Monopole-fermion Scattering and the Solution to the Semiton/Unitarity Problem

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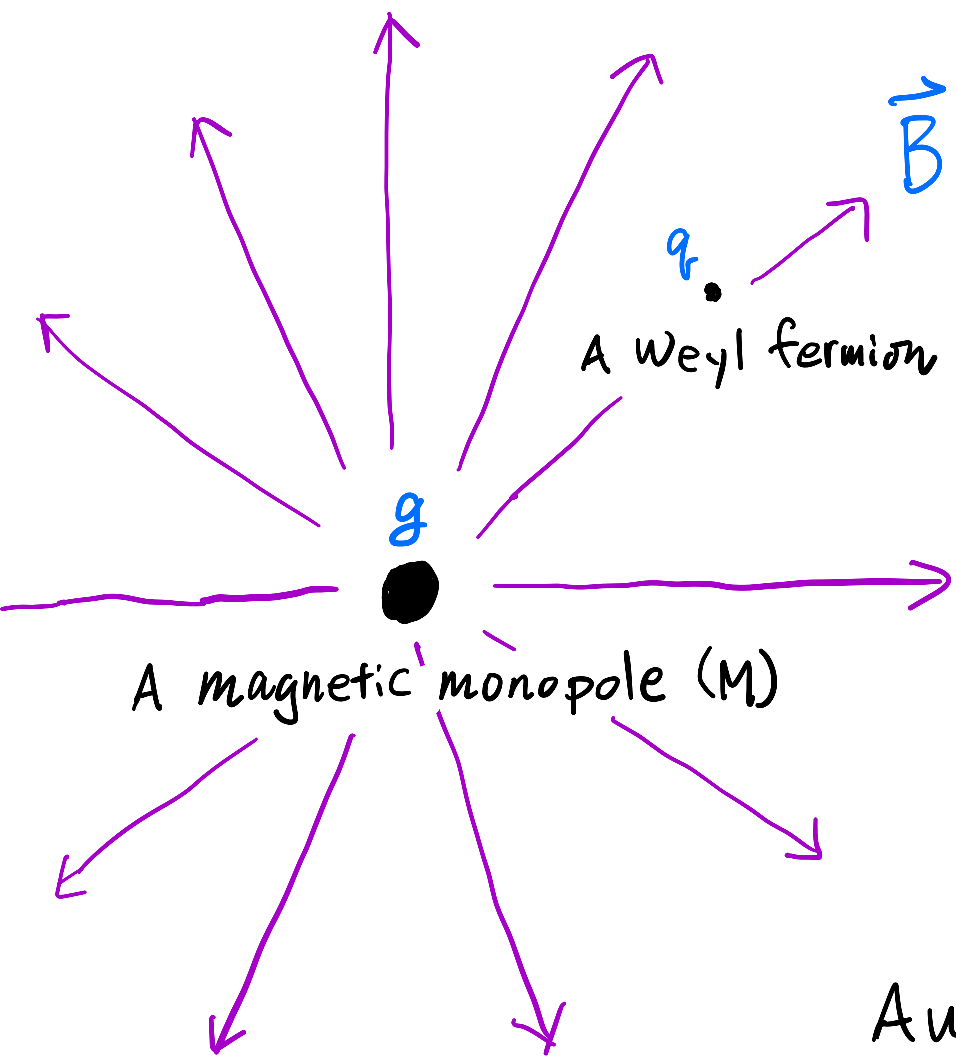
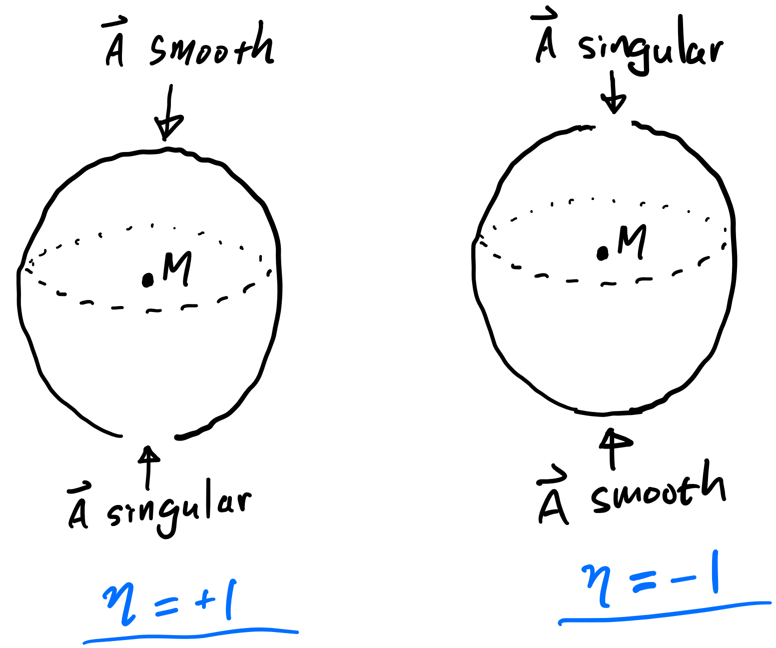
Dirac Quantization



$$\vec{B} = g\,\frac{\hat{r}}{r^2} \Longleftarrow \vec{A} = g\,\frac{\eta - \cos\theta}{r\sin\theta}\,\hat{\phi}$$

$\eta = \pm 1$



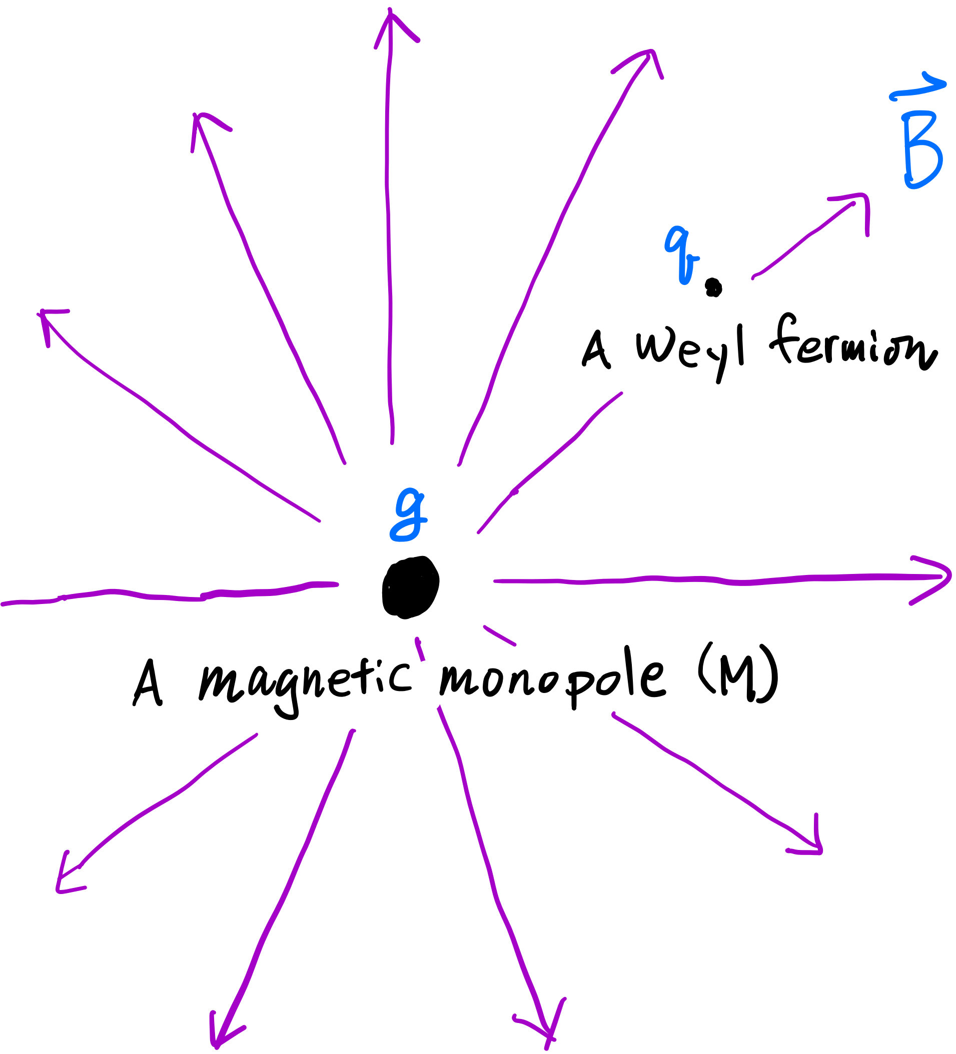
Away from poles, gauge trans. relate them:

