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QM 115B Lec. 5

spectroscopic notation

l	0	1	2	3	4	5
letter	s	p	d	f	g	h

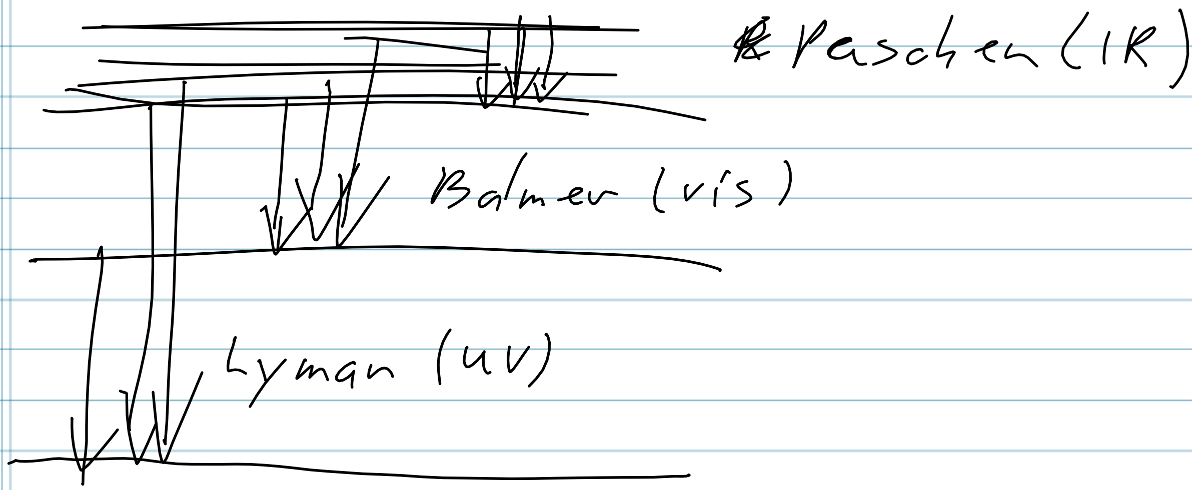
$$E_\gamma = E_i - E_f = -13.6\,eV\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

$$= h\nu = h\frac{c}{\lambda}$$

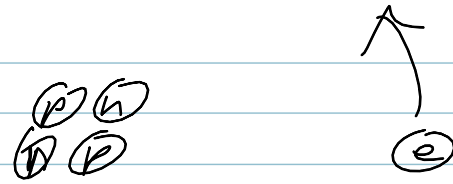
$$\frac{1}{\lambda} = -\frac{\alpha^2 mc^2}{2hc}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

$$R = \frac{\alpha^2 mc}{2(2\pi\hbar)} = \frac{\alpha}{a\,4\pi} \quad \text{Rydberg}$$

$$= 1.097\times10^7\,m^{-1}$$



Helium



$$V_H = -\frac{e^2}{4\pi\epsilon_0}\frac{1}{r} = -\hbar c\frac{\alpha}{r}$$

$$V_{He} = -\frac{(2e)e}{4\pi\epsilon_0}\frac{1}{r} = -\hbar c\frac{2\alpha}{r}$$

Z protons

$$V = -\hbar c\frac{Z\alpha}{r}$$

$$a = \frac{\hbar}{\alpha m c} \rightarrow \frac{\hbar}{Z\alpha m c}$$

$$E = -\frac{\alpha^2 m c^2}{2n^2} \rightarrow -\frac{Z\alpha^2 m c^2}{2n^2}$$

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Angular Momentum

$$\vec{L} = \vec{r} \times \vec{p}$$

$$L_x = y p_z - z p_y \qquad L_y = z p_x - x p_z \qquad L_z = x p_y - y p_x$$

$$p_x \to \frac{\hbar}{i}\frac{\partial}{\partial x} \quad \cdots$$

$$[L_x, L_y] = [y p_z - z p_y,\ z p_x - x p_z]$$

$$= [y p_z, z p_x] - [y p_z, x p_z]$$

$$- [z p_y, z p_x] + [z p_y, x p_z]$$

$$= y p_x [p_z, z] + p_y x [z, p_z]$$

$$= y p_x (-i\hbar) + p_y x\, i\hbar$$

$$= i\hbar (x p_y - y p_x) = i\hbar L_z$$

$$[L_y, L_z] = i\hbar L_x \qquad [L_z, L_x] = i\hbar L_y$$

$$L^2 = L_x^2 + L_y^2 + L_z^2$$

$$[L^2, L_x] = [L_x^2, L_x] + [L_y^2, L_x] + [L_z^2, L_x]$$

$$= L_y [L_y, L_x] + [L_y, L_x] L_y$$

$$+ L_z [L_z, L_x] + [L_z, L_x] L_z$$

$$= L_y (-i\hbar L_z) + (-i\hbar L_z) L_y$$

$$+ L_z (i\hbar L_y) + (i\hbar L_y) L_z$$

$$= 0$$

$$[L^2, L_y] = 0 \qquad [L^2, L_z] = 0$$

$$[L^2, \vec{L}] = 0$$

$$L^2 \phi = \lambda \phi \qquad\qquad L_z \phi = \mu \phi$$

raising / lowering $\qquad L_\pm = L_x \pm i L_y$

$$[L_z, L_\pm] = [L_z, L_x] \pm i [L_z, L_y]$$

$$= i\hbar L_y \pm i(-i\hbar L_x)$$

$$= \pm \hbar L_\pm$$

$$[L^2, L_\pm] = 0$$

$$L^2 \underbrace{L_\pm \phi}_{\text{eigenfunction}} = L_\pm L^2 \phi = L_\pm \lambda \phi = \lambda \underbrace{L_\pm \phi}_{\text{eigenfunction}}$$

$$L_z L_\pm \phi = (L_z L_\pm - L_\pm L_z)\phi + L_\pm L_z \phi$$

$$= \pm\hbar L_\pm \phi + L_\pm \cancel{L_z} \mu \phi$$

$$= \underbrace{(\mu \pm \hbar)}_{\text{eigenvalue}} \underbrace{L_\pm \phi}_{\text{eigenfunction}}$$

$\mu^2 < \lambda^2$ $\qquad$ top of the ladder

$$L_+ \phi_t = 0$$

$$L_z \phi_t = \hbar \underset{\text{def}}{\ell} \phi_t \qquad\qquad L^2 \phi_t = \lambda \phi_t$$