Larmor Precession

$$\vec{B} = B_0 \hat{z} \qquad \vec{\mu} = \gamma \vec{S} \qquad \gamma \approx -\frac{e}{m}$$

$$H = -\vec{\mu} \cdot \vec{B} = -\gamma B_0 S_z = \frac{\omega \hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$i\hbar \frac{\partial}{\partial t} \psi(t) = H \psi(t)$$

$$\psi(t) = \begin{pmatrix} \cos(\alpha/2) e^{-i\omega t/2} \\ \sin(\alpha/2) e^{+i\omega t/2} \end{pmatrix}$$

Larmor Precession

$$
\begin{aligned}
\langle\psi|S_z|\psi\rangle &= \left(\cos(\alpha/2)e^{i\omega t/2} \;\; \sin(\alpha/2)e^{-i\omega t/2}\right) \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\
&\quad \times \begin{pmatrix} \cos(\alpha/2)e^{-i\omega t/2} \\ \sin(\alpha/2)e^{i\omega t/2} \end{pmatrix} \\
&= \frac{\hbar}{2}\left(\cos^2(\alpha/2) - \sin^2(\alpha/2)\right) \\
&= \frac{\hbar}{2}\cos(\alpha)
\end{aligned}
$$

Larmor Precession

$$
\begin{aligned}
\langle\psi|S_x|\psi\rangle &= \left(\cos(\alpha/2)e^{i\omega t/2} \;\; \sin(\alpha/2)e^{-i\omega t/2}\right)\frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\
&\quad \times \begin{pmatrix} \cos(\alpha/2)e^{-i\omega t/2} \\ \sin(\alpha/2)e^{i\omega t/2} \end{pmatrix} \\
&= \frac{\hbar}{2}\cos(\alpha/2)\sin(\alpha/2)\left(e^{i\omega t} + e^{-i\omega t}\right) \\
&= \frac{\hbar}{2}\sin(\alpha)\cos(\omega t)
\end{aligned}
$$

