QM115B Lec 27

$$\nabla^2 \phi(r) = -\frac{\rho(\vec{r})}{\epsilon}$$

$$\rho(r) = 4\pi e\,\delta(\vec{r}) - 4\pi e\,(n(\vec{r}) - n_0)$$

ave equilibrium Vale

guess $\phi(\vec{r}) = -\frac{e}{\epsilon r} g(r)$

$$\nabla^2 \phi(\vec{r}) = -\frac{e}{\epsilon}\left(4\pi\delta(\vec{r})g(r) + \frac{4\pi}{r}\frac{\partial^2 g}{\partial r^2}\right)$$

$$-\frac{1}{r}\frac{\partial^2 g}{\partial r^2} = (n(\vec{r}) - n_0)$$

$$n(\vec{r}) = n_0 e^{+e\phi/k_BT}$$

$$\simeq n_0\left(1 + \frac{e\phi}{k_BT} + \cdots\right)$$

$$-\frac{1}{r}\frac{\partial^2 g}{\partial r^2} \approx n_0 \frac{e}{k_BT}\left(-\frac{e}{\epsilon r} g(r)\right)$$

$$g(r) = e^{-q_D r}$$

Debye screening length $\frac{1}{q_D}$

$$q_D^2 = \frac{n_0 e^2}{\varepsilon k_BT}$$

take the approx that electrons are in a Fermi gas

$$E_F = \frac{\hbar^2 k_F^2}{2 m_*} \approx \frac{3}{2} k_B T$$

$$n_0 = \frac{k_F^3}{3\pi^2}$$

$$q_D^2 = \frac{n_0 e^2}{\varepsilon k_B T} = \frac{k_F^3 e^2}{3\pi^2 \varepsilon \left(\frac{2}{3} \frac{\hbar k_F^2}{2 m_*} \right)}$$

$$= \frac{k_F m_* e^2}{\varepsilon \pi^2 \hbar^2} \equiv q_{TF}^2$$

$$f(\theta) = \frac{2 m \alpha c}{\epsilon_r \hbar (q_{TF}^2 + K^2)}$$

$$K = 2k \sin(\theta/2)$$

cf Coulomb $f(\theta) = \frac{2 m \alpha c}{\hbar K^2}$

$$\epsilon_{eff} = \epsilon_r \epsilon_0 \left(1 + \frac{q_{TF}^2}{K^2} \right)$$ dielectric function

R4

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 = \frac{4m^2\alpha^2c^2}{\epsilon_r^2\hbar^2(q_{TF}^2+K^2)^2}$$

$$\sigma = \int \frac{d\sigma}{d\Omega}\,d\Omega = \frac{16\pi m_*^2\alpha^2c^2}{\hbar^2\epsilon_r^2 q_{TF}^2(q_{TF}^2+4k^2)}$$

$$= \frac{4\pi^2}{k_F a_*(q_{TF}^2+4k^2)}$$

$n = \frac{10^{18}}{cm^3}$ $\quad k = k_F = \frac{1}{3nm}$ $\quad q_{TF} = \frac{1}{5nm}$

$$\sigma = 24\,nm^2$$

$$L = \frac{1}{n\cdot\sigma} = \frac{1}{\frac{10^{18}}{cm^3}\left(\frac{100cm}{1m}\frac{1m}{10^9nm}\right)^3 24\,nm^2}$$

$$= 42\,nm$$

Hypefine triplet -> singlet lifetime

$$\Delta E = \frac{4 g_p \hbar^4}{3 m_p m_e^2 a^4 c^2} \qquad \omega_0 = \frac{\Delta E}{\hbar}$$

EM energy density $u = \frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}$ $\qquad \epsilon_0 \mu_0 = \frac{1}{c^2}$

electric dipole $H' = \vec{p} \cdot \vec{E}$ $\qquad$ magnetic $H' = -\frac{\gamma\hbar}{2} \vec{\sigma} \cdot \vec{B}$

$$A_{dipole} = \frac{\omega_0^3 |\langle \vec{p} \rangle|^2}{3\pi \epsilon_0 \hbar c^3} \qquad A_{magnetic} = \frac{\omega_0^3 \mu_0 |\langle \uparrow | -\frac{\gamma\hbar}{2} \vec{\sigma} | \downarrow \rangle|^2}{3\pi \hbar c^3}$$

$$\langle \uparrow | \vec{\sigma} | \downarrow \rangle = (1\ 0) \begin{pmatrix} \hat{z} & \hat{x} - i\hat{y} \\ \hat{x} + i\hat{y} & -\hat{z} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \hat{x} - i\hat{y}$$

$$|\langle \uparrow | -\frac{\gamma\hbar}{2} \vec{\sigma} | \downarrow \rangle|^2 = \frac{\gamma^2 \hbar^2}{4} (1^2 + |i|^2) = \frac{\gamma^2 \hbar^2}{2} = \frac{e^2 \hbar^2}{2 m_e^2}$$

$$= \frac{4\pi \epsilon_0 \hbar^3 c \alpha}{2 m_e^2}$$

$$A_{magnetic} = \frac{\omega_0^3 4\pi \epsilon_0 \hbar^3 c \alpha \mu_0}{2 m_e^2 \, 3\pi \hbar c^3} = \left(\frac{\Delta E}{\hbar}\right)^3 \frac{4 \hbar^2 \alpha}{6 m_e^2 c^2} \frac{1}{c^2}$$

$$= (5.9 \times 10^{-6} eV)^3 \frac{2}{3} \frac{1}{137} \frac{1}{(511 \times 10^3 eV)^2} \frac{1}{\hbar}$$

$$= \frac{3.8 \times 10^{-30} eV}{6.6 \times 10^{-16} eV \cdot s} = 5.8 \times 10^{-15}\ 1/s$$

$$\tau = \frac{1}{A} = 1.7 \times 10^{4} s \left(\frac{1 yr}{\pi \times 10^7 s}\right) = 5.5 \times 10^{6} yr$$