$$\vec{A}_{(\eta=+1)} - \vec{A}_{(\eta=-1)} = \vec{\nabla}(2g\phi)$$

Dirac '31

$$\Rightarrow\ f_{(\eta=+1)} = e^{iq\cdot 2g\phi} f_{(\eta=-1)} \Rightarrow 2qg\cdot 2\pi = 2\pi n \Rightarrow \boxed{qg = \frac{n}{2}}$$

Orbital Angular Momentum around Monopole



$$\vec{B} = g\,\frac{\hat{r}}{r^2} \;\Longleftarrow\; \vec{A} = g\,\frac{\eta - \cos\theta}{r\sin\theta}\,\hat{\phi}$$

$\eta = \pm 1$

$$\frac{d}{dt}\left(\vec{r}\times\vec{p}\right) = \underbrace{\vec{r}\times\Big(\underbrace{q\,\vec{v}\times\Big(g\,\frac{\hat{r}}{r^2}\Big)}_{\text{Lorentz force}}\Big)}_{\text{torque}}$$

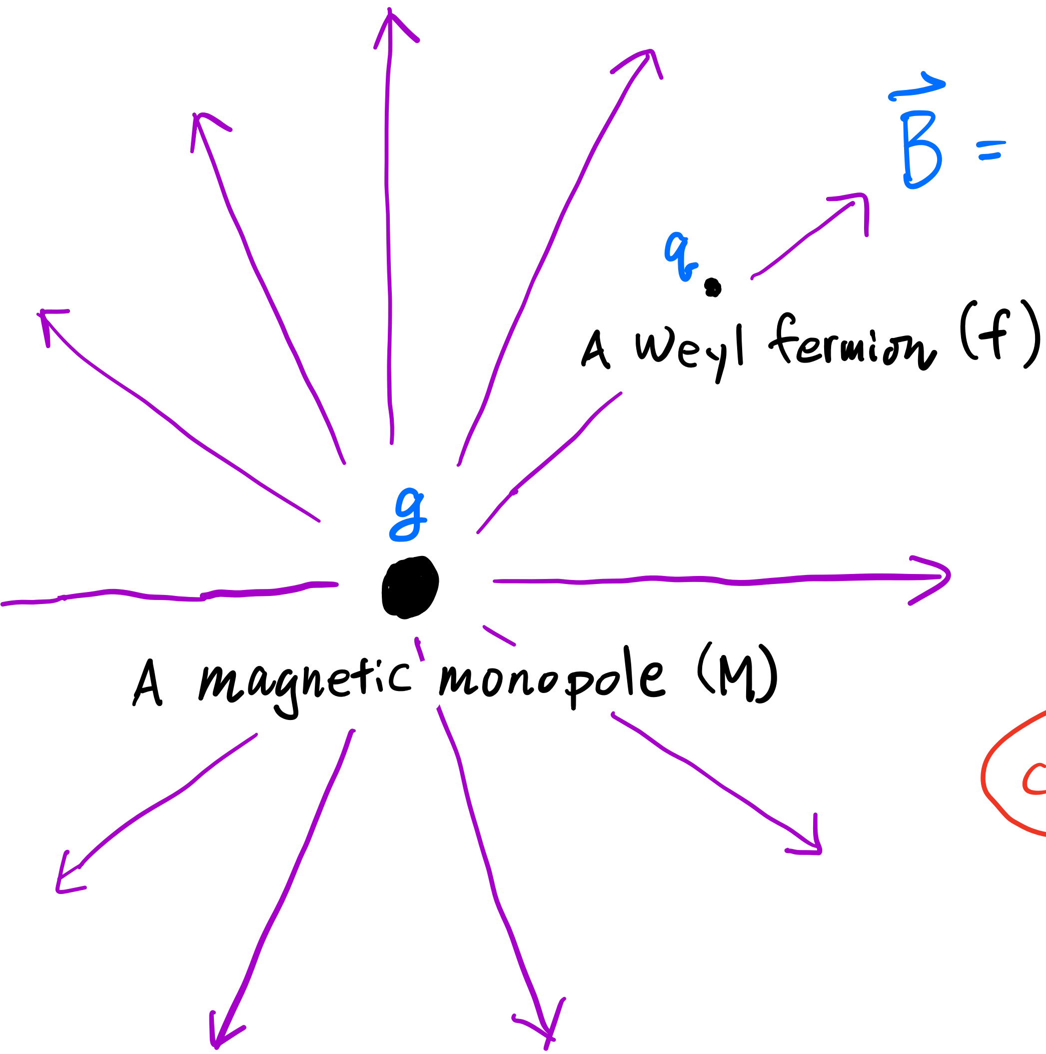
$$= qg\left(\frac{\vec{v}}{r} - \frac{(\vec{r}\cdot\vec{v})\,\vec{r}}{r^3}\right)$$

$$= \frac{d}{dt}\left(qg\,\hat{r}\right)$$

Classically, conserved orbital angular momentum is not $\vec{r}\times\vec{p}$

but $\vec{r}\times\vec{p} - qg\,\hat{r}$!

Orbital Angular Momentum around Monopole



$$\vec{B} = g\frac{\hat{r}}{r^2} \Longleftarrow \vec{A} = g\,\frac{\eta - \cos\theta}{r\sin\theta}\,\hat{\phi}$$

