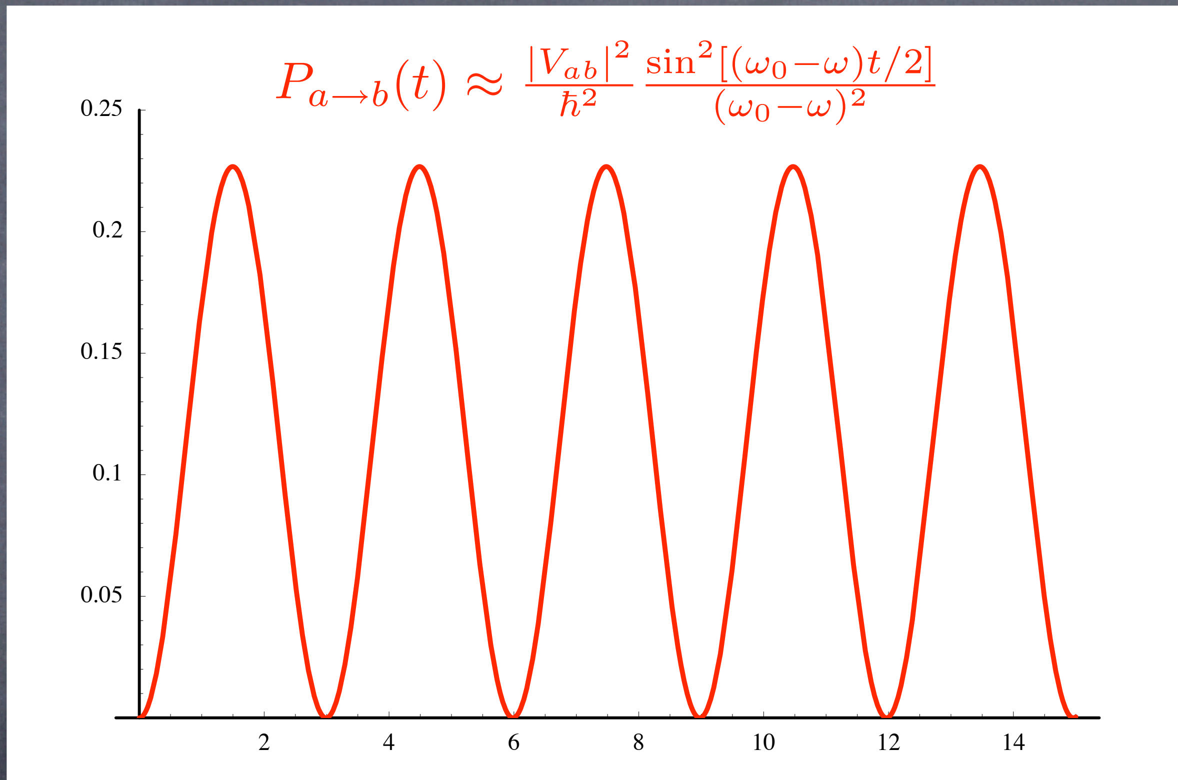
$$P_{a\to b}(t)$$

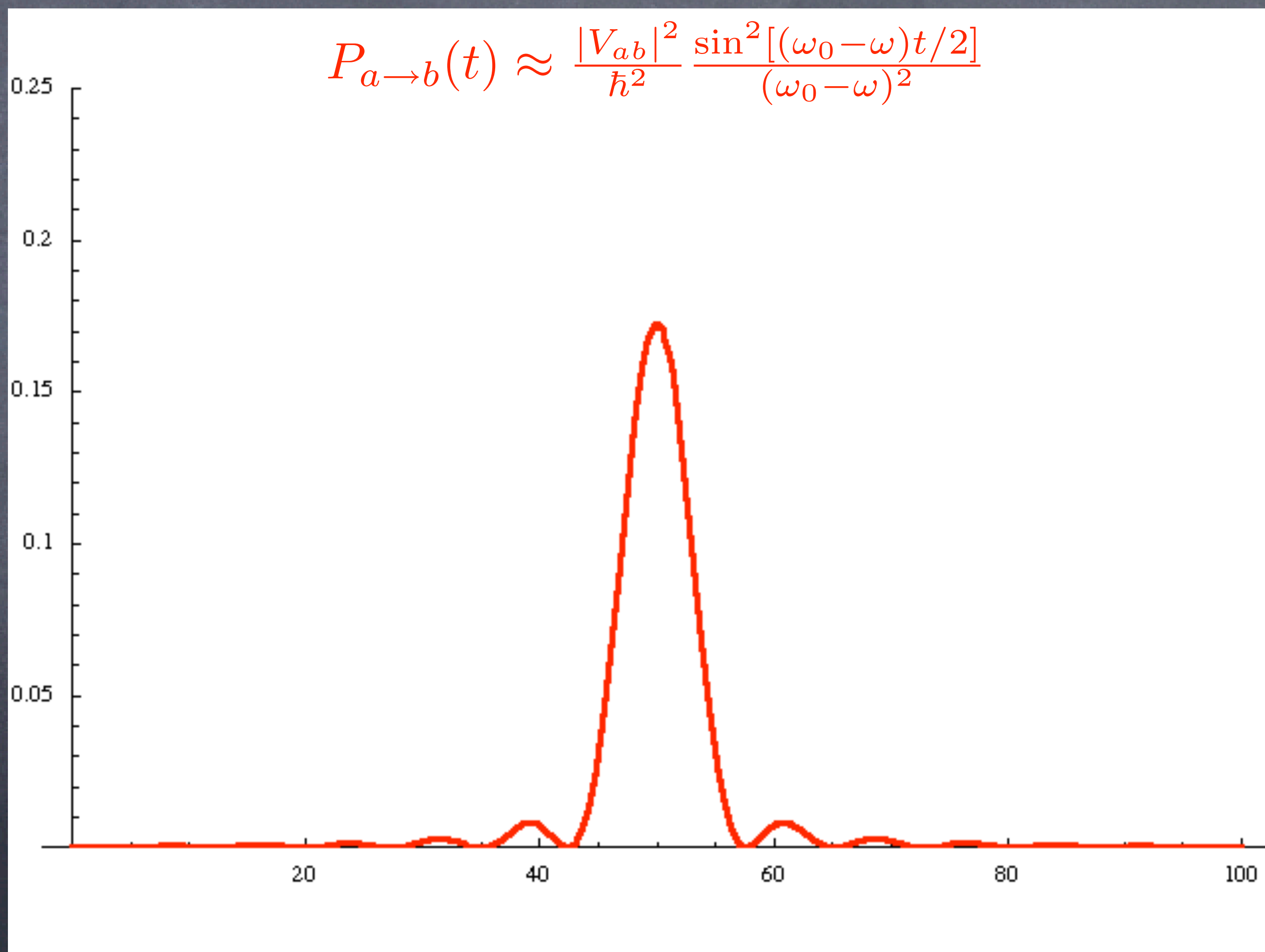


$P_{a\to b}(t) \approx \frac{|V_{ab}|^2}{\hbar^2} \frac{\sin^2[(\omega_0 - \omega)t/2]}{(\omega_0 - \omega)^2}$
0.25
0.2
0.15
0.1
0.05
2
4
6
8
10
12
14

$P_{a\to b}(\omega)$

$$P_{a\to b}(t) \approx \frac{|V_{ab}|^2}{\hbar^2}\frac{\sin^2[(\omega_0-\omega)t/2]}{(\omega_0-\omega)^2}$$

$$P_{a \to b}(\omega)$$



$P_{a\to b}(t) \approx \frac{|V_{ab}|^2}{\hbar^2} \frac{\sin^2[(\omega_0-\omega)t/2]}{(\omega_0-\omega)^2}$
0.25
0.2
0.15
0.1
0.05
20
40
60
80
100

Quantum Teleportation

Time Evolution

energy eigenstate: $e^{-i\,E\,t/\hbar}\,\psi(x)$

small Δt: $e^{-i\,H\,\Delta t/\hbar}\,\psi(x)$

$$e^{-i\,H\,\Delta t_2/\hbar}\,e^{-i\,H\,\Delta t_1/\hbar}\,\psi(x)$$

in general: $T\,e^{-i\,\int_0^t H\,dt/\hbar}\,\psi(x)$

↑

Unitary Matrix $U^\dagger U = 1$

Entangled States

two electrons in spin zero state

$$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

take one electron light years away

as soon one spin is measured the other is determined

Einstein hated this
“spooky action at a distance”

