Dyonic Bound States

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Monopoles

Obviously a good idea

$$\partial_\mu F^{\mu\nu} = J^\nu_E \qquad \partial_\mu \tilde{F}^{\mu\nu} = J^\nu_M$$

Maxwell Equation
Symmetric

Quantization of Charge

Q_E ● Q_M ●

$$L = \frac{\hbar}{2} Q_E Q_M$$

Monopoles

Obviously a good idea

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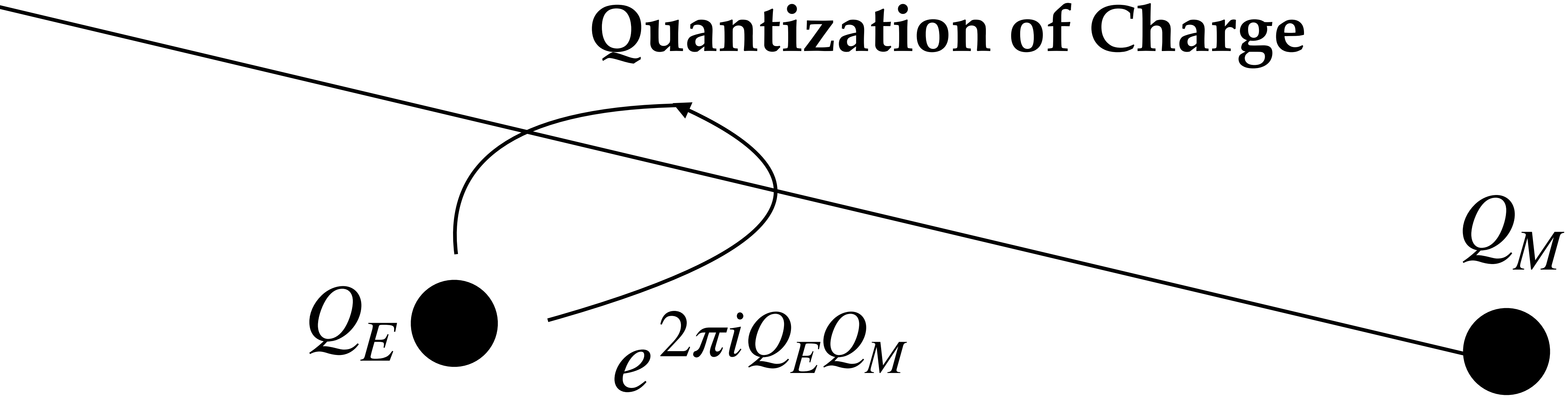
Maxwell Equation
Symmetric

Quantization of Charge



Monopoles

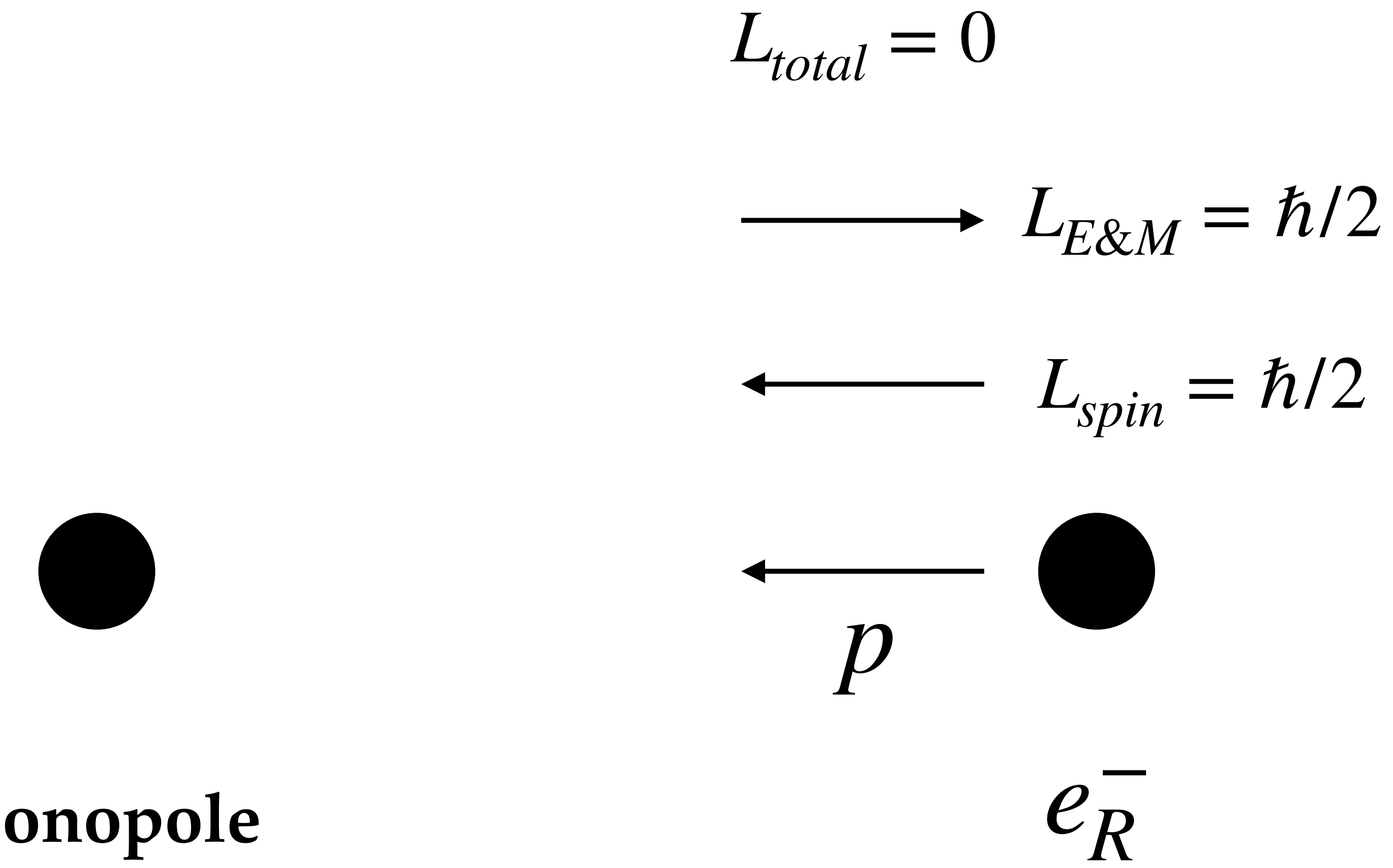
Two additional interesting facts

1. **Monopoles break symmetries in a way that is UV dependent and unsuppressed by the UV scale**

2. **Witten Effect**
   - Monopoles come with dyons

Monopoles break symmetries

Callan-Rubakov Effect



$L_{total} = 0$

$\longrightarrow$ $L_{E\&M} = \hbar/2$

$\longleftarrow$ $L_{spin} = \hbar/2$

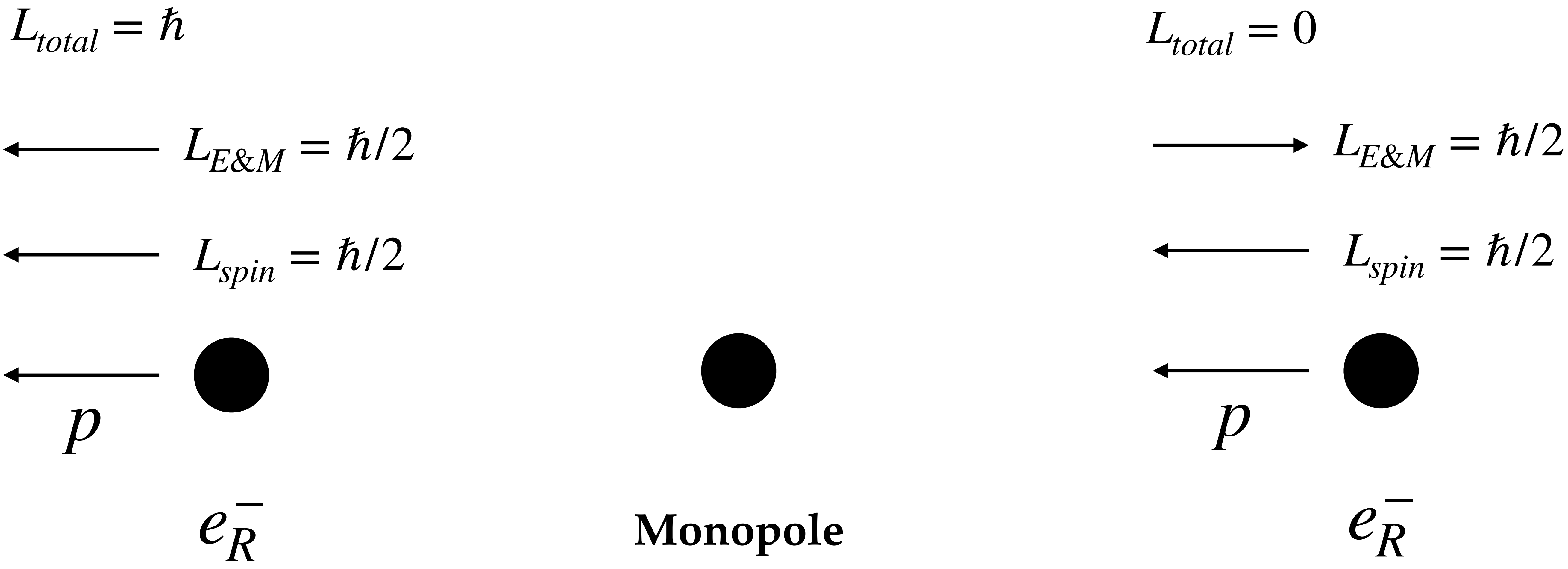
$\longleftarrow$ p

Monopole

e_R^-

Monopoles break symmetries

Callan-Rubakov Effect



$L_{total} = \hbar$

$\longleftarrow$ $L_{E\&M} = \hbar/2$

$\longleftarrow$ $L_{spin} = \hbar/2$

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e_R^-

Monopole

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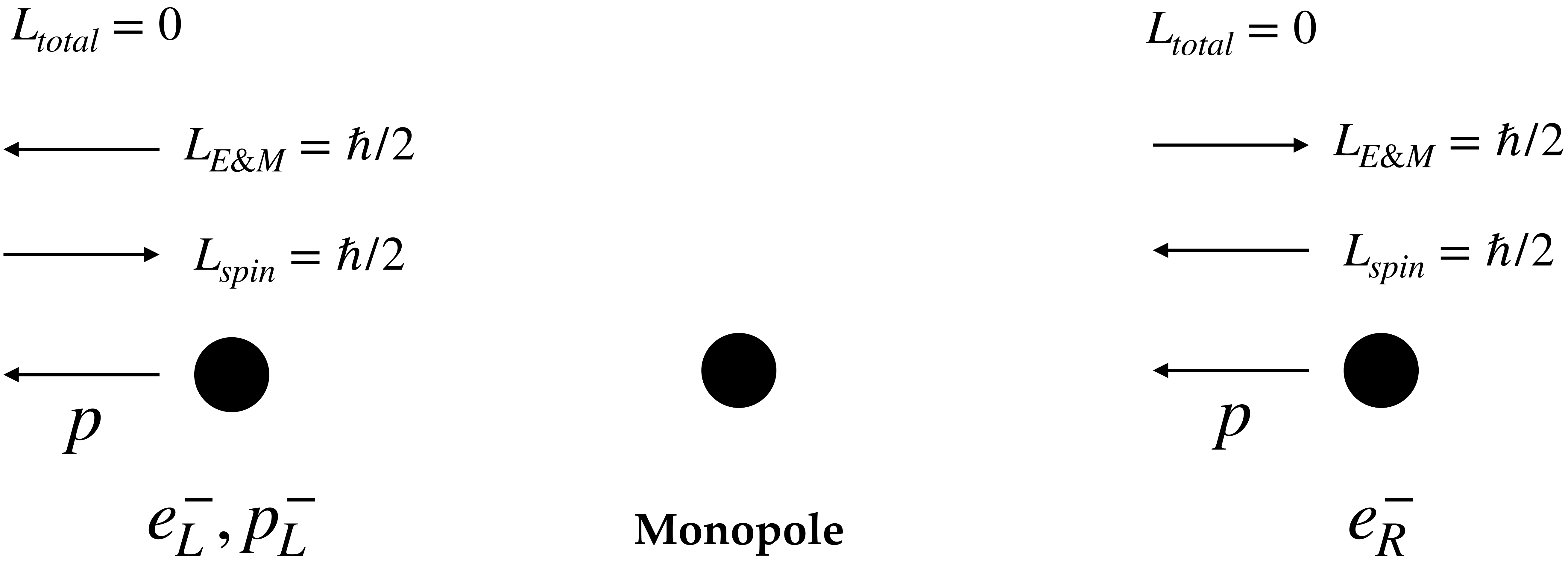
e_R^-

Monopoles break symmetries

Boundary conditions must break chiral symmetry or Baryon/Lepton number!



