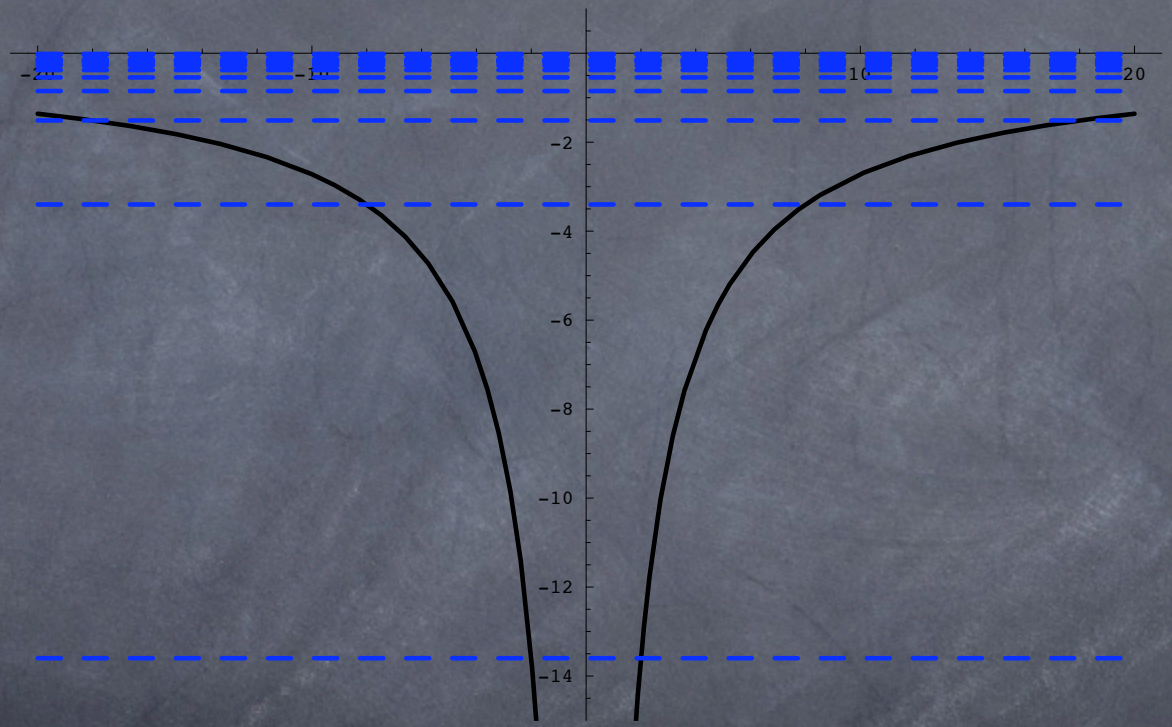
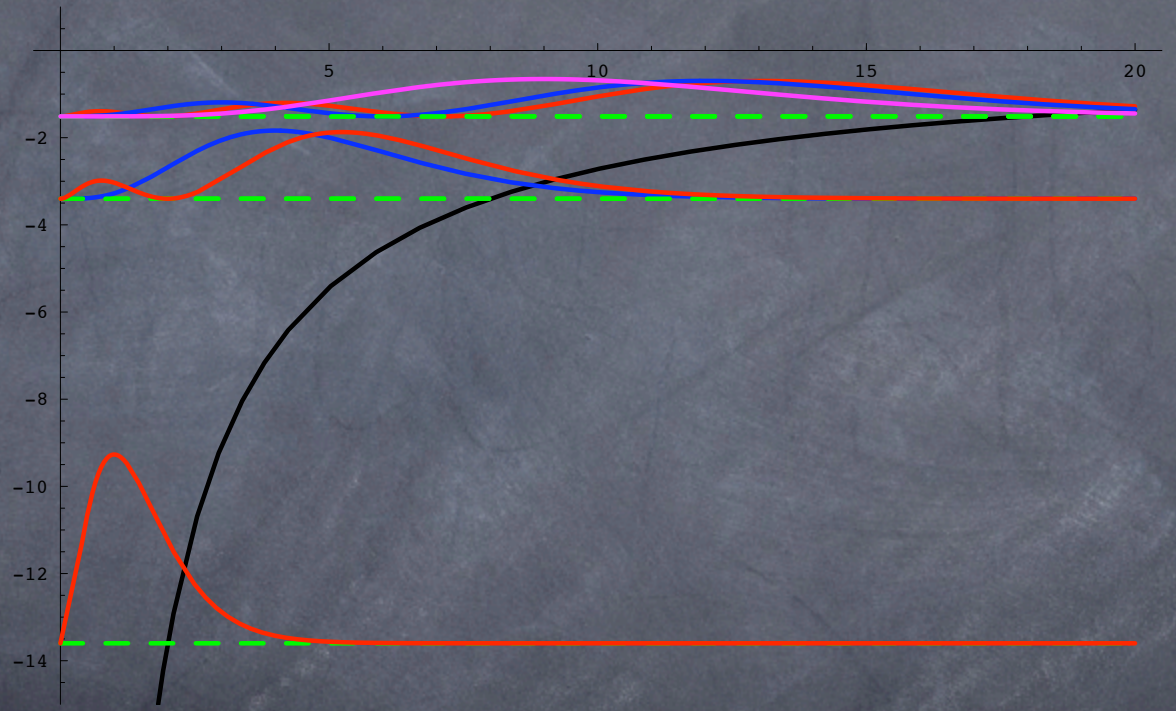
Hydrogen Energy Levels



hydrogen wavefunctions



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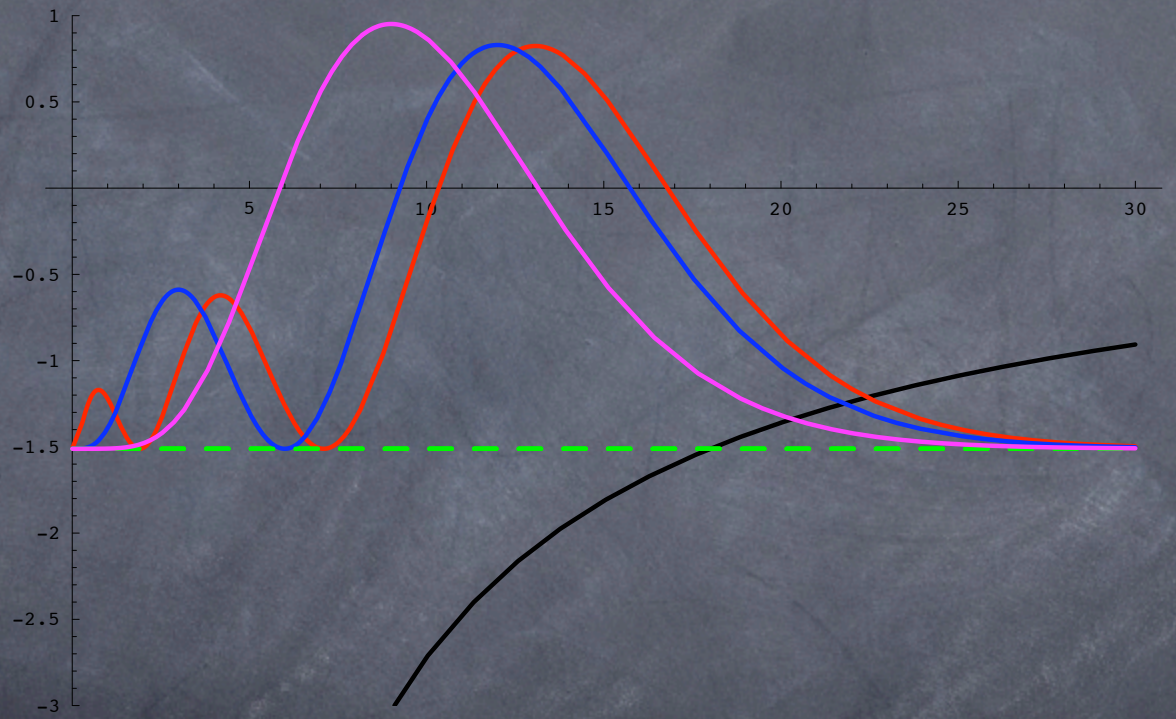

