

# Larmor Precession

$$\vec{B} = B_0 \hat{z} \quad \vec{\mu} = \gamma \vec{S} \quad \gamma \approx -\frac{e}{m}$$

$$H = -\vec{\mu} \cdot \vec{B} = -\gamma B_0 S_z = \frac{\omega \hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$i\hbar \frac{\partial}{\partial t} \psi(t) = H\psi(t)$$

$$\psi(t) = \begin{pmatrix} \cos(\alpha/2)e^{-i\omega t/2} \\ \sin(\alpha/2)e^{+i\omega t/2} \end{pmatrix}$$

# Larmor Precession

$$\begin{aligned}\langle \psi | S_z | \psi \rangle &= \begin{pmatrix} \cos(\alpha/2)e^{i\omega t/2} & \sin(\alpha/2)e^{-i\omega t/2} \end{pmatrix} \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ &\quad \times \begin{pmatrix} \cos(\alpha/2)e^{-i\omega t/2} \\ \sin(\alpha/2)e^{i\omega t/2} \end{pmatrix} \\ &= \frac{\hbar}{2} (\cos^2(\alpha/2) - \sin^2(\alpha/2)) \\ &= \frac{\hbar}{2} \cos(\alpha)\end{aligned}$$

# Larmor Precession

$$\begin{aligned}\langle \psi | S_x | \psi \rangle &= \begin{pmatrix} \cos(\alpha/2)e^{i\omega t/2} & \sin(\alpha/2)e^{-i\omega t/2} \end{pmatrix} \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ &\quad \times \begin{pmatrix} \cos(\alpha/2)e^{-i\omega t/2} \\ \sin(\alpha/2)e^{i\omega t/2} \end{pmatrix} \\ &= \frac{\hbar}{2} \cos(\alpha/2) \sin(\alpha/2) (e^{i\omega t} + e^{-i\omega t}) \\ &= \frac{\hbar}{2} \sin(\alpha) \cos(\omega t)\end{aligned}$$

# Larmor Precession

$$\begin{aligned}\langle\psi|S_y|\psi\rangle &= \begin{pmatrix} \cos(\alpha/2)e^{i\omega t/2} & \sin(\alpha/2)e^{-i\omega t/2} \end{pmatrix} \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \\ &\quad \times \begin{pmatrix} \cos(\alpha/2)e^{-i\omega t/2} \\ \sin(\alpha/2)e^{i\omega t/2} \end{pmatrix} \\ &= \frac{\hbar}{2} \cos(\alpha/2) \sin(\alpha/2) i (-e^{i\omega t} + e^{-i\omega t}) \\ &= \frac{\hbar}{2} \sin(\alpha) \sin(-\omega t)\end{aligned}$$

# Larmor Precession

$$\langle \psi | S_x | \psi \rangle = \frac{\hbar}{2} \sin(\alpha) \cos(\omega t)$$