$L_{total} = 0$

$L_{E\&M} = \hbar/2$

$L_{spin} = \hbar/2$

p

e_L^-, p_L^-

Monopole

$L_{total} = 0$

$L_{E\&M} = \hbar/2$

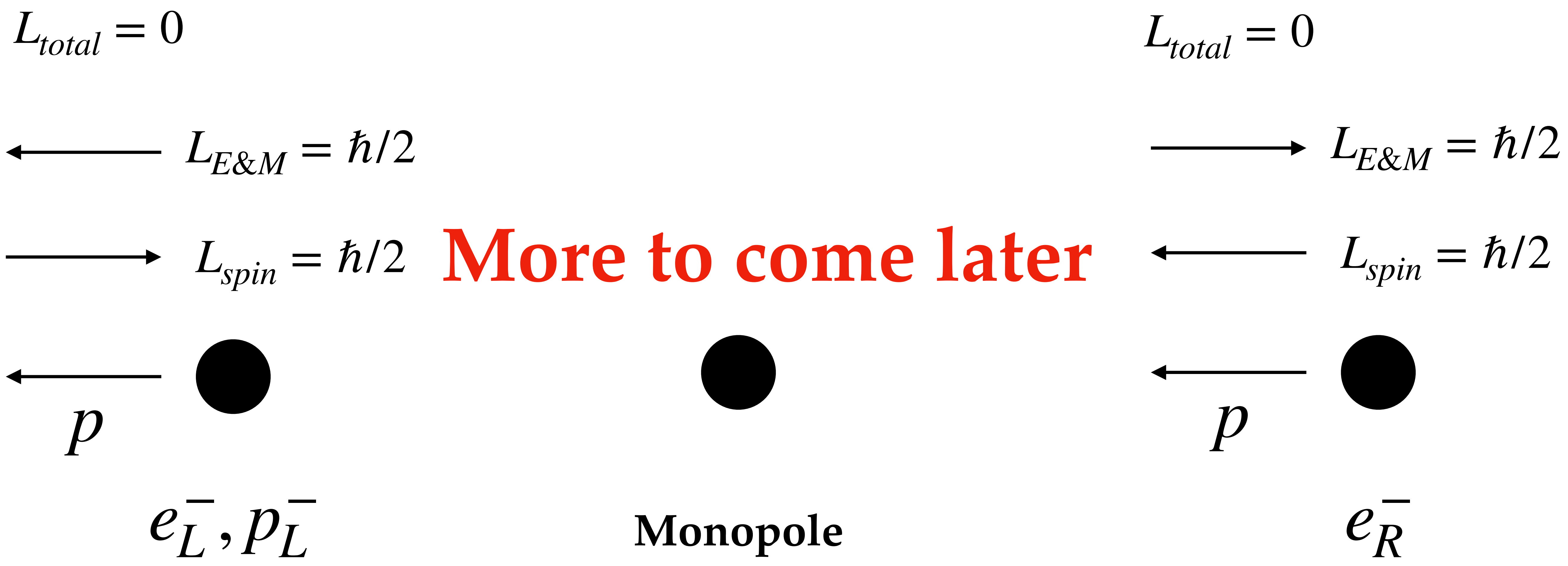
$L_{spin} = \hbar/2$

p

e_R^-

Monopoles break symmetries

Boundary conditions must break chiral symmetry or Baryon/Lepton number!



$L_{total} = 0$

$L_{E\&M} = \hbar/2$

$L_{spin} = \hbar/2$

p

e^-_L, p^-_L

More to come later

Monopole

$L_{total} = 0$

$L_{E\&M} = \hbar/2$

$L_{spin} = \hbar/2$

p

e^-_R

Witten Effect

Monopoles carry charge $\theta/2\pi$

In EFT after all charged particles are integrated out

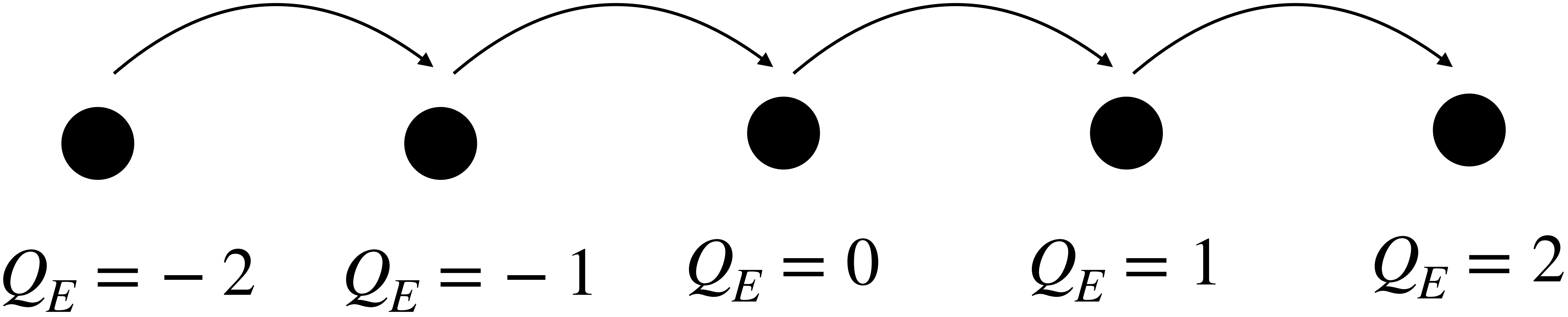
$$\delta\mathscr{L} = \frac{e^2\theta}{16\pi^2}F\tilde{F} = \frac{e^2\theta}{4\pi^2}E\cdot B = \frac{e^2\theta}{4\pi^2}\nabla A_0\cdot B$$

$$-\frac{e^2\theta}{4\pi^2}A_0\nabla\cdot B = -\frac{e^2\theta}{4\pi^2}\frac{2\pi}{e}\delta^3(r)A_0 = -\frac{e\theta}{2\pi}\delta^3(r)A_0$$

Dyons

$$\theta = \theta + 2\pi$$

Because θ is 2π periodic, monopole must be accompanied by a tower of dyons



Dyons

’t Hooft Polyakov Monopole

	$SU(2)$
Φ	Adj

Explicit monopole solution whenever $\Pi_2(G/H) \neq 0$

$$\Phi_a = v\,\hat{r}_a Q(r)$$

$$A_i^a = -\,\epsilon_{iab}\hat{r}_b \frac{A(r)}{e\,r}$$

$$A_0^a = 0$$

Solve for functions $A(r), Q(r)$ numerically

$$A(r) = Q(r) = 0 \qquad r \ll r_M$$

$$A(r) = Q(r) = 1 \qquad r \gg r_M$$

Dyons

● ● ● ● ●

$Q_E = -2$ $Q_E = -1$ $Q_E = 0$ $Q_E = 1$ $Q_E = 2$

$$A_0^a = \dot{\lambda}\,\hat{r}_a$$

Quantize the charge rotator degree of freedom λ to get dyons

Since only charged states are the W bosons, dyons often called bound states of W boson and Monopole

$$\Delta M \sim \alpha\, m_W \ll m_W$$

If interpreted as bound states, $\mathcal{O}(m_W)$ binding energy

Bound States

Are there other bound states?

1. **Specify UV theory to obtain boundary conditions**
    - Depending on the bound states you're interested in, this may not be needed

2. **Solve Dirac Equation ala Hydrogen Atom**
    - Important caveat, need to take into account the anomaly!
    - I will only consider s-wave solutions

Outline

- **Witten Effect with light fermions**
- **A plethora of bound states**
- **The massless limit**
- **Conclusion**

Witten Effect with Light Fermions

What is the ground state?

	$SU(2)$	$U(1)_{B-L}$
Φ	Adj	0
$\psi_1 = \begin{pmatrix} b \\ l^c \end{pmatrix}$	$\square$	1
$\psi_2 = \begin{pmatrix} l \\ b^c \end{pmatrix}$	$\square$	-1

$$\delta\mathcal{L} = -\frac{g^2\theta}{32\pi^2}G\tilde{G} - m\psi_1\psi_2$$

Witten Effect with Light Fermions

To find the ground state monopole solution
Just solve Dirac equation (kinda) and
Maxwell's equation

1. **Reduce to the s-wave**
2. **Derive boundary conditions**
3. **Turn into 2D problem on half plane**
4. **Bosonize**
5. **Numerically solve**

Witten Effect with Light Fermions

Step 1 - Reduce to the s-wave

$$\Phi_a = v\,\hat{r}_a Q(r)$$

$$A_i^a = -\,\epsilon_{iab}\hat{r}_b \frac{A(r)}{e\,r}$$

Break SU(2)xSU(2) to $SU(2)_D$

$$e^{-i\vec{J}\cdot\vec{\theta}} = e^{-i(\vec{L}+\vec{S})\cdot\theta} e^{i\frac{\vec{\tau}}{2}\cdot(-\theta)} = e^{-i(\vec{L}+\vec{\sigma}/2+\vec{\tau}/2)\cdot\theta}$$

Decompose fermion into two types spin 0 states

$$\psi_{J=0} = g(r,t)\psi_{J=0;L=0} + p(r,t)\psi_{J=0;L=1}$$

$$= \frac{g(r,t)+p(r,t)(\hat{r}\cdot\vec{\sigma})}{\sqrt{8\pi r^2}} \overset{\text{Isospin}}{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}} \ \text{Spin}$$

Witten Effect with Light Fermions

Step 2 - Boundary Conditions

$$\psi_{J=0} = g(r,t)\psi_{J=0;L=0} + p(r,t)\psi_{J=0;L=1}$$

$$= \frac{g(r,t) + p(r,t)(\hat{r}\cdot\vec{\sigma})}{\sqrt{8\pi r^2}} \overset{\text{Isospin}}{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}} \text{Spin}$$

Near monopole, symmetry restored and angular momentum of the L=1 state reappears

$$p(r,t) = 0$$

Note that this mixes left handed baryons and right handed leptons

Witten Effect with Light Fermions

Step 2 - Boundary Conditions

$$\psi_{J=0} = g(r,t)\psi_{J=0;L=0} + p(r,t)\psi_{J=0;L=1}$$

$$= \frac{g(r,t) + p(r,t)(\hat{r}\cdot\vec{\sigma})}{\sqrt{8\pi r^2}} \overset{\text{Isospin}}{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}} \ \text{Spin}$$

Fix gauge near monopole $A_0^a = 0$

Dirac Equation

$$g' = \frac{g}{r} \qquad p' = -\frac{p}{r}$$

$$g \propto r \qquad p \propto \frac{1}{r}$$

Witten Effect with Light Fermions

Step 3 - Simplify to 2D theory on half plane

2D fermions

$$\xi_b = \frac{1}{\sqrt{2}}\begin{pmatrix} g_1 - p_1 \\ g_2^* + p_2^* \end{pmatrix} \quad \xi_l = \frac{1}{\sqrt{2}}\begin{pmatrix} g_2 - p_2 \\ g_1^* + p_1^* \end{pmatrix}$$

$$\mathscr{L}_{\text{Fermion}} = \sum_{i=b,\ell} \int_0^\infty dr\, i\bar{\xi}_i \bar{\gamma}^\mu \partial_\mu \xi_i + \frac{e}{2}\dot{\lambda}\bar{\xi}_i \bar{\gamma}^1 \xi_i - m\bar{\xi}_i \xi_i$$

$$\mathscr{L}_{\text{Gauge}} = \int_0^\infty dr 2\pi r^2 \dot{\lambda}'^2 + \frac{\bar{\theta} e}{2\pi}\dot{\lambda}'^2$$

Witten Effect with Light Fermions

Step 4a - Why Bosonize?

1. **Computers don’t like Grassman numbers**

2. **Dirac equation doesn’t depend on the phase of the fermion mass**
    - Misses the anomaly!
    - Critical for the Witten effect

Witten Effect with Light Fermions

Step 4b - Bosonize

Quantum Duality! **Only of the radial and time coordinates though**

Critical Aspect : 1-loop quantum anomaly is now tree level classical physics

Tree level physics uses bosons and sees the anomaly
Ready for computers!

Witten Effect with Light Fermions

Step 4b - Bosonize

Fermions are exponential of scalars

$$\psi_{2D} = \begin{pmatrix} b(r-t) \\ l^c(r+t) \end{pmatrix} \sim \begin{pmatrix} e^{i\phi(r-t)} \\ e^{i\phi(r+t)} \end{pmatrix}$$

$$\xi_i(r,t) = Z^{1/2}(r) : \begin{pmatrix} e^{i\sqrt{\pi}\left(\phi(r,t)-\int_0^r dx\dot{\phi}(x,t)\right)} \\ e^{i\alpha}e^{-i\sqrt{\pi}\left(\phi(r,t)+\int_0^r dx\dot{\phi}(x,t)\right)} \end{pmatrix} :$$

Normal ordered exponential of integral of scalar have exactly the same correlators as fermions

Witten Effect with Light Fermions

Step 4b - Bosonize

Lagrangian

$$\mathscr{L} = \frac{1}{8\pi}\sum_{i=B,L}(\partial\phi_i)^2 + \frac{\pi m^2}{16}\cos(\phi_i + \theta_m) - \frac{g^2}{8\pi r^2}(\frac{\phi_B + \phi_L}{4\pi} - \frac{\theta}{2\pi})^2$$

Boundary Conditions

$$\phi_B(0) = \phi_L(0) \qquad \partial\phi_B(0) = -\partial\phi_L(0)$$

Witten Effect with Light Fermions

Step 4b - Bosonize

Lagrangian

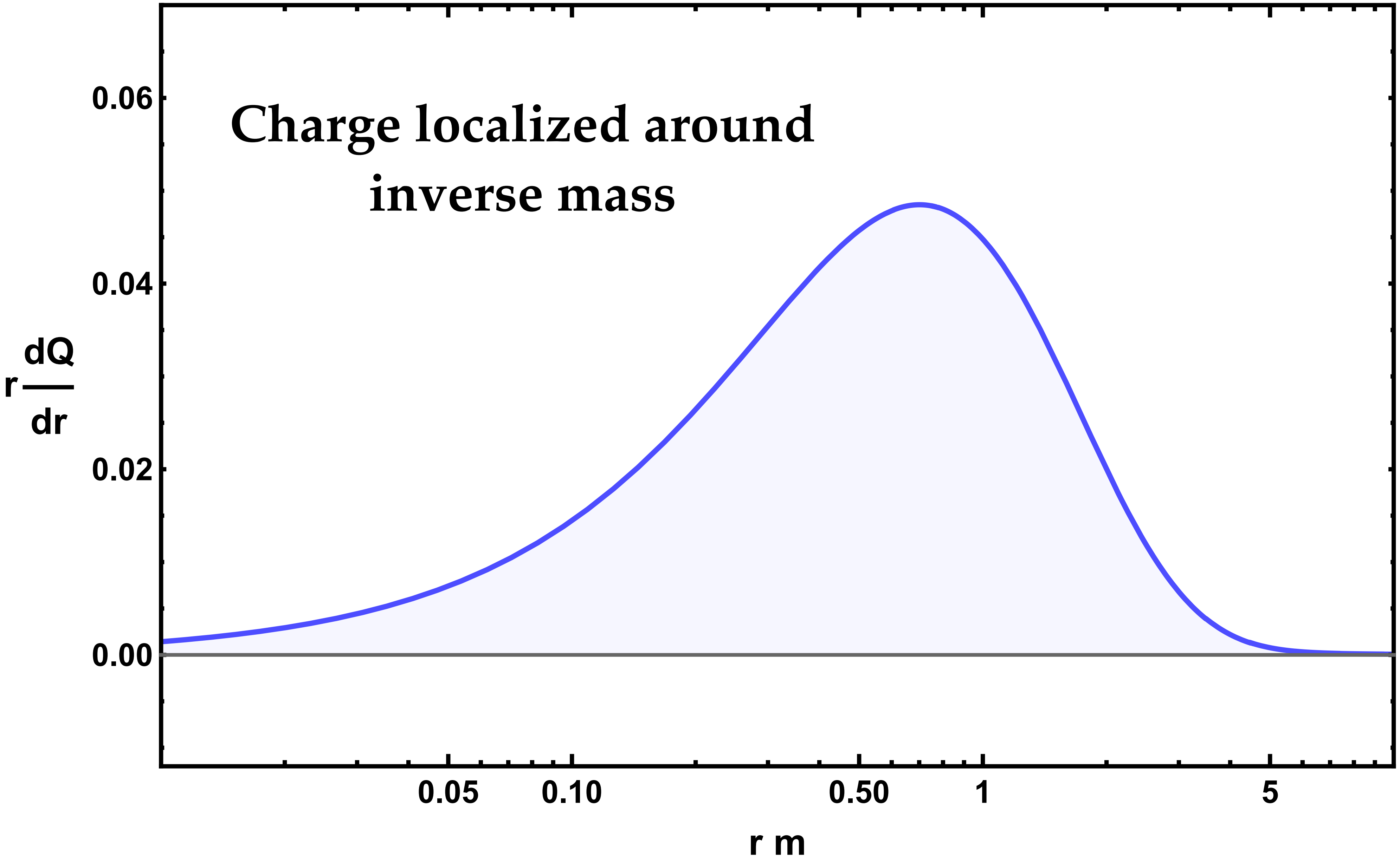
$$\mathscr{L} = \frac{1}{8\pi} \sum_{i=B,L} (\partial \phi_i)^2 + \frac{\pi m^2}{16} \cos \phi_i - \frac{g^2}{8\pi r^2} (\frac{\phi_B + \phi_L}{4\pi} - \frac{\theta}{2\pi})^2$$

