Oct. 10th, 2022, UC Davis

Parity symmetry breaking scale and SM parameters

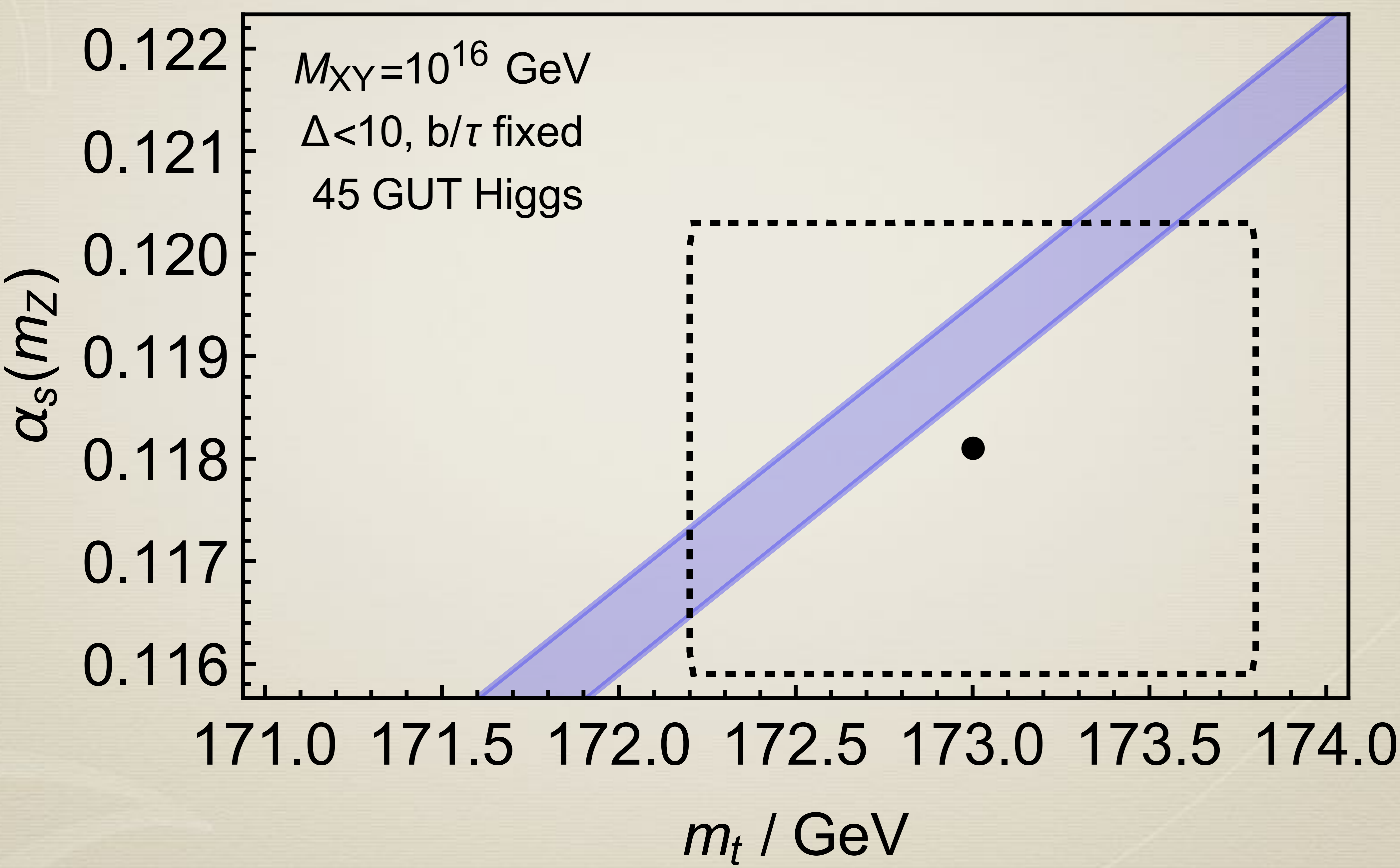
Keisuke Harigaya (UChicago)