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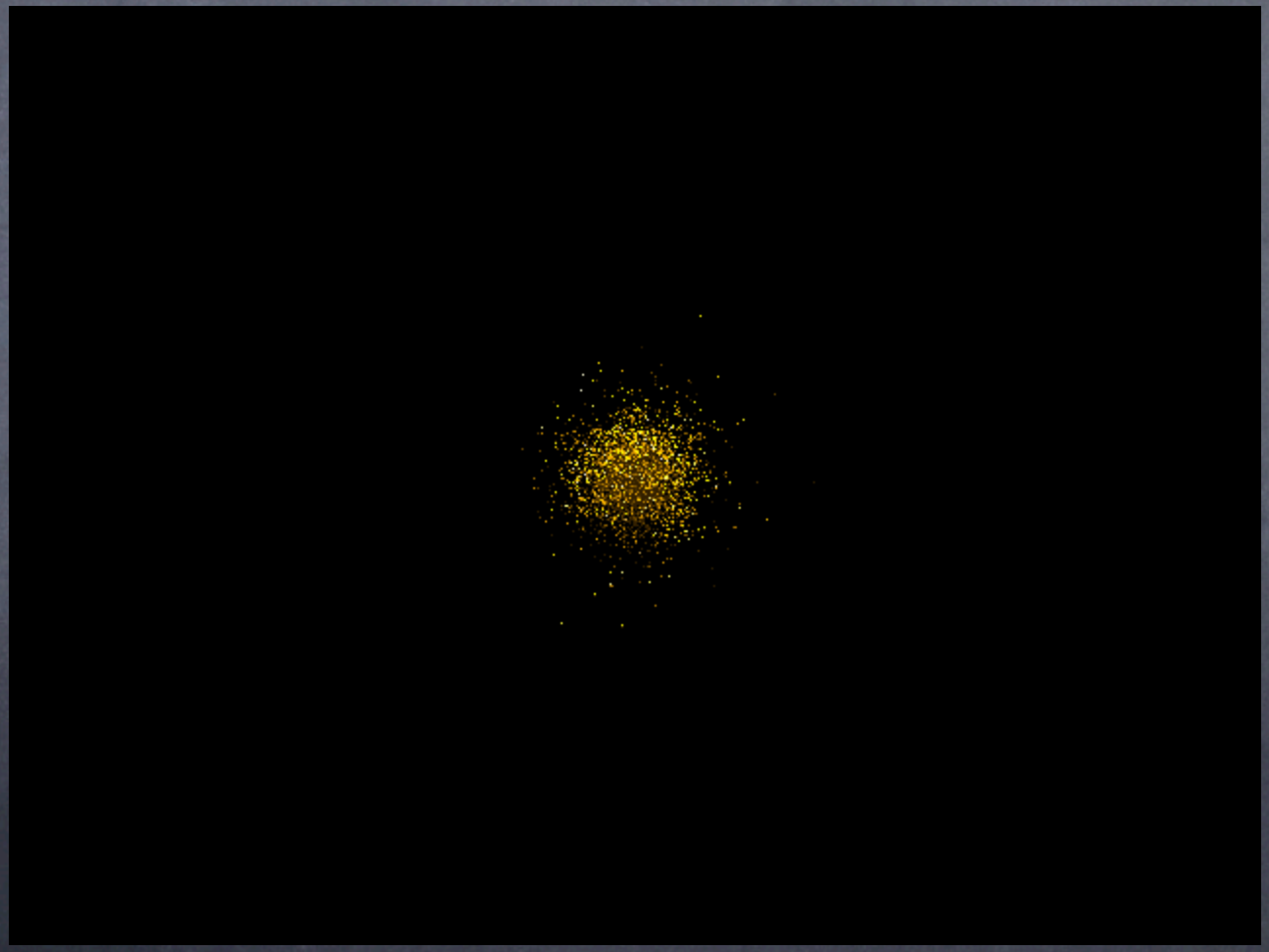