Boundary Conditions

$$\phi_B(0) = \phi_L(0) \qquad \partial \phi_B(0) = - \partial \phi_L(0)$$

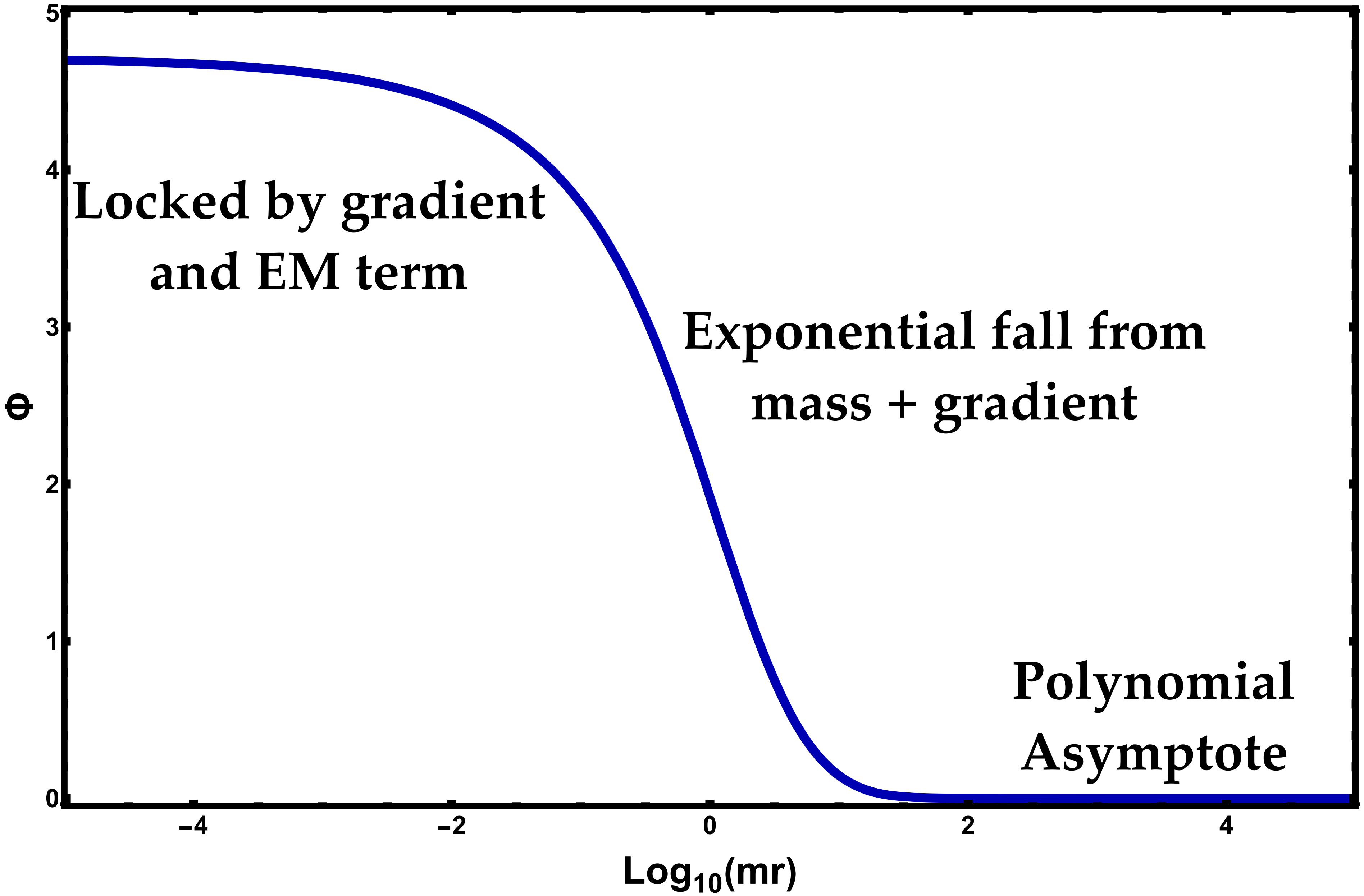
Witten Effect with Light Fermions

Step 5 - Numerical Solution



Witten Effect with Light Fermions

$$\partial_r^2\phi = \frac{\pi^2 m^2}{4}\sin\phi + \frac{\alpha}{2\pi r^2}(\phi - \theta)$$



Locked by gradient and EM term

Exponential fall from mass + gradient

Polynomial Asymptote

$$\phi \approx \frac{2\alpha\theta}{\pi^3 m^2 r^2}$$

Witten Effect with Light Fermions

$$\rho = -\frac{1}{8\pi^2 r^2}\partial_r \phi = \frac{\theta\alpha}{2\pi^5 m^2 r^5} \neq e^{-mr}$$

Non-Exponential Polynomial fall off is required by anomaly equation

$$\partial_\mu J_5^\mu \sim \alpha F\tilde{F} + m\overline{\psi}\gamma_5\psi$$

$$\mathrm{Im}(bb^c) \sim \mathrm{Im}(\ell\ell^c) \sim \frac{\alpha\theta}{mr^4}$$

Witten Effect with Light Fermions

Massless limit

Witten effect : Total electric charge is always $g\theta/2\pi$

Massless limit : θ is unphysical

Witten Effect with Light Fermions

Massless limit

Witten effect : Total electric charge is always $g\theta/2\pi$

Massless limit : θ is unphysical

Observer sits a fixed distance away from monopole

$r_{\text{obs}} \gg 1/m$ **Total electric charge is $g\theta/2\pi$**

$r_{\text{obs}} \ll 1/m$ **Total electric charge is 0**

Resolution is order of limits issue as well as what is called a monopole

Outline

✓ **Witten Effect with light fermions**

- **A plethora of bound states**
- **The massless limit**
- **Conclusion**

UV Theories

	$SU(2)$	$SU(N_f)$
Φ	Adj.	
ψ	$\square$	$\square$

Mass term breaks Flavor symmetry

$$\delta\mathscr{L} = -\, m\, \psi_a^i \psi_b^j\, \epsilon^{ab} \epsilon_{ij} \qquad SU(N_f)/Sp(N_f)$$

$$\delta\mathscr{L} = -\, y\, \psi_a^i\, \Phi_b^a\, \psi_c^j\, \epsilon^{ab} \delta_{ij} \qquad SU(N_f)/SO(N_f)$$

UV Theories

Difference Visually

$SO(2)$ Flavor Symmetry

	$SU(2) \;\to\; U(1)_{EM}$	$U(1)_{B-L}$
$\begin{pmatrix} b \\ b^c \end{pmatrix}$	$\square \;\to\; \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	1 -1
$\begin{pmatrix} \ell \\ \ell^c \end{pmatrix}$	$\square \;\to\; \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	-1 1

$Sp(2)$ Flavor Symmetry

	$SU(2) \;\to\; U(1)_{EM}$	$U(1)_{B-L}$
$\begin{pmatrix} b \\ \ell^c \end{pmatrix}$	$\square \;\to\; \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	1 1
$\begin{pmatrix} \ell \\ b^c \end{pmatrix}$	$\square \;\to\; \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	-1 -1

UV Theories

Bosonized Theories only slightly different

$$4\pi\mathcal{L} = \frac{1}{2}\sum_i (\partial\phi_i)^2 + \left(\frac{\pi m(r)}{2}\right)^2 \sum_i \cos(\phi_i) - \frac{\alpha}{2\pi r^2}\left(\sum_i \frac{1}{2}\phi_i - \theta\right)^2$$

SO

$$m(r) = e^{K_0(\pi^2 e^{-\gamma} mr/4)/2} m$$

Identical far from the monopole where you just see the identical IR

Sp

$$m(r) = m$$

$$\partial_r \phi_i(0) = 0$$

Forces quantization of U(1) subgroups of Sp(N$_f$)

$$\partial_r \phi_{b,i}(0) = -\partial_r \phi_{\ell,i}(0)$$

$$\phi_{b,i}(0) = \phi_{\ell,i}(0)$$

Getting our feet wet : $N_f = 1$

Find all stationary solutions to EOM

$$\partial_r^2 \phi = \frac{\pi^2 m^2}{4} \sin\phi + \frac{\alpha}{4\pi r^2}(\phi - \theta)$$

$$\partial_r \phi(0) = 0$$

$$\phi(0) = \theta$$

$$\phi(\infty) = 2\pi q$$

Finite Energy

Every minimum energy solution is
labeled by a single integer q

Getting our feet wet : $N_f = 1$

Find all stationary solutions to EOM

$$\partial_r^2 \phi = \frac{\pi^2 m^2}{4} \sin\phi + \frac{\alpha}{4\pi r^2}(\phi - \theta)$$

$$\partial_r \phi(0) = 0$$

$$\phi(0) = \theta$$

$$\phi(\infty) = 2\pi q$$

Finite Energy

$$Q_{\mathrm{EM}} = q - \theta/2\pi = -\theta_{\mathrm{eff}}/2\pi$$

Really finding Dyon solutions!

Getting our feet wet : N$_f$ = 1

Find all solutions to EOM

$$\phi(\infty) = 2\pi q$$

A solution is ALWAYS monopole ground state + q solitons at infinity

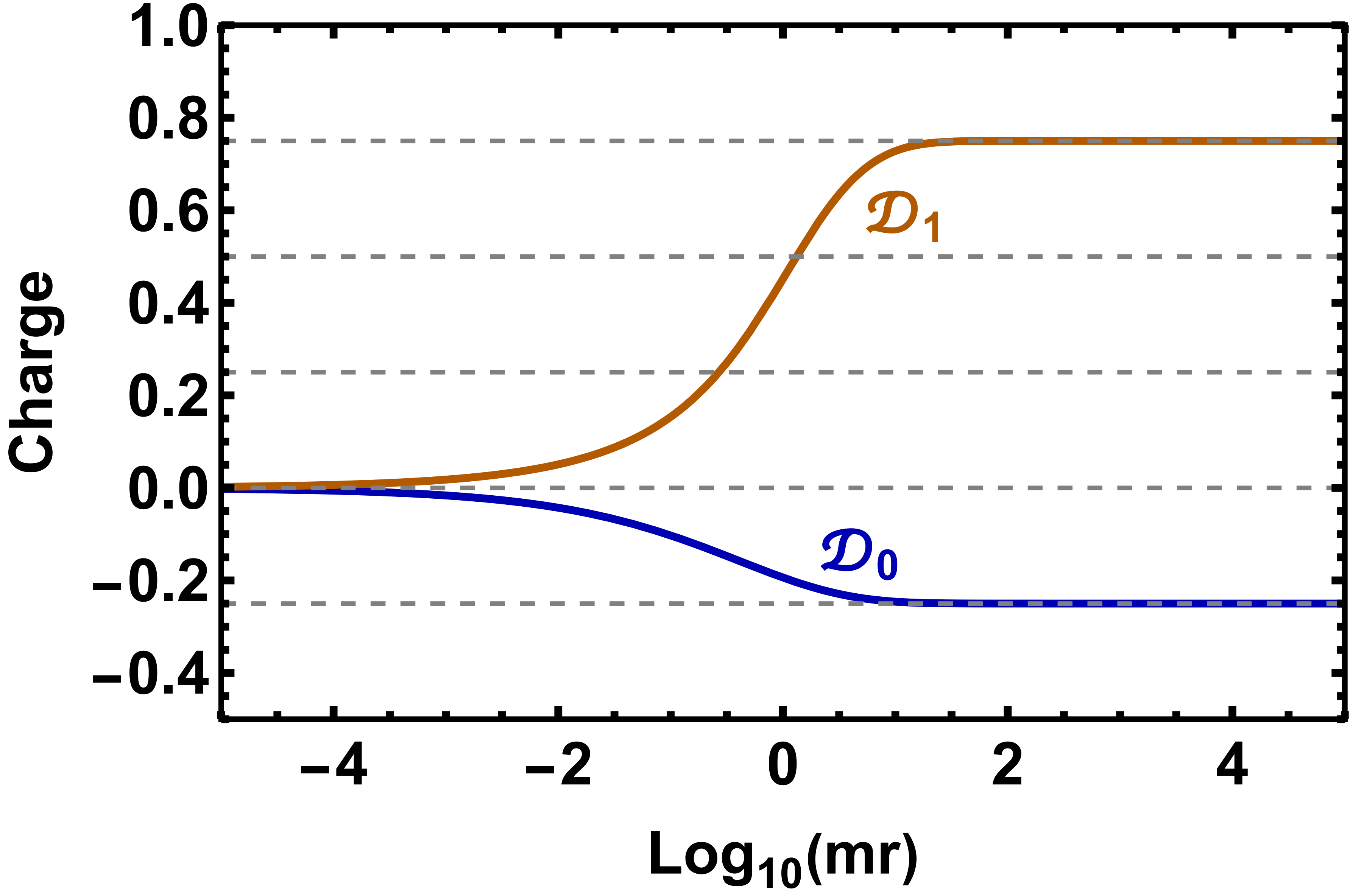
If $|Q_{\text{EM}}| > 1$, this is actually always the minimum energy configuration