$\eta = \pm 1$

Quantum mechanically,

wrong! $\vec{L} = \vec{r}\times(-i\vec{D})$

$\vec{D} = \vec{\nabla} - iq\vec{A}$

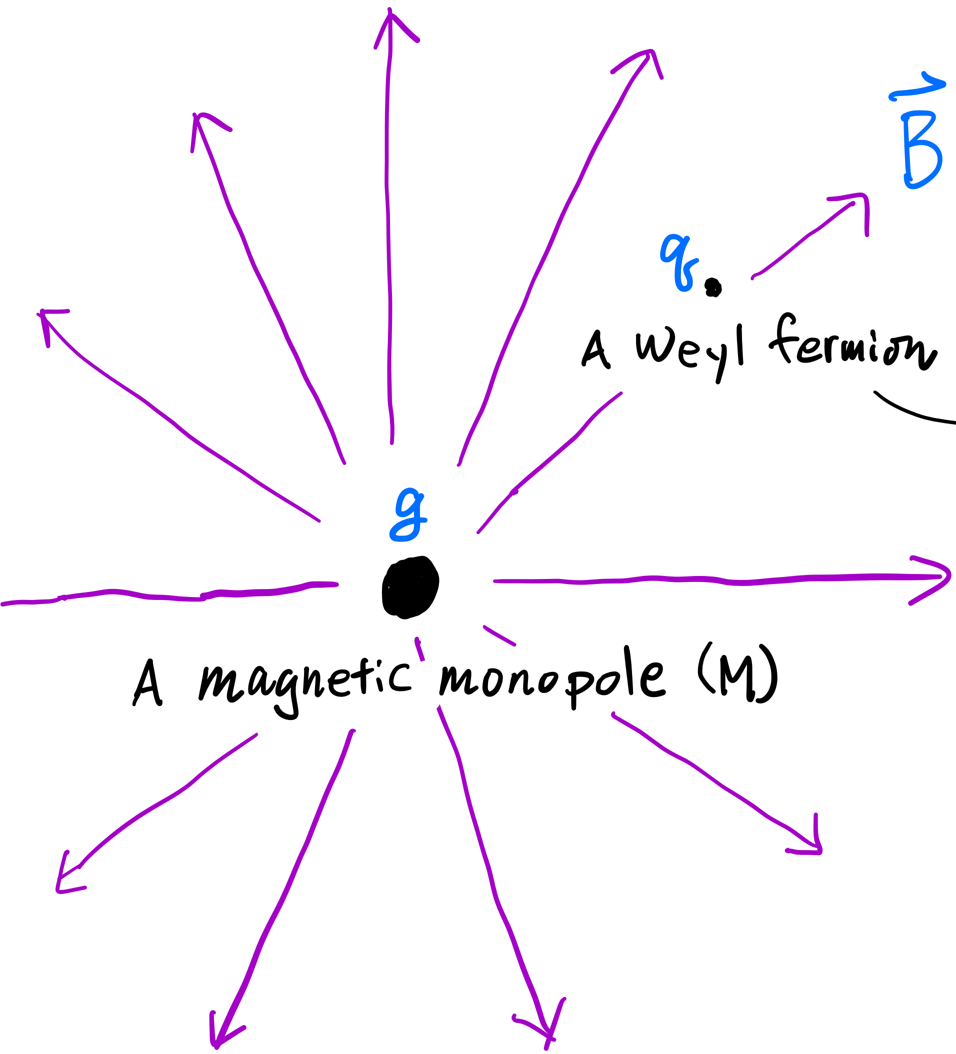
correct! $\vec{L} = \vec{r}\times(-i\vec{D}) - qg\hat{r}$

$\longrightarrow [L_i, L_j] = i\,\epsilon_{ijk}L_k$ ✓

$$\Longrightarrow L^2 = \vec{L}\cdot\vec{L} \geq q^2 g^2$$

$$\Longrightarrow \ell \geq \ell_{min} = |qg| = \frac{|n|}{2}$$

The Lowest Partial Wave



$\eta = \pm 1$

$$\vec{B} = g\,\frac{\hat{r}}{r^2} \Longleftarrow \vec{A} = g\,\frac{\eta - \cos\theta}{r\sin\theta}\,\hat{\phi}$$

$$\partial_t f = \vec{\sigma}\cdot\vec{D} f$$ (Dirac eq. in monopole bkgd)

L.H. spinor without loss of generality

Let

$$f_\alpha(t,r,\theta,\phi) = \frac{\chi(t,r)}{r}\,\Phi_\alpha(\theta,\phi)$$

L.H. spinor index ($\alpha = 1,2$)

Now, suppose Φ is in the lowest j:

$$\vec{J} = \vec{L} + \frac{\vec{\sigma}}{2} \quad \text{with} \quad j = j_{min} = \ell_{min} - \frac{1}{2} = |qg| - \frac{1}{2}$$

The Lowest Partial Wave

Then, $\partial_t f = \vec{\sigma} \cdot \vec{D} f \implies \partial_t \chi(t,r) = \text{sgn}(qg)\, \partial_r \chi(t,r)$! (*)

An example:

if $|qg| = \frac{1}{2}$ (a "minimal monopole") $\implies$ $j_{min} = 0$

with $\Phi(\theta,\phi) = \frac{1}{\sqrt{2}} \begin{pmatrix} Y^{(\pm)}_{\frac{1}{2},-\frac{1}{2}}(\theta,\phi) \\ -Y^{(\pm)}_{\frac{1}{2},\frac{1}{2}}(\theta,\phi) \end{pmatrix} \Longleftrightarrow \dfrac{\overset{L\,S}{|\uparrow\downarrow\rangle} - \overset{L\,S}{|\downarrow\uparrow\rangle}}{\sqrt{2}}$

"$Y_{\ell m}(\theta,\phi)$" for $\ell = \frac{1}{2}$ and $qg = \pm\frac{1}{2}$

But (*) is true for any j_{min} !

The Lowest Partial Wave

Then, $\partial_t f = \vec{\sigma}\cdot\vec{D} f \implies \partial_t \chi(t,r) = \mathrm{sgn}(qg)\,\partial_r \chi(t,r)$!

Two surprises:

(1) [Mildly Surprising]

χ = a free wave $\implies$ The lowest-j partial wave overlaps with the monopole core at $\vec{r}=0$ without a centrifugal barrier despite $\ell \neq 0$!

(2) [Really Surprising]

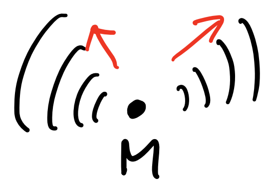
The wave's direction is fixed by $\mathrm{sgn}(qg)$!

$qg>0$

only ingoing!

missing outgoing waves!

$qg<0$



only outgoing!

missing ingoing waves!

The Semiton/Unitarity Puzzle

Rubakov '82 ; Callan '82, '83

To be concrete, take $|qg| = \frac{1}{2}$ with $g = -1$.

And imagine N "doublets":

$$\chi_k = \begin{pmatrix} \chi_k^+ \\ \chi_k^- \end{pmatrix} \begin{matrix} \longleftarrow q = +\frac{1}{2} & \Rightarrow qg < 0 & \text{outgoing only!} \\ \longleftarrow q = -\frac{1}{2} & \Rightarrow qg > 0 & \text{ingoing only!} \end{matrix}$$

$(k = 1, 2, \cdots, N)$

(A popular "real-life" example:

The SU(5) grand-unified theory (the SU(5) GUT)

$$\chi_{1,2,3,4} = \begin{pmatrix} e \\ d_3^c \end{pmatrix}, \begin{pmatrix} d^3 \\ e^c \end{pmatrix}, \begin{pmatrix} u_1^c \\ u^2 \end{pmatrix}, \begin{pmatrix} u_2^c \\ u^1 \end{pmatrix}$$

(Only one generation for simplicity. All left-handed.))

The Semiton/Unitarity Puzzle

Rubakov '82 ; Callan '82, '83

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$(k = 1, 2, \cdots, N)$

Two symmetries:

- $U(1)$ for "electromagnetism" with $Q = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ acting on each doublet
- $SU(N)$ "flavor" symmetry rotating N doublets among themselves

$\Rightarrow$ $N-1$ diagonal generators: $\begin{pmatrix} 1 & & & & \\ & -1 & & & \\ & & 0 & & \\ & & & 0 & \\ & & & & 0 \\ & & & & & \ddots \end{pmatrix}, \begin{pmatrix} 0 & & & & \\ & 1 & & & \\ & & -1 & & \\ & & & 0 & \\ & & & & 0 \\ & & & & & \ddots \end{pmatrix}, \begin{pmatrix} 0 & & & & \\ & 0 & & & \\ & & 1 & & \\ & & & -1 & \\ & & & & 0 \\ & & & & & \ddots \end{pmatrix}, \cdots$

The Semiton/Unitarity Puzzle

Rubakov '82 ; Callan '82, '83

[Example 1]

$N=2$ with $\begin{pmatrix} \chi_1^+ \\ \chi_1^- \end{pmatrix}, \begin{pmatrix} \chi_2^+ \\ \chi_2^- \end{pmatrix}$ out only / in only

initial state $= \chi_1^-$ given

final state $= A\,\chi_1^+ + B\,\chi_2^+$ must use $\chi_{1,2}^+$, can't use $\chi_{1,2}^-$

$\Rightarrow$ • $U(1)$ conservation: $-1 = A + B$

• $SU(2)$ " : $1 = A - B$

$\Rightarrow$ $A=0, \quad B=-1$ unique final state of incoming χ_1^- !