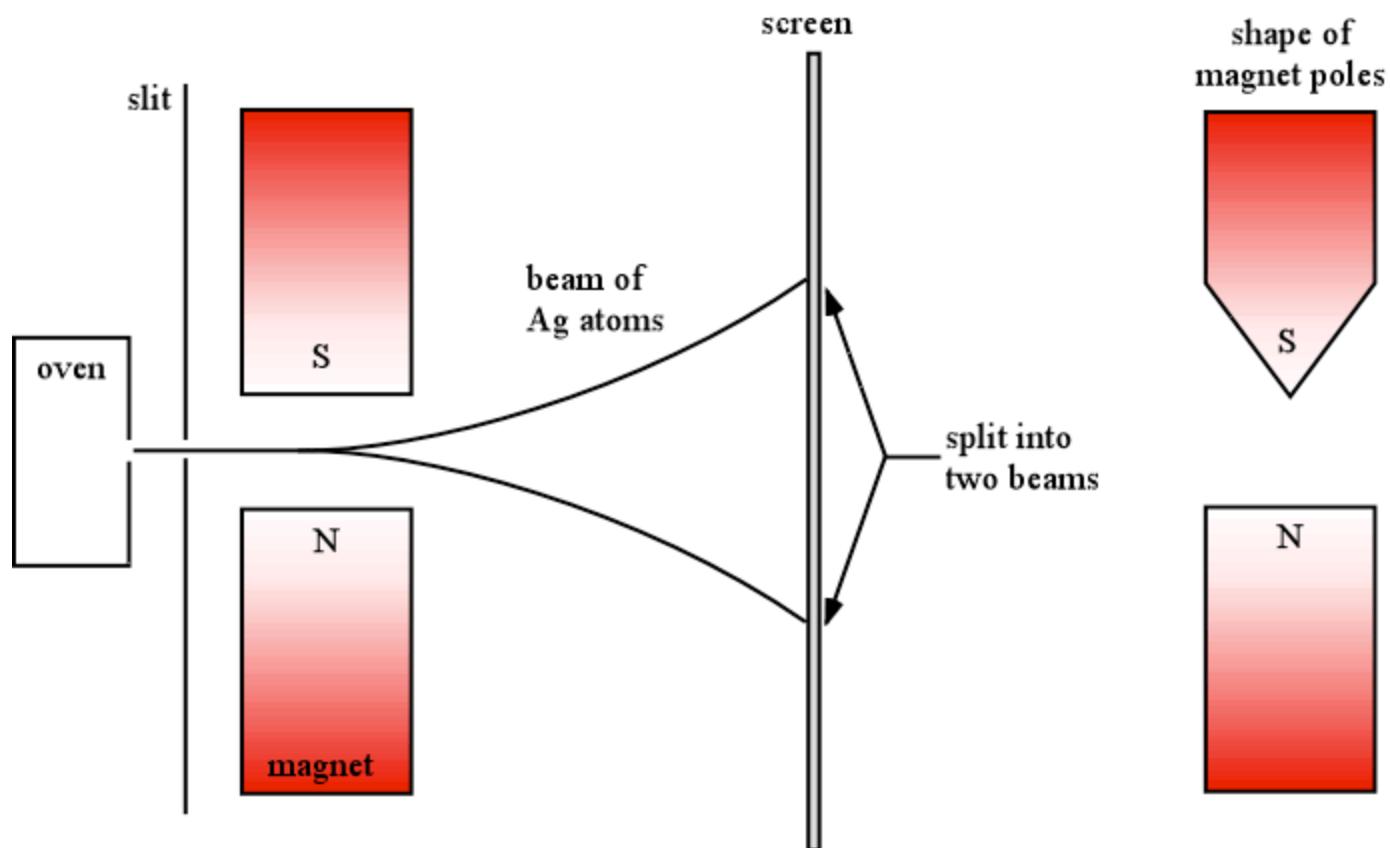
$$\langle \psi | S_y | \psi \rangle = \frac{\hbar}{2} \sin(\alpha) \sin(-\omega t)$$

$$\langle \psi | S_z | \psi \rangle = \frac{\hbar}{2} \cos(\alpha)$$

$$\omega = -\gamma B_0$$

# Stern-Gerlach

## Nobel Prize 1943



STERN-GERLACH EXPERIMENT

# Stern-Gerlach

$$H = -\gamma \vec{B} \cdot \vec{S}$$

$$\vec{B} = -\alpha x \hat{x} + (B_0 + \alpha z) \hat{z} \quad \vec{\nabla} \cdot \vec{B} = 0$$

silver atom rest frame,  $x=0$

$$H = \begin{cases} 0 & t < 0 \\ -\gamma(B_0 + \alpha z)S_z & 0 \leq t \leq T \\ 0 & T < t \end{cases}$$

$$\chi(t < 0) = a\chi_{\uparrow} + b\chi_{\downarrow}$$

$$\chi(0 < t < T) = a\chi_{\uparrow}e^{-i\omega t/2 + i\gamma\alpha zt/2} + b\chi_{\downarrow}e^{i\omega t/2 - i\gamma\alpha zt/2}$$

