Early Universe

Universe expands with scale factor $a(t)$

Einstein:

$$H^2 = \left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi}{3} G\rho$$

hot:

$$\begin{aligned} k_B T &\gg mc^2 \\ pc &\gg mc^2 \end{aligned}$$

neglect μ

Early Universe

$$\rho_b = \frac{d_s}{2\pi^2}\int_0^\infty \frac{(\hbar k c)k^2 dk}{e^{\hbar k c/k_B T}-1}$$
$$\rho_f = \frac{d_s}{2\pi^2}\int_0^\infty \frac{(\hbar k c)k^2 dk}{e^{\hbar k c/k_B T}+1}$$

$$\int_0^\infty dx \frac{x^{\nu-1}}{e^{ax}-1} = a^{-\nu}\,\Gamma(\nu)\zeta(\nu)$$
$$\int_0^\infty dx \frac{x^{\nu-1}}{e^{ax}+1} = (1-2^{1-\nu})\,a^{-\nu}\,\Gamma(\nu)\zeta(\nu)$$

$$\rho_b = \frac{d_s \pi^2}{30} T^4$$
$$\rho_f = \frac{7}{8}\,\frac{d_s \pi^2}{30} T^4$$

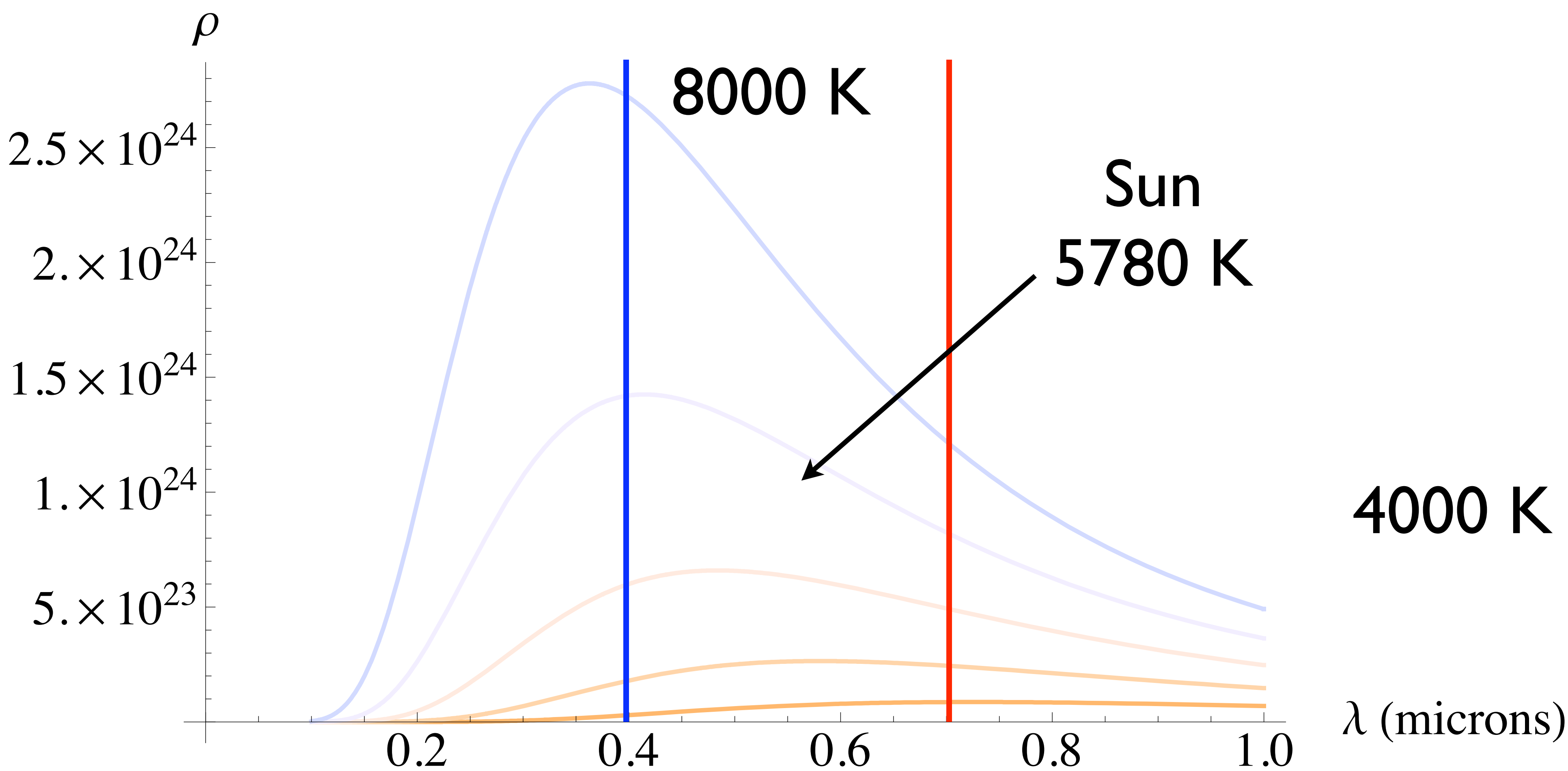
energy density $\Rightarrow$ expansion rate