$\Rightarrow$ $\chi_1^- + M \longrightarrow \overline{\chi_2^+} + M$ occurs with 100 % probability!

M: monopole

$\overline{\chi_2^+}$: an "anti-χ_2^+"

(An example of "Callan-Rubakov process" (CR))

The Semiton/Unitarity Puzzle

Rubakov '82 ; Callan '82, '83

[Example 2]

$N=4$ with $\begin{pmatrix}\chi_1^+\\ \chi_1^-\end{pmatrix}, \begin{pmatrix}\chi_2^+\\ \chi_2^-\end{pmatrix}, \begin{pmatrix}\chi_3^+\\ \chi_3^-\end{pmatrix}, \begin{pmatrix}\chi_4^+\\ \chi_4^-\end{pmatrix}$ out only / in only

initial state = χ_1^-

final state = $A\chi_1^+ + B\chi_2^+ + C\chi_3^+ + D\chi_4^+$

$\Rightarrow$ • U(1) conservation : $-1 = A+B+C+D$

• SU(4) " : $1 = A-B$

$0 = B-C$

$0 = C-D$

$\Rightarrow A=\frac{1}{2},\ B=C=D=-\frac{1}{2}$ unique final state!

$\Rightarrow \chi_1^- + M \rightarrow \frac{1}{2}\chi_1^+ + \frac{1}{2}\overline{\chi_2^+} + \frac{1}{2}\overline{\chi_3^+} + \frac{1}{2}\overline{\chi_4^+} + M$ 100 %

(Another CR process)

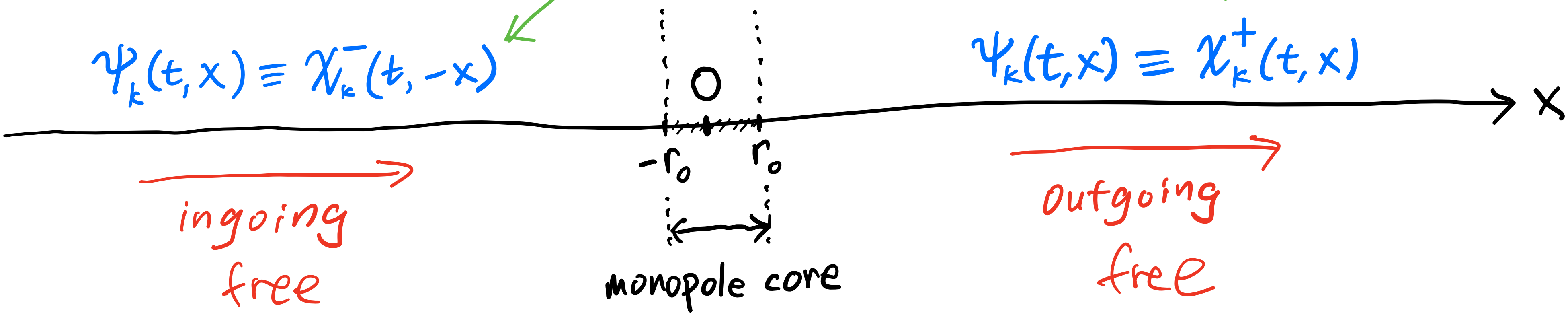
half (anti) particles "semitons"

But there are no semitons in the Fock space! "unitarity problem"

Polchinski's Fermion-rotor model

Polchinski '84

$$\begin{pmatrix} \chi_k^+(t,r) \\ \chi_k^-(t,r) \end{pmatrix} \begin{array}{l} \text{only outgoing} \\ \text{only ingoing} \end{array}$$



$\psi_i(t,x)$ = a free massless right-moving fermion in (1+1)D

$\Longrightarrow$ Away from the core, the action is:

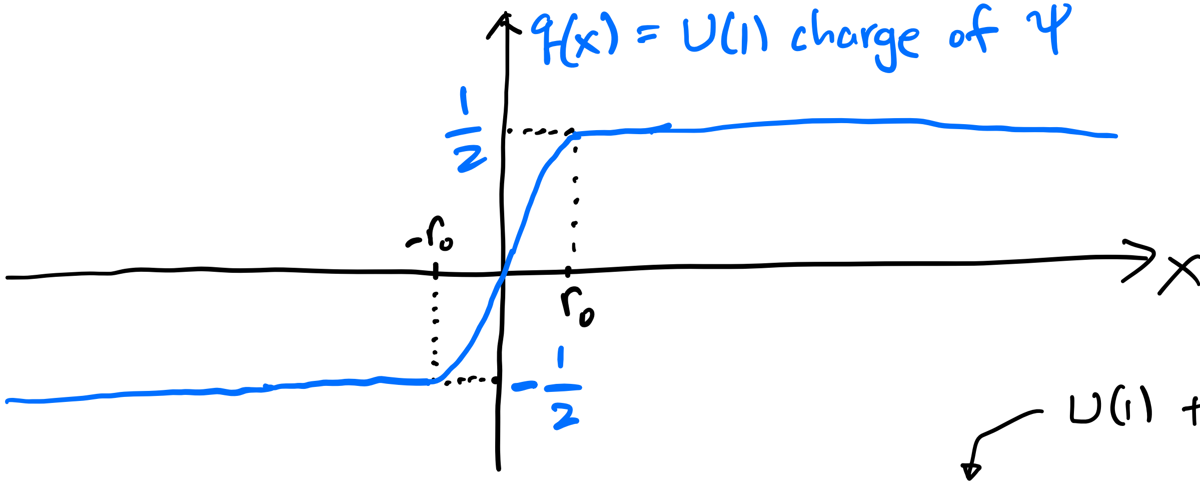
$$S = \int_{-\infty}^{\infty} dt\, dx \sum_{k=1}^{N} \psi_k^\dagger\, i(\partial_t + \partial_x)\psi_k + \int_{-r_0}^{r_0} dt\, dx\, [\, ?\,]$$

What happens here?

Polchinski's Fermion-rotor model

Polchinski '84

U(1) "electromagnetism" must be an exact symmetry.



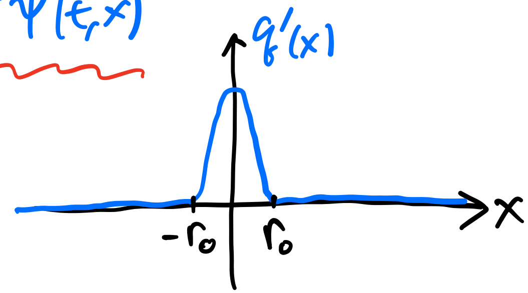
U(1) transformation parameter independent of t, x

(gauge interaction is higher order in semi-classical)

U(1) transformation : $\psi \rightarrow \psi'(t,x) = e^{i\theta q(x)}\psi(t,x)$

But the ψ kinetic term isn't invariant inside the core:

$$\psi^\dagger i(\partial_t+\partial_x)\psi(t,x) \longrightarrow \psi^\dagger i(\partial_t+\partial_x)\psi(t,x) - \theta q'(x)\psi^\dagger\psi(t,x)$$



We need another degree of freedom $\alpha(t)$ "rotor" with

U(1) transformation : $\alpha \rightarrow \alpha'(t) = \alpha(t) - \theta$

and add $-\alpha(t)q'(x)\psi^\dagger\psi(t,x)$ to the Lagrangian for U(1) invariance.

