

Relativistic Corrections

$$E_{\text{rel}}^1 \sim \frac{Z^2 \alpha^2 m c^2}{2 n^2}$$

unusual metals with large Z :
gold (79) and mercury (80)



Relativistic Corrections

	Atom	Average atomic mass	Ground state configuration
79	Au	196.9665	[Kr] 4d ¹⁰ 4f ¹⁴ 5s ² 5p ⁶ 5d ¹⁰ 6s ¹
80	Hg	200.59	[Kr] 4d ¹⁰ 4f ¹⁴ 5s ² 5p ⁶ 5d ¹⁰ 6s ²
81	Tl	204.383	[Kr] 4d ¹⁰ 4f ¹⁴ 5s ² 5p ⁶ 5d ¹⁰ 6s ² 6p ¹

relativistic corrections shrink s-wave orbitals
electrons not shared, almost chemically inert

Shrinking s-wave

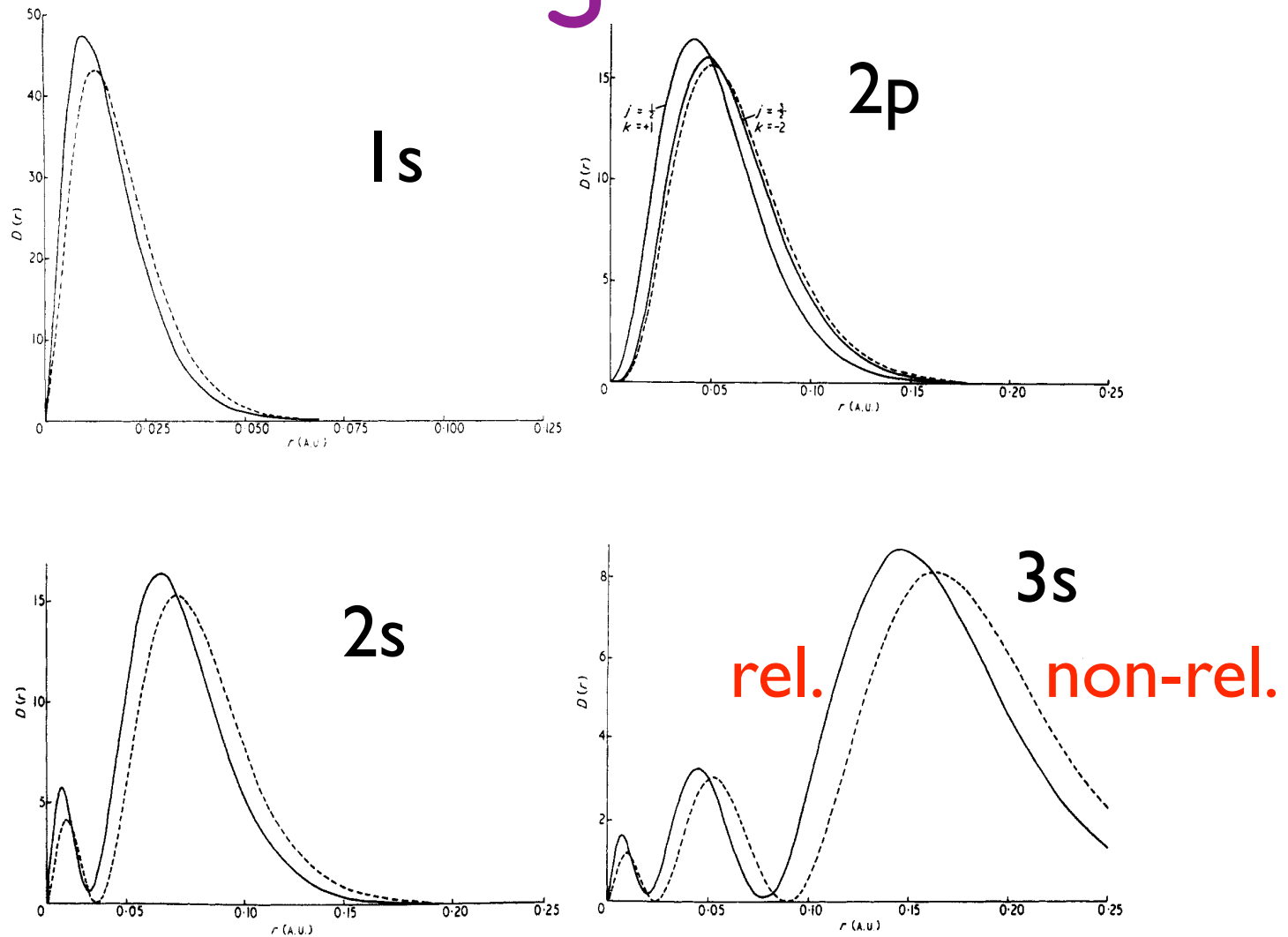
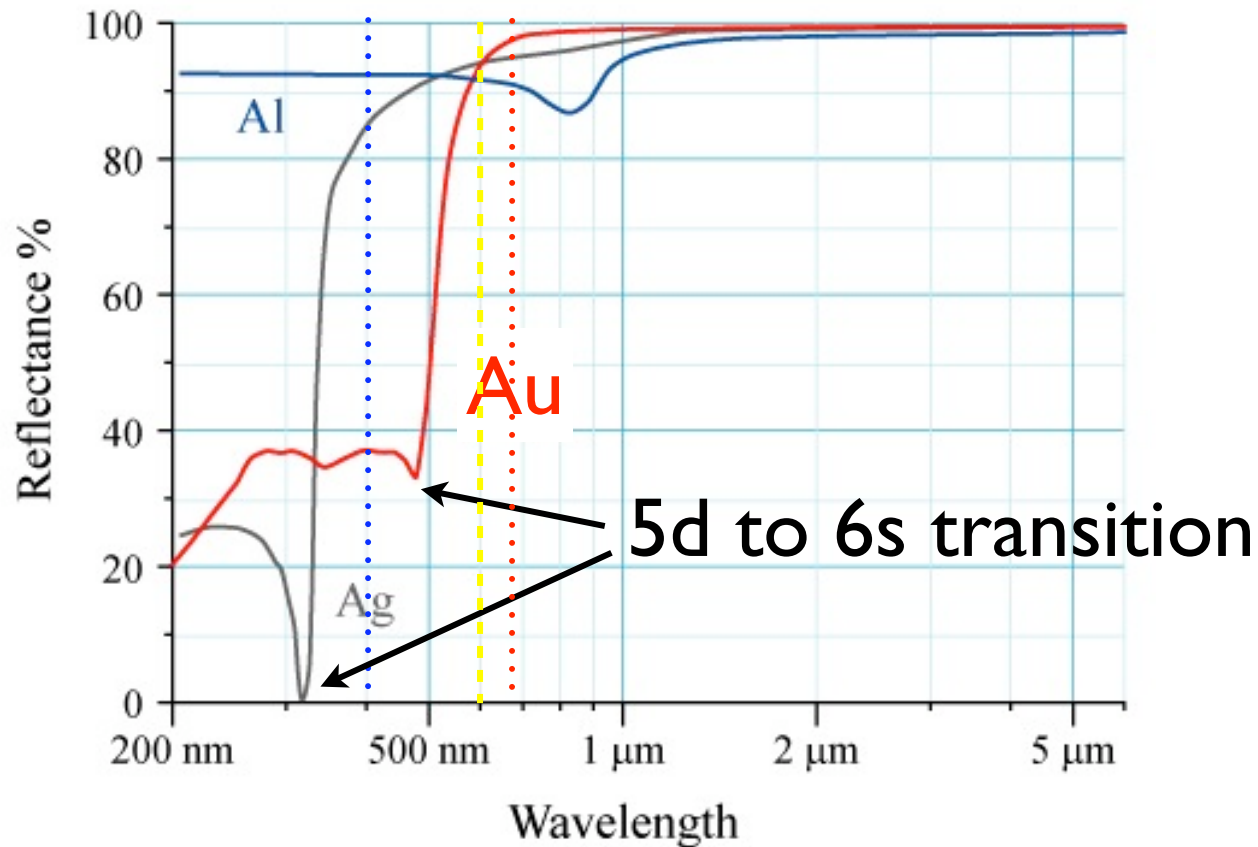


