Larmor Precession

$$\vec{B} = B_0\hat{z} \qquad \vec{\mu} = \gamma\vec{S} \qquad \gamma \approx -\frac{e}{m}$$

$$H = -\vec{\mu}\cdot\vec{B} = -\gamma B_0 S_z = \frac{\omega\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$i\hbar\frac{\partial}{\partial t}\psi(t) = H\psi(t)$$

$$\psi(t) = \begin{pmatrix} \cos(\alpha/2)e^{-i\omega t/2} \\ \sin(\alpha/2)e^{+i\omega t/2} \end{pmatrix}$$

Larmor Precession

$$\begin{aligned}
\langle\psi|S_z|\psi\rangle &= \left(\cos(\alpha/2)e^{i\omega t/2} \;\; \sin(\alpha/2)e^{-i\omega t/2}\right)\frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\
&\quad \times \begin{pmatrix} \cos(\alpha/2)e^{-i\omega t/2} \\ \sin(\alpha/2)e^{i\omega t/2} \end{pmatrix} \\
&= \frac{\hbar}{2}\left(\cos^2(\alpha/2) - \sin^2(\alpha/2)\right) \\
&= \frac{\hbar}{2}\cos(\alpha)
\end{aligned}$$

Larmor Precession

$$\begin{aligned}
\langle\psi|S_x|\psi\rangle &= \left(\cos(\alpha/2)e^{i\omega t/2} \;\; \sin(\alpha/2)e^{-i\omega t/2}\right)\frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\
&\quad \times \begin{pmatrix} \cos(\alpha/2)e^{-i\omega t/2} \\ \sin(\alpha/2)e^{i\omega t/2} \end{pmatrix} \\
&= \frac{\hbar}{2}\cos(\alpha/2)\sin(\alpha/2)\left(e^{i\omega t} + e^{-i\omega t}\right) \\
&= \frac{\hbar}{2}\sin(\alpha)\cos(\omega t)
\end{aligned}$$

Larmor Precession

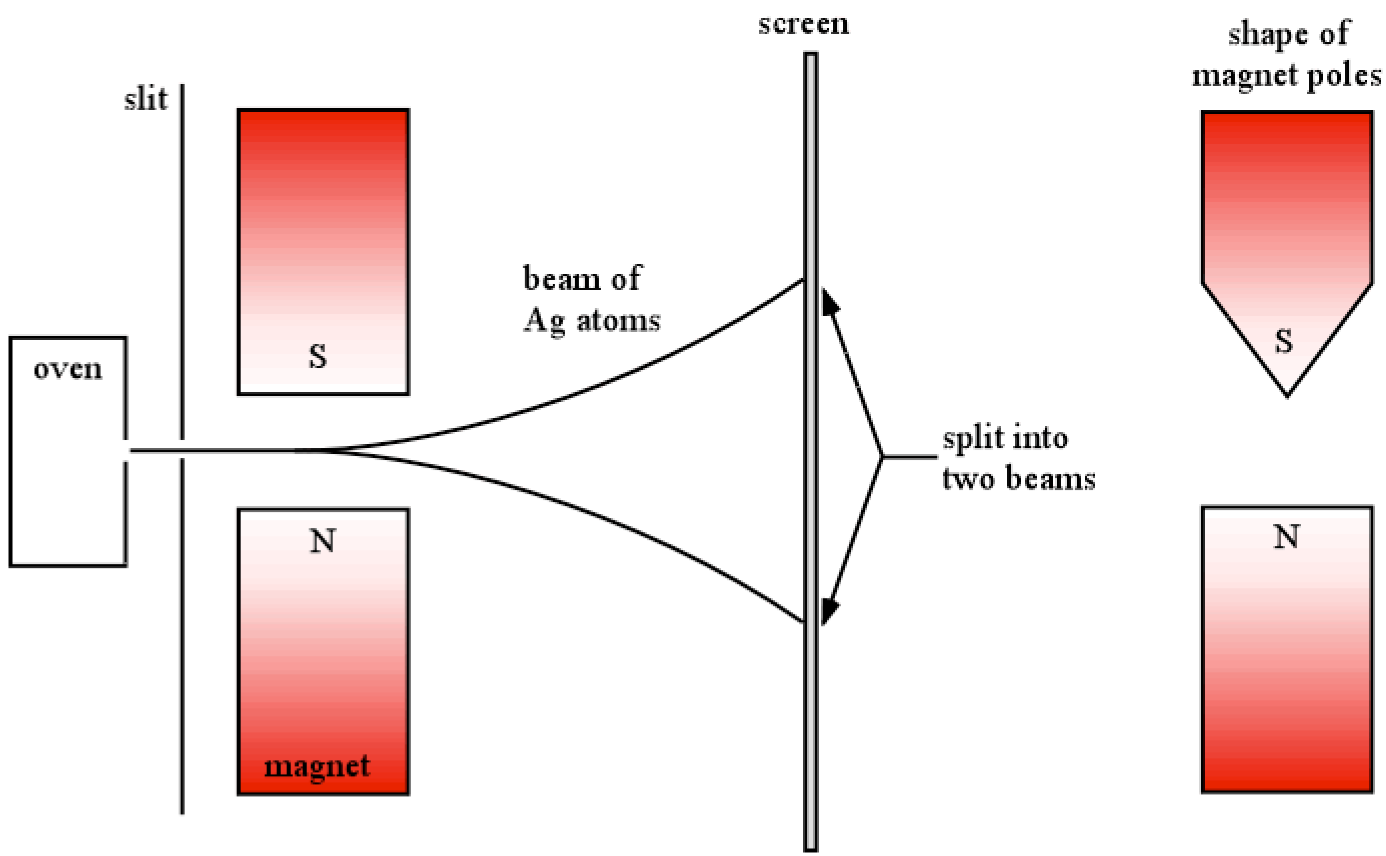
$$
\begin{aligned}
\langle\psi|S_y|\psi\rangle &= \left(\cos(\alpha/2)e^{i\omega t/2} \;\; \sin(\alpha/2)e^{-i\omega t/2}\right)\frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \\
&\quad \times \begin{pmatrix} \cos(\alpha/2)e^{-i\omega t/2} \\ \sin(\alpha/2)e^{i\omega t/2} \end{pmatrix} \\
&= \frac{\hbar}{2}\cos(\alpha/2)\sin(\alpha/2)\, i\, \left(-e^{i\omega t} + e^{-i\omega t}\right) \\
&= \frac{\hbar}{2}\sin(\alpha)\sin(-\omega t)
\end{aligned}
$$