$\Rightarrow$ relative abundance of He, D, Li

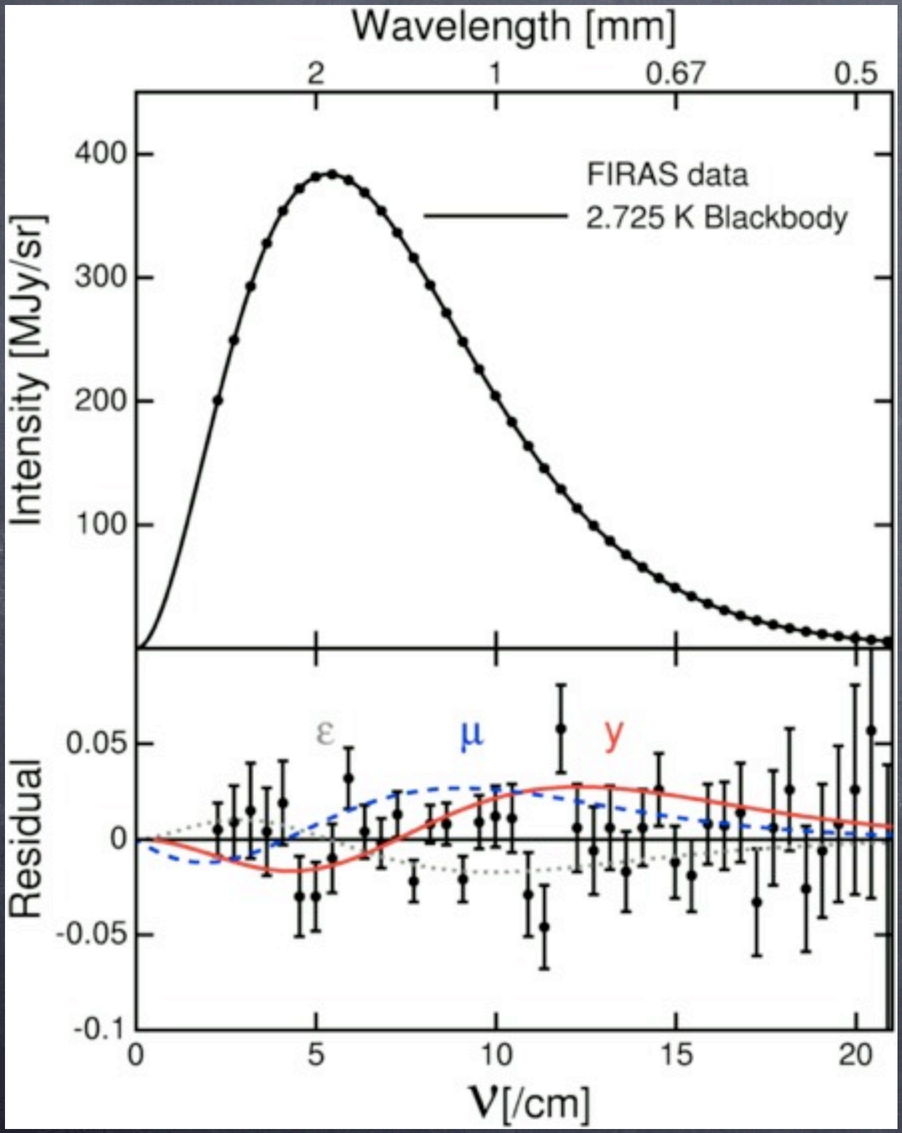
requires $N_\nu = 3 \pm 0.1$

LEP found $N_\nu = 2.994 \pm 0.012$

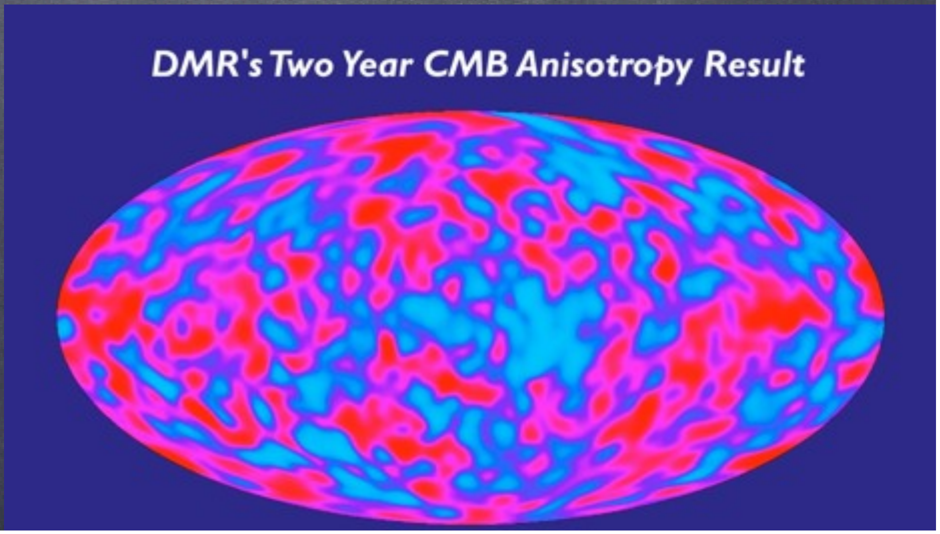
Planck Distribution



Cosmic Microwave Background



FIRAS experiment on COBE



2006 Mather and Smoot

Bose-Einstein Condensation

$$n_b(\epsilon) = \frac{1}{e^{(\epsilon-\mu)/k_BT} - 1} > 0$$

$$e^{(\epsilon-\mu)/k_BT} > 1$$

$$(\epsilon - \mu) > 0$$

for any allowed ϵ

$$\epsilon > \mu\ (T)$$

for free particles

$$\epsilon = \frac{\hbar^2 k^2}{2m} \geq 0$$

$$\mu(T) < 0$$

Bose-Einstein Condensation

fixed N, V and vary T

$$\frac{N}{V} = \frac{1}{2\pi^2} \int_0^\infty \frac{k^2\, dk}{e^{(\hbar^2 k^2/(2m) - \mu)/k_B T} - 1}$$