Polchinski's Fermion-rotor model

Polchinski '84

The rotor's kinetic term is U(1) invariant:

$$\frac{I}{2}\left[\dot{\alpha}(t)\right]^2 \xrightarrow{U(1)} \frac{I}{2}\left[\dot{\alpha}(t)\right]^2$$

$I =$ "moment of inertia"

but the rotor must be "massless":

$$[\alpha(t)]^2 \longrightarrow [\alpha(t)-\theta]^2 \neq [\alpha(t)]^2$$ not U(1) invariant

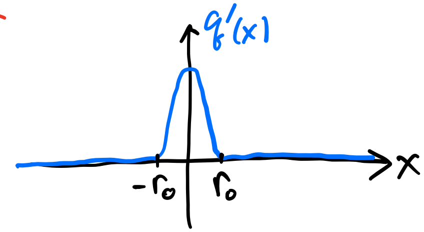
The action of the fermion-rotor model:

regulator-dependent counterterms

$$S = \int dt\, \frac{I}{2}\left[\dot{\alpha}(t)\right]^2 + \int dt\, dx \sum_{k=1}^{N} \psi_k^\dagger\left[i(\partial_t+\partial_x) - \alpha(t)\, q'(x)\right]\psi_k + S_{ct}$$

rotor

localized at core



Polchinski's Fermion-rotor model

Polchinski '84

The rotor carries U(1) charge:

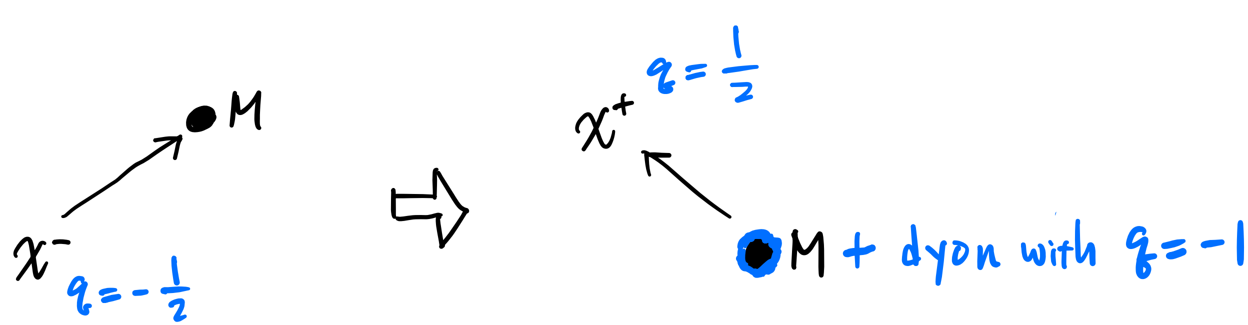
$$Q = I\dot{\alpha}(t) + \sum_{k=1}^{N} \int_{-\infty}^{\infty} dx\; q(x)\, \psi_k^\dagger \psi_k(t,x)$$

(So, the rotor is clearly not the U(1) gauge field — it's charged!)

Then,

$$\text{rotor energy} \sim I\dot{\alpha}^2 \sim \frac{(I\dot{\alpha})^2}{I} \sim \frac{1}{I} \quad \text{for charge} \sim O(1)$$

What's this physically? It's a dyon's excitation energy!



$$E_{\text{dyon}} \sim \frac{e^2}{r_0}$$
$$\parallel$$
$$E_{\text{rotor}} \sim \frac{1}{I}$$

$$\Longrightarrow \quad \frac{1}{I} \sim \frac{e^2}{r_0}$$

Scales and range of validity of EFT

The rotor EFT assumes semi-classical expansion in e^2:

$$S = S^{(0)} + S^{(1)} + S^{(2)} + \cdots$$

$S^{(0)}$: $O\left(\frac{1}{e^2}\right)$ — classical monopole field, monopole mass

$S^{(1)}$: $O(e^0)$ — quantized fermions in mono. bkgd. + rotor (+ photon's $F_{\mu\nu}F^{\mu\nu}$)

$S^{(2)}$: $O(e^2)$ — photon-fermion interactions → Ignored

$$M_{mono} \sim \frac{g^2}{r_0} \sim \frac{1}{e^2 r_0} \gg \frac{1}{r_0} \gg \frac{e^2}{r_0} \sim \frac{1}{I} = E_{dyon}$$

and the low-energy limit, in particular, $E \ll E_{dyon}$

Energy scale of scattering

So the dyon excitation in the previous slide is highly virtual (off-shell) lasting only for $\Delta t \sim I \ll E^{-1}$!

So, the theory is an accurate effective theory provided that

$$\left(m_f, \Lambda_{QCD} \ll\right) \; E \ll \frac{1}{I} \ll \frac{1}{r_0} \; \left(\ll M_{mono}\right)$$

These scales are not in the fermion-rotor EFT itself

In particular, can't take $I \to 0$ before $r_0 \to 0$!

Solution to the Semiton/Unitarity Puzzle

Executive Summary

Traditional Callan-Rubakov

- "Semitonic" CR

  (e.g.) $N=4$

  $$\chi_1^- \longrightarrow \frac{1}{2}\chi_1^+ + \frac{1}{2}\overline{\chi_2^+} + \frac{1}{2}\overline{\chi_3^+} + \frac{1}{2}\overline{\chi_4^+}$$

  ("+M" understood)

- "Non-semitonic" CR

  (e.g.) $N=2$

  $$\chi_1^- \longrightarrow \overline{\chi_2^+}$$

  (e.g.) $N=4$

  $$\chi_1^- + \chi_2^- \longrightarrow \overline{\chi_3^+} + \overline{\chi_4^+}$$

Our results

$\Rightarrow$ Just free propagation!

$$\chi_1^- \longrightarrow \chi_1^-$$

No semitons!

$\Rightarrow$ Same as traditional!

Solution to the Semiton/Unitarity Puzzle

Begin with semitonic case.

In traditional CR,

$$\underset{\text{only ingoing}}{\chi_1^-} \longrightarrow \overset{M}{\bullet} \qquad \cancel{\longrightarrow \chi_1^-}$$

can't come out!

Free propagation is not even possible!

What did they miss?

" $\chi_K^-(t,r)$ contains only ingoing waves" CORRECT ✓

" $\chi_K^+(t,r)$ contains only outgoing waves" CORRECT ✓

$\Rightarrow$ " So, we must use $(\chi_K^+)^\dagger$ to create an outgoing fermion" WRONG!

" So, the Fock space contains no outgoing χ_K^- " WRONG!

There are composite rotor-fermion operators that can create outgoing χ_K^-!

Solution to the Semiton/Unitarity Puzzle

$$U(1): \begin{cases} \chi_k^- \to e^{-i\frac{\theta}{2}} \chi_k^- \\ \chi_k^+ \to e^{+i\frac{\theta}{2}} \chi_k^+ \end{cases}$$

χ_k^- carries charge $-\frac{1}{2}$

χ_k^+ " $+\frac{1}{2}$

but we also have $\alpha \to \alpha - \theta$ under $U(1)$, so

$$e^{i\alpha} \chi_k^+ \longrightarrow e^{-i\frac{\theta}{2}} e^{i\alpha} \chi_k^+$$

" $e^{i\alpha} \chi_k^+$ " carries charge $-\frac{1}{2}$ just like χ_k^- and contains outgoing waves!

