QM 115B Lec. 13

Time-Independent Non-Degenerate Perturbation Theory

$$H = H^0 + \underset{\overset{\shortparallel}{1}}{\lambda} H'$$

we want $H\psi_n = E_n\psi_n$ $\langle \psi_n^0 | \psi_m^0 \rangle = \delta_{mn}$

we have $H^0\psi_n^0 = E_n^0\psi_n^0$

$$\psi_n = \psi_n^0 + \lambda\psi_n^1 + \lambda^2\psi_n^2 + \ldots$$

$$E_n = E_n^0 + \lambda E_n^1 + \lambda^2 E_n^2 + \ldots.$$

$$(H^0 + \lambda H')(\psi_n^0 + \lambda\psi_n^1 + \lambda^2\psi_n^2 + \ldots)$$

$$= (E_n^0 + \lambda E_n^1 + \lambda^2 E_n^2 + \ldots)(\psi_n^0 + \lambda\psi_n^1 + \lambda^2\psi_n^2 + \ldots)$$

$$H^0\psi_n^0 + \lambda(\overset{H^0}{\cancel{E_n}}\psi_n^1 + H'\psi_n^0) + \lambda^2(H^0\psi_n^2 + H'\psi_n^1 + \ldots$$

$$= E_n^0\psi_n^0 + \lambda(E_n^0\psi_n^1 + E_n^1\psi_n^0) + \lambda^2(E_n^0\psi_n^2 + E_n^1\psi_n^1 + E_n^2\psi_n^0) + \ldots$$

λ^0: $H^0 \cancel{E}\psi_n^0 = E_n^0\psi_n^0$

λ^1: $H^0\psi_n^1 + H'\psi_n^0 = E_n^0\psi_n^1 + E_n^1\psi_n^0$

λ^2: $H^0\psi_n^2 + H'\psi_n^1 = E_n^0\psi_n^2 + E_n^1\psi_n^1 + E_n^2\psi_n^0$

set $\lambda = 1$

* first order eq. $H^0\psi_n' + H'\psi_n^0 = E_n^0\psi_n' + E_n'\psi_n^0$

inner product

$$\underset{\parallel}{\langle\psi_n^0|H^0\psi_n'\rangle} + \langle\psi_n^0|H'\psi_n^0\rangle = E_n^0\langle\psi_n^0|\psi_n'\rangle + E_n'\underset{\overset{\parallel}{1}}{\langle\psi_n^0|\psi_n^0\rangle}$$

$$\langle H^0\psi_n^0|\psi_n'\rangle = E_n^0\langle\psi_n^0|\psi_n'\rangle$$

$$E_n' = \langle\psi_n^0|H'|\psi_n^0\rangle$$

"most import Eq" in Q.M.

* $$(H^0 - E_n^0)\psi_n' = -(H' - E_n')\psi_n^0$$

$$\psi_n' = \sum_{m\neq n} c_m^{(n)}\psi_m^0$$

↑

$$(H^0 - E_n^0)\psi_n^0 = 0$$

$$\sum_{m\neq n}(E_m^0 - E_n^0)\cancel{\Sigma}c_m^{(n)}\psi_m^0 = -(H' - E_n')\psi_n^0$$

inner prod with ψ_l^0

$$\sum_{m\neq n}(E_m^0 - E_n^0)c_m^{(n)}\langle\psi_l^0|\psi_m^0\rangle = -\langle\psi_l^0|H'\psi_n^0\rangle + E_n'\langle\psi_l^0|\psi_n^0\rangle$$

$l=n$ $$0 = -\langle\psi_n^0|H'|\psi_n^0\rangle + E_n'$$

$l\neq n$: $$(E_l^0 - E_n^0)c_l^{(n)} = -\langle\psi_l^0|H'|\psi_n^0\rangle$$

$$c_m^{(n)} = \frac{\langle\psi_m^0|H'|\psi_n^0\rangle}{E_n^0 - E_m^0}$$

so $$\psi_n' = \sum_{m\neq n}\frac{\langle\psi_m^0|H'|\psi_n^0\rangle}{E_n^0 - E_m^0}\psi_m^0$$

