QM 115B Lec 9

Combine $|l,m\rangle$ $|s,s_m\rangle$

highest state

$j = l+s$ $|j = l+s, j_m = l+s\rangle = |l,l\rangle|s,s\rangle$

$J_- = L_- + S_-$ J_-

$|j = l+s, l+s-1\rangle = \dfrac{a|l,l-1\rangle|s,s\rangle + b|l,l\rangle|s,s-1\rangle}{\sqrt{a^2+b^2}}$

orthogonal

$\vdots$

lowest $|l,-l\rangle|s,-s\rangle$

$|j = l+s-1, l+s-1\rangle = \dfrac{1}{\sqrt{a^2+b^2}}\left(-b|l,l-1\rangle|s,s\rangle + a|l,l\rangle|s,s-1\rangle\right)$

J_- $|j = l+s-1, l+s-2\rangle = \dfrac{1}{\sqrt{c^2+d^2+e^2}}\left(c|l,l-2\rangle|s,s\rangle + d|l,l-1\rangle|s,s-1\rangle + e|l,l\rangle|s,s-2\rangle\right)$

$\vdots$

$|j = l+s-1, -l-s+1\rangle$

$j = |l-s|$

$$(2l+1)(2s+1) = \sum_{j,j_m} 1$$

$j = |l-s|, |l-s|+1, \ldots \ldots l+s$

$j_m = \begin{cases} |l-s| \\ \vdots \\ -|l-s| \end{cases}$ $\qquad j_m = \begin{cases} l+s \\ \vdots \\ -l-s \end{cases}$

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two particle states

$\psi(r_1, r_2, t)$ $\qquad i\hbar \frac{\partial \psi}{\partial t} = H\psi$

$$H = -\frac{\hbar^2}{2m_1}\vec{\nabla}_1^2 - \frac{\hbar^2}{2m_2}\vec{\nabla}_2 + V(r_1, r_2, t)$$

$$\int \psi^2 \, d^3r_1 \, d^3r_2 = 1$$

if particle $\frac{1}{2}$ is state $\psi_a(r_1)$, $\psi_b(r_2)$

$$\psi(r_1, r_2) = \psi_a(r_1)\psi_b(r_2)$$

identical particles $\psi(r_1, r_2) = A\left(\psi_a(r_1)\psi_b(r_2) \pm \psi_b(r_2)\psi_a(r_1)\right)$

bosons $+$ symmetric
fermions $-$ antisymmetric

integer spin → Higgs 0, photon 1, graviton? 2

$\frac{1}{2}$ integer spin → electrons $\frac{1}{2}$, quarks $\frac{1}{2}$, neutrinos $\frac{1}{2}$

Pauli exclusion: $\psi_a = \psi_b$ $\quad \psi = 0$

in general exchange op P : $P^2 = 1$
eigenvalues ± 1

identical particles $[P, H] = 0$

$$\psi(r_1, r_2) = \pm \psi(r_2, r_1)$$

bosons ($+$) fermions ($-$)

$$\psi = \psi_{space} \, \chi_{spin}$$

triplet (spin 1)

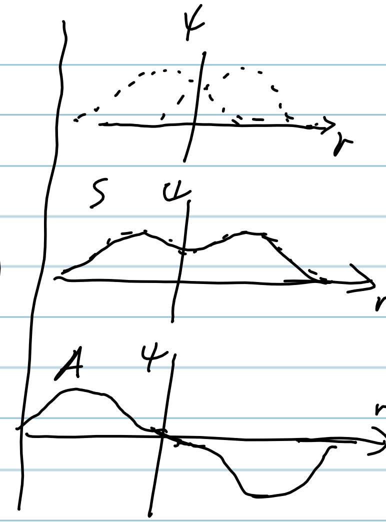
for fermions $\quad A \cdot S = A$

$\qquad\qquad\qquad S \cdot A = A$

singlet (spin 0)

for bosons $\quad S \cdot S = S$

$\qquad\qquad\quad A \cdot A = S$



Atoms

Z electrons

$$H = \sum_{j=1}^{Z} \left\{ \frac{-\hbar^2 \vec{\nabla}_j^2}{2M} - \frac{\hbar c Z \alpha}{r_j} \right\} + \underbrace{\frac{1}{2} \sum_{j \neq k} \frac{\hbar c \alpha}{|\vec{r}_j - \vec{r}_k|}}_{\text{neglect}}$$

Helium $\quad Z = 2$

$$\psi(\vec{r}_1, \vec{r}_2) = \psi_{n\ell m}(\vec{r}_1)\, \psi_{n'\ell' m'}(\vec{r}_2)$$

ground state $\quad \psi = \psi_{100}(\vec{r}_1)\, \psi_{100}(\vec{r}_2)$

spin is antisymmetric $\quad$ singlet

para

$$E_{nn'} = 4(E_n + E_{n'})$$

$$E_{11} = 8(-13.6\,eV) = -109\,eV$$

$$E_{exp} = -79\,eV$$

excited para $\quad \psi_{n\ell m}(\vec{r}_1)\psi_{100}(\vec{r}_2) + \psi_{100}(\vec{r}_1)\psi_{n\ell m}(\vec{r}_2)$

ortho $\quad$ spatial is anti $\quad$ spin is symm $\quad$ triplet

$$\psi_{n\ell m}(\vec{r}_1)\, \psi_{100}(\vec{r}_2) - \psi_{100}(\vec{r}_1)\, \psi_{n\ell m}(\vec{r}_2)$$

e further apart

lower energy