Can $[e^{i\alpha(\tau)} \chi_k^+(t,r)]^\dagger$ create an outgoing χ_k^- particle?

Yes, if and only if $\tau = t - r$!

(Similarly, $e^{-i\alpha(\tau)} \chi_k^-(t,r)$ can create an ingoing χ_k^+ particle iff $\tau = t + r$!)

We can show this by a direct calculation as follows:

Solution to the Semiton/Unitarity Puzzle

Polchinski already calculated purely fermionic Green's function exactly:

$$\left\langle \prod_{i=1}^{n} \psi_{k_i}(t_i, x_i) \prod_{j=1}^{n'} [\psi_{k_j'}(t_j', x_j')]^\dagger \right\rangle = Z\, W_0 \qquad \text{Polchinski '84}$$

where

W_0 = Green's function of the literally free theory of the fermions without the rotor

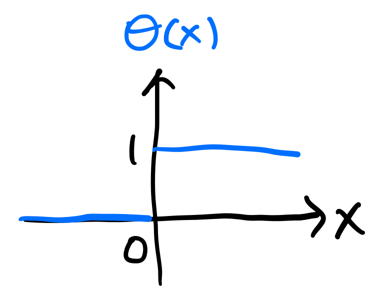
$$Z = \int [d\alpha] \exp\left[\int_{\mu}^{\Lambda} \frac{d\omega}{2\pi} \left\{ -\frac{N\omega}{4\pi} \alpha(\omega)\alpha(-\omega) - iA\alpha(\omega) + iB\alpha(-\omega) \right\} \right]$$

$$= \exp\left[\frac{2}{N} \int_{\mu}^{\Lambda} \frac{d\omega}{\omega} AB \right]$$

(• Λ = UV cutoff $\sim \frac{1}{I} \to \infty$.
• μ = IR cutoff $\to 0$ but subtle!)

$$A = \sum_{i=1}^{n} \theta(x_i)\, e^{-i\omega(t_i - x_i)} - \sum_{j=1}^{n'} \theta(x_j')\, e^{-i\omega(t_j' - x_j')}$$

$$B = \sum_{i=1}^{n} \theta(-x_i)\, e^{i\omega(t_i - x_i)} - \sum_{j=1}^{n'} \theta(-x_j')\, e^{i\omega(t_j' - x_j')}$$



in $r_0 \to 0$ limit

Solution to the Semiton/Unitarity Puzzle

For example,

$$\langle \psi_i(t,x)[\psi_j(t',x')]^\dagger \rangle = Z\, W_0$$

where

$$W_0 = \delta_{ij}\, G_0(t-t', x-x') \quad \text{with} \quad G_0(t,x) = \frac{1}{2\pi i}\,\frac{t+x}{t^2-x^2+i\varepsilon}$$

$$Z = \int [d\alpha]\, \exp\left[\int_\mu^\Lambda \frac{d\omega}{2\pi}\left\{-\frac{N\omega}{4\pi}\,\alpha(\omega)\,\alpha(-\omega) - iA\,\alpha(\omega) + iB\,\alpha(-\omega)\right\}\right]$$

$$= \exp\left[\frac{2}{N}\int_\mu^\Lambda \frac{d\omega}{\omega} AB\right]$$

(• Λ = UV cutoff $\sim \frac{1}{I} \to \infty$.
• μ = IR cutoff $\to 0$ but subtle!)

$$A = \theta(x)\, e^{-i\omega(t-x)} - \theta(x')\, e^{-i\omega(t'-x')}$$

$$B = \theta(-x)\, e^{i\omega(t-x)} - \theta(-x')\, e^{i\omega(t'-x')}$$

Solution to the Semiton/Unitarity Puzzle

Let's now insert our rotor exponentials:

$$\langle \psi_i(t,x)\, e^{i\alpha(t)\theta(x)} \left[\psi_j(t',x')\, e^{i\alpha(t')\theta(x')}\right]^\dagger \rangle = Z\, W_0$$

where

$$W_0 = \delta_{ij}\, G_0(t-t', x-x') \quad \text{with} \quad G_0(t,x) = \frac{1}{2\pi i}\frac{t+x}{t^2-x^2+i\varepsilon}$$

$$Z = \int [d\alpha]\, e^{i\alpha(t)\theta(x) - i\alpha(t')\theta(x')} \exp\left[\int_\mu^\Lambda \frac{d\omega}{2\pi}\left\{-\frac{N\omega}{4\pi}\alpha(\omega)\alpha(-\omega) - iA\alpha(\omega) + iB\alpha(-\omega)\right\}\right]$$

$$= \exp\left[\frac{2}{N}\int_\mu^\Lambda \frac{d\omega}{\omega}\tilde{A}\tilde{B}\right]$$

(• Λ = UV cutoff $\sim \frac{1}{I} \to \infty$.
• μ = IR cutoff $\to 0$ but subtle!)

$$\tilde{A} = \theta(x)\, e^{-i\omega(t-x)} - \theta(x')\, e^{-i\omega(t'-x')} - \theta(x)\, e^{-i\omega\tau} + \theta(x')\, e^{-i\omega\tau'}$$

$$\tilde{B} = \theta(-x)\, e^{i\omega(t-x)} - \theta(-x')\, e^{i\omega(t'-x')} + \theta(x)\, e^{i\omega\tau} - \theta(x')\, e^{i\omega\tau'}$$

Solution to the Semiton/Unitarity Puzzle

Let's now insert our rotor exponentials:

$$\langle \psi_i(t,x)\, e^{i\alpha(\tau)\theta(x)} \left[\psi_j(t',x')\, e^{i\alpha(\tau')\theta(x')}\right]^\dagger \rangle = \mathcal{Z}\, W_0$$

where

$$W_0 = \delta_{ij}\, G_0(t-t', x-x') \quad \text{with} \quad G_0(t,x) = \frac{1}{2\pi i}\,\frac{t+x}{t^2-x^2+i\varepsilon}$$

$$\mathcal{Z} = \int [d\alpha]\, e^{i\alpha(\tau)\theta(x) - i\alpha(\tau')\theta(x')} \exp\left[\int_\mu^\Lambda \frac{d\omega}{2\pi}\left\{-\frac{N\omega}{4\pi}\alpha(\omega)\alpha(-\omega) - iA\alpha(\omega) + iB\alpha(-\omega)\right\}\right]$$

$$= \exp\left[\frac{2}{N}\int_\mu^\Lambda \frac{d\omega}{\omega}\tilde{A}\tilde{B}\right]$$

$\Big(\cdot\ \Lambda =$ UV cutoff $\sim \frac{1}{I} \to \infty$.
$\cdot\ \mu =$ IR cutoff $\to 0$ but subtle!$\Big)$

$\tilde{A} = 0$ iff $\tau = t - x$ (within "time resolution" $\sim O(I)$)

$\Rightarrow \mathcal{Z} = 1 \Rightarrow$ The $\langle \cdots\cdots \rangle$ is just W_0 iff $\tau = t-x$!