Larmor Precession

$$\langle\psi|S_x|\psi\rangle = \frac{\hbar}{2}\sin(\alpha)\cos(\omega t)$$
$$\langle\psi|S_y|\psi\rangle = \frac{\hbar}{2}\sin(\alpha)\sin(-\omega t)$$
$$\langle\psi|S_z|\psi\rangle = \frac{\hbar}{2}\cos(\alpha)$$

$$\omega = -\gamma B_0$$

Stern-Gerlach
Nobel Prize 1943



STERN-GERLACH EXPERIMENT

Stern-Gerlach

$$H = -\gamma \vec{B} \cdot \vec{S}$$

$$\vec{B} = -\alpha\, x\, \hat{x} + (B_0 + \alpha z)\hat{z} \qquad \vec{\nabla} \cdot \vec{B} = 0$$

silver atom rest frame, x=0

$$H = \begin{cases} 0 & t < 0 \\ -\gamma(B_0 + \alpha z)S_z & 0 \leq t \leq T \\ 0 & T < t \end{cases}$$

$$\chi(t < 0) = a\chi_\uparrow + b\chi_\downarrow$$

$$\chi(0 < t < T) = a\chi_\uparrow e^{-i\omega t/2 + i\gamma\alpha z t/2} + b\chi_\downarrow e^{i\omega t/2 - i\gamma\alpha z t/2}$$