Larmor Precession

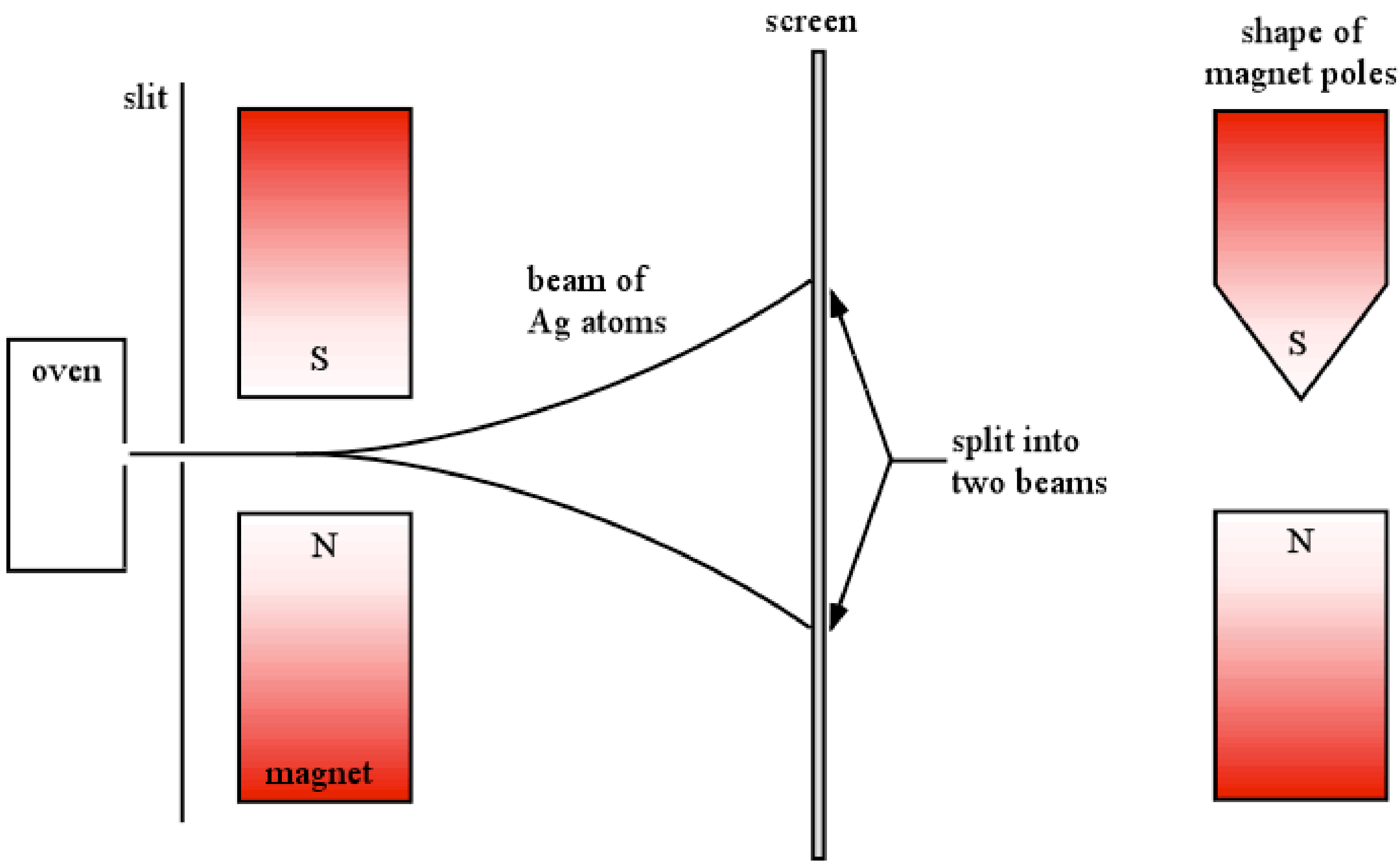
$$\begin{aligned}
\langle\psi|S_y|\psi\rangle &= \left(\cos(\alpha/2)e^{i\omega t/2} \;\; \sin(\alpha/2)e^{-i\omega t/2}\right)\frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \\
&\quad \times \begin{pmatrix} \cos(\alpha/2)e^{-i\omega t/2} \\ \sin(\alpha/2)e^{i\omega t/2} \end{pmatrix} \\
&= \frac{\hbar}{2}\cos(\alpha/2)\sin(\alpha/2)\, i\, \left(-e^{i\omega t} + e^{-i\omega t}\right) \\
&= \frac{\hbar}{2}\sin(\alpha)\sin(-\omega t)
\end{aligned}$$

Larmor Precession

$$\begin{aligned}
\langle\psi|S_x|\psi\rangle &= \frac{\hbar}{2}\sin(\alpha)\cos(\omega t) \\
\langle\psi|S_y|\psi\rangle &= \frac{\hbar}{2}\sin(\alpha)\sin(-\omega t) \\
\langle\psi|S_z|\psi\rangle &= \frac{\hbar}{2}\cos(\alpha)
\end{aligned}$$

$$\omega = -\gamma B_0$$

Stern-Gerlach
Nobel Prize 1943



STERN-GERLACH EXPERIMENT

Stern-Gerlach

$$H = -\gamma \vec{B} \cdot \vec{S}$$

$$\vec{B} = -\alpha\, x\, \hat{x} + (B_0 + \alpha z)\hat{z} \qquad \vec{\nabla} \cdot \vec{B} = 0$$

silver atom rest frame, x=0

$$H = \begin{cases} 0 & t < 0 \\ -\gamma(B_0 + \alpha z)S_z & 0 \le t \le T \\ 0 & T < t \end{cases}$$

$$\chi(t < 0) = a\chi_\uparrow + b\chi_\downarrow$$

$$\chi(0 < t < T) = a\chi_\uparrow e^{-i\omega t/2 + i\gamma\alpha z t/2} + b\chi_\downarrow e^{i\omega t/2 - i\gamma\alpha z t/2}$$

$$\omega = -\gamma B_0$$

Stern-Gerlach

$$\chi(t > T) = a\chi_\uparrow e^{-i\omega T/2 + i\gamma\alpha z T/2} + b\chi_\downarrow e^{i\omega T/2 - i\gamma\alpha z T/2}$$

$$p_z = \pm\frac{\alpha\gamma T\hbar}{2}$$