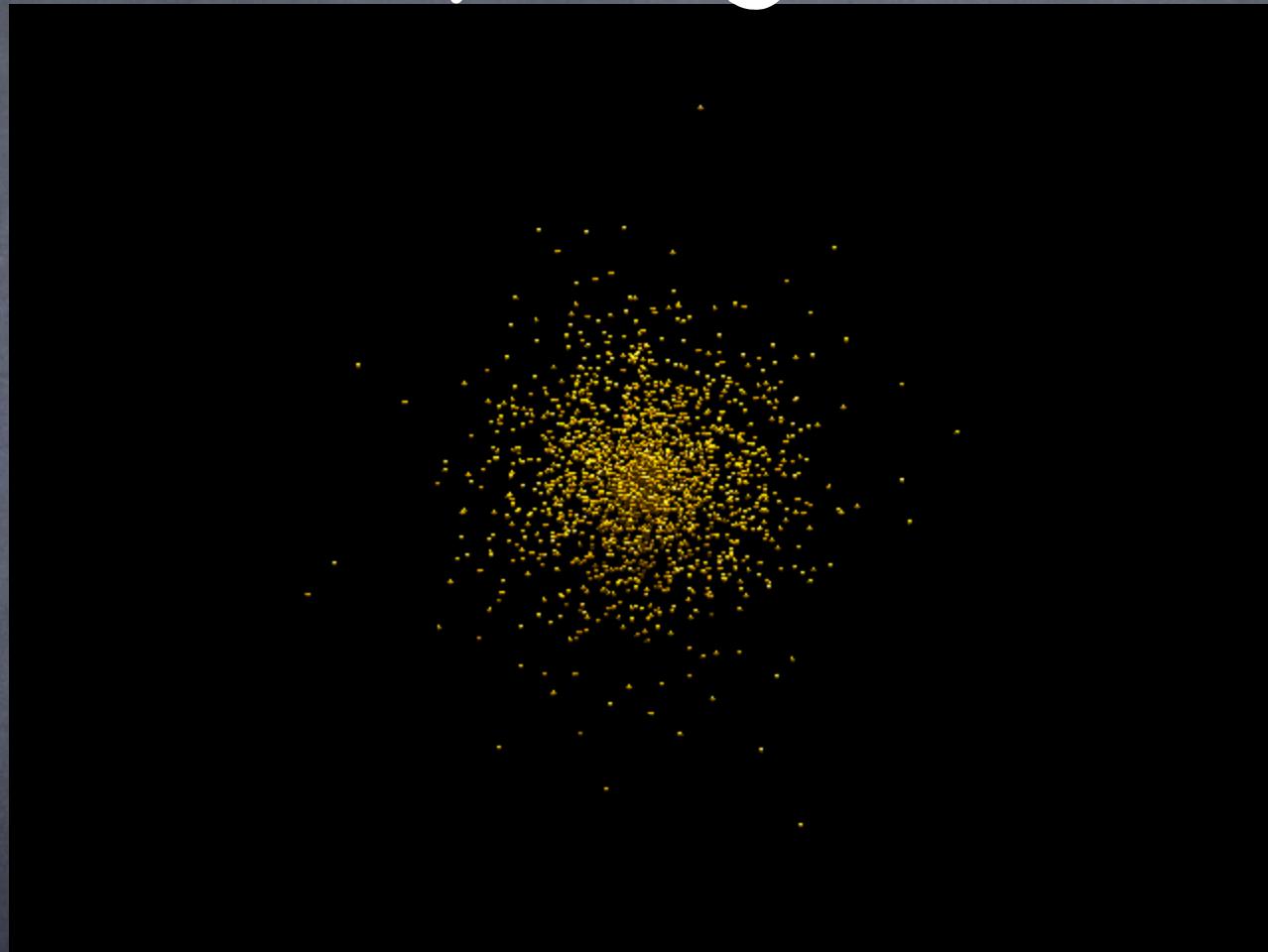
$$\omega = -\gamma B_0$$

# Stern-Gerlach

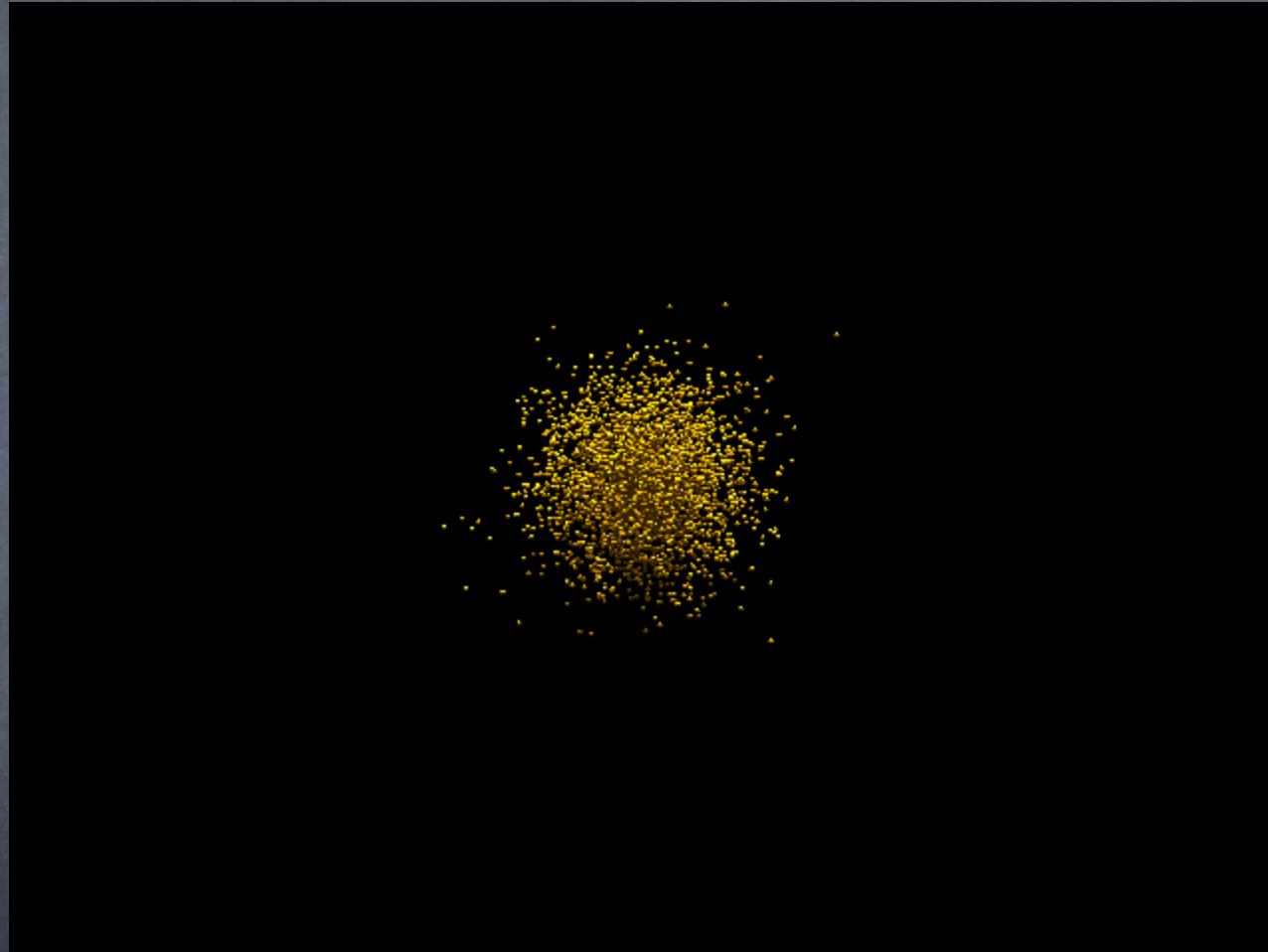
$$\chi(t > T) = a\chi_{\uparrow}e^{-i\omega T/2 + i\gamma\alpha zT/2} + b\chi_{\downarrow}e^{i\omega T/2 - i\gamma\alpha zT/2}$$