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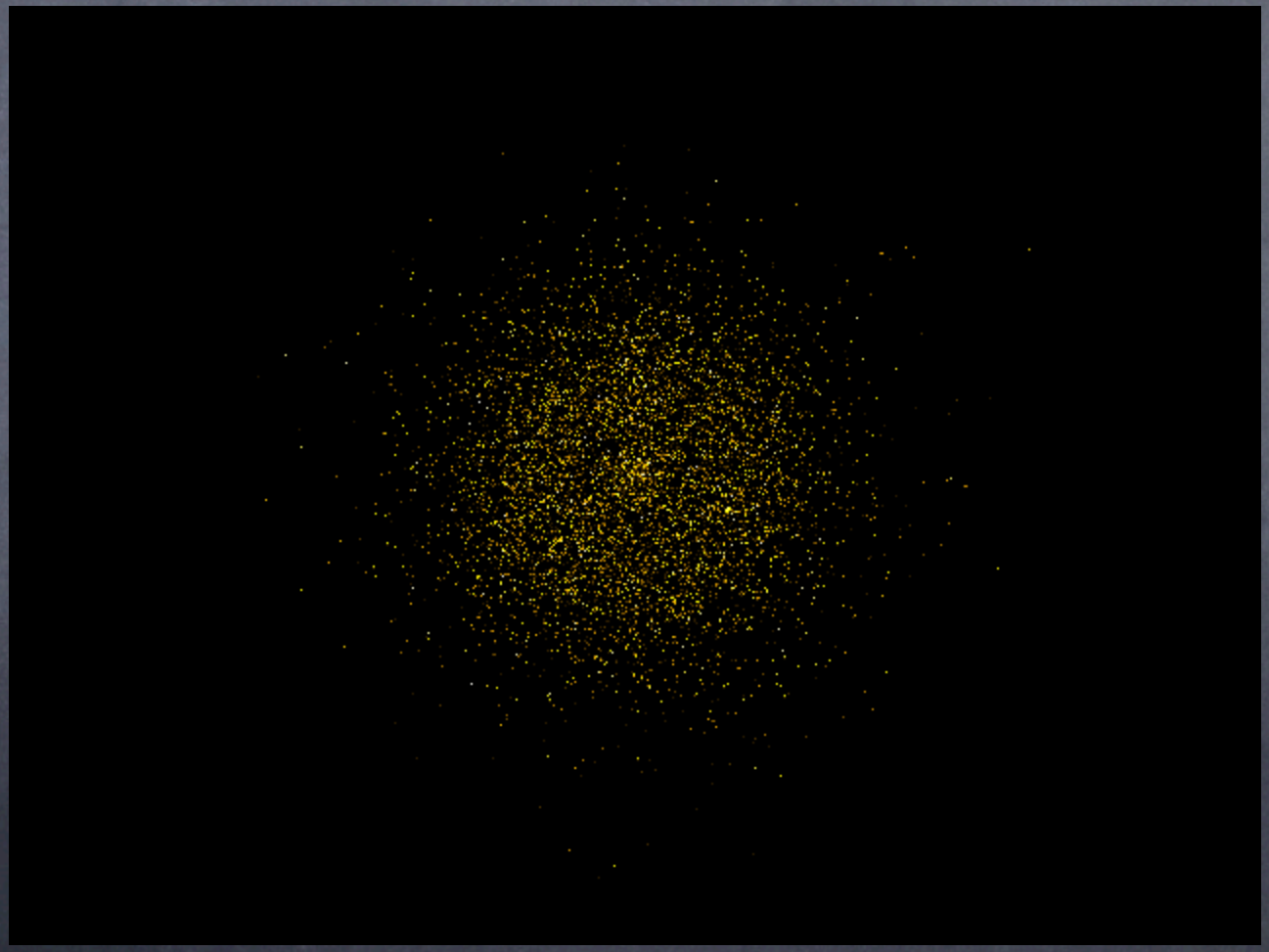

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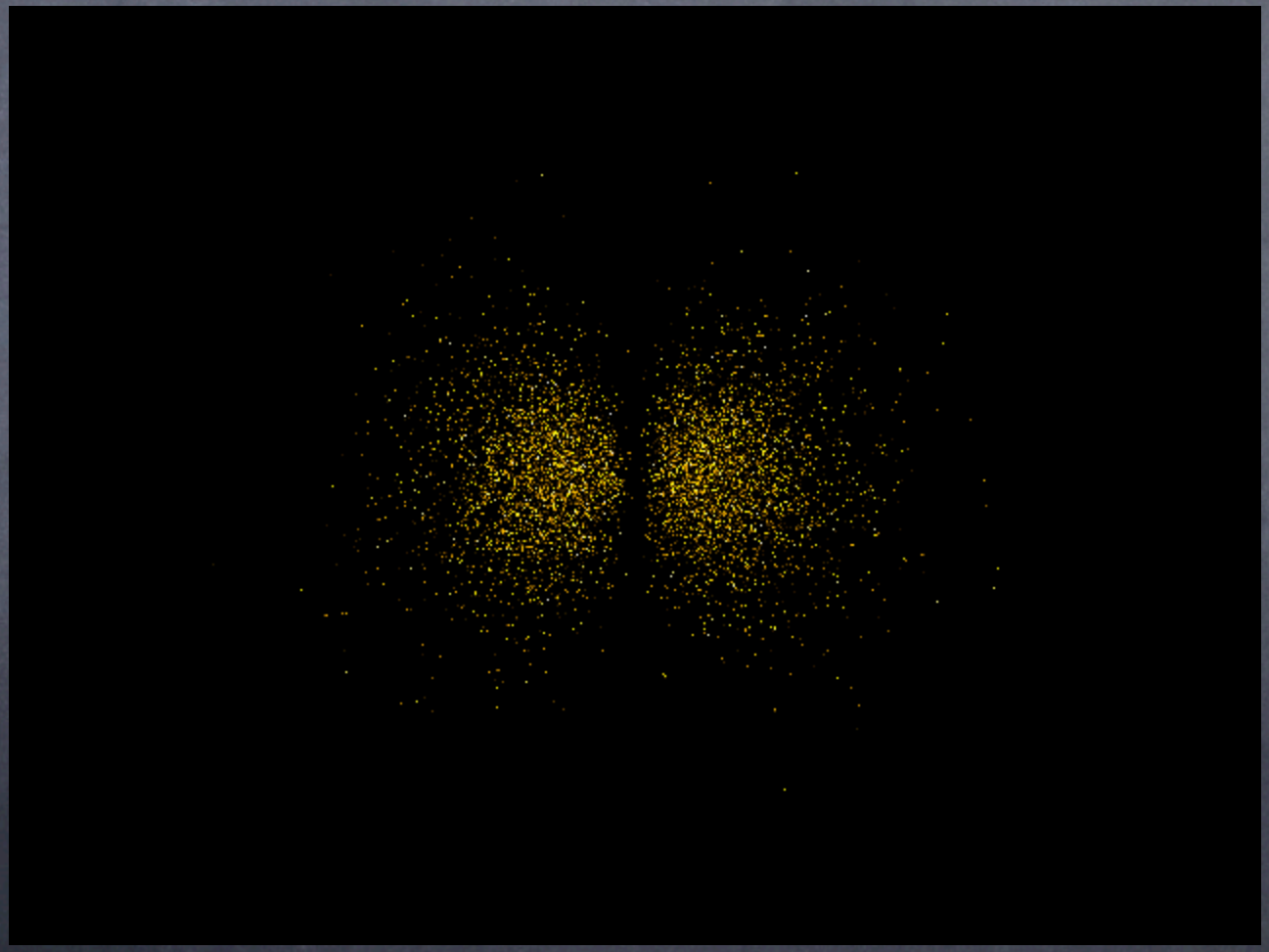

