$$

$$\left. + \langle 0 | J^0_\alpha(0) | n \rangle \langle n | J^+_\alpha(0) | 0 \rangle + \langle 0 | J^{+\nu}_{\dot\alpha}(0) | n \rangle \langle n | J^0_\alpha(0) | 0 \rangle \right)$$

$$= \sum_n (2\pi)^3 \delta^{(3)}(\vec{p}_n) \left(-i \int_0^\infty dt \, e^{-iE_n} E_n \, f(E_n) + f(E_n) \right)$$

$$\int_0^\infty dt \, e^{-iE_n t} E_n f(E_n) = 0$$

$f(E_n) \neq 0 \Rightarrow$ ~~broken susy~~

$$f(E_n) \propto \delta(E_n)$$

$\exists$ a fermionic state with $\vec{p} = 0$, $E = 0$