In the small α limit, can analytically find the energy

$$E(q, \alpha = 0, m) = \sin^2(\frac{\pi}{2} Q_{\text{EM}})\, m$$

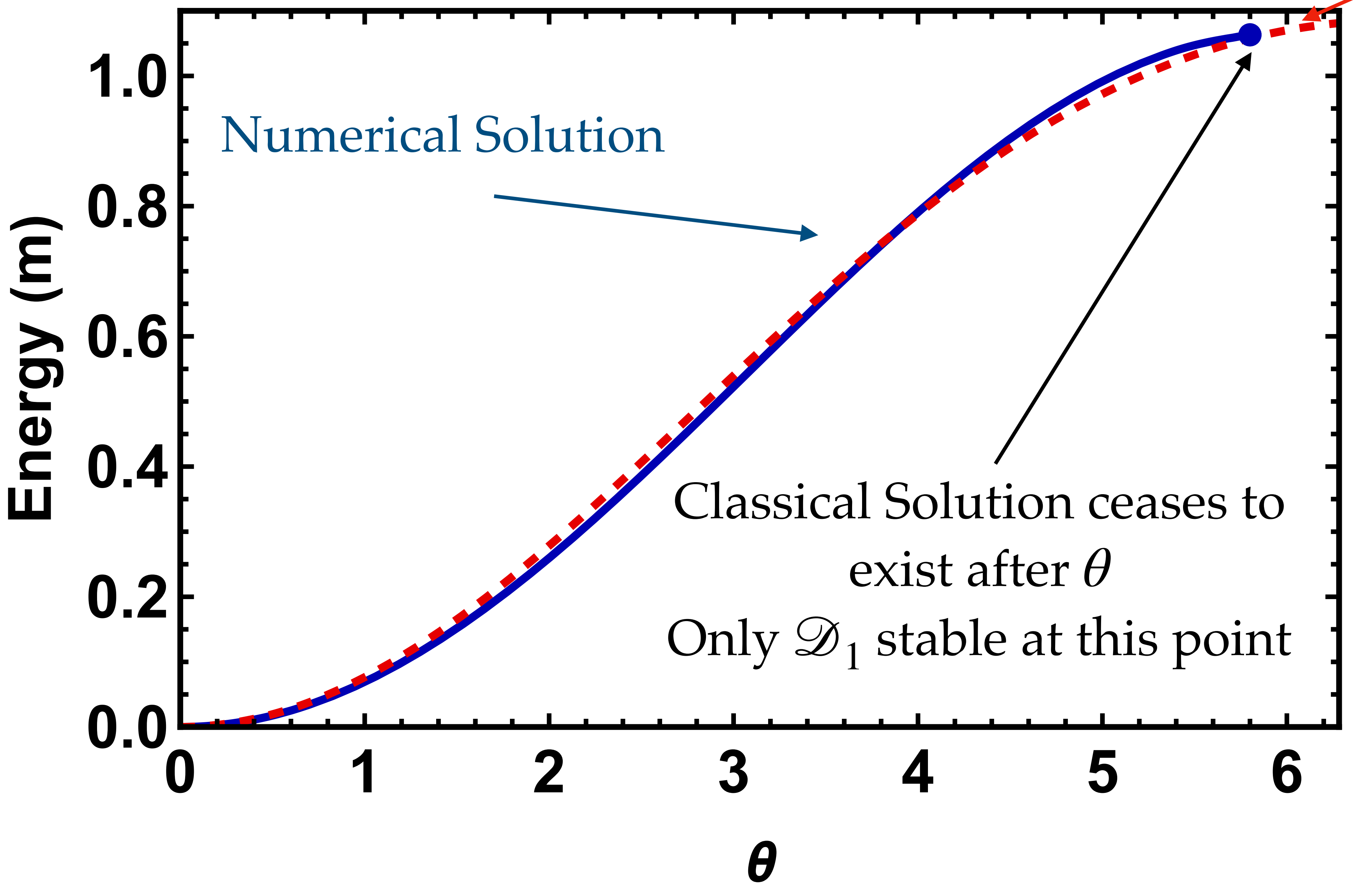
Getting our feet wet : $N_f = 1$

Only two possibly stable solutions q=0,1



Getting our feet wet : $N_f = 1$

$\alpha = 0.6$



Energy of $\mathscr{D}_0$

Analytic Approximation

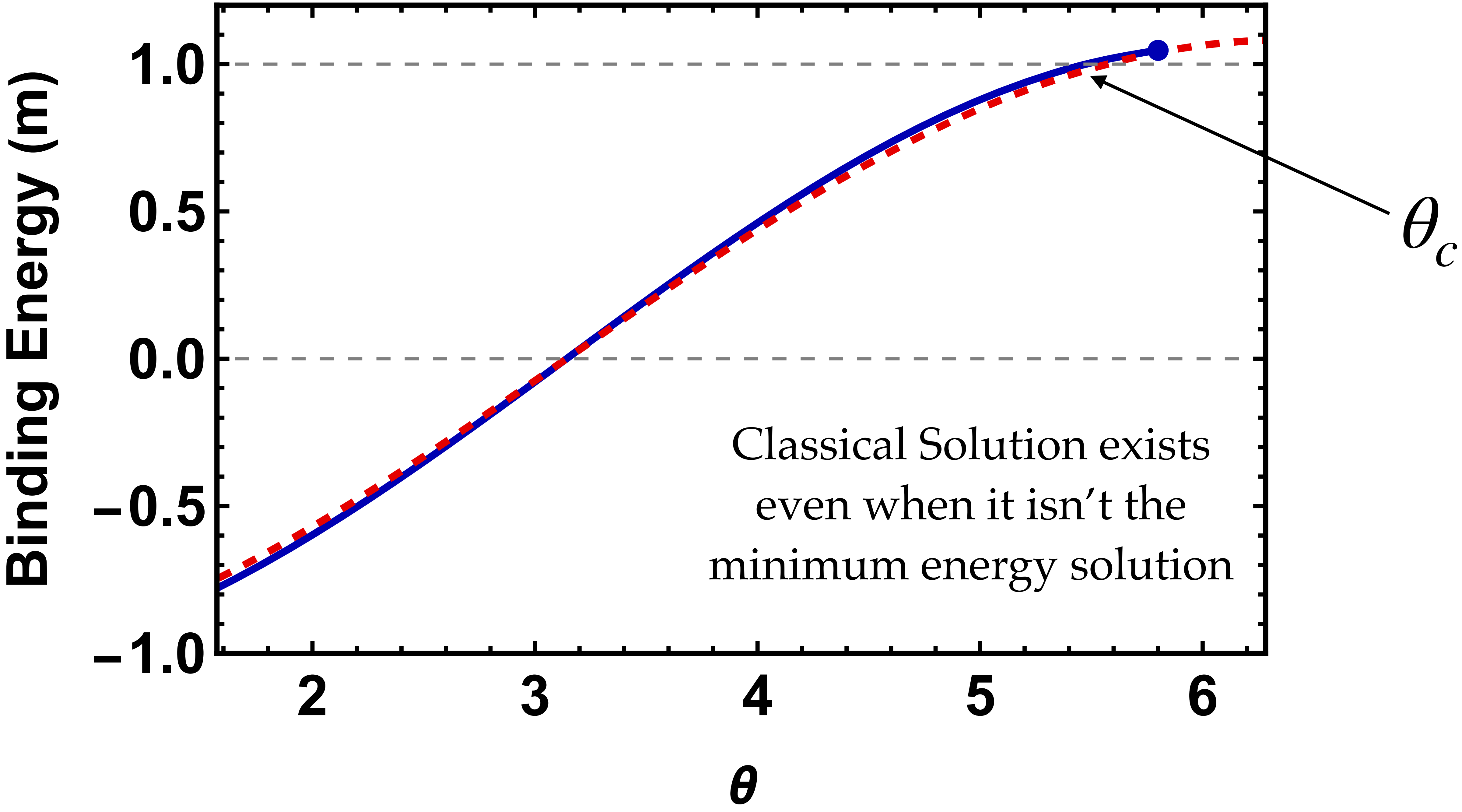
Numerical Solution

Classical Solution ceases to exist after θ

Only $\mathscr{D}_1$ stable at this point

Getting our feet wet : $N_f = 1$

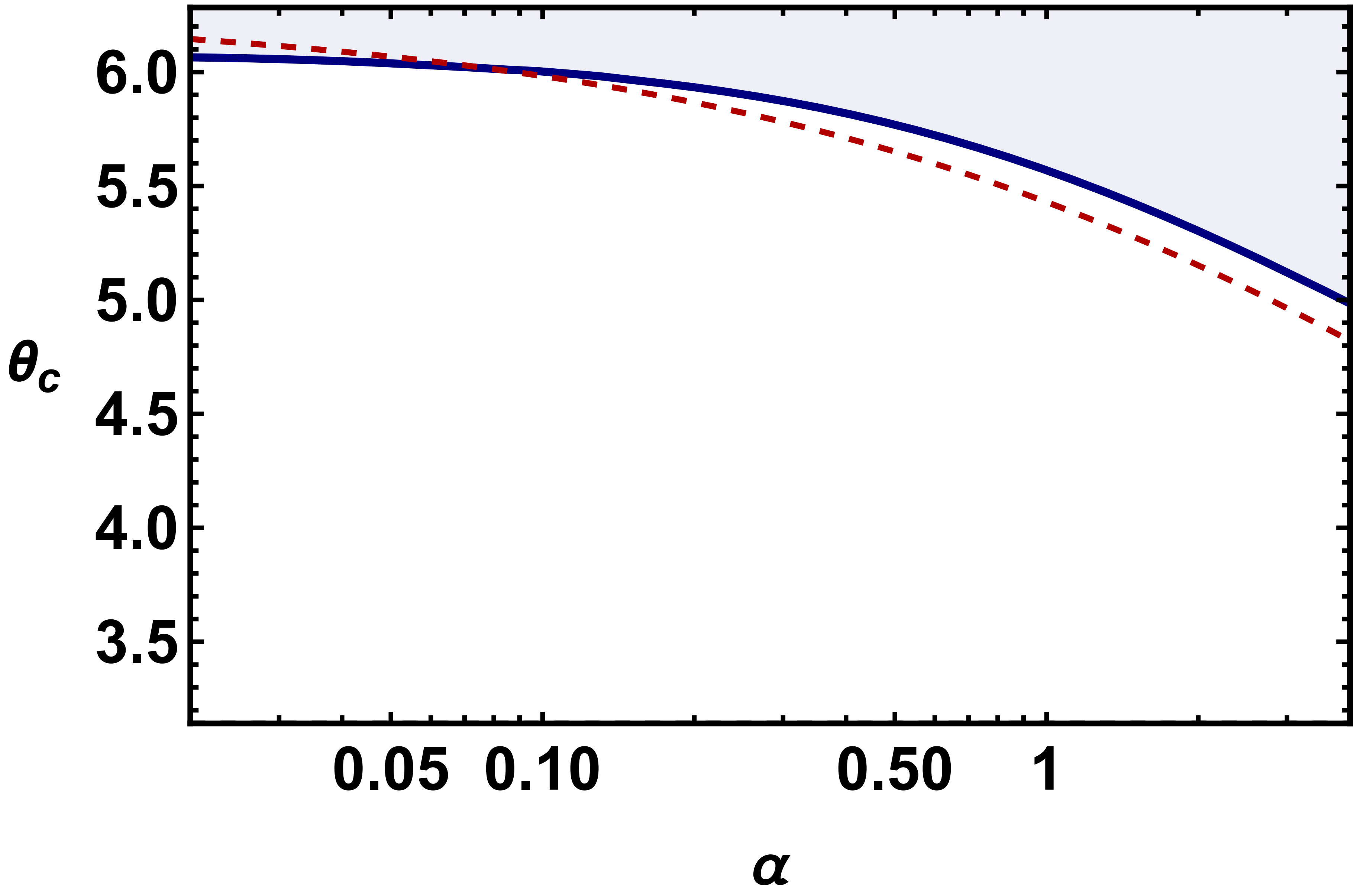
Interpret $\mathscr{D}_0 = \mathscr{D}_1$ + fermion



Classical Solution exists even when it isn't the minimum energy solution

Getting our feet wet : $N_f = 1$

Interpret $\mathscr{D}_0 = \mathscr{D}_1$ + fermion



Sp($N_f = 2$)

$$
\begin{aligned}
L =& \frac{1}{4\pi}\int_0^\infty dr\ \frac{1}{2}\left(\partial_\mu\phi_\ell\partial^\mu\phi_\ell + \partial_\mu\phi_b\partial^\mu\phi_b\right) - \left(\frac{\pi m_\ell(r)}{2}\right)^2 \left(1-\cos(\phi_\ell)\right) \\
&- \left(\frac{\pi m_b(r)}{2}\right)^2 \left(1-\cos(\phi_b)\right) - \frac{\alpha}{8\pi r^2}\left(\phi_\ell + \phi_b - 2\theta\right)^2
\end{aligned}
$$

Flavor symmetry is Sp(2) if masses equal

	$SU(2) \ \to \ U(1)_{EM}$	$U(1)_{B-L}$
$\begin{pmatrix} b \\ \ell^c \end{pmatrix}$	$\Box \ \to \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	1 1
$\begin{pmatrix} \ell \\ b^c \end{pmatrix}$	$\Box \ \to \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	-1 -1