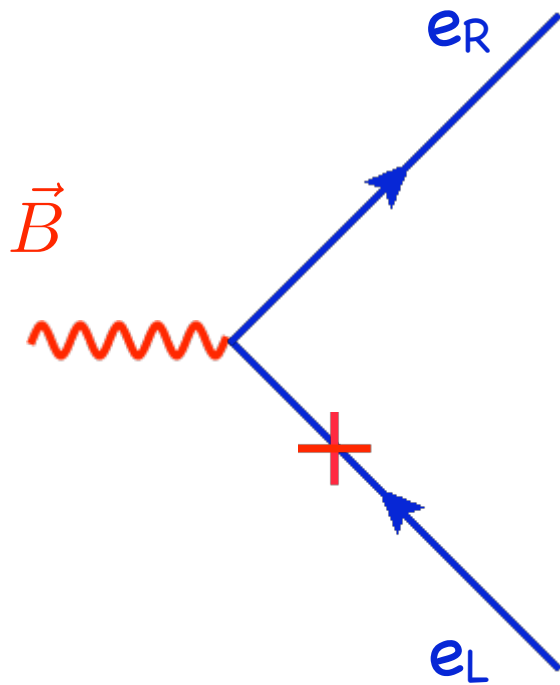
Figure 1. Radial densities for the 1s, 2s, 3s, and 2p states of a hydrogen-like atom with $Z = 80$. The dashed curves are nonrelativistic (NR) and the full curves relativistic. The contractions for 1s, 2s, $2p_{1/2}$, and 3s are of the same order of magnitude while that for $2p_{3/2}$ is much smaller. Reproduced with permission from Burke and Grant.^{23b}

Relativistic Corrections



relativistic corrections shrink 5d-6s splitting
gold reflects yellow, but absorbs shorter wavelengths