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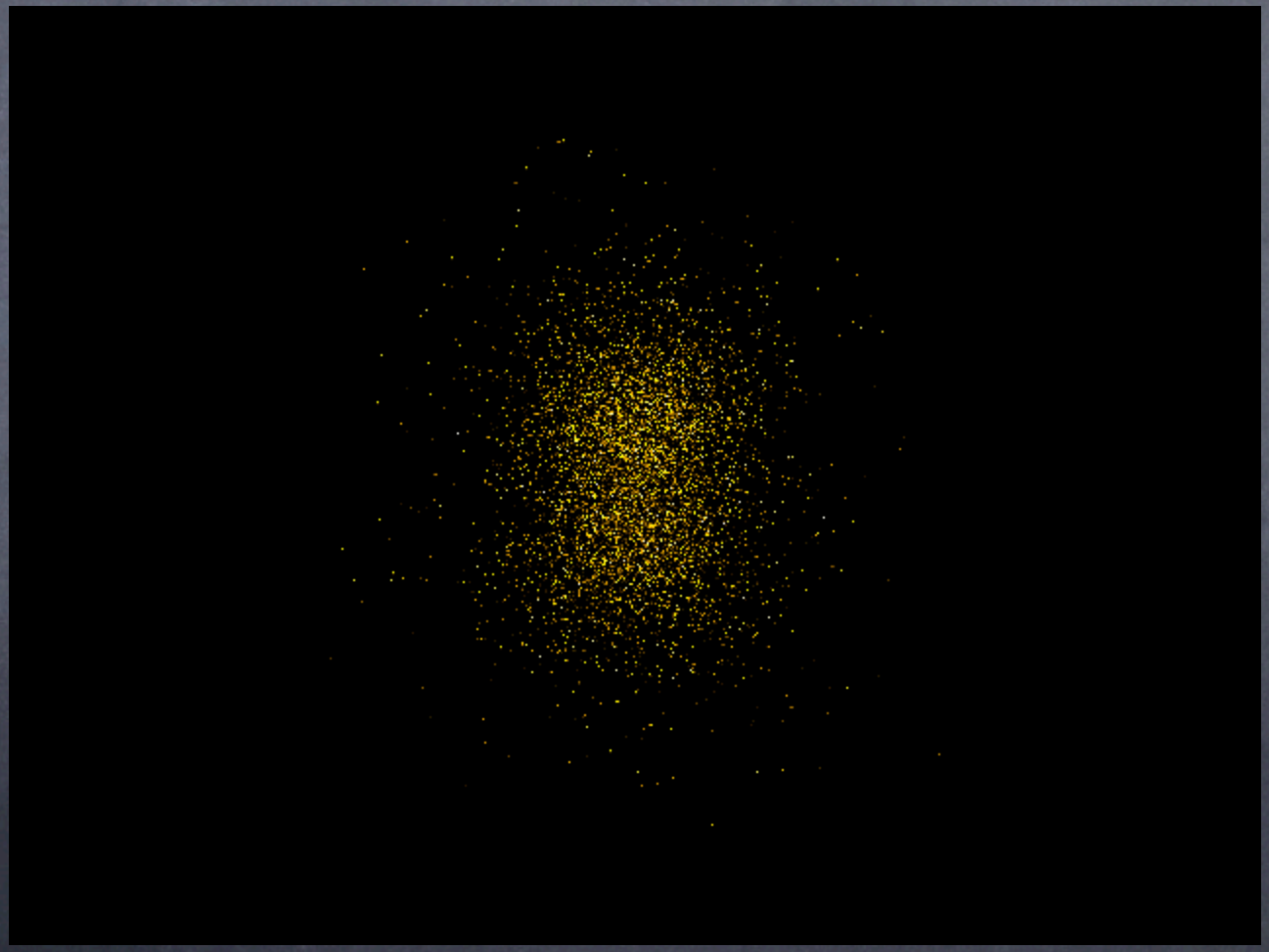

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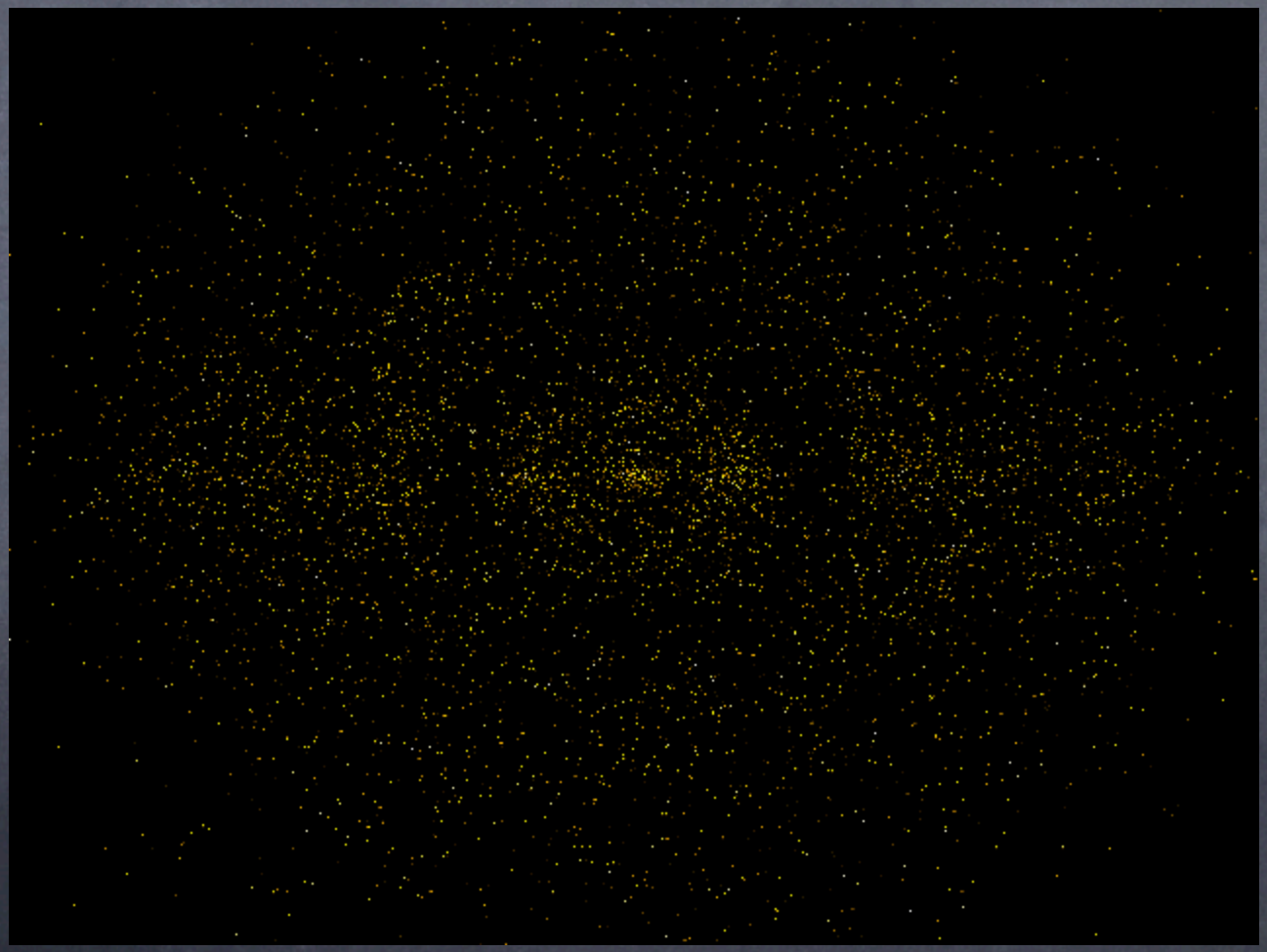