$$p_z = \pm \frac{\alpha \gamma T \hbar}{2}$$

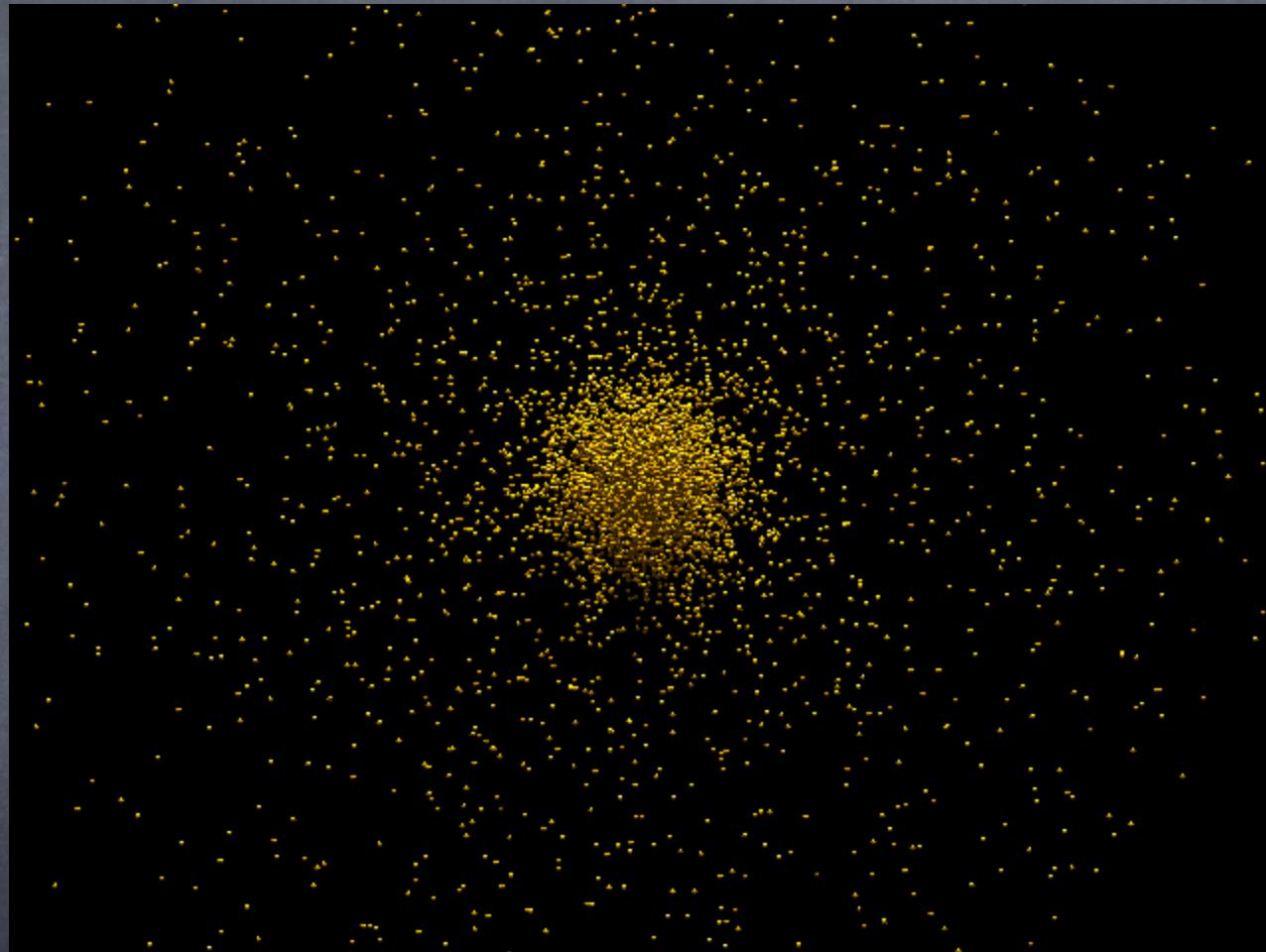
# Hydrogen



# Helium



# Lithium



# Elements

$Z$  protons

ignoring repulsion,  $(n, l, m)$  orbitals

2 electrons per orbital  $\uparrow\downarrow$  (spin singlet)

$n^2$ -fold degeneracy

$n = 1$     2 electrons

$n = 2$      $2^2 \cdot 2 = 8$  electrons

$n = 3$      $3^2 \cdot 2 = 18$  electrons

$n = 4$      $4^2 \cdot 2 = 32$  electrons

$n = 5$      $5^2 \cdot 2 = 50$  electrons

$\vdots$

$\vdots$

# Periodic Table of the Elements

	IA	Periodic Table of the Elements																		O		
1	H	IIA																		He		
2	Li	Be																		Ne		
3	Na	Mg	IIIIB	IVB	VB	VIB	VIIIB	VII		IB	IIB											
4	K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	31	32	33	34	35	36	Kr			
5	Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	54	Xe			
6	Cs	Ba	*La	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	86	Rn			
7	Fr	Ra	+Ac	Rf	Ha	Sg	Ns	Hs	Mt	110	111	112	113	113								
*	Lanthanide Series	58	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu						
+	Actinide Series	90	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr						

2	2
8	8
8	18
18	32
18	50

\* Lanthanide Series

+ Actinide Series

# Elements

Z	Element	outer electrons	
1	Hydrogen	1 in (1,0,0)	
2	Helium	2 in (1,0,0)	
3	Lithium	1 in (2,0,0)	
4	Beryllium	2 in (2,0,0)	
5	Boron	1 in (2,1,m)	
6	Carbon	2 in (2,1,m)	
7	Nitrogen	3 in (2,1,m)	
8	Oxygen	4 in (2,1,m)	
9	Fluorine	5 in (2,1,m)	
10	Neon	6 in (2,1,m)	
11	Sodium	1 in (3,0,0)	
12	Magnesium	2 in (3,0,0)	
13	Aluminum	1 in (3,1,m)	
14	Silicon	2 in (3,1,m)	
15	Phosphorous	3 in (3,1,m)	
16	Sulfur	4 in (3,1,m)	
17	Chlorine	5 in (3,1,m)	
18	Argon	6 in (3,1,m)	
19	Potassium	1 in (4,0,0)	
20	Calcium	2 in (4,0,0)	
21	Scandium	1 in (3,2,m)	
:	:	:	

# Elements

$^{2s+1}L_J$

36 Kr Krypton [Ar](4s)<sup>2</sup>(3d)<sup>10</sup>(4p)<sup>6</sup>  $^1S_0$

47 Ag Silver [Kr](4d)<sup>10</sup>(5s)<sup>1</sup>  $^2S_{1/2}$