Solution to the Semiton/Unitarity Puzzle

So, we have shown that

$$\left\langle \psi_i(t,x)\, e^{i\alpha(t-x)\theta(x)} \left[\psi_j(t',x')\, e^{i\alpha(t'-x')\theta(x')}\right]^\dagger \right\rangle = \delta_{ij}\, G_0(t-t', x-x')$$

exactly. Since

$$\psi_i(t,x)\, e^{i\alpha(t-x)\theta(x)} = \begin{cases} \chi_i^+(t,r)\, e^{i\alpha(t-r)} & \text{for } x=r>0 \\ \chi_i^-(t,r) & \text{for } x=-r<0 \end{cases}$$

and since G_0 is exactly the free massless right-mover's propagator, $(\chi_i^-)^\dagger$ creates a free particle at $x'<0$, which is an exact 1-particle state of the full hamiltonian, and this same particle is annihilated by $\chi_i^+ e^{i\alpha}$!

$$\Longrightarrow \quad \left[\chi_i^+(t,r)\, e^{i\alpha(t-r)}\right]^\dagger \text{ creates an outgoing } \chi_i^- \text{ particle!}$$

Being an exact 1-particle state, this free propagation occurs 100%!

(Similarly, $\left[\chi_i^-(t,r)\, e^{-i\alpha(t+r)}\right]^\dagger$ creates an ingoing χ_i^+ particle.)

Solution to the Semiton/Unitarity Puzzle

Therefore,

Traditional CR (wrong)		Correct Picture
$\chi_1^- \longrightarrow \overset{M}{\bullet} \cancel{\longrightarrow \chi_1^-}$	$\Rightarrow$	$\chi_1^- \longrightarrow \overset{M}{\bullet} \longrightarrow \chi_1^-$
only ingoing; can't come out!		created by $(\chi_1^-)^\dagger$; created by $[\chi_1^+ e^{i\alpha}]^\dagger$
Free propagation is not even possible. Forced into semitonic final states.		Free propagation. No scattering. No semitons.

- "Semitonic CR" is just free propagation!
- That's possible because with the rotor-fermion composite operators we have a full spectrum of both ingoing and outgoing modes for every fermion regardless of sgn(qg)!

(• But what's "$e^{i\alpha}$" in (3+1)D? Related to the "abelian vortex with fractional winding number" in Csáki et al. (2406.13738)? Future Project!)

Solution to the Semiton/Unitarity Puzzle

Move on to <u>non-semitonic</u> CR processes.

- Take $N=2$, for example. Then, traditionally, $\chi_1^- \to \overline{\chi_2^+}$, 100%.
  No semitons, no unitarity problem.

  But, now, our calculation shows $\chi_1^- \to \chi_1^-$, 100%.
  So, we now have a unitarity problem!?

- Take $N=4$, for example. Then, traditionally, $\chi_1^- + \chi_2^- \to \overline{\chi_3^+} + \overline{\chi_4^+}$, 100%.
  No semitons, no unitarity problem.

  But, now, our calculation shows $\chi_1^- + \chi_2^- \to \chi_1^- + \chi_2^-$, 100%.
  So, we now have a unitarity problem!?

Solution to the Semiton/Unitarity Puzzle

Take $N=2$ case. Our method shows that for $x_1, x_2, x_3, x_4 > 0$,

$$\langle \psi_i(1)\, e^{i\alpha(1)}\, \psi_j(2)\, [\psi_k(3)\, e^{i\alpha(3)}\, \psi_l(4)]^\dagger \rangle$$

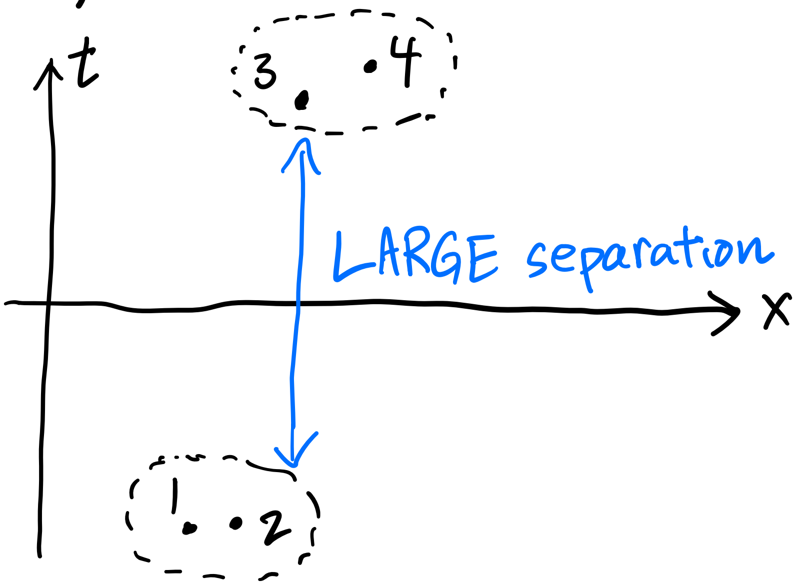
($\psi_i(1)\, e^{i\alpha(1)} \equiv \psi_i(t_1,x_1)\, e^{i\alpha(t_1-x_1)}$, etc.)

$$= \underbrace{\frac{S_{23}\, S_{14}}{S_{12}\, S_{34}}}_{Z} \underbrace{\left(\delta_{il}\delta_{jk}\langle 14\rangle\langle 23\rangle - \delta_{ik}\delta_{jl}\langle 13\rangle\langle 24\rangle\right)}_{W_0}$$

($S_{12} \equiv t_1 - t_2 - (x_1 - x_2)$, etc.)

($\langle 14\rangle \equiv G_0(1-4) = \frac{1}{2\pi i}\frac{1}{S_{14}}$, etc.)

Now, separate the "12" and "34" clusters A LOT:



Then,

$$\text{LHS} \longrightarrow \langle \psi_i(1)\, e^{i\alpha(1)}\, \psi_j(2)\rangle \cdot \langle \psi_k(3)\, e^{i\alpha(3)}\, \psi_l(4)\rangle^*$$

$$\text{RHS} \longrightarrow \langle 12\rangle\langle 34\rangle\left(\delta_{il}\delta_{jk} - \delta_{ik}\delta_{jl}\right)$$

$$= \langle 12\rangle\langle 34\rangle\, \epsilon_{ij}\, \epsilon_{lk}$$

Solution to the Semiton/Unitarity Puzzle

Take $N=2$ case. Our method shows that for $x_1, x_2, x_3, x_4 > 0$,

$$\langle \psi_i(1)\, e^{i\alpha(1)}\, \psi_j(2)\, [\psi_k(3)\, e^{i\alpha(3)}\, \psi_\ell(4)]^\dagger \rangle \qquad (\psi_i(1)\, e^{i\alpha(1)} \equiv \psi_i(t_1,x_1)\, e^{i\alpha(t_1-x_1)},\ \text{etc.})$$

$$= \underbrace{\frac{S_{23}\, S_{14}}{S_{12}\, S_{34}}}_{Z} \underbrace{\Big(\delta_{i\ell}\,\delta_{jk}\,\langle 14\rangle\langle 23\rangle - \delta_{ik}\,\delta_{j\ell}\,\langle 13\rangle\langle 24\rangle\Big)}_{W_0}$$

$(S_{12} \equiv t_1 - t_2 - (x_1 - x_2),\ \text{etc.})$

$\left(\langle 14\rangle \equiv G_0(1-4) = \frac{1}{2\pi i}\frac{1}{S_{14}},\ \text{etc.}\right)$