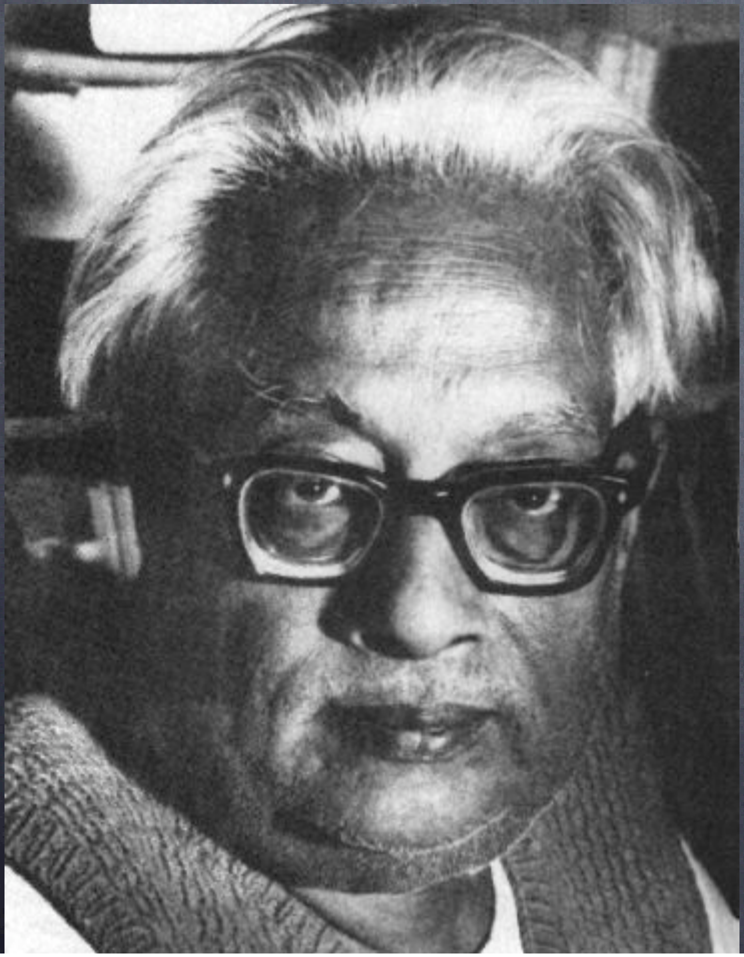
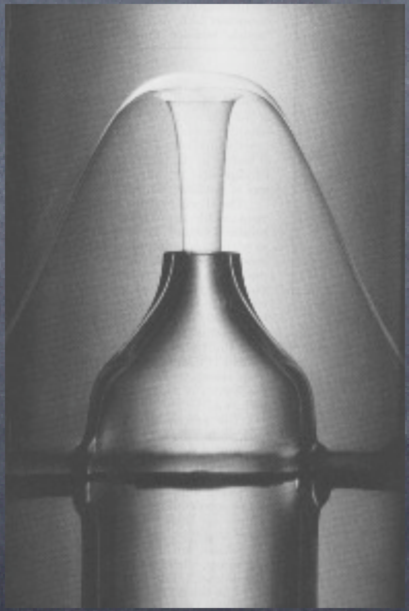
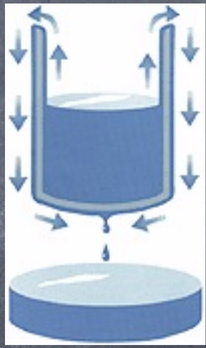
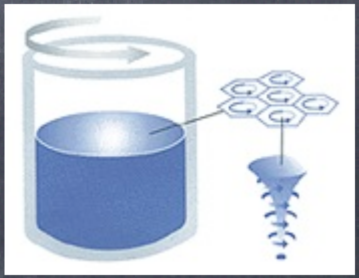
$$T \downarrow \text{ requires } \left(\frac{\hbar^2 k^2}{2m} - \mu\right) \downarrow$$

$$T \downarrow \text{ requires } \mu \uparrow$$

But $\mu(T) < 0$

at T_c, $\mu(T_c) = 0$

Bose-Einstein Condensation

Bose-Einstein Condensation

$$\frac{N}{V} = \frac{1}{2\pi^2}\int_0^\infty \frac{k^2\,dk}{e^{(\hbar^2 k^2/(2m))/k_B T} - 1}$$

$$\begin{aligned} k &= \frac{\sqrt{2mk_BT_c}\sqrt{x}}{\hbar} \\ dk &= \frac{\sqrt{mk_BT_c}}{\sqrt{2}\,\hbar}\frac{dx}{\sqrt{x}} \end{aligned}$$

$$\begin{aligned} \frac{N}{V} &= \frac{1}{4\pi^2}\left(\frac{2mk_BT_c}{\hbar^2}\right)^{3/2}\int_0^\infty \frac{\sqrt{x}\,dx}{e^x - 1} \\ &= \frac{1}{4\pi^2}\left(\frac{2mk_BT_c}{\hbar^2}\right)^{3/2}\Gamma\left(\tfrac{3}{2}\right)\zeta\left(\tfrac{3}{2}\right) \\ &= \frac{1}{8}\left(\frac{2mk_BT_c}{\pi\hbar^2}\right)^{3/2} 2.612 \end{aligned}$$

$$T_c = \frac{2\pi\hbar^2}{mk_B}\left(\frac{N}{2.612V}\right)^{2/3}$$

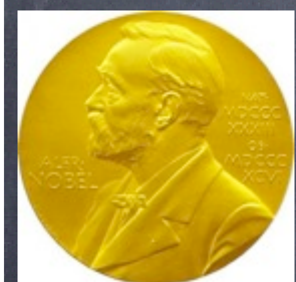
for ^4He:

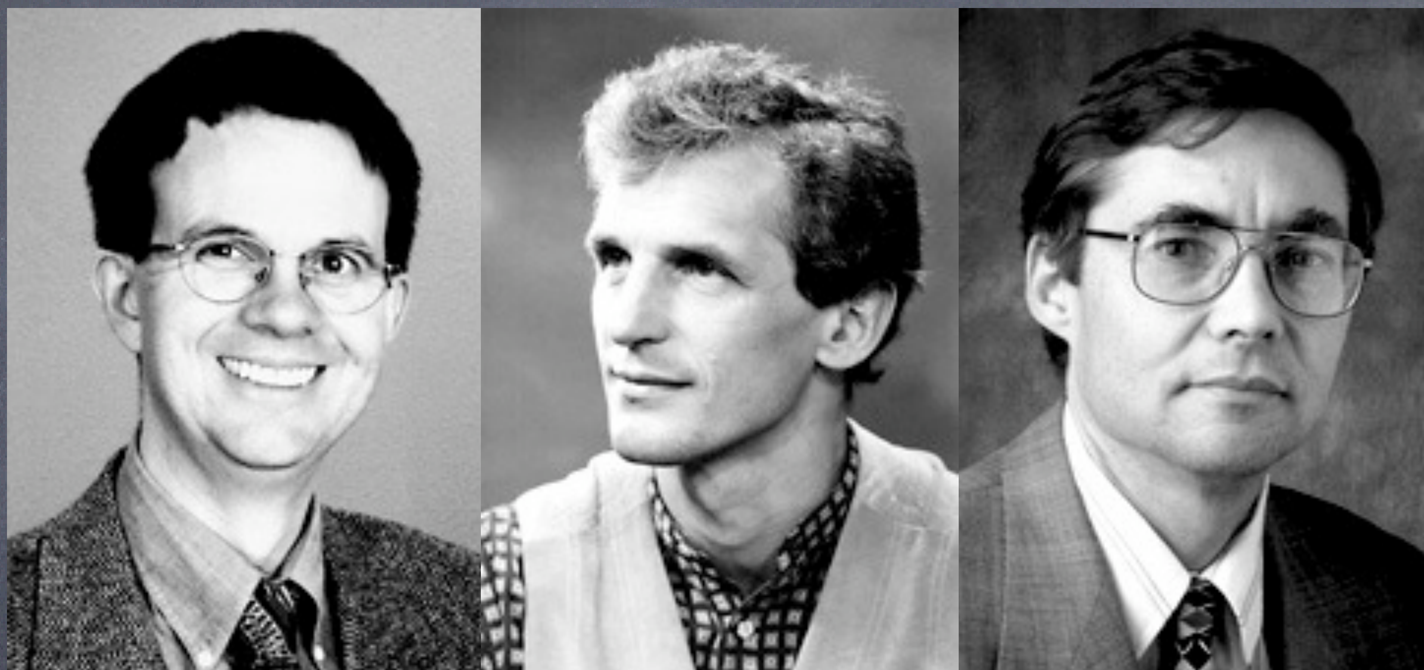
$$\frac{N}{V} = 0.15\,g/cm^3\,\frac{1}{4\cdot 1.67\times 10^{-27}kg/\text{atom}} = 2.2\times 10^8\;1/m^3$$

$$T_c = 3.1K$$

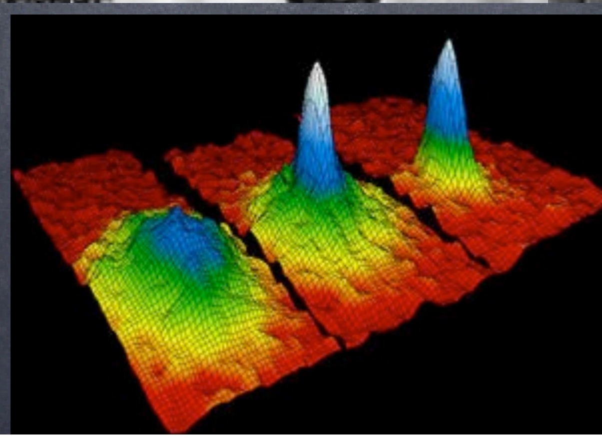
$$T_{c,exp} = 2.17K$$

2001 Cornell, Ketterle, Wieman





velocity distribution



dilute gas of rubidium atoms