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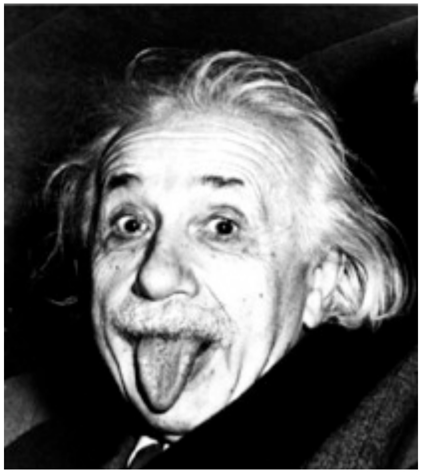

Preparing to Teleport

Alice has an electron is some quantum state

$$|\psi_1\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$$

she prepares electrons 2 and 3

in a spin zero state

$$|\psi_{23}^{(-)}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

she give electron 3 to Bob who goes to the teleport destination

$$\begin{aligned} |\Gamma_{123}\rangle &= |\psi_1\rangle \otimes |\psi_{23}^{(-)}\rangle \\ &= \frac{\alpha}{\sqrt{2}}\left(|\uparrow\uparrow\downarrow\rangle - |\uparrow\downarrow\uparrow\rangle\right) + \frac{\beta}{\sqrt{2}}\left(|\downarrow\uparrow\downarrow\rangle - |\downarrow\downarrow\uparrow\rangle\right) \end{aligned}$$

Quantum Teleportation

Alice performs a measurement with 4 possible outcomes

$$|\psi_{12}^{(\pm)}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle\right)$$

$$|\phi_{12}^{(\pm)}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle\right)$$

$$1 = |\psi_{12}^{(-)}\rangle\langle\psi_{12}^{(-)}| + |\psi_{12}^{(+)}\rangle\langle\psi_{12}^{(+)}| + |\phi_{12}^{(-)}\rangle\langle\phi_{12}^{(-)}| + |\phi_{12}^{(+)}\rangle\langle\phi_{12}^{(+)}|$$

new state:

$$|\Gamma_{123}\rangle = \frac{1}{2}\left[\begin{array}{c} |\psi_{12}^{-}\rangle\left(-\alpha\,|\uparrow\rangle - \beta\,|\downarrow\rangle\right) + |\psi_{12}^{+}\rangle\left(-\alpha\,|\uparrow\rangle + \beta\,|\downarrow\rangle\right) \\ + |\phi_{12}^{-}\rangle\left(\alpha\,|\downarrow\rangle + \beta\,|\uparrow\rangle\right) + |\phi_{12}^{+}\rangle\left(\alpha\,|\downarrow\rangle - \beta\,|\uparrow\rangle\right) \end{array}\right]$$

Quantum Teleportation

4 possible states each with probability 1/4

if Alice finds $|\psi_{12}^{(-)}\rangle$ for example, she tells Bob:

$$|\psi_3\rangle = -\alpha\,|\uparrow\rangle - \beta\,|\downarrow\rangle$$

Bob does unitary transformation:

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -\alpha \\ -\beta \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

Bob's electron is now in first electron's initial state:

$$|\psi_3\rangle = \alpha\,|\uparrow\rangle + \beta\,|\downarrow\rangle$$

Quantum Teleportation

similarly

$$|\psi_{12}^{(+)}\rangle : \quad \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -\alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$|\phi_{12}^{(-)}\rangle : \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \beta \\ \alpha \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$|\phi_{12}^{(+)}\rangle : \quad \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -\beta \\ \alpha \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$|\psi_3\rangle = \alpha\,|\uparrow\rangle + \beta\,|\downarrow\rangle$$

Quantum Teleportation

electron 3 is in original state
electron 1 is in some new state
no one ever found out what α and β are