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QM 115 B Lec. 23

$$H = -\gamma \vec{B} \cdot \vec{S} \quad \text{spin } \frac{1}{2}$$

$$\vec{B} = B_{rf} \cos(\omega t) \hat{x} - B_{rf} \sin(\omega t) \hat{y} + B_0 \hat{z}$$

$$\chi(t) = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

$$a(t) = e^{\frac{i\omega t}{2}} \left(a_0 \cos\left(\frac{\omega' t}{2}\right) + \frac{i}{\omega'} (a_0(\omega_0 - \omega) + b_0 \Omega) \sin\left(\frac{\omega' t}{2}\right) \right)$$

$$b(t) = e^{-\frac{i\omega t}{2}} \left(b_0 \cos\left(\frac{\omega' t}{2}\right) + \frac{i}{\omega'} (-b_0(\omega_0 - \omega) + a_0 \Omega) \sin\left(\frac{\omega' t}{2}\right) \right)$$

$$\omega_0 = \gamma B_0 \quad \Omega = \gamma B_{rf}$$

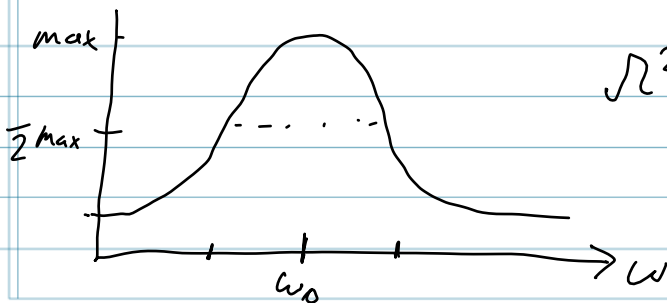
$$\omega' = \sqrt{(\omega - \omega_0)^2 + \Omega^2}$$

Start with particle spin up at $t=0$ $a_0=1$
 $b_0=0$

$$P_{\uparrow \rightarrow \downarrow}(t) = |b(t)|^2 = \left| \frac{i}{\omega'} \Omega \sin\left(\frac{\omega' t}{2}\right) e^{-\frac{i\omega t}{2}} \right|^2$$

$$= \frac{\Omega^2}{(\omega - \omega_0)^2 + \Omega^2} \sin^2\left(\frac{\omega' t}{2}\right)$$

Resonance curve $P_{\uparrow \rightarrow \downarrow}(\omega) = \frac{\Omega^2}{(\omega - \omega_0)^2 + \Omega^2}$



$$\Omega^2 = \frac{1}{2} ((\omega - \omega_0)^2 + \Omega^2)$$
$$\frac{1}{2} \Omega^2 = \frac{1}{2} (\omega - \omega_0)^2$$

$$\omega = \omega_0 \pm \Omega$$

$$\Delta\omega = 2\Omega$$

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for a proton $\gamma = \frac{g_p e}{2m_p}$ $g_p = 5.58$

$$B_0 = 1 \text{ T} \quad B_{rf} = 10^{-6} \text{ T}$$

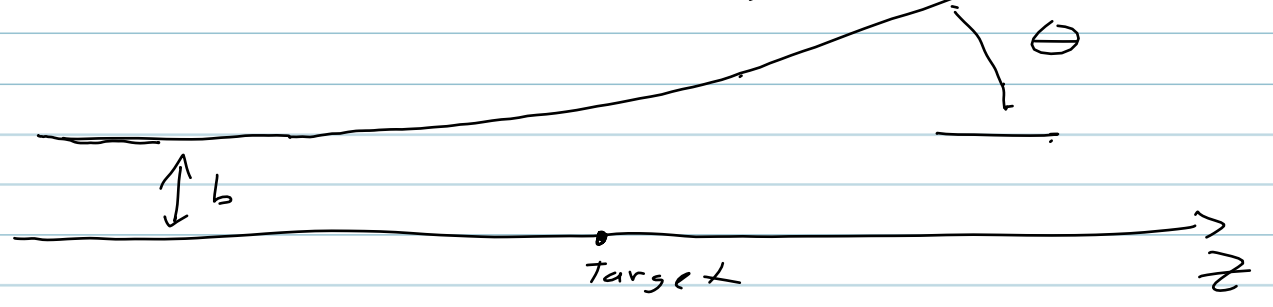
$$\nu_{res} = \frac{\omega_0}{2\pi} = \frac{g_p e B_0}{4\pi m_p} = 42 \text{ MHz}$$

$$\Delta\nu = \frac{\Delta\omega}{2\pi} = \frac{\hbar \gamma}{\pi} = \frac{\gamma B_{rf}}{\pi} = \frac{\gamma B_0}{\pi} \frac{B_{rf}}{B_0}$$