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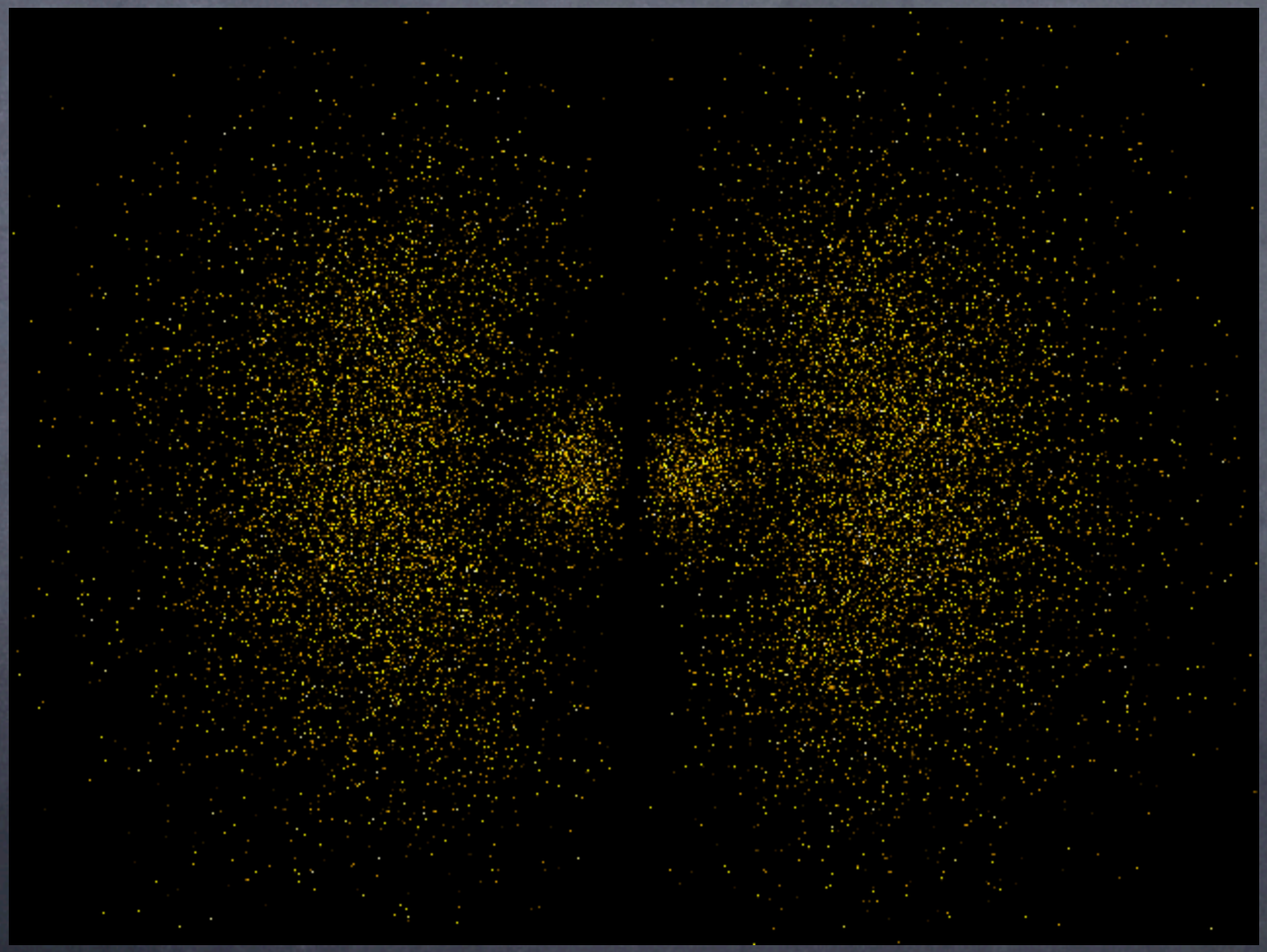

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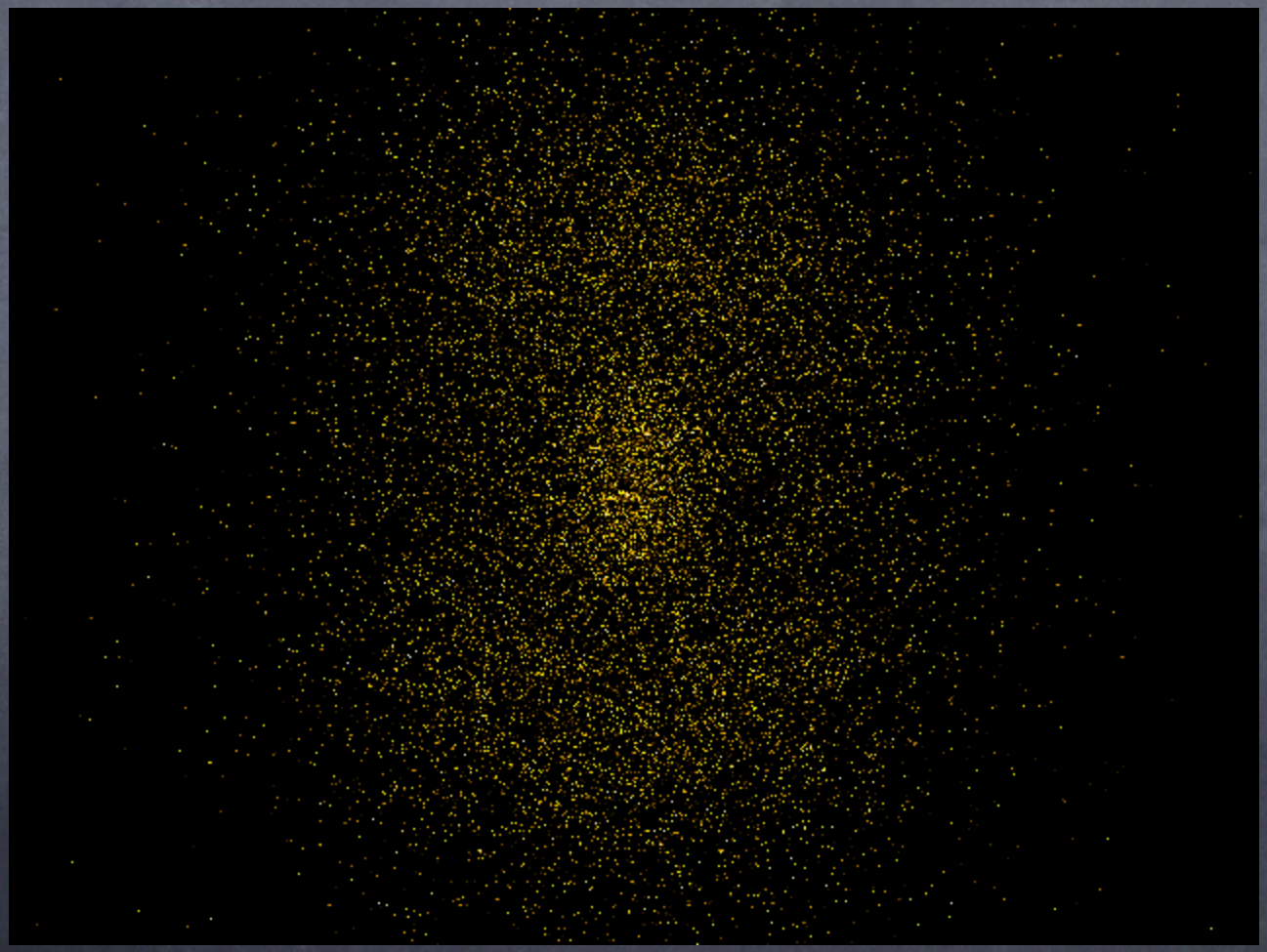