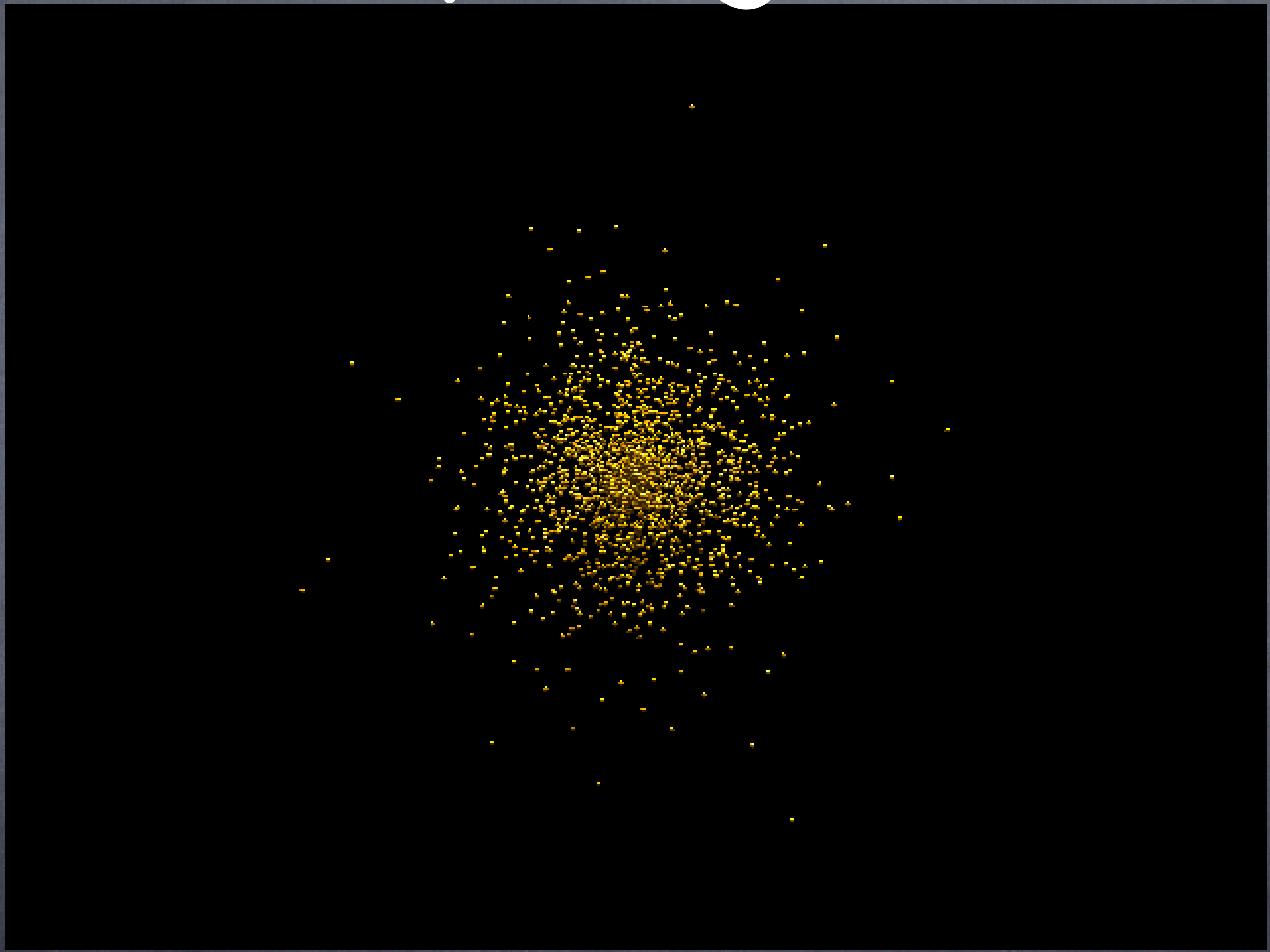
$$\omega = -\gamma B_0$$

Stern-Gerlach

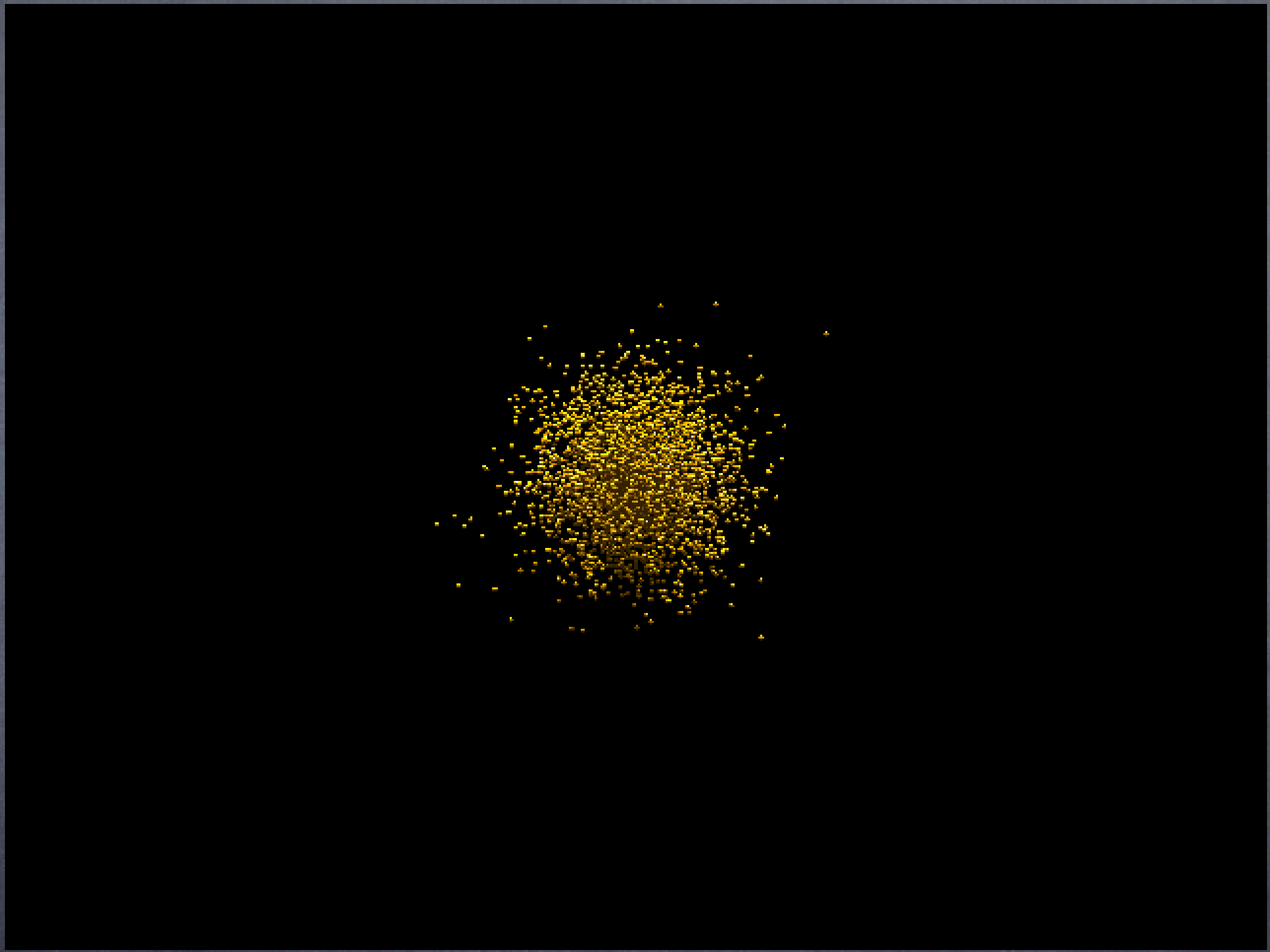
$$\chi(t > T) = a\chi_{\uparrow}e^{-i\omega T/2 + i\gamma\alpha z T/2} + b\chi_{\downarrow}e^{i\omega T/2 - i\gamma\alpha z T/2}$$