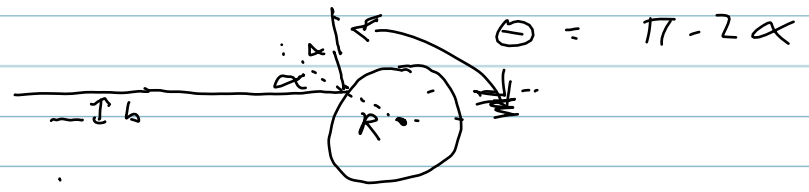
$$= 2 \nu_{res} \frac{B_{rf}}{B_0} = \nu_{res} \cdot 2 \times 10^{-6}$$

$$= 85 \text{ Hz}$$

Classical Scattering



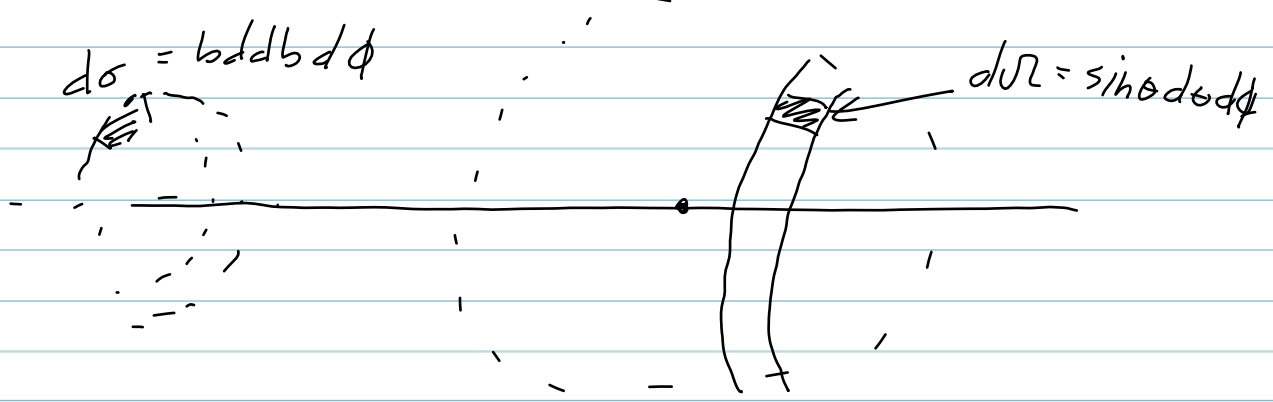
eg. hard sphere



$$b = R \sin \alpha$$

$$= R \sin \left(\frac{\pi - \theta}{2} \right) = R \cos \left(\frac{\theta}{2} \right)$$

$$\theta = \begin{cases} 2 \cos^{-1} (b/R) & b \leq R \\ 0 & b > R \end{cases}$$



differential-scattering cross-section

$$d\sigma = D(\theta) d\Omega$$

$$D(\theta) = \frac{d\sigma}{d\Omega} = \frac{b db d\phi}{\sin\theta d\theta d\phi} = \frac{b}{\sin\theta} \left| \frac{db}{d\theta} \right|$$

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hard sphere $b = R \cos \frac{\theta}{2}$

$$\frac{db}{d\theta} = -\frac{1}{2} R \sin \frac{\theta}{2}$$

$$D(\theta) = \frac{R \cos \frac{\theta}{2}}{\sin \theta} \left(\frac{1}{2} R \sin \frac{\theta}{2} \right) = \frac{R^2}{4}$$

total cross-section

$$\sigma = \int D(\theta) d\Omega = \frac{R^2}{4} \int d\Omega = \frac{R^2 4\pi}{4} = \pi R^2$$

cross-sectional area of a sphere

beam of particles with uniform "luminosity"

$\mathcal{L} \equiv \#$ of incident particles per unit area per unit time

number enter $d\sigma$ scatter to $d\Omega$

$$dN = \mathcal{L} d\sigma = \mathcal{L} D(\theta) d\Omega$$

$$D(\theta) = \frac{1}{\mathcal{L}} \frac{dN}{d\Omega}$$

count number per unit time

divide by $d\Omega$

divide by Luminosity

nuclear physics $1 \text{ barn} = 10^{-28} \text{ m}^2$

LHC one year of running $10 \frac{1}{\text{fb}}$

$$\mathcal{L} = 10 \frac{1}{\text{fb year}}$$

to get 1000 events

$$\frac{d\sigma}{d\Omega} = \frac{1}{\mathcal{L}} \cdot \frac{1000}{d\Omega}$$

$$\sigma = \frac{1000 \text{ fb}}{\mathcal{L}} = \frac{1000 \cdot 10 \text{ fb}}{10}$$

$$= 1000 \text{ fb} = 10^{12} \text{ b}$$

$$= 10^{-40} \text{ m}^2$$

direct detection of dark matter

$$\sigma \sim 10^{-46} \text{ m}^2 - 10^{-50} \text{ m}^2$$