$\mathrm{Sp}(N_f = 2)$

Find all solutions

$$\phi_b(0) = \phi_\ell(0) = \theta$$

Finite at origin

$$\phi_b(\infty) = 2\pi n_b$$
$$\phi_\ell(\infty) = 2\pi n_\ell$$

Finite energy

All solutions labeled by two integers

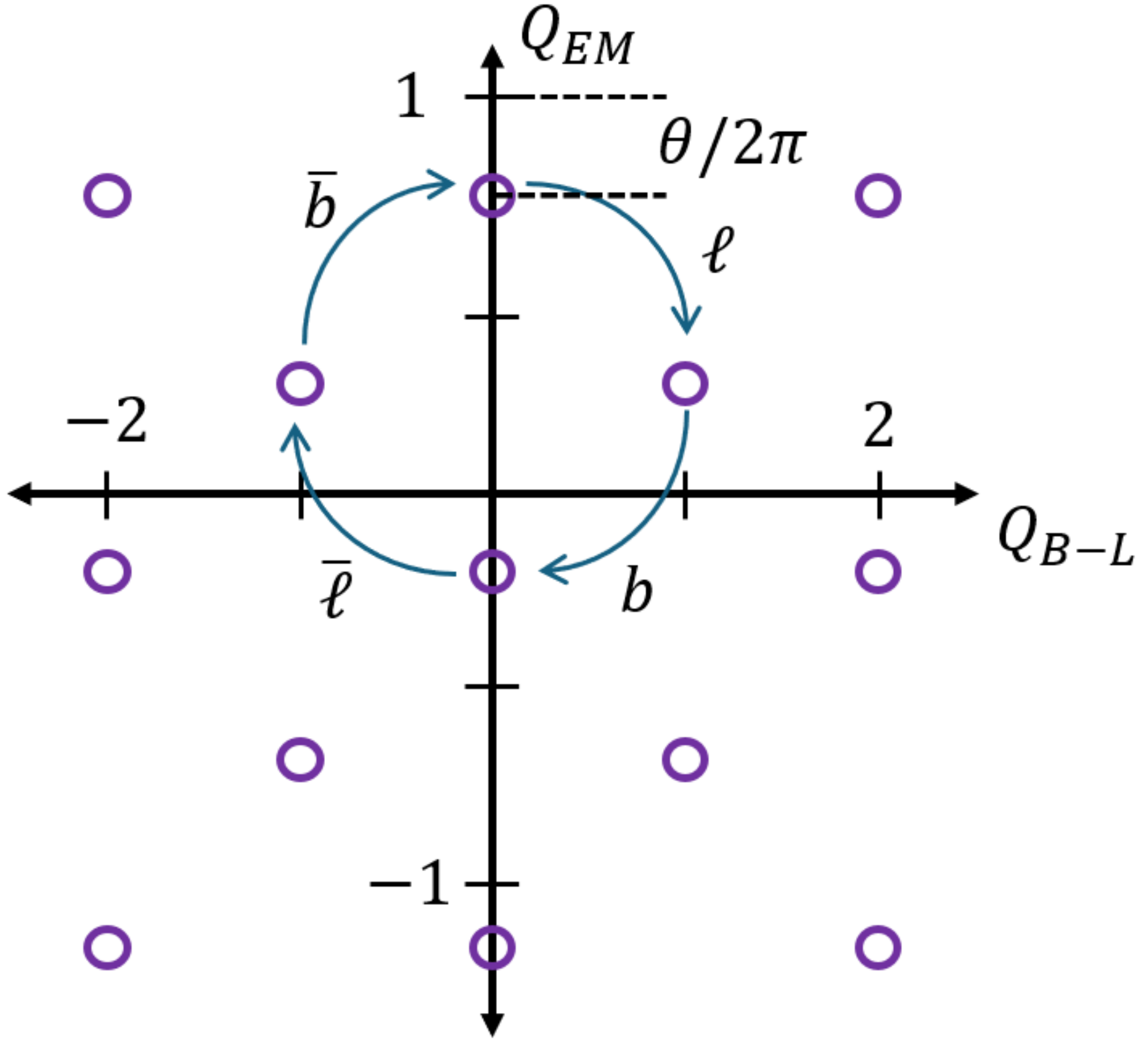
$$Q_{B-L} = n_b - n_\ell \qquad Q_{\mathrm{EM}} = \frac{n_b + n_\ell}{2} - \frac{\theta}{2\pi} = \frac{q}{2} - \frac{\theta}{2\pi}$$

Sp($N_f = 2$)

All solutions

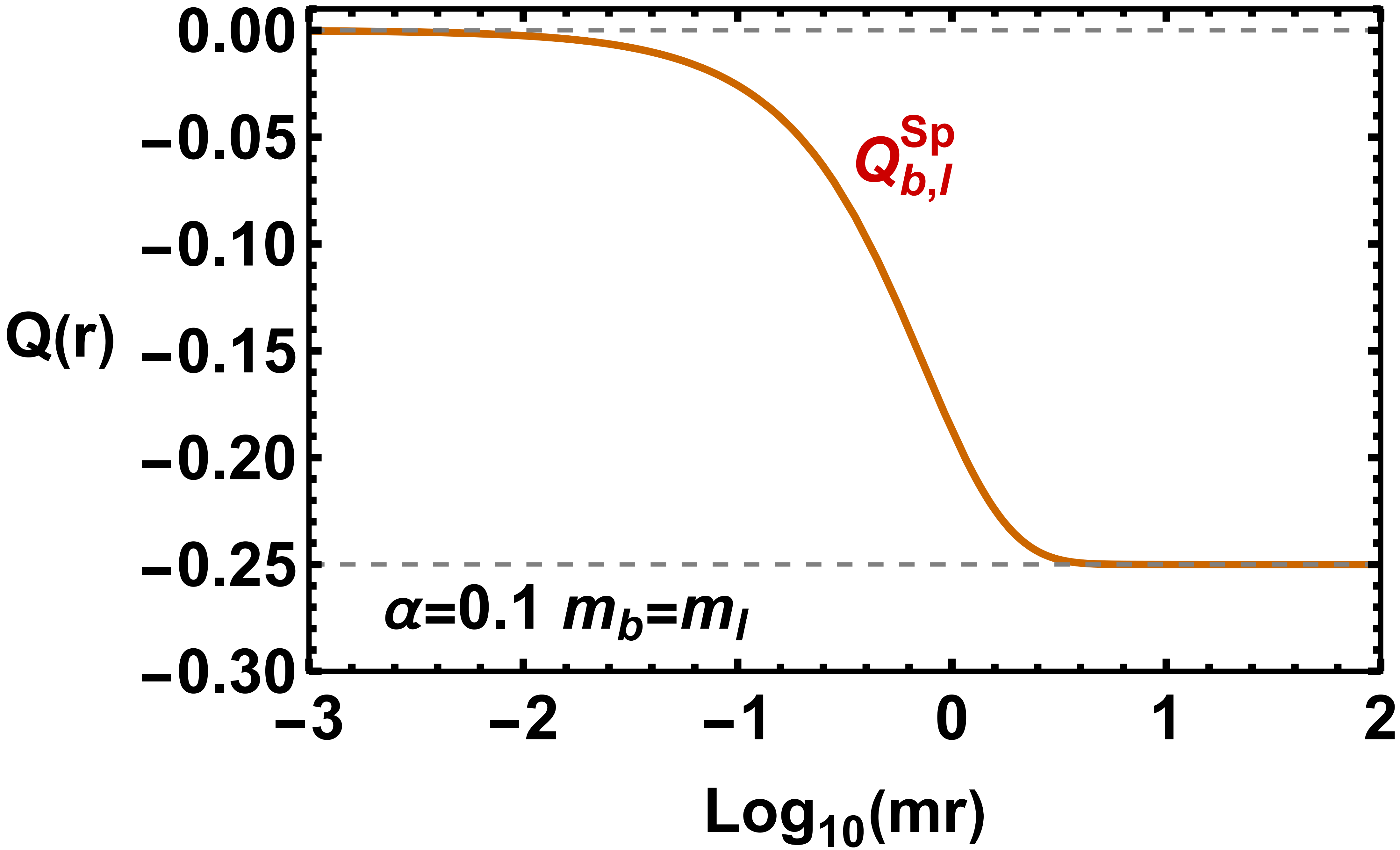
$$\mathscr{D}_{q,n_b - n_l}$$

$n_b = n_\ell$ are dyons
$n_b \neq n_\ell$ are other bound states



$\mathrm{Sp}(N_f = 2)$: Some solutions

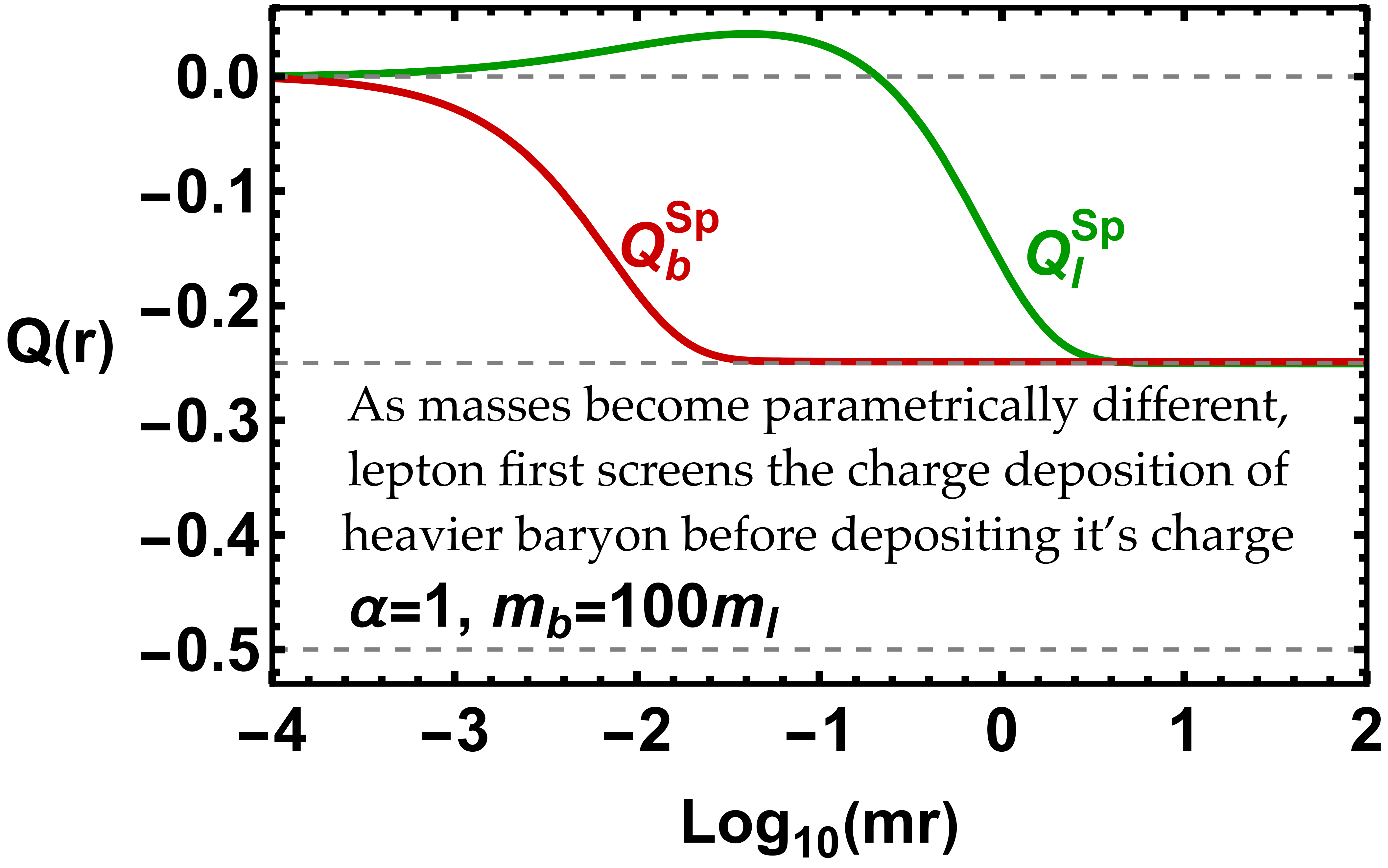
$\mathscr{D}_{0,0}$



$\mathrm{Sp}(N_f = 2)$: Some solutions

$\mathscr{D}_{0,0}$

$Q_\ell(r) \approx -Q_b^{\text{tot}}(1-(m_b r)^{-\alpha/4\pi})$



As masses become parametrically different, lepton first screens the charge deposition of heavier baryon before depositing it’s charge