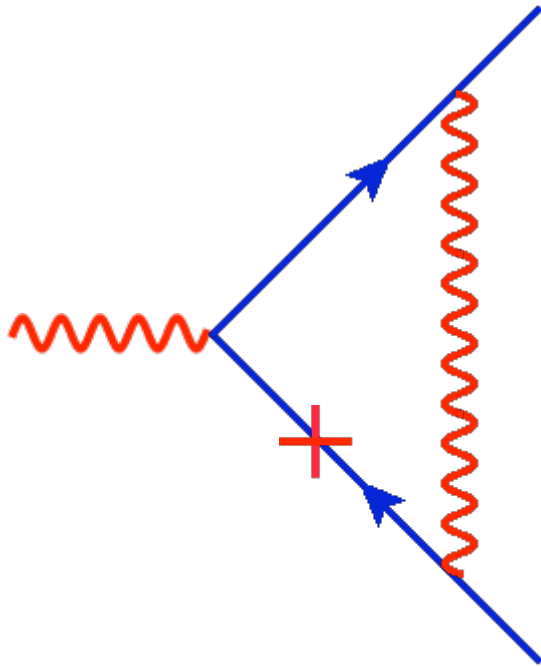
Anomalous Magnetic Moment



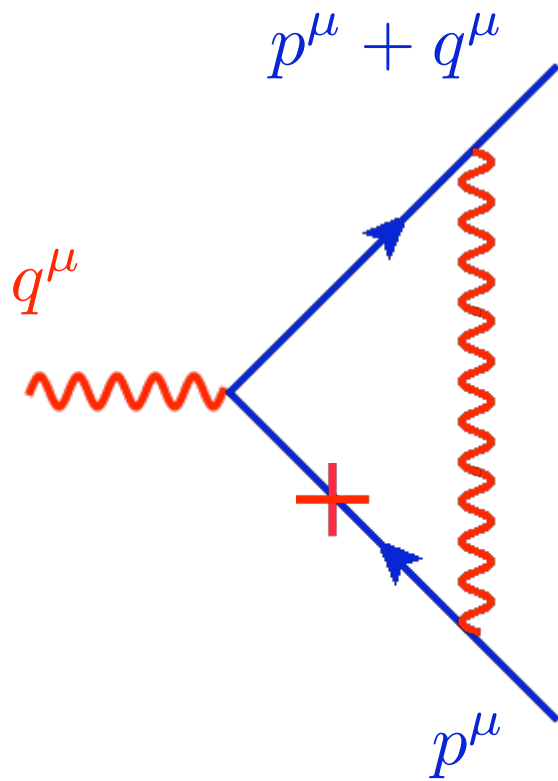
$$\vec{M} = g \frac{e}{2m} \vec{S}$$

$$g = 2$$

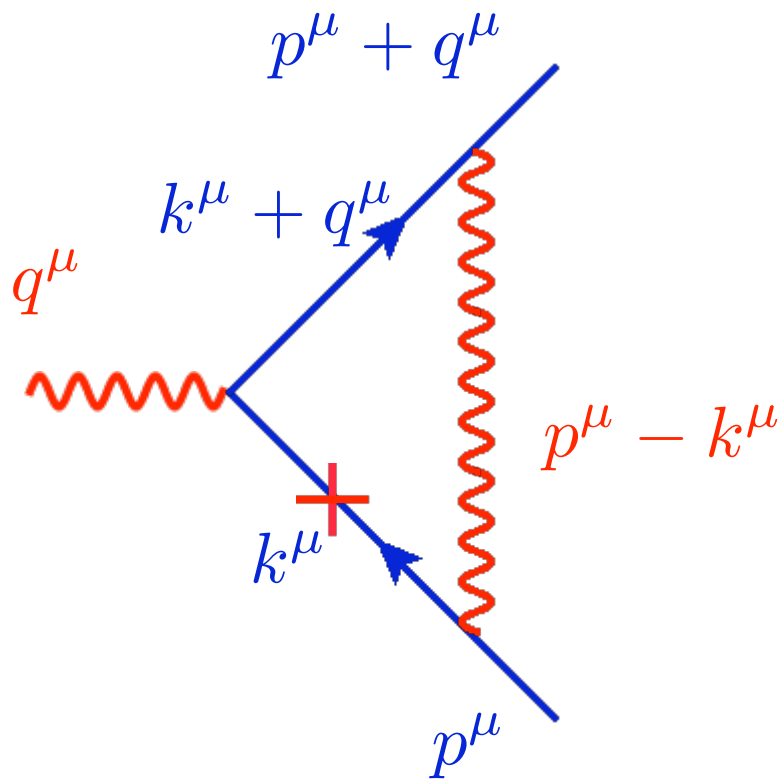
Anomalous Magnetic Moment



Anomalous Magnetic Moment



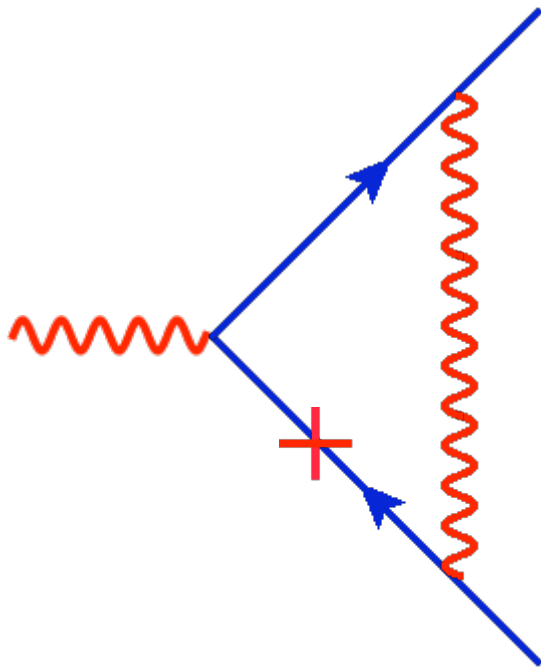
Anomalous Magnetic Moment



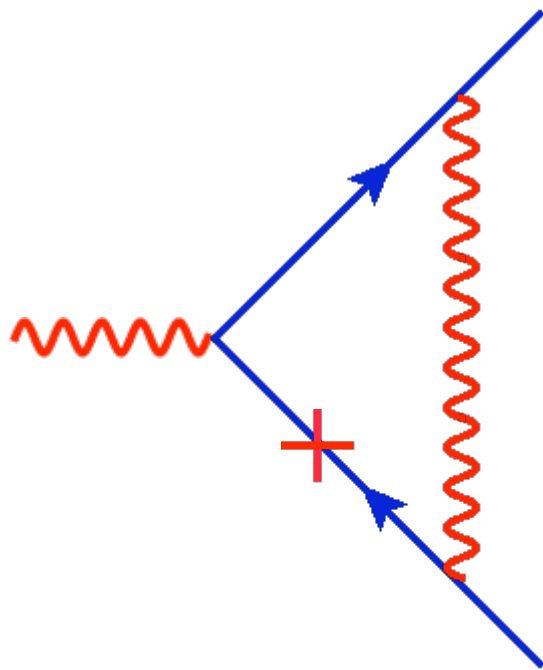