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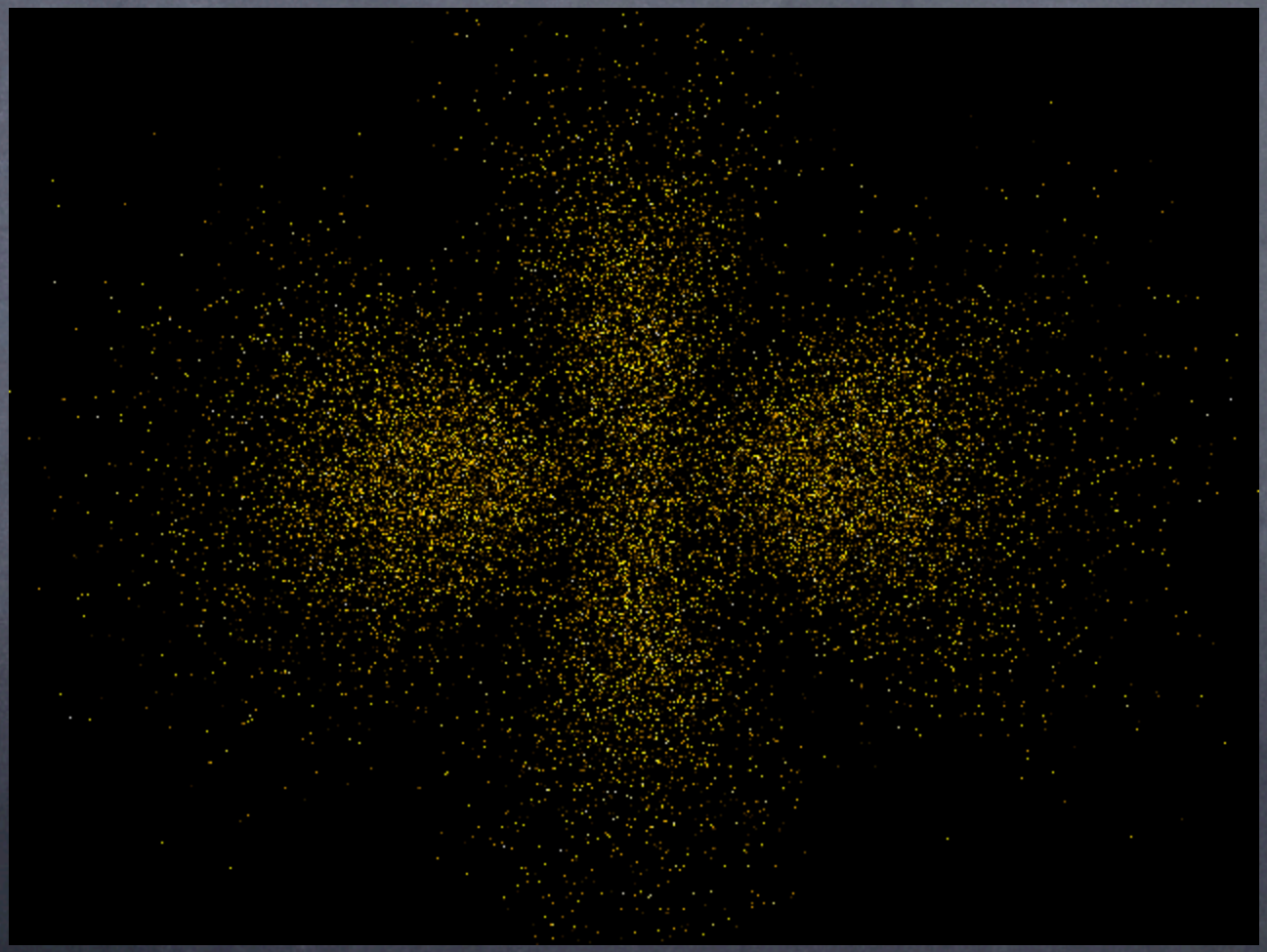

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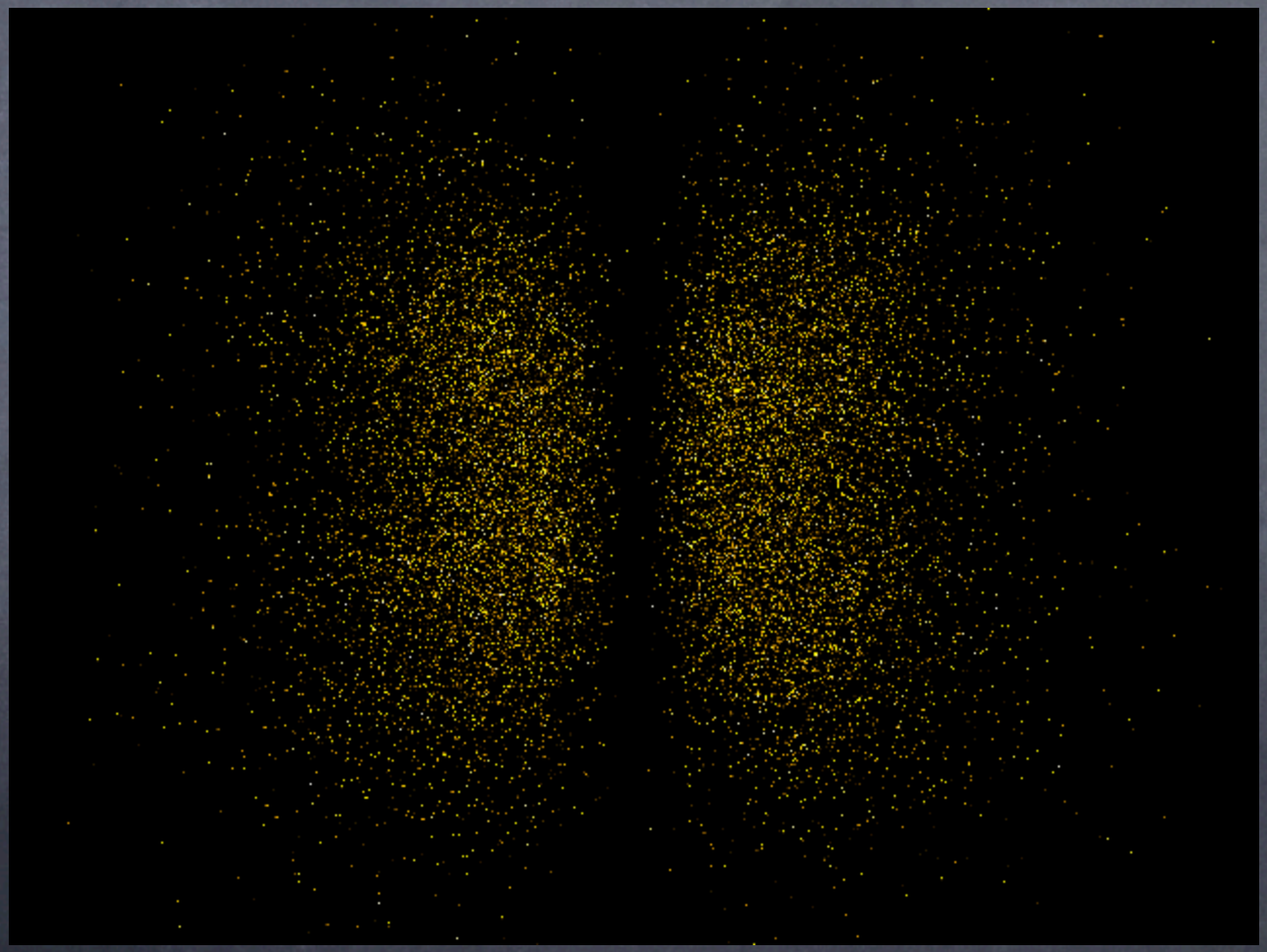