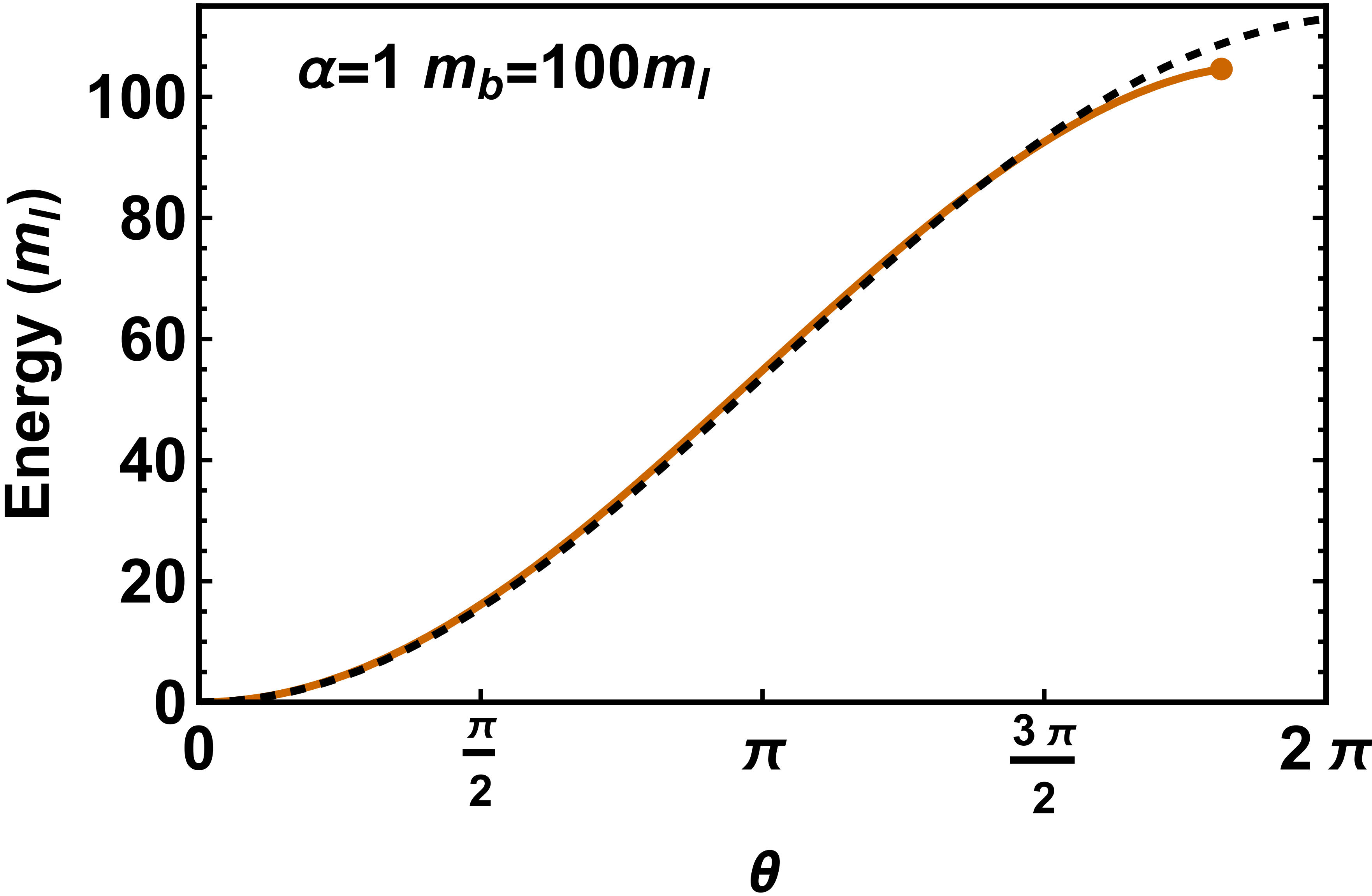
α=1, m_b=100m_l

$\mathrm{Sp}(\mathrm{N_f} = 2)$: Some solutions

$\mathscr{D}_{0,0}$

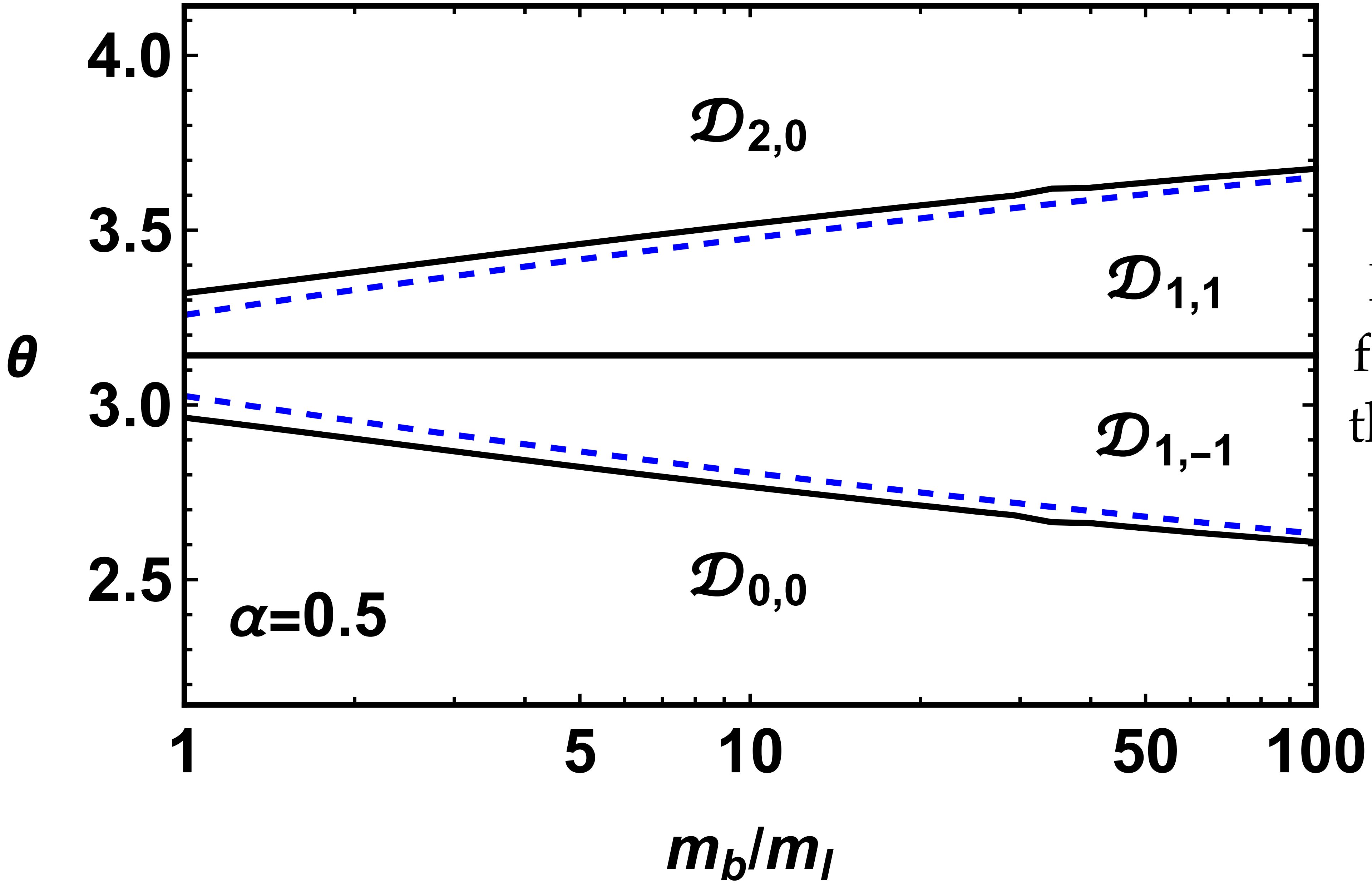
$$Q_\ell(r) \approx -Q_b^{\text{tot}}(1 - (m_b r)^{-\alpha/4\pi})$$

- Witten effect vanishes in the massless limit but extremely slowly
- θ dependence is driven by heavier particle
- B-L neutral dyon/monopole must excite heavier baryon even if lepton massless



$\mathrm{Sp}(N_f = 2)$: Ground State

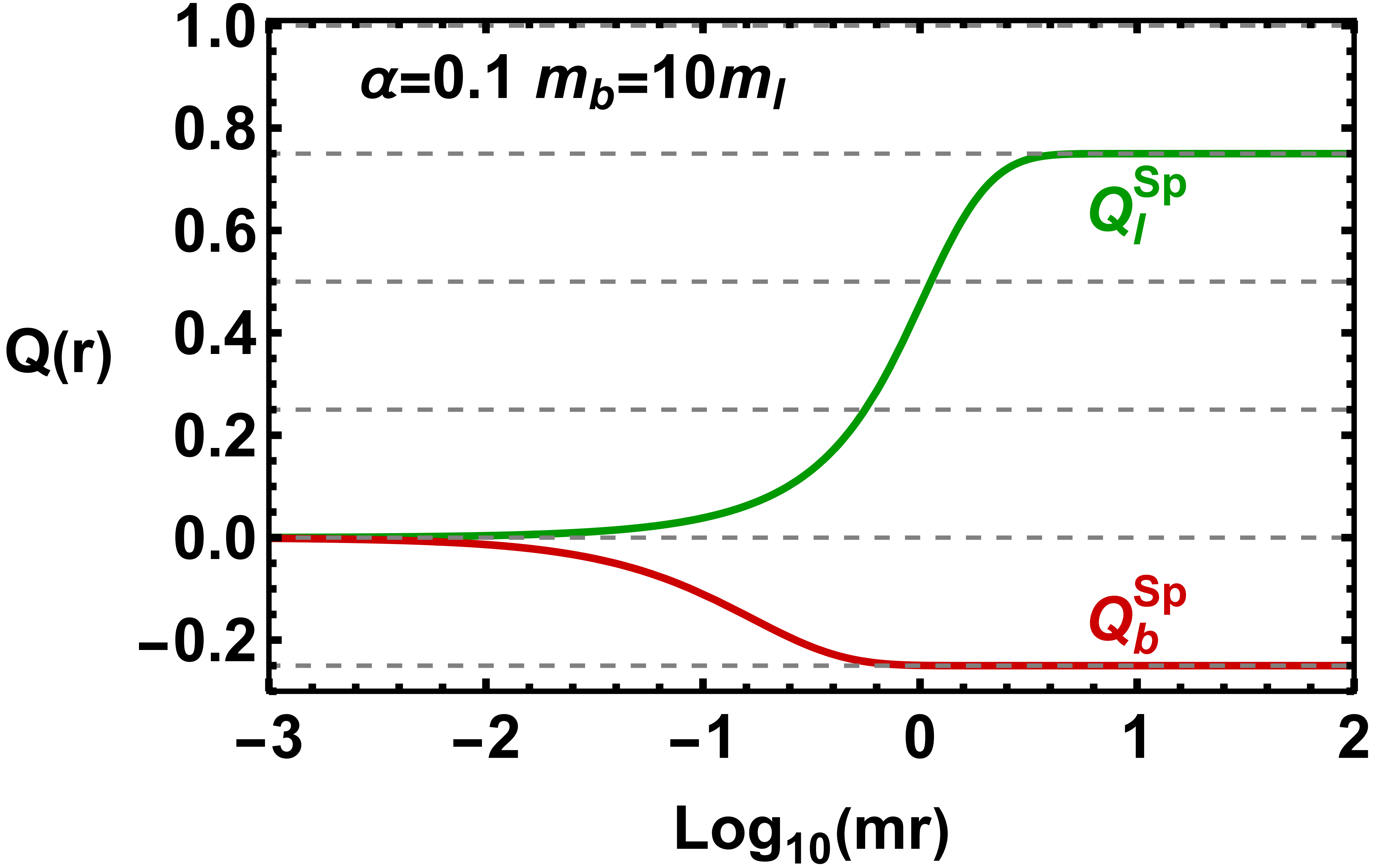
What is the lowest energy magnetically charged state?



Monopole bound with fermion is lower energy than monopole by itself!

$\mathrm{Sp}(N_f = 2)$: Some solutions

$\mathscr{D}_{1,-1}$



$\mathrm{Sp}(N_f = 2)$: Ground State

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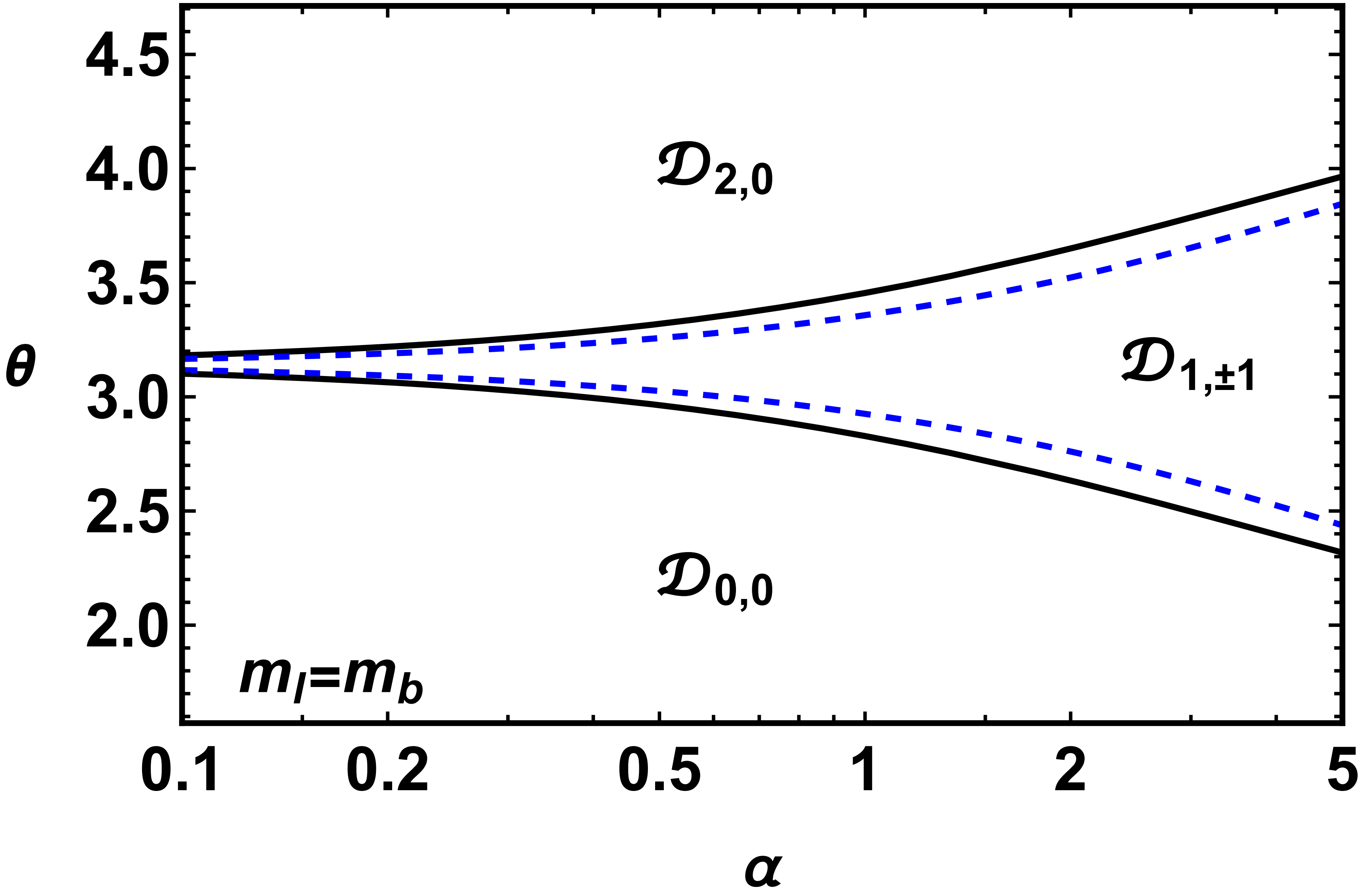


This is entirely due to EM effects!