Anomalous Magnetic Moment

integrate over momentum:

$$g = 2 + \frac{\alpha}{\pi}$$



Anomalous Magnetic Moment

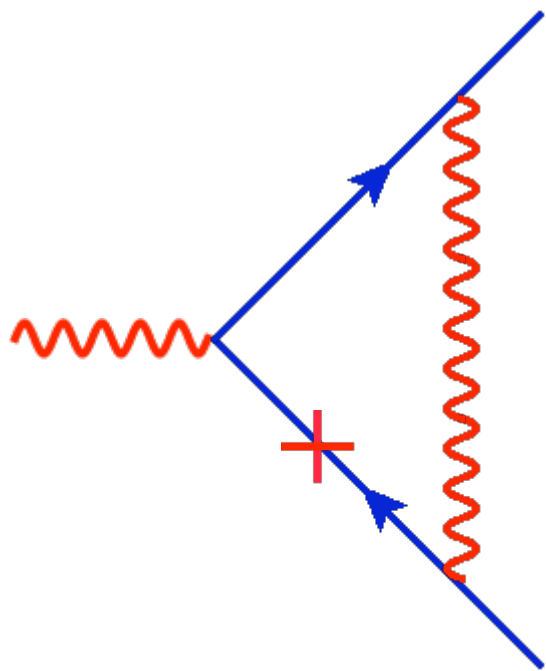


integrate over momentum:

$$g = 2 + \frac{\alpha}{\pi}$$

$$g = 2 + \frac{\alpha}{\pi} + c_2 \alpha^2 + c_3 \alpha^3 + c_4 \alpha^4$$

Anomalous Magnetic Moment



integrate over momentum:

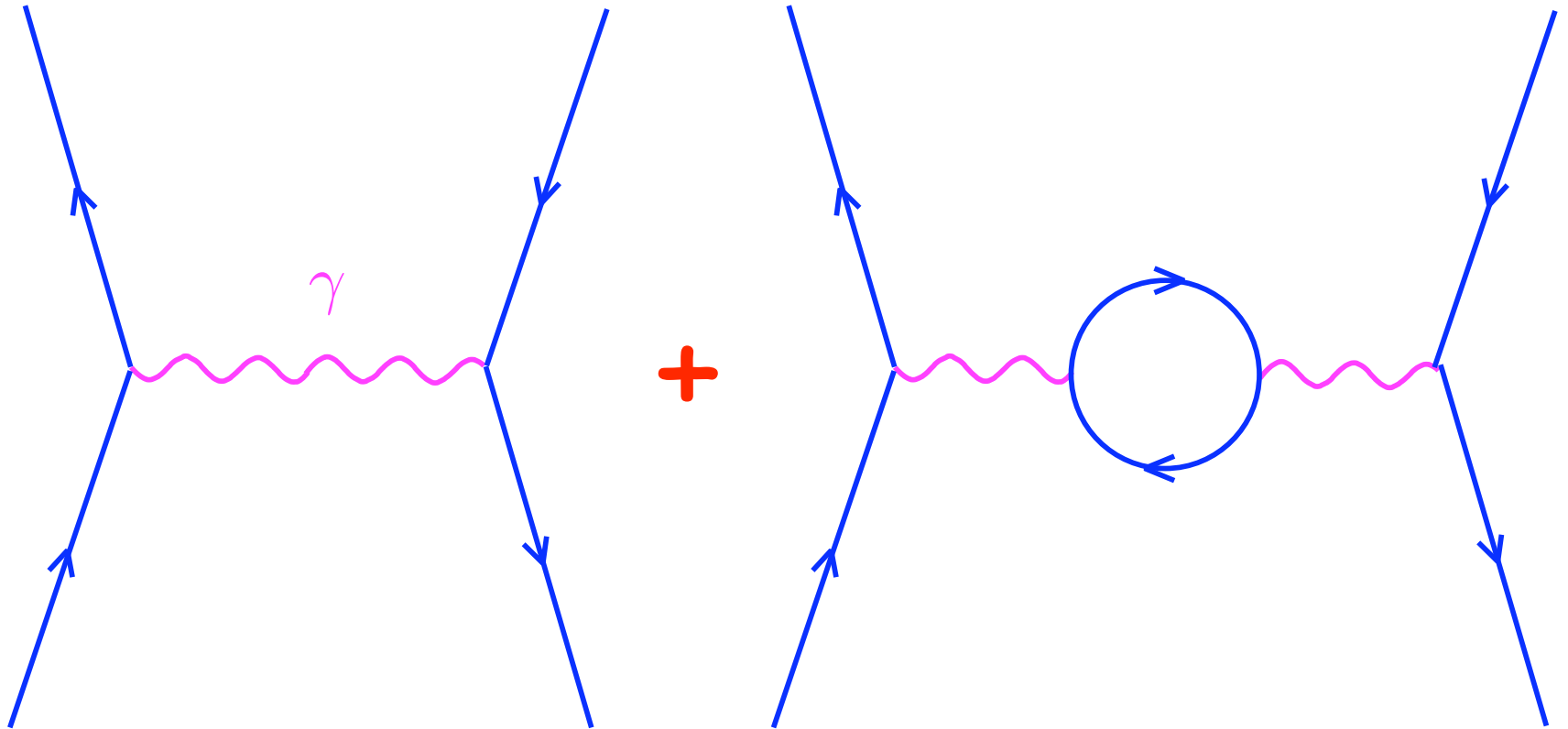
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$$\alpha^{-1} = 137.035999070(98)$$

spectroscopy: $\alpha^{-1} = 137.03599878(91)$.

Quantum Corrections



coupling is distance dependent

$$\alpha(10^{-10} \text{ m}) \approx \frac{1}{137}$$

$$\alpha(10^{-17} \text{ m}) \approx \frac{1}{128}$$