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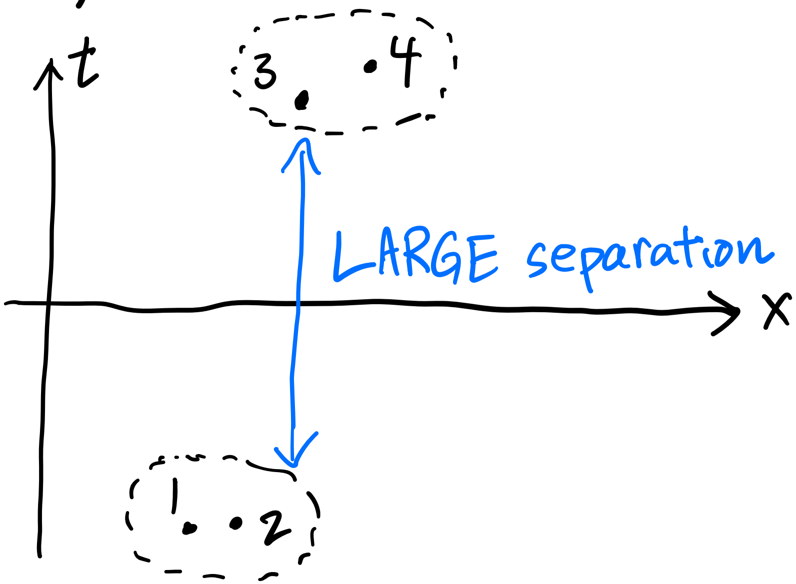

Thus,

parametrizes degenerate vacua

$$\langle \psi_i(1)\, e^{i\alpha(1)}\, \psi_j(2)\rangle = \epsilon_{ij}\,\langle 12\rangle\, e^{i\theta}$$

$\Rightarrow$ $[\chi_1^- e^{i\alpha}]^\dagger$ creates the same outgoing particle as χ_2^+, which by definition is $\overline{\chi_2^+}$! No unitarity problem, and confirms traditional CR!

Solution to the Semiton/Unitarity Puzzle

Wait! Doesn't our method also say

$$\langle \psi_i(1)\, e^{i\alpha(1)}\, \psi_{\bar{j}}(2) \rangle = Z\, W_0 \quad \text{with} \quad W_0 = 0$$

because

$$W_0 = \langle \psi_i(1)\, \psi_{\bar{j}}(2) \rangle \text{ in literally free theory} = 0 \text{ ?}$$

Actually, it doesn't due to an IR subtlety.

Our IR cutoff $\int_\mu \frac{d\omega}{\omega} (\cdots)$ removes low-frequency modes

$\Rightarrow$ Evolution of $\alpha(t)$ with $\alpha(t_f) \neq \alpha(t_i)$ excluded.

But that's inconsistent with the anomaly:

$\psi_k \to e^{i\alpha} \psi_k$ simultaneously for all k $\Rightarrow$ $\int \prod_k [d\psi_k][d\bar{\psi}_k] \to \int \prod_k [d\psi_k][d\bar{\psi}_k]\, e^{-\frac{iN}{2\pi}\int d^2x\, \gamma q'_\delta(x)\, \dot{\alpha}(t)}$

$\Rightarrow$ $\underline{\Delta N_\psi} = \frac{N}{2\pi} \Delta\alpha$ Change in tot. # of fermions

Solution to the Semiton/Unitarity Puzzle

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But that's inconsistent with the anomaly: $\Delta N_\psi = \frac{N}{2\pi} \Delta\alpha = 0$

$\Rightarrow$ Our method valid only when $N_\psi = N_{\psi^\dagger}$ inside $\langle \cdots\cdots \rangle$!

In particular, not valid for $\langle \psi_i(1)\, \psi_j(2) \rangle$,

but valid for $\langle \psi_1 e^{i\alpha_1} \psi_2 [\psi_3 e^{i\alpha_3} \psi_4]^\dagger \rangle$! Perfect!

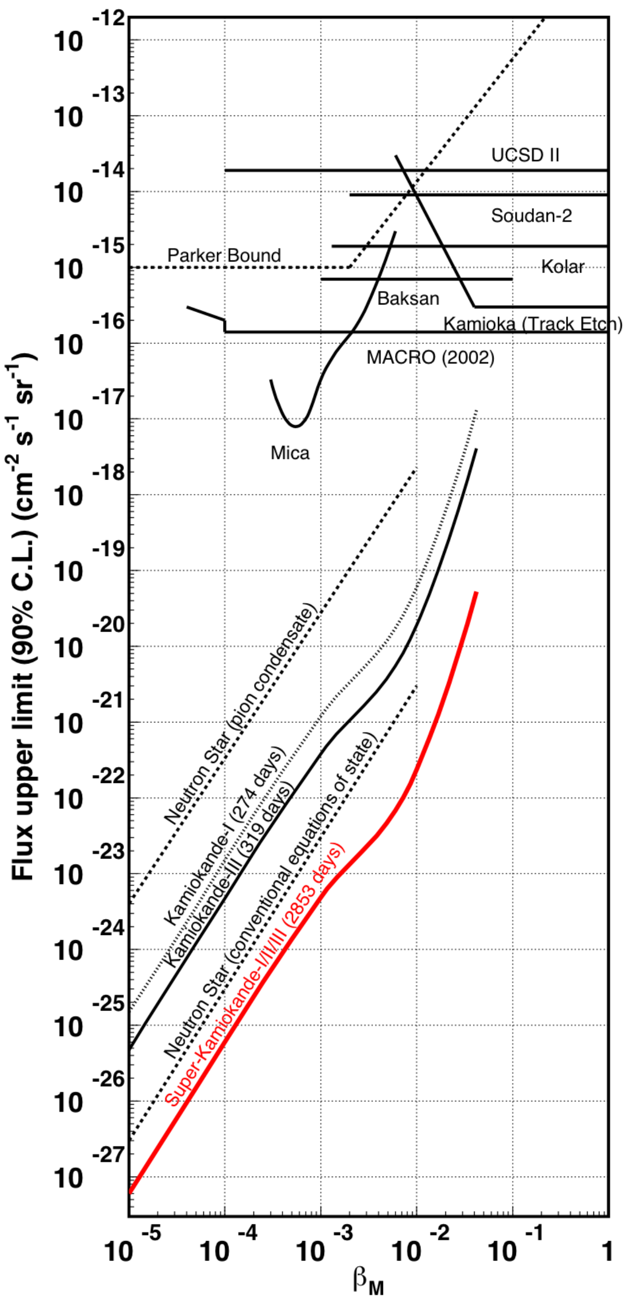
Solution to the Semiton/Unitarity Puzzle

A similar cluster analysis also confirms the traditional CR results for $N > 2$. So, all problems solved!

Summary

- The "missing modes" in the lowest-j partial waves are "recovered" by our composite rotor-fermion operators.
- That allows fermions to freely propagate past the monopole, and indeed one-fermion states do so for $N>2$. No semitons!
- Traditionally non-semitonic CR processes are still correct, where the fermions "freely morph" into the CR final state.
- These are shown by exact evaluation of path integral of fermion-rotor model, an accurate EFT in $e \to 0$ limit.

Does it matter?



← Experimental bounds on the monopole flux

Super-K hep-ex/1203.0940

These strongest bounds all come from the CR effects! E.g.,

(1) $u + u + M \rightarrow e^+ + \bar{d} + M$

(2) $u + M \rightarrow \frac{1}{2}(u + \bar{u} + \bar{d} + e^+) + M$

They both violate baryon & lepton numbers 100%, being CR! "Monopole-catalyzed nucleon decay"

But (2) is actually free propagation!

How do the bounds change? We need to include:

- fermion masses

  straightforward in rotor model, a big advantage over BCFT approach by van Beest et al.!

  2306.07318, 2312.17746

- confinement

  more challenging but I have some ideas...

Stay tuned!