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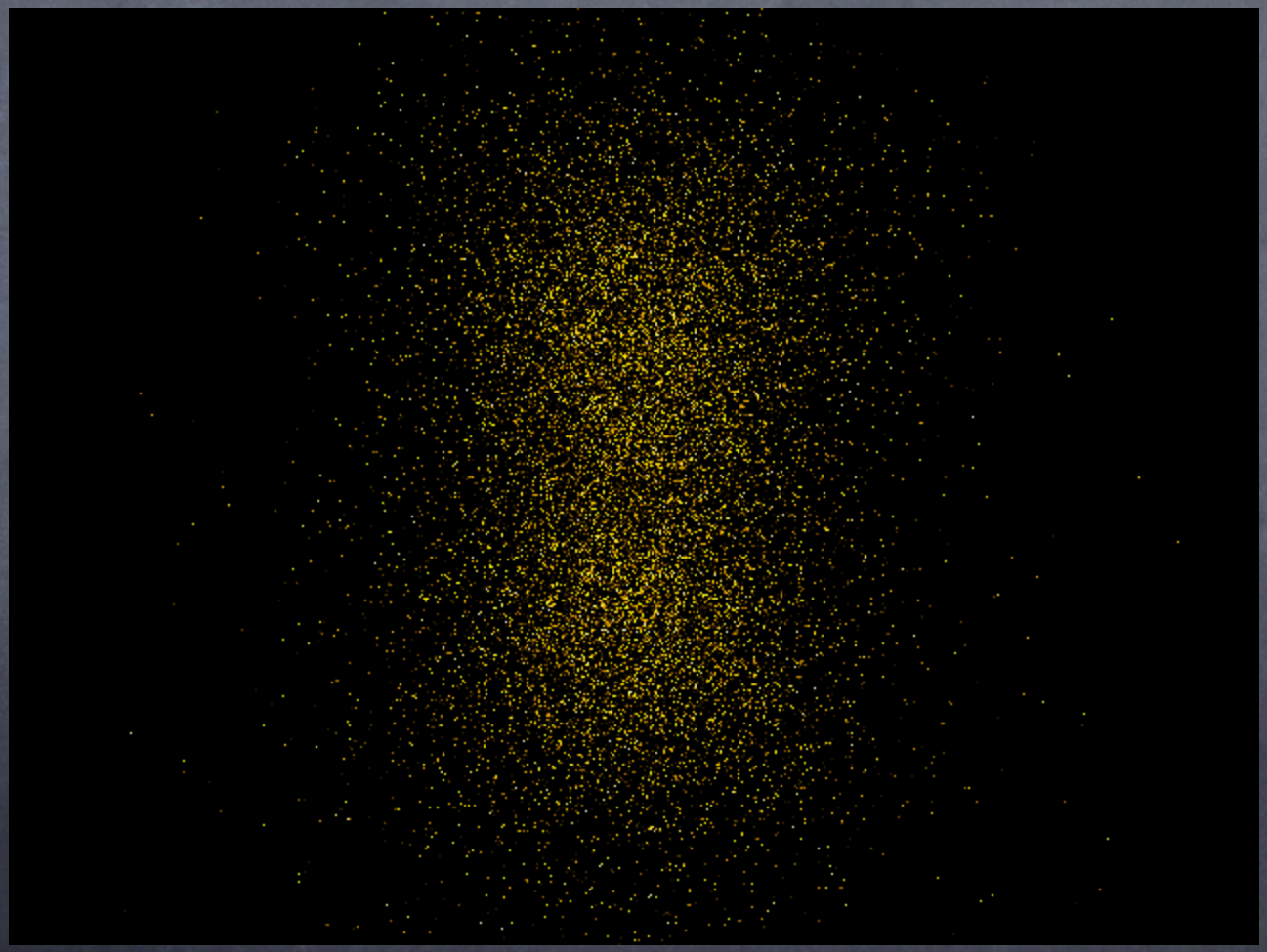
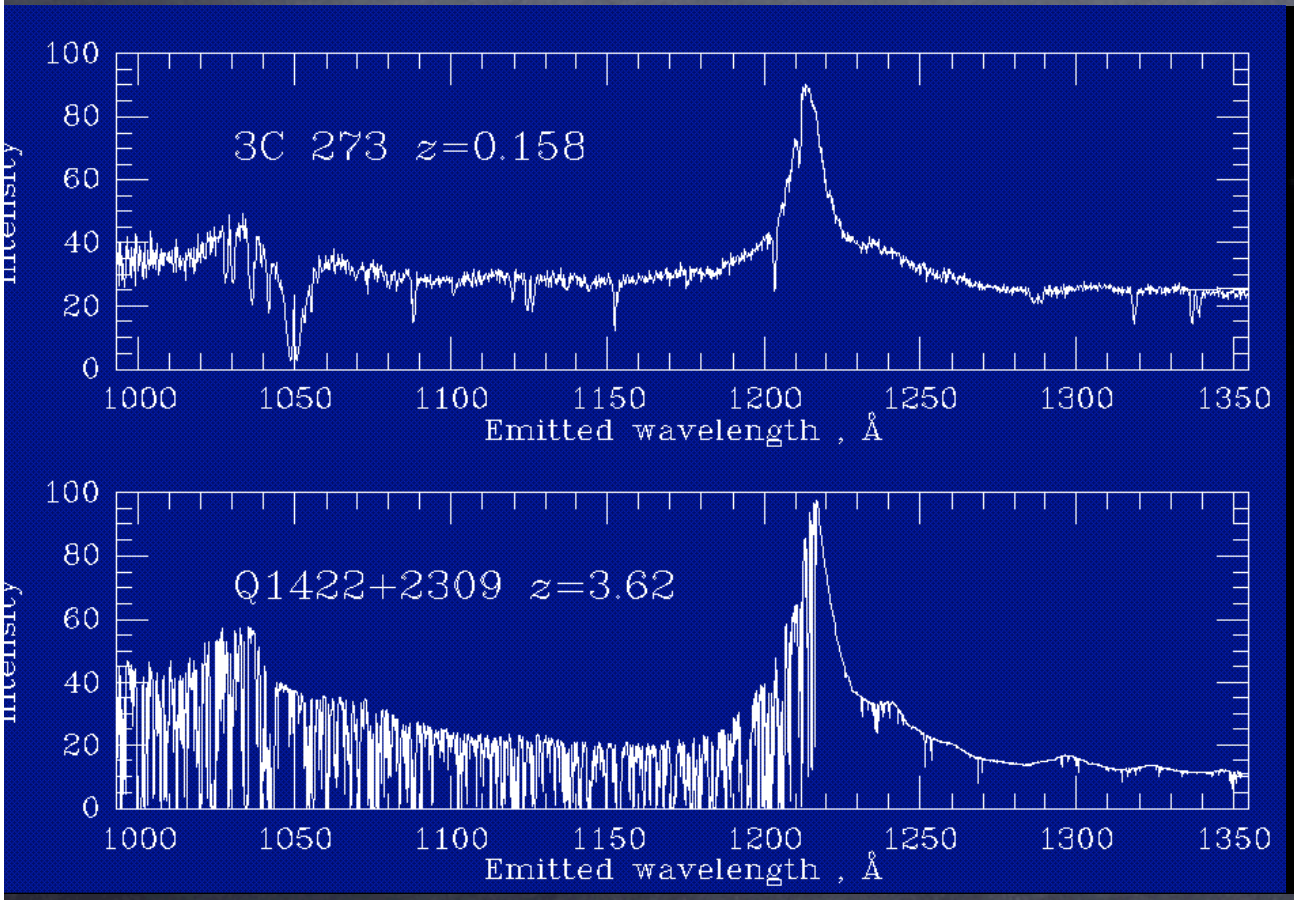
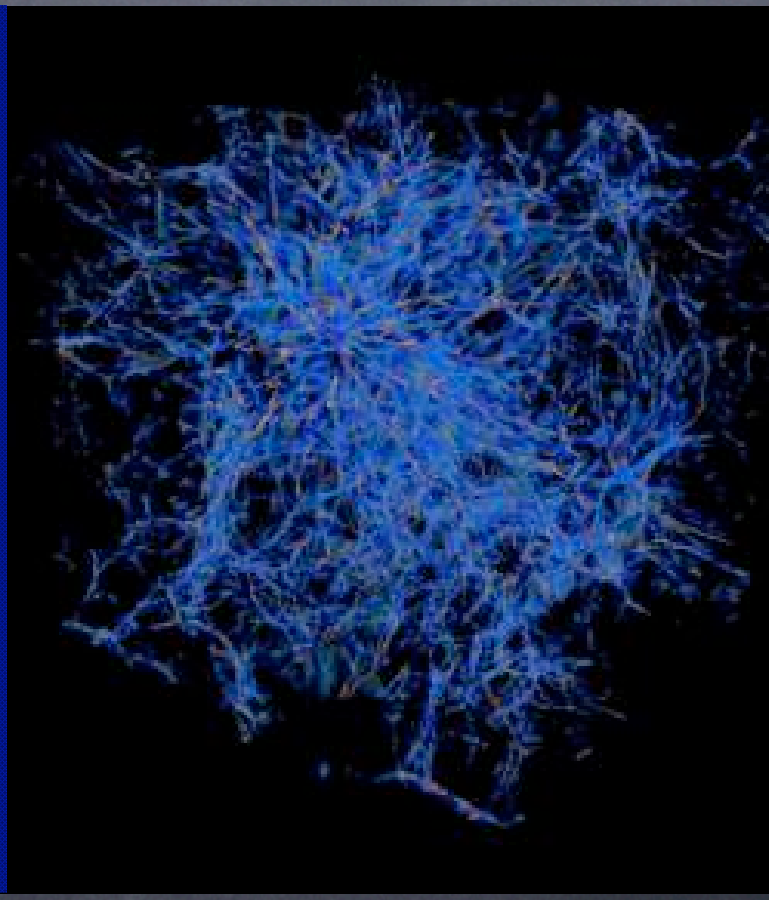


