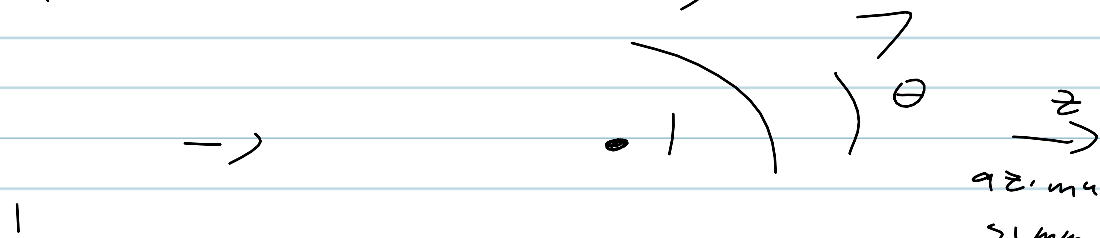
QM 115B Lec 24

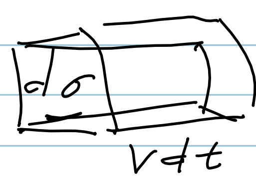
Quantum Scattering



azimuthal symmetry
elastic

$$\psi(r,\theta) \simeq A\left(e^{ikz} + f(\theta)\frac{e^{ikr}}{r}\right) \quad \text{for large } r$$

$$k = \frac{\sqrt{2mE}}{\hbar} \qquad p_z = \hbar k$$



$$dP_{in} = |\psi_{incident}|^2 dV = |A|^2 v\,dt\,d\sigma$$

$$dP_{out} = |\psi_{scattered}|^2 dV = \left|\frac{Af}{r^2}\right|^2 v\,dt\,r^2 d\Omega$$

$$d\sigma = |f|^2 d\Omega$$

$$D(\theta) = \frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

Integral Schrödinger Eq

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi$$

$$(\nabla^2 + k^2)\psi = Q$$

$$k = \frac{\sqrt{2mE}}{\hbar} \qquad Q = \frac{2m}{\hbar^2}V\psi$$

almost Helmholtz eq.

Green's functions $\quad (\nabla^2+k^2)G(r) = \delta^{(3)}(\vec{r})$

$$\psi(\vec{r}) = \int G(\vec{r}-\vec{r}_0)\,Q(\vec{r}_0)\,d^3r_0$$

$$(\nabla^2+k^2)\psi(\vec{r}) = \int (\nabla^2+k^2)G(\vec{r}-\vec{r}_0)\,Q(\vec{r}_0)\,d^3r_0$$

$$= \int \delta^{(3)}(\vec{r}-\vec{r}_0)\,Q(\vec{r}_0)\,d^3r_0$$

$$= Q(\vec{r})$$

$G(\vec{r})$ is "response" to a δ-function source
propagator

$$G(\vec{r}) = -\frac{e^{ikr}}{4\pi r}$$

(111)

Prob 11.8

$$\nabla^2 G(\vec{r}) = -\frac{e^{ikr}}{4\pi} \nabla^2 \frac{1}{r} - \frac{1}{4\pi r} \nabla^2 e^{ikr}$$
$$- \frac{2}{4\pi} \left(\vec{\nabla} \frac{1}{r} \right) \cdot \left(\vec{\nabla} e^{ik_1 r} \right)$$

$$= -\frac{e^{ikr}}{4\pi} \left(-4\pi \delta^{(3)}(\vec{r}) \right) - \frac{1}{4\pi r} \left(\frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} e^{ikr} \right)$$
$$- \frac{1}{2\pi} \left(\frac{\partial}{\partial r} \frac{1}{r} \right) \frac{\partial}{\partial r} e^{ikr}$$

$$= e^{ikr} \delta^{(3)}(\vec{r}) - \frac{1}{4\pi r^3} \frac{\partial}{\partial r} r^2 ik e^{ikr}$$
$$- \frac{1}{2\pi} \left(\frac{-1}{r^2} \right) \left(ik e^{ikr} \right)$$

$$= \delta^{(3)}(\vec{r}) - \frac{1}{4\pi r^3} \left(2rik + r^2 (ik)^2 \right) e^{ikr}$$
$$+ \frac{ik}{2\pi r^2} e^{ikr}$$

$$= \delta^{(3)}(\vec{r}) + \frac{k^2 e^{ikr}}{4\pi r} - \frac{ik}{2\pi r^2} e^{ikr} + \frac{ik}{2\pi r^2} e^{ikr}$$

$$= \delta^{(3)}(\vec{r}) - k^2 G(\vec{r})$$

$$(\nabla^2 + k^2) \left(\frac{-e^{ikr}}{4\pi r} \right) = \delta^{(3)}(\vec{r})$$

$$\psi(\vec{r}) = \int G(\vec{r}-\vec{r}_0)\, Q(\vec{r}_0)\, d^3 r_0$$

$$G(\vec{r}) = \frac{-e^{ikr}}{4\pi r} + G_0(\vec{r})$$

free solution

$$(\nabla^2 + k^2)\, G_0(\vec{r}) = 0 \qquad (\nabla^2 + k^2)\, \psi_0 = 0$$

$$\psi(\vec{r}) = \psi_0(\vec{r}) + \int \left(\frac{-e^{ik|\vec{r}-\vec{r}_0|}}{4\pi |\vec{r}-\vec{r}_0|} \right) \frac{2mV(\vec{r}_0)}{\hbar^2}\, \psi(\vec{r}_0)\, d^3 r_0$$

incoming wave

$$= \psi_0(\vec{r}) - \frac{m}{2\pi\hbar^2} \int \frac{e^{ik|\vec{r}-\vec{r}_0|}}{|\vec{r}-\vec{r}_0|}\, V(\vec{r}_0)\, \psi(\vec{r}_0)\, d^3 r_0$$

$$\psi \approx \psi_0$$

$$\approx \psi_0 + \int G V \psi_0$$

$$\approx \psi_0 + \int\int G V G V \psi_0$$

$$= \psi_0 + \int\int\int G V G V G V \psi_0$$