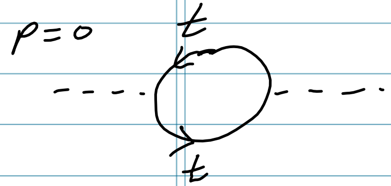
Introduction to Supersymmetry

$$\mathcal{L}_{int} = -\frac{y_t}{\sqrt{2}} H^0 \bar{t}_L t_R \quad + h.c.$$

$$H^0 = \langle H^0 \rangle + h \qquad m_t = \frac{y_t}{\sqrt{2}} \langle H^0 \rangle$$



$$= N_c(-1)\int \frac{d^4k}{(2\pi)^4}\, Tr\left(\frac{-i y_t}{\sqrt{2}} \frac{i}{\not{k} - m_t}\left(\frac{-i y_t^*}{\sqrt{2}}\right)\frac{i}{\not{k} - m_t}\right)$$

$$= -\frac{N_c}{2} |y_t|^2 \int \frac{d^4k}{(2\pi)^4}\, Tr\left(\frac{(\not{k} + m_t)(\not{k} + m_t)}{(k^2 - m_t^2)^2}\right)$$

$$= -\frac{N_c}{2} |y_t|^2 \int \frac{d^4k}{(2\pi)^4} \frac{4(k^2 + m_t^2)}{(k^2 - m_t^2)^2}$$

$$= -2\frac{N_c}{\cancel{2}} |y_t|^2 \int \frac{i\, d^4k_E}{(2\pi)^4} \frac{(-k_E^2 + m_t^2)}{(-k_E^2 - m_t^2)^2}$$

$$= +2iN_c |y_t|^2 \int_0^{\Lambda^2} \frac{2\pi^2}{(2\pi)^4} \frac{1}{2} k_E^2\, dk_E^2 \frac{(k_E^2 - m_t^2)}{(k_E^2 + m_t^2)^2}$$

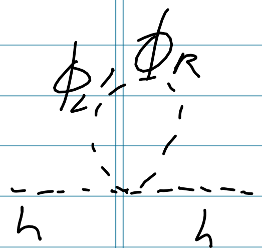
$$= \frac{2iN_c}{16\pi^2} |y_t|^2 \int_{m_t^2}^{\Lambda^2 + m_t^2} dx \frac{(x - m_t^2)(x - 2m_t^2)}{x^2}$$

$$= \frac{2iN_c}{16\pi^2} |y_t|^2 \int_{m_t^2}^{\Lambda^2 + m_t^2} dx \left(1 - 3\frac{m_t^2}{x} + 2\frac{m_t^4}{x^2}\right)$$

$$= \frac{2iN_c}{16\pi^2} |y_t|^2 \left[x - 3m_t^2 \ln x \cancel{+} - 2\frac{m_t^4}{x}\right]_{m_t^2}^{\Lambda^2 + m_t^2}$$

$$= \frac{2iN_c}{16\pi^2} |y_t|^2 \left(\Lambda^2 - 3m_t^2 \ln\left(\frac{\Lambda^2 + m_t^2}{m_t^2}\right) - \text{finite}\right)$$

$$\mathcal{L}_{int} = -\frac{\lambda}{2} h^2 \left(|\phi_L|^2 + |\phi_R|^2\right) - h\left(\mu_L |\phi_L|^2 + \mu_R |\phi_R|^2\right) - m_L^2 |\phi_L|^2 - m_R^2 |\phi_R|^2$$



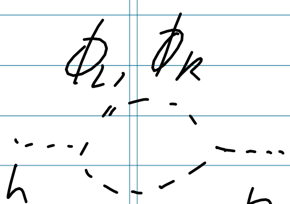
$$= -i\lambda N \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m_L^2} + \frac{i}{k^2 - m_R^2}$$

$$= -i\lambda N \int \frac{i\, d^4k_E}{(2\pi)^4} \frac{i}{-k_E^2 - m_L^2} + \frac{i}{-k_E^2 - m_R^2}$$

$$= -i\lambda N \int_0^{\Lambda^2} \frac{2\pi^2}{(2\pi)^4} \frac{1}{2} k_E^2\, dk_E^2 \left(\frac{1}{k_E^2 + m_L^2} + \frac{1}{k_E^2 + m_R^2}\right)$$