Lyman α forest





Angular Momentum

$$\vec{L} = \vec{r} \times \vec{p}$$

$$L_x = yp_z - zp_y, \quad L_y = zp_x - xp_z, \quad L_z = xp_y - yp_x$$

$$p_x \to \frac{\hbar}{i}\frac{\partial}{\partial x} \quad p_y \to \frac{\hbar}{i}\frac{\partial}{\partial y} \quad p_z \to \frac{\hbar}{i}\frac{\partial}{\partial z}$$

$$\begin{aligned}
[L_x, L_y] &= [yp_z - zp_y, zp_x - xp_z] \\
&= [yp_z, zp_x] - [yp_z, xp_z] - [zp_y, zp_x] + [zp_y, xp_z] \\
&= yp_x\,[p_z, z] + p_y x\,[z, p_z] \\
&= i\hbar\,(xp_y - yp_x) \\
&= i\hbar\,L_z
\end{aligned}$$

$$[L_y, L_z] = i\hbar\,L_x, \quad [L_z, L_x] = i\hbar\,L_y$$

Angular Momentum

$$\vec{L} \cdot \vec{L} = L^2 = L_x^2 + L_y^2 + L_z^2$$

$$\begin{aligned}
\left[L^2, L_x\right] &= \left[L_x^2, L_x\right] + \left[L_y^2, L_x\right] + \left[L_z^2, L_x\right] \\
&= L_y \left[L_y, L_x\right] + \left[L_y, L_x\right] L_y \\
&\quad + L_z \left[L_z, L_x\right] + \left[L_z, L_x\right] L_z \\
&= L_y \left(-i\hbar L_z\right) + \left(-i\hbar L_z\right) L_y \\
&\quad + L_z \left(i\hbar L_y\right) + \left(i\hbar L_y\right) L_z \\
&= 0
\end{aligned}$$

$$\left[L^2, L_y\right] = 0, \quad \left[L^2, L_z\right] = 0$$

$$\left[L^2, \vec{L}\right] = 0$$

$$\begin{aligned}
[AB, C] &= ABC - CAB = ABC - ACB + ACB - CAB \\
&= A[B, C] + [A, C]B
\end{aligned}$$