Sp(N$_f$ = 2) : Ground State

$\alpha = 0$ limit

$$E_{0,0}(\theta, \alpha = 0, m_b = m_\ell) = 2m \sin^2(\theta/4)$$

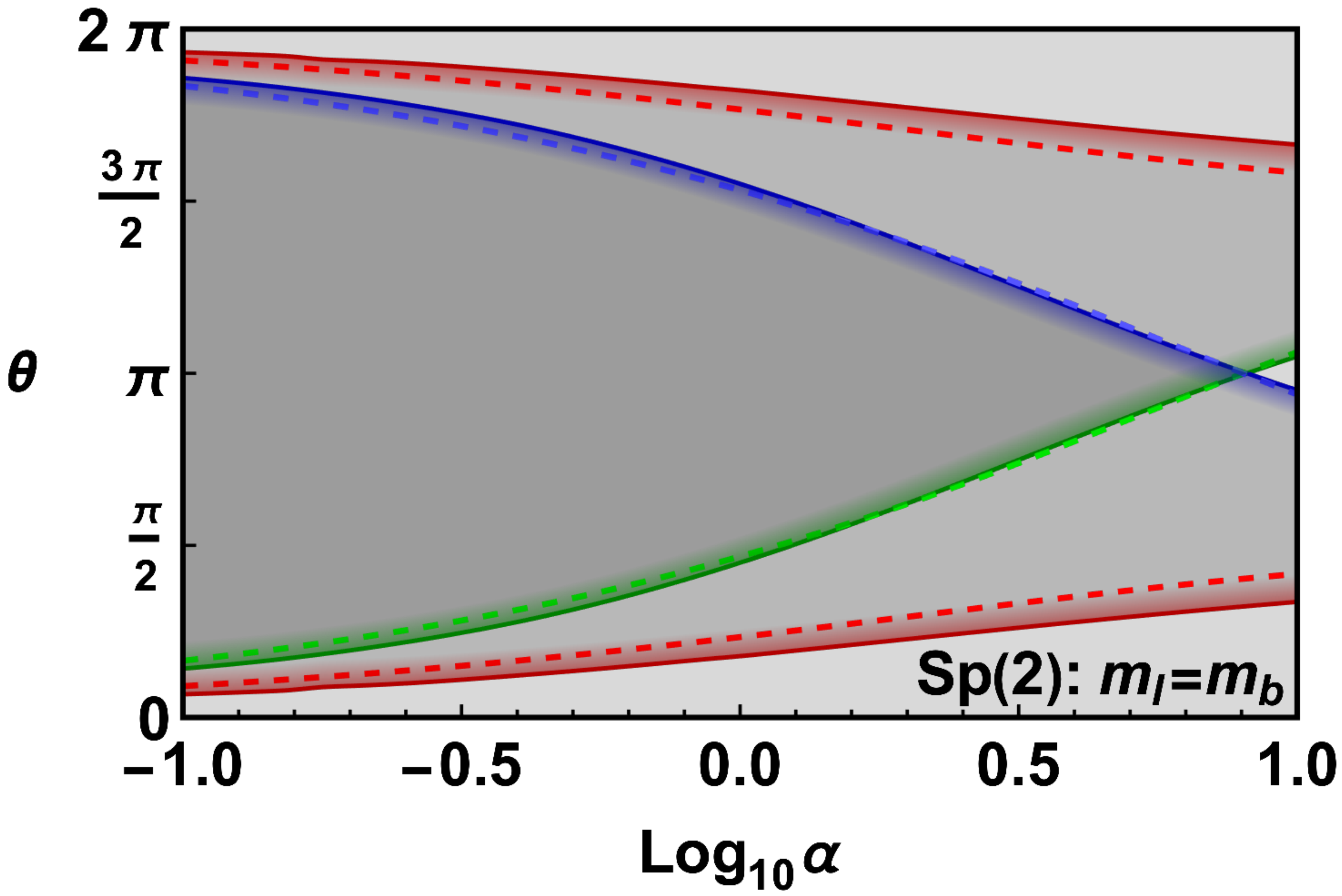
$$E_{1,-1}(\theta, \alpha = 0, m_b = m_\ell) = m$$

Monopole is always more stable until dyon is more stable

EM effects eventually stabilize near $\theta = \pi$ as
the bound state has smaller total charge

Sp($N_f = 2$) : Stable States

How many stable states at any given time?



Stability

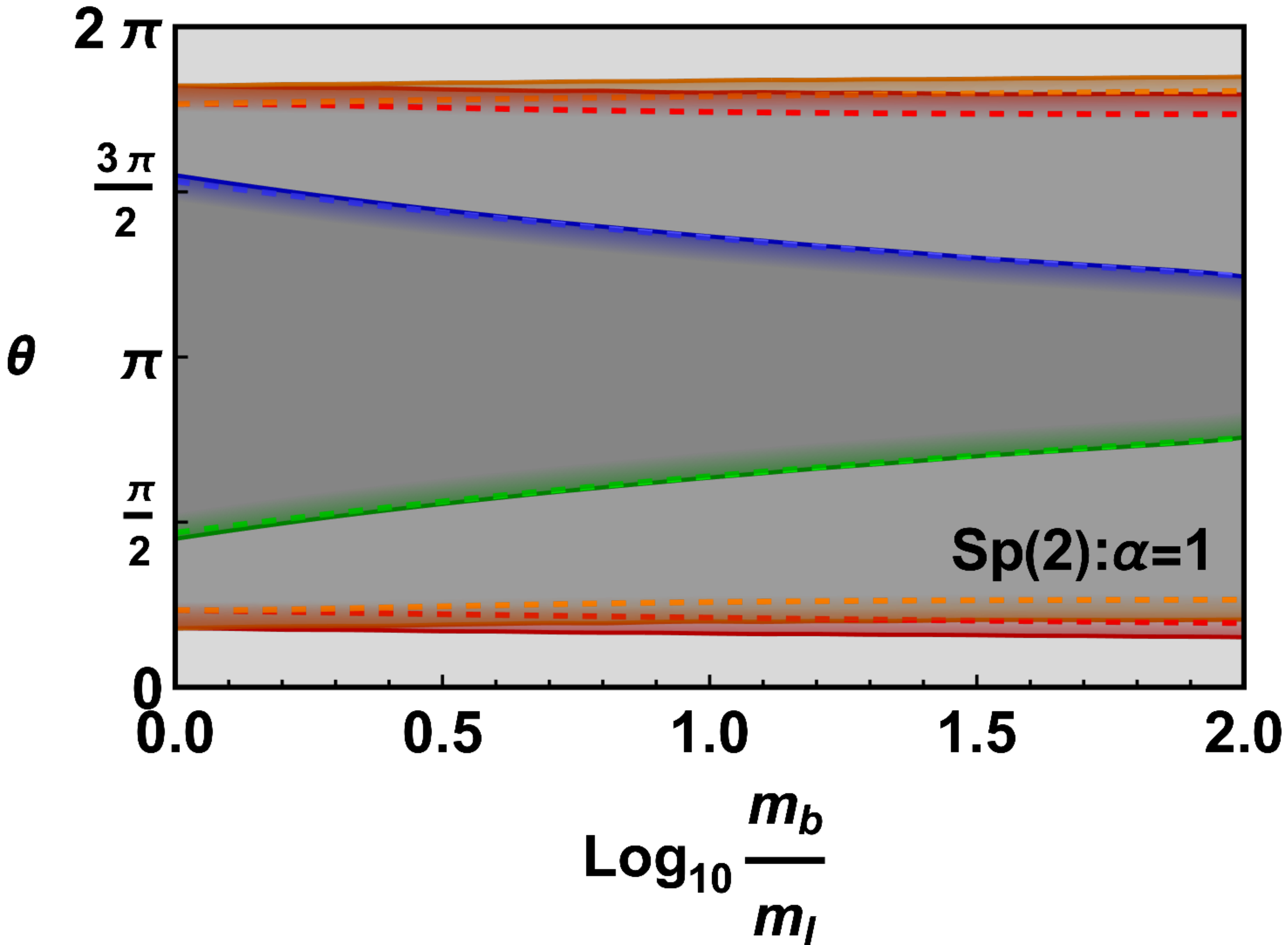
Monopole : below blue

Dyon : above green

Monopole + fermion : inside red

Sp($N_f = 2$) : Stable States

Larger Mass Splitting destabilizes monopole/dyon



Stability

Monopole : below blue

Dyon : above green

Monopole + fermion : inside red

Sp / SO($N_f \geq 4$)

Too complicated to study in general

Equal mass

Number of stable states at $\theta = 0$

θ_c when monopole becomes unstable

Sp / SO($N_f \geq 4$)

Separate into N_f/2 identical N_f = 2 scalars

$$H = \frac{N_f}{2}\frac{1}{4\pi}\int_0^\infty dr \frac{{\phi_b'}^2}{2} + \frac{{\phi_\ell'}^2}{2} + \left(\frac{\pi m(r)}{2}\right)^2 (2 - \cos(\phi_b)) - \cos(\phi_\ell))$$
$$+ \frac{\alpha N_f/2}{8\pi r^2}\left(\phi_\ell + \phi_b - \frac{4\theta}{N_f}\right)^2$$

Reduces to the same N_f = 2 system we studied before with slightly different parameters

$\alpha_{\text{eff}} = \alpha N_f/2$

$\theta_{\text{eff}} = 2\theta/N_f$

$$E_0^{N_f}(\alpha, \theta) = \frac{N_f}{2} E_0^{N_f=2}(\alpha N_f/2, 2\theta/N_f)$$

Sp / SO($N_f \geq 4$)

Dyonic bound states are just lots of copies of Witten effect solutions

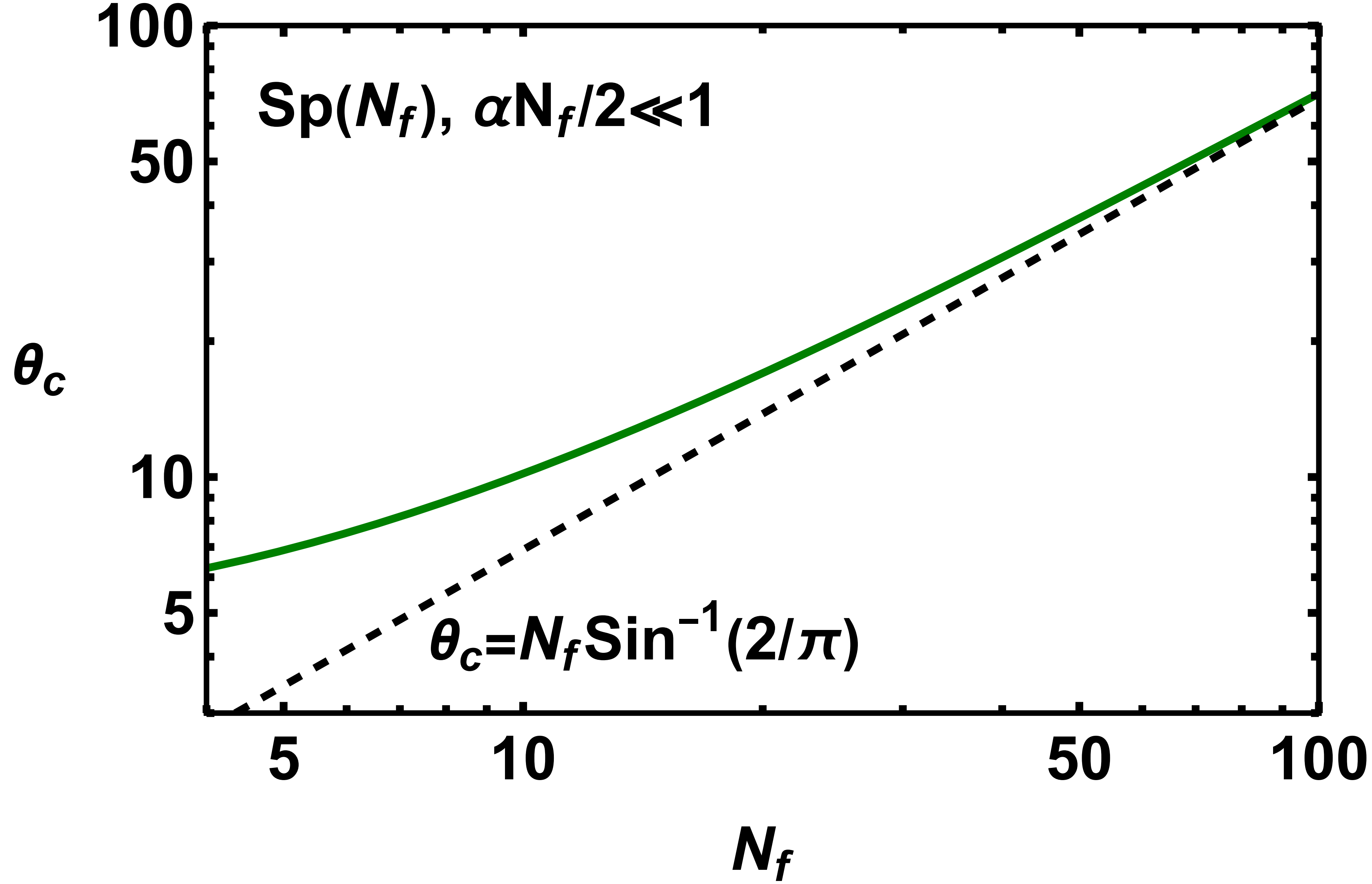
$$\alpha_{\text{eff}} = \alpha N_f/2 \qquad \theta_{\text{eff}} = 2\theta/N_f$$

$$E_0^{N_f}(\alpha, \theta) = \frac{N_f}{2} E_0^{N_f=2}(\alpha N_f/2, 2\theta/N_f)$$

Sp(N_f) also has the N_f $\mathscr{D}_{1,\pm\vec{1}}$ bound states, but they are unstable around $\theta = 0$

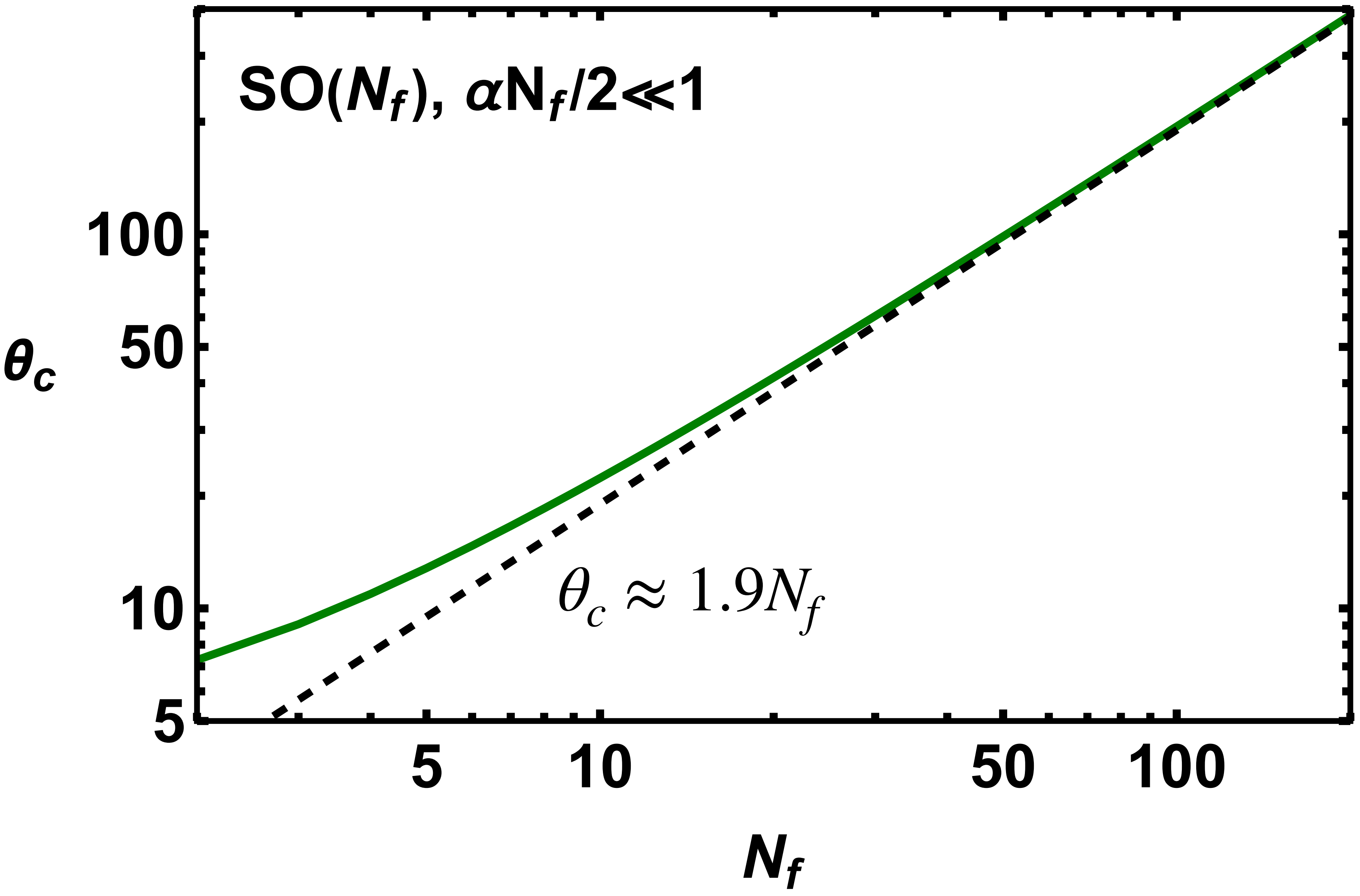
$Sp(N_f \geq 4)$

Mapped to numerically solved situation



SO($N_f \geq 4$)