$$= \frac{-i\lambda N}{16\pi^2}\left(\int_{m_L^2}^{\Lambda^2 + m_L^2} dx\, \frac{(x - m_L^2)}{x} + \int_{m_R^2}^{\Lambda^2 + m_R^2} dx\, \frac{(x - m_R^2)}{x}\right)$$

$$= \frac{-i\lambda N}{16\pi^2}\left(2\Lambda^2 - m_L^2 \ln\left(\frac{\Lambda^2 + m_L^2}{m_L^2}\right) - m_R^2 \ln\left(\frac{\Lambda^2 + m_R^2}{m_R^2}\right) + \text{finite}\right)$$



$$= N \int \frac{d^4k}{(2\pi)^4} (-i\mu_L)^2 \left(\frac{i}{k^2 - m_L^2}\right)^2 + (-i\mu_R)^2 \left(\frac{i}{k^2 - m_R^2}\right)^2$$

$$= iN \int \frac{d^4k_E}{(2\pi)^4} \frac{\mu_L^2}{(k_E^2 + m_L^2)^2} + \frac{\mu_R^2}{(k_E^2 + m_R^2)^2}$$

$$= \frac{iN}{16\pi^2}\left(\int_{m_L}^{\Lambda^2 + m_L^2} dx\, \frac{\mu_L^2 (x - m_L^2)}{x^2} + \mu_L \leftrightarrow \mu_R\right)$$

$$= \frac{iN}{16\pi^2}\left(\mu_L^2 \ln\left(\frac{\Lambda^2 + m_L^2}{m_L^2}\right) + \mu_R^2 \ln\left(\frac{\Lambda^2 + m_R^2}{m_R^2}\right) + \text{finite}\right)$$

if $N = N_c$ and $\lambda = |y_t|^2 \Rightarrow \Lambda^2$ terms cancel

if $m_t = m_L = m_R$ and $\mu_L^2 = \mu_R^2 = 2\lambda m_t^2 \Rightarrow \ln \Lambda^2$ terms cancel

To cancel Λ^2 order by order
we need a symmetry

$SU(6) \to$ Coleman & Mandula $\Rightarrow$ Poincaré $\times$ Internal
$SU(6)$ sucks

Golfand Likhtman $\to$ spinor generators

$$\text{Boson} \underset{}{\overset{Q_\alpha}{\rightleftarrows}} \text{Fermion}$$

extended by Haag, Lopuszanski, Sohnius

$$\{Q_\alpha, Q^\dagger_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha} P_\mu \, , \quad g^{\mu\nu} = g^{\mu\nu}(+1,-1,-1,-1)$$

spinor index 1,2 $\quad \{Q_\alpha, Q_\beta\} = 0 \quad \{Q^\dagger_{\dot\alpha}, Q^\dagger_{\dot\beta}\} = 0$

$$[P^\mu, Q_\alpha] = 0 \qquad [P^\mu, Q^\dagger_{\dot\beta}] = 0$$

$U(1)_R$ symmetry $\quad [Q_\alpha, R] = Q_\alpha \quad [Q^\dagger_{\dot\alpha}, R] = -Q^\dagger_{\dot\alpha}$

Q_α is anticommuting spinor

P_μ is momentum (generator of translations)

$$\sigma^\mu_{\alpha\dot\beta} = (1, \sigma^i)$$

$$\bar\sigma^{\mu\dot\alpha\beta} = (1, -\sigma^i)$$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$Q_1 Q_1^\dagger + Q_1^\dagger Q_1 = 2\sigma^0_{11} P_0 + 2\sigma^3_{11} P_3$$
$$Q_2 Q_2^\dagger + Q_2^\dagger Q_2 = 2\sigma^0_{22} P_0 + 2\sigma^3_{22} P_3$$

$$\delta^{\alpha\dot\alpha}\left(Q_\alpha Q_{\dot\alpha}^\dagger + Q_{\dot\alpha}^\dagger Q_\alpha\right) = 4P_0$$

$$H = \frac{1}{4}\left(Q_\alpha Q_{\dot\alpha}^\dagger + Q_{\dot\alpha}^\dagger Q_\alpha\right)\delta^{\alpha\dot\alpha} \geq 0$$