↑ non-degenerate

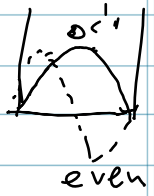
(4/9)

delta function pert. infinite sq. well $0 < x < a$

$$H' = \alpha\,\delta(x - a/2)$$

$$\psi_n^0 = \sqrt{\frac{2}{a}}\,\sin\left(\frac{n\pi}{a}x\right)$$

$$E_n' = \langle \psi_n^0 | H' | \psi_n^0 \rangle$$
$$= \frac{2}{a}\alpha \int_0^a \sin^2\left(\frac{n\pi}{\cancel{a}}x\right)\delta(x - a/2)\,dx$$
$$= \frac{2\alpha}{a}\sin^2\left(\frac{n\pi}{\cancel{2}a}\frac{a}{2}\right) = \frac{2\alpha}{a}\sin^2\left(\frac{n\pi}{\cancel{2}}\right)$$
$$= \begin{cases} 0 & n \text{ even} \\ \frac{2\alpha}{a} & n \text{ odd} \end{cases}$$



$$\psi_1' = \sum_{k \neq 1} \frac{\langle \psi_k^0 | H' | \psi_1^0 \rangle}{E_1^0 - E_k^0}\,\psi_k^0$$

$$\langle \psi_k^0 | H' | \psi_1^0 \rangle = \frac{2\alpha}{a}\int_0^a \sin\left(\frac{k\pi x}{a}\right)\delta(x - a/2)\sin\left(\frac{\pi x}{a}\right)$$
$$= \frac{2\alpha}{a}\sin\left(\frac{k\pi a}{2a}\right)\sin\left(\frac{\pi a}{2a}\right)$$
$$= \frac{2\alpha}{a}\sin\left(\frac{k\pi}{2}\right)$$

$$E_1^0 - E_k^0 = \frac{\pi^2\hbar^2}{2ma^2}(1 - k^2) \qquad k = 3, 5, 7 \ldots$$

$$\psi_1' = \frac{2\alpha}{a}\frac{2ma^2}{\pi^2\hbar^2}\left(\frac{-1}{1-9}\psi_3^0 + \frac{1}{1-25}\psi_5^0 - \frac{1}{1-49}\psi_7^0 + \ldots\right)$$

$$= \frac{4ma\alpha}{\pi^2\hbar^2}\left(\frac{1}{8}\sin\left(\frac{3\pi x}{a}\right) - \frac{1}{24}\sin\left(\frac{5\pi x}{a}\right) + \frac{1}{48}\sin\left(\frac{7\pi x}{\ldots}\right.\right.$$

Second Order

$$H^0\psi_n^2 + H'\psi_n^1 = E_n^0\psi_n^2 + E_n^1\psi_n^1 + E_n^2\psi_n^0$$

inner prod with ψ_n^0

$$\langle\psi_n^0|H^0\psi_n^2\rangle + \langle\psi_n^0|H'\psi_n^1\rangle \cancel{=}$$

$$= E_n^0\langle\psi_n^0|\psi_n^2\rangle + E_n^1\langle\psi_n^0|\psi_n^1\rangle + E_n^2\underset{\substack{\| \\ 1}}{\langle\psi_n^0|\psi_n^0\rangle}$$

$$\langle\psi_n^0|H^0\psi_n^2\rangle = \langle H^0\psi_n^0|\psi_n^2\rangle = E_n^0\langle\psi_n^0|\psi_n^2\rangle$$

$$E_n^2 = \langle\psi_n^0|H'|\psi_n^1\rangle - E_n^1\langle\psi_n^0|\psi_n^1\rangle$$

$$\langle\psi_n^0|\psi_n^1\rangle = \sum_{m\neq n} \underset{\substack{\| \\ n}}{c_m^{(n)}}\langle\psi_n^0|\psi_m^0\rangle = 0$$

$$E_n^2 = \langle\psi_n^0|H'|\psi_n^1\rangle = \sum_{m\neq n} C_m^{(n)}\langle\psi_n^0|H'\psi_m^0\rangle$$

$$= \sum_{m\neq n} \frac{\langle\psi_m^0|H'|\psi_n^0\rangle}{E_n^0 - E_m^0}\langle\psi_n^0|H'|\psi_m^0\rangle$$

$$= \sum_{m\neq n} \frac{|\langle\psi_m^0|H'|\psi_n^0\rangle|^2}{E_n^0 - E_m^0}$$