$$p_z = \pm\frac{\alpha\gamma T\hbar}{2}$$

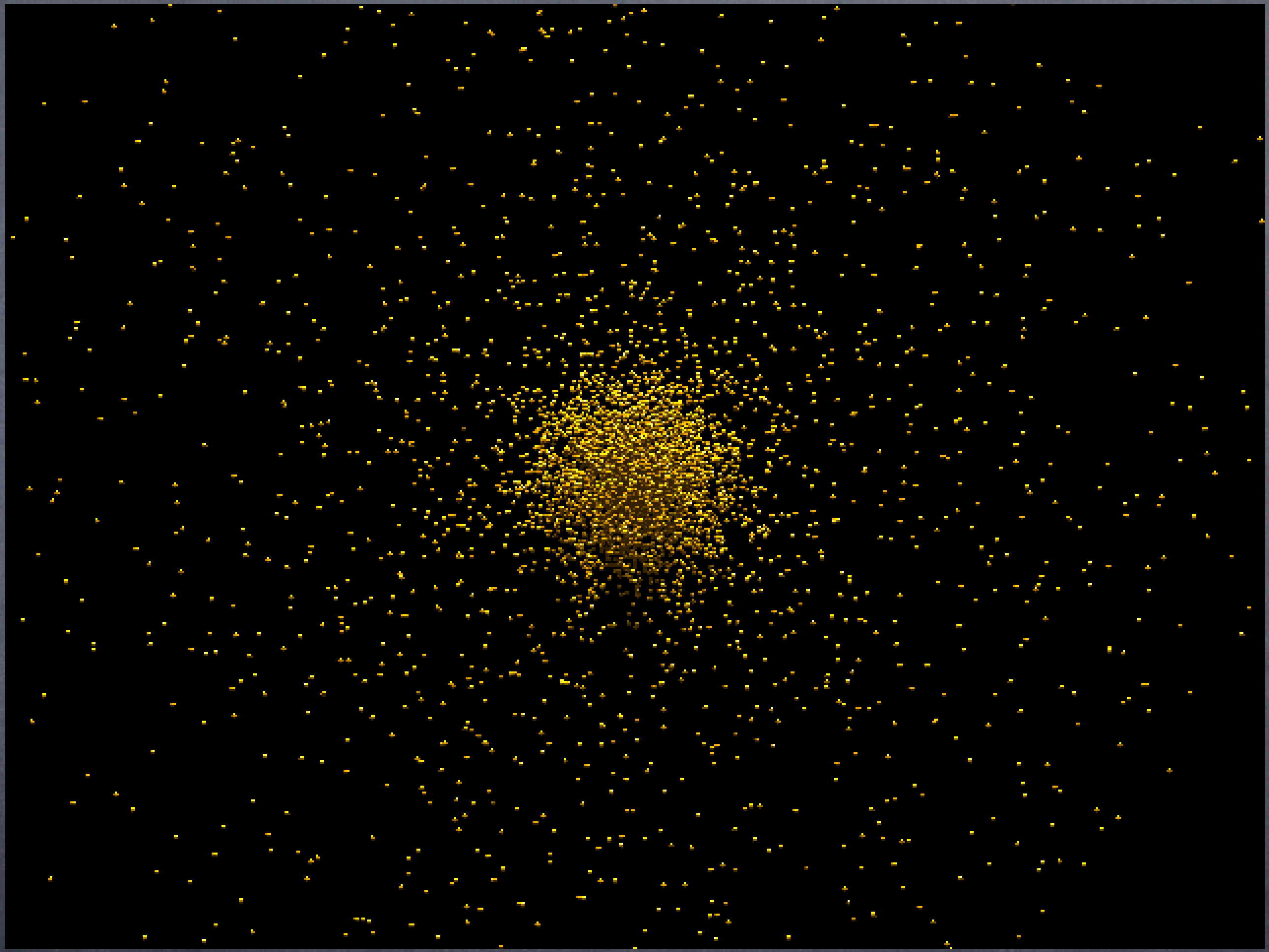
Hydrogen



Helium



Lithium



Elements

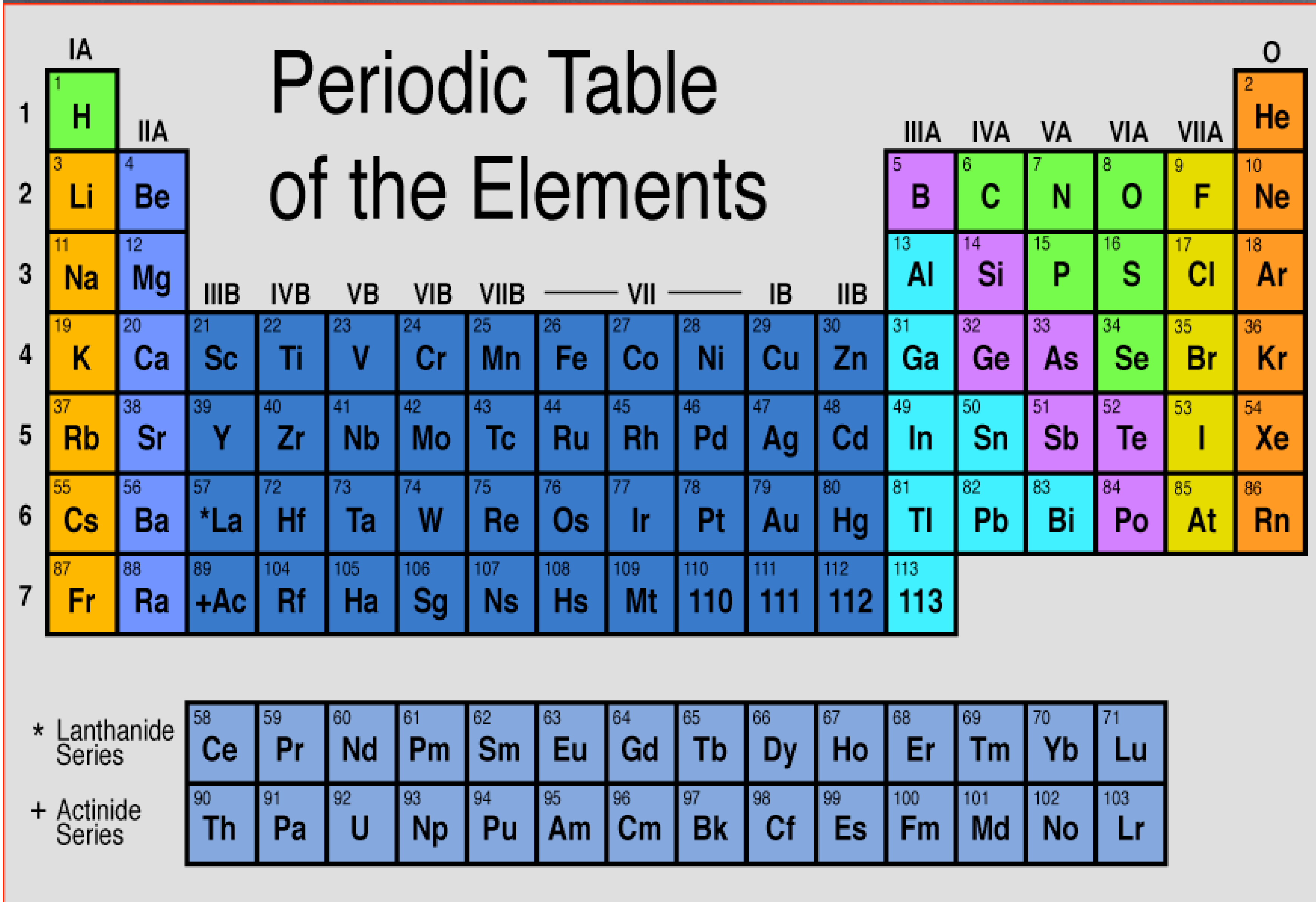
Z protons

ignoring repulsion, (n, l, m) orbitals

2 electrons per orbital $\uparrow\downarrow$ (spin singlet)

n^2-fold degeneracy

$$
\begin{array}{ll}
n = 1 & 2 \text{ electrons} \\
n = 2 & 2^2 \cdot 2 = 8 \text{ electrons} \\
n = 3 & 3^2 \cdot 2 = 18 \text{ electrons} \\
n = 4 & 4^2 \cdot 2 = 32 \text{ electrons} \\
n = 5 & 5^2 \cdot 2 = 50 \text{ electrons} \\
\vdots & \vdots
\end{array}
$$



2	2
8	8
8	18
18	32
18	50

Elements

Z	Element	outer electrons
1	Hydrogen	1 in (1,0,0)
2	Helium	2 in (1,0,0)