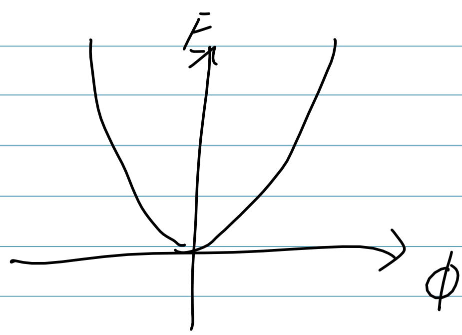
$|0\rangle$ is supersymmetric $\Rightarrow Q_\alpha|0\rangle = 0$

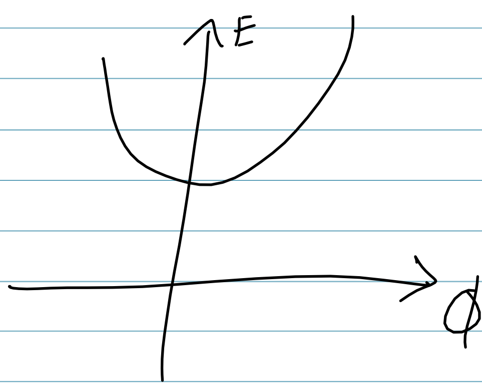
$$\Downarrow$$

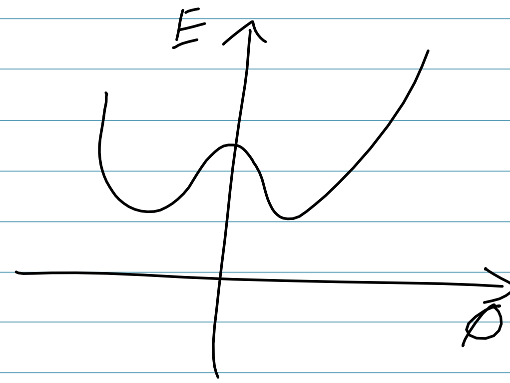
$$\langle 0|H|0\rangle = 0$$

$|0\rangle$ is not supersymmetric $\Rightarrow Q_\alpha|0\rangle \neq 0$

$$\langle 0|H|0\rangle > 0$$







single particle states into irreps of the SUSY algebra call <u>supermultiplets</u>

fermions and bosons in one s.m.
they are called superpartners
P^2 commute with $Q, Q^\dagger$ so all masses in s.m. are the same
gauge generators commute with $Q, Q^\dagger$ → same gauge rep.

$$(-1)^F |\text{boson}\rangle = +1 |\text{boson}\rangle$$
$$(-1)^F |\text{fermion}\rangle = -1 |\text{fermion}\rangle$$
$$\{(-1)^F, Q_\alpha\} = 0$$

consider $|i\rangle$ in s.m. subspace with same p^μ

$$\sum_i |i\rangle\langle i| = 1 \quad \text{in s.m.}$$

$$Tr\left((-1)^F p^0\right) = p^0\, Tr\left((-1)^F\right)$$

$$= \sum_i \langle i | (-1)^F p^0 | i\rangle = \frac{1}{4} \sum_i \langle i | (-1)^F \left(Q_\alpha Q_\alpha^\dagger + Q_\alpha^\dagger Q_\alpha\right) | i\rangle$$

$$= \frac{1}{4}\left(\sum_i \langle i | (-1)^F Q_\alpha Q_\alpha^\dagger | i\rangle + \sum_i \sum_j \langle i | (-1)^F Q^\dagger | j\rangle\langle j | Q_\alpha | i\rangle\right)$$

$$= \frac{1}{4}\left(\sum_i \langle i | (-1)^F Q_\alpha Q_\alpha^\dagger | i\rangle + \sum_i \sum_j \langle j | Q_\alpha | i\rangle \langle i | (-1)^F Q^\dagger | j\rangle\right)$$

$$= \frac{1}{4}\left(\sum_i \langle i | (-1)^F Q_\alpha Q_\alpha^\dagger | i\rangle + \sum_j \langle j | Q_\alpha (-1)^F Q_\alpha^\dagger | j\rangle\right)$$

$$= \frac{1}{4}\left(\sum_i \langle i | (-1)^F Q_\alpha Q_\alpha^\dagger | i\rangle - \sum_j \langle j | (-1)^F Q_\alpha Q_\alpha^\dagger | j\rangle\right)$$

$$= 0$$

$n_B = n_F$ in each s.m. with $p^0 \neq 0$