Mapped to numerically solved situation



Sp / SO($N_f \geq 4$)

Number of stable states : SO

$$N \approx \frac{3.8 N_f}{\pi} \qquad N_f \gg 1 \qquad \alpha N_f/2 \ll 1$$

Number of stable states : Sp

$$N \approx \frac{\sin^{-1}(2/\pi)\, N_f}{\pi} \qquad N_f \gg 1 \qquad \alpha N_f/2 \ll 1$$

Outline

- ✓ **Witten Effect with light fermions**
- ✓ **A plethora of bound states**
- **The massless limit**
- **Conclusion**

Callan Rubakov with $N_f \geq 4$

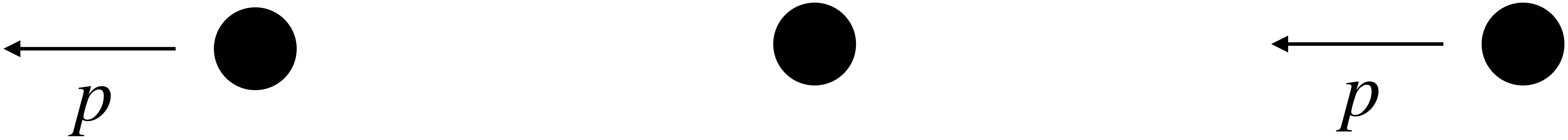
	$SU(2)$	$SU(N_f)$
Φ	Adj.	
$\Psi = \begin{pmatrix} \psi \\ \chi \end{pmatrix}$	$\square$	$\square$

Monopole and two sets of N$_f$ massless fermions

$$\chi = (-1, \square)$$
$$\chi^\dagger = (1, \overline{\square})$$

$$\psi = (1, \square)$$
$$\psi^\dagger = (-1, \overline{\square})$$

Monopole



$$(1, \square)$$

No s-wave final state that preserves all symmetries!

$$\psi_i$$

Callan Rubakov with $N_f \geq 4$

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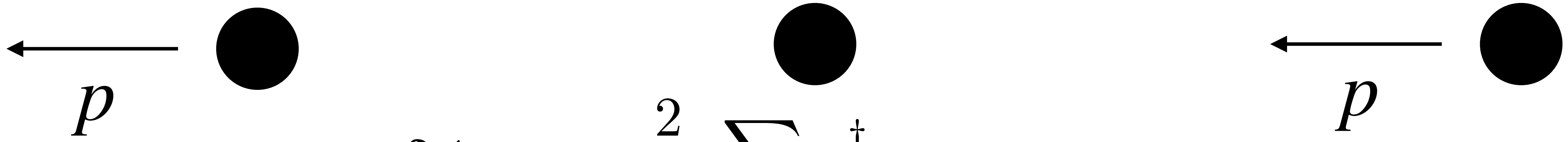


χ_i $\quad \frac{2}{N_f}\sum_i \chi_i^\dagger$ $\quad \psi_i$

$$(1, \square) = (-1, \square) + (2, \ )$$

Fractional state - semiton

Semiton

$$\frac{2}{N_f}\sum_i \chi_i^\dagger$$

Charge ±2, no flavor symmetries

I knew my final states (standard quantized fermions), how did this new possible final state come into the story?

Semiton

Ala massless particles and Witten effect, lets take massive fermions then take massless limit

	$SU(2)$	$Sp(N_f)$
Φ	Adj.	
$\Psi = \begin{pmatrix} \psi \\ \chi \end{pmatrix}$	$\square$	$\square$

$$\delta \mathscr{L} = - m\, \Psi^i_a\, \Psi^j_b\, \epsilon^{ab} \epsilon_{ij}$$

$$\theta = 0$$

We will be interested in the dyonic bound state $\mathscr{D}_{\pm 2,0}$

$\mathscr{D}_{\pm 2,0}$

We can solve this the same way as before, but there is a simpler picture

$$n_{b,i} - n_{\ell,i} = 0 \qquad \sum_i \frac{n_{b,i} + n_{\ell,i}}{2} - \frac{\theta}{2\pi} = \pm 1$$

Many gauge equivalent solutions

$$n_{b,i} = n_{\ell,i} = 0 \qquad \theta = \mp 2\pi$$

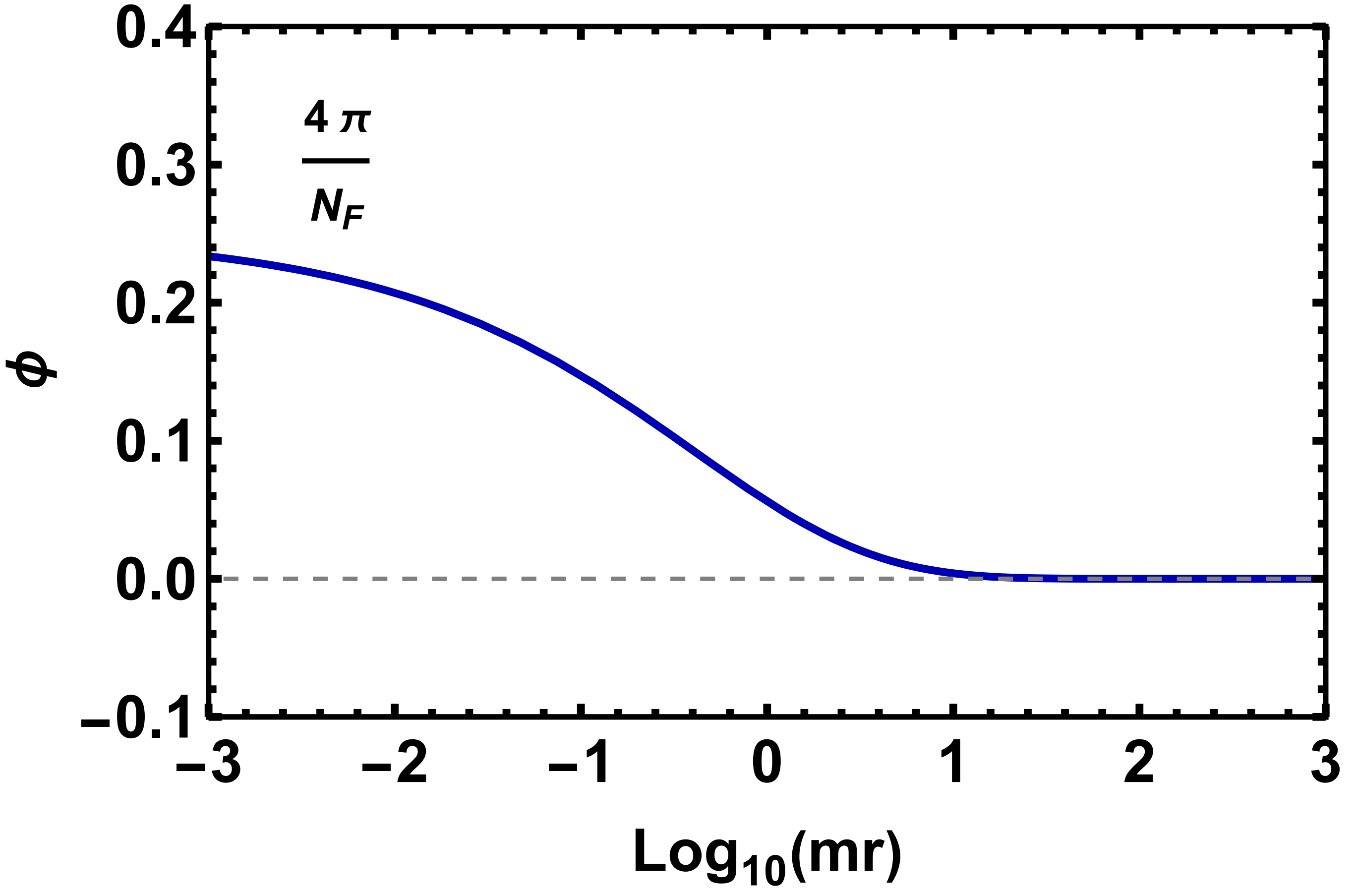
$\mathscr{D}_{\pm 2,0}$

Solution is clearly symmetric $\phi_{b,i} = \phi_{\ell,i} = \phi$

$$L = N_f \frac{1}{4\pi}\left(\frac{(\partial_r \phi)^2}{2} + (\frac{\pi m}{2})^2 \cos\phi \right) + \frac{\alpha N_f/2}{2\pi r^2}(\phi - \frac{2}{N_f}(\pm 2\pi))^2$$

N_f **Witten effects**

with $\theta = \pm \frac{4\pi}{N_f}$

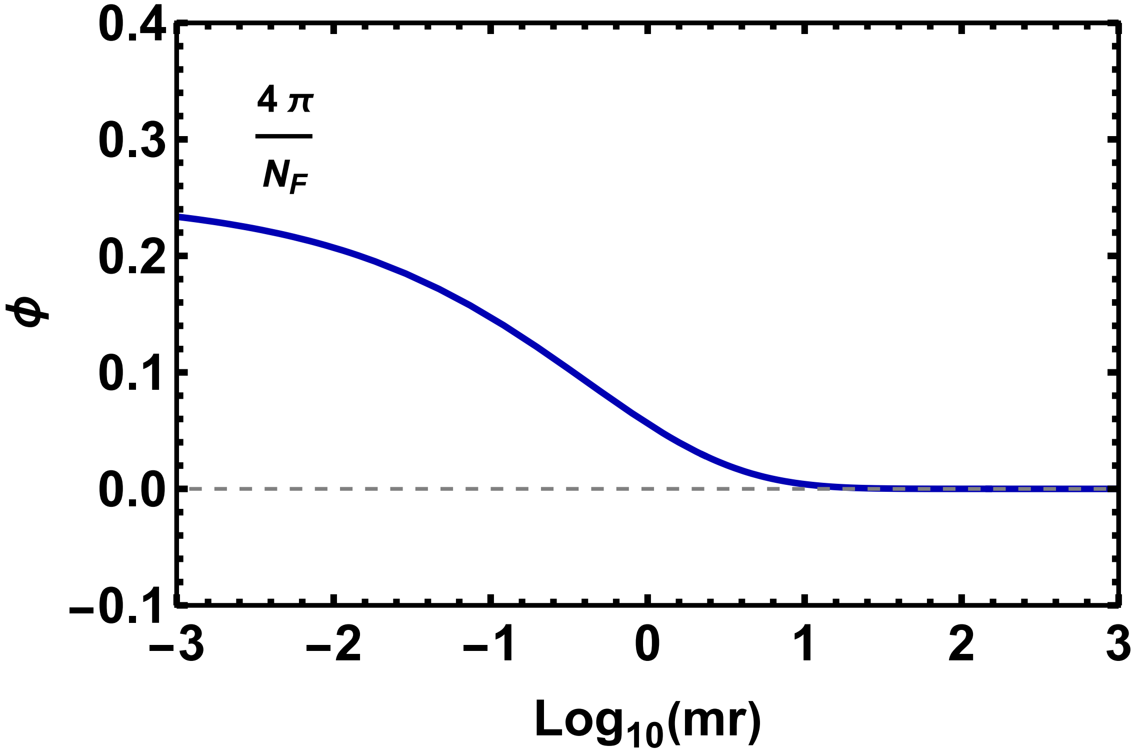


$\mathscr{D}_{\pm 2,0}$

Large Binding energy $E \approx N_f m \sin^2(\frac{\pi}{N_f}) \approx \frac{m\pi^2}{N_f} \ll 2m$

Radii $r \sim 1/m$

Quantum numbers Charge ±2, no flavor symmetries



Massless Limit - Monopole

Observer sits at finite radius r_O

Monopole + polarized fermion vacuum of size $r \sim 1/m$

Decrease fermion mass until $r_O \approx 1/m$

Charge sweeps over observer and enclosed charge transitions to 0

Definition of monopole changes from size $r \sim 1/m$ to $r \sim 1/m_W$

Massless Limit - Dyon

Observer sits at finite radius r_O

Asymptotic s-wave states are quantized fermions

$$\chi = (-1, \Box) \quad \chi^\dagger = (1, \overline{\Box}) \qquad \psi = (1, \Box) \qquad \psi^\dagger = (-1, \overline{\Box})$$

Decrease fermion mass until $r_O \approx 1/m$

$\mathscr{D}_2$ transitions into an asymptotic state + monopole core

New asymptotic state has quantum numbers of semiton!

Massless Limit - Dyon

Caveat

Explains existence of semitons

If cross sections to produce go to zero in the massless limit
then it may not matter that they exist

Conclusion

Monopoles are fun

There are often a whole plethora of bound states

These are stabilized by boundary terms at monopole and their existence depends on UV symmetries, not IR properties

Binding energy O(m)
Bound state radii ~ 1/m

In the massless limit, due to order of limit, bound states become new asymptotic states