Hall and KH :1803.08119, 1905.12722
Dunsky, Hall and KH :1902.07726, 1908.02756

Summary

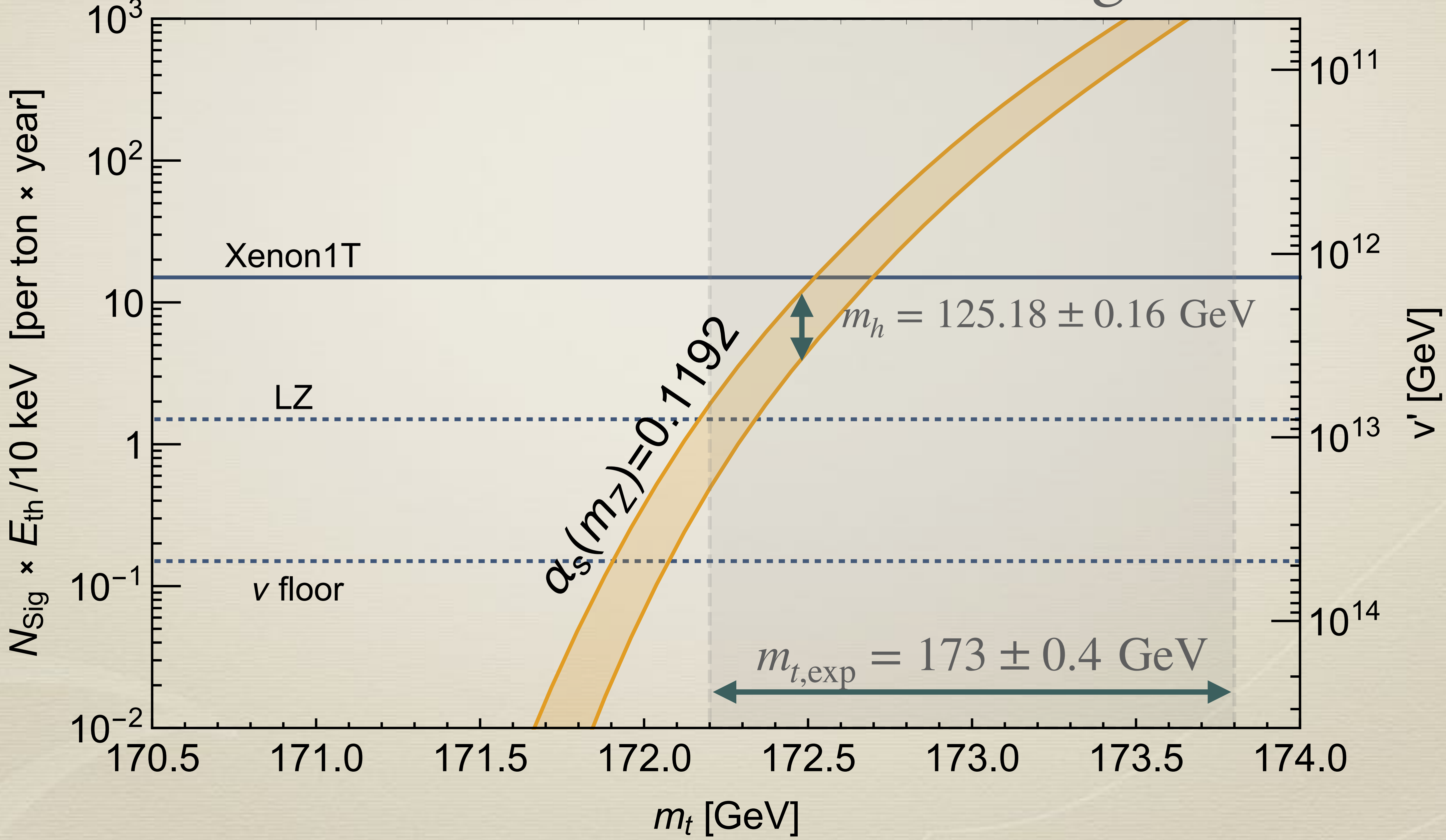
* The strong CP problem may be solved by parity symmetry
* In models with the minimal Higgs content, the SM Higgs quartic coupling vanishes at the scale where parity is spontaneously broken
* BSM signals and/or success of precise gauge coupling unification may be correlated with SM parameters

Unification



Dark matter detection

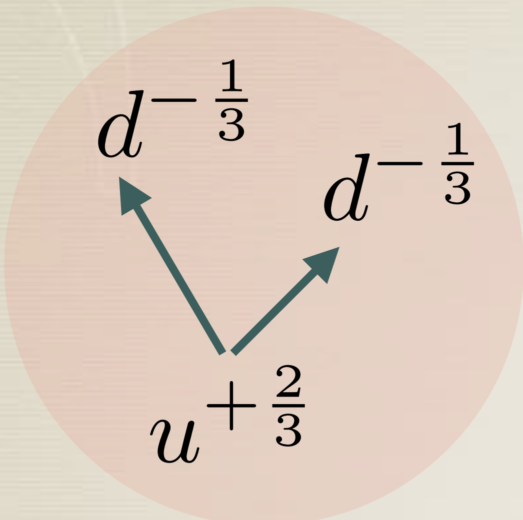
Direct detection rate for Xe targets



Outline

* Strong CP problem and solutions to it
* Parity solutions with minimal fermions or Higgses
* Parity symmetry breaking scale
* Unification
* Dark matter

The strong CP problem



Neutron Electric Dipole Moment

$$H = d_n \vec{E} \cdot \vec{S}$$

$$d_n/e \sim 0.1 \text{ fm} \sim 10^{-14} \text{ cm ?}$$

$$d_n/e < 2.9 \times 10^{-26} \text{ cm}$$

Baker et.al (2006)

Suggests CP symmetry

$$d_n \vec{E} \cdot \vec{S} \overset{\text{CP}}{\Longrightarrow} -d_n \vec{E} \cdot \vec{S} \quad \Rightarrow \quad d_n = 0$$

CP is not preserved in SM

$$\mathscr{L} = y^u_{ij} H^\dagger q_i \bar{u}_j + y^d_{ij} H q_i \bar{d}_j + \frac{\theta_{\rm QCD}}{32\pi^2} G\tilde{G}$$

CP violation

observed $\theta_{\rm CKM} = O(1)$ → $y^{u,d}$ must have O(1) complex phases

→ CP is violated

$d_n/e \simeq 5 \times 10^{-16}\bar{\theta}$ cm   Crewther, Vecchia and Witten (1979)

$$\bar{\theta} = \mathrm{argdet}(m^u m^d) + \theta_{\rm QCD} < 10^{-10}$$

The strong CP problem

Known Solutions

* QCD axion
* Spontaneously broken CP
* Spontaneously broken parity

Known Solutions

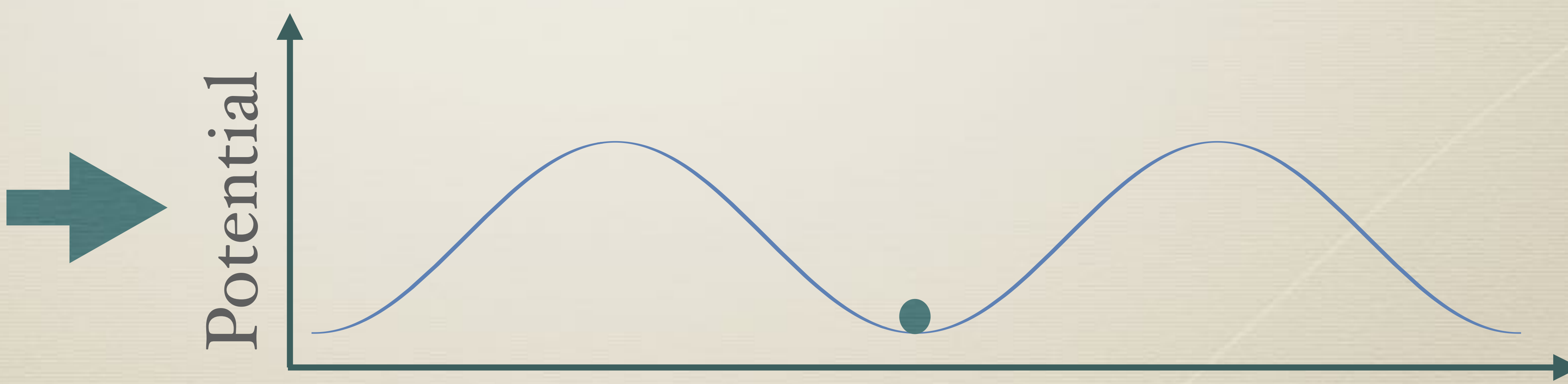
- QCD axion
- Spontaneously broken CP
- Spontaneously broken parity

Peccei and Quinn (1977)
Weinberg (1978), Wilczek (1978)

shift symmetry

$$a \to a + \delta$$

$$\frac{1}{32\pi^2}\frac{a}{f_a}G\tilde{G}$$



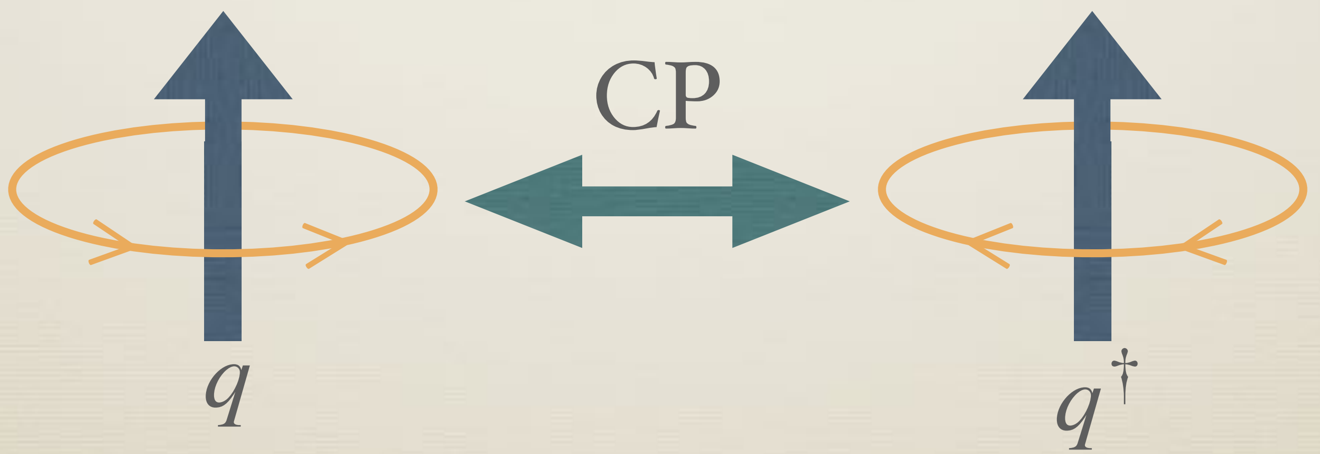
$$\theta_{\rm QCD} = 0$$

$$d_n = 0$$

Known Solutions

Nelson (1984), Barr (1984)

* QCD axion
* Spontaneously broken CP
* Spontaneously broken parity