3	Lithium	1 in (2,0,0)
4	Beryllium	2 in (2,0,0)
5	Boron	1 in (2,1,m)
6	Carbon	2 in (2,1,m)
7	Nitrogen	3 in (2,1,m)
8	Oxygen	4 in (2,1,m)
9	Fluorine	5 in (2,1,m)
10	Neon	6 in (2,1,m)
11	Sodium	1 in (3,0,0)
12	Magnesium	2 in (3,0,0)
13	Aluminum	1 in (3,1,m)
14	Silicon	2 in (3,1,m)
15	Phosphorous	3 in (3,1,m)
16	Sulfur	4 in (3,1,m)
17	Chlorine	5 in (3,1,m)
18	Argon	6 in (3,1,m)
19	Potassium	1 in (4,0,0)
20	Calcium	2 in (4,0,0)
21	Scandium	1 in (3,2,m)
⋮	⋮	⋮

Boron–Neon (Z = 5–10): $(2\ell+1)\times 2=6$

Aluminum–Argon (Z = 13–18): $(2\ell+1)\times 2=6$

Elements

			$^{2s+1}L_J$
36 Kr	Krypton	$[Ar](4s)^2(3d)^{10}(4p)^6$	$^1S_0$
47 Ag	Silver	$[Kr](4d)^{10}(5s)^1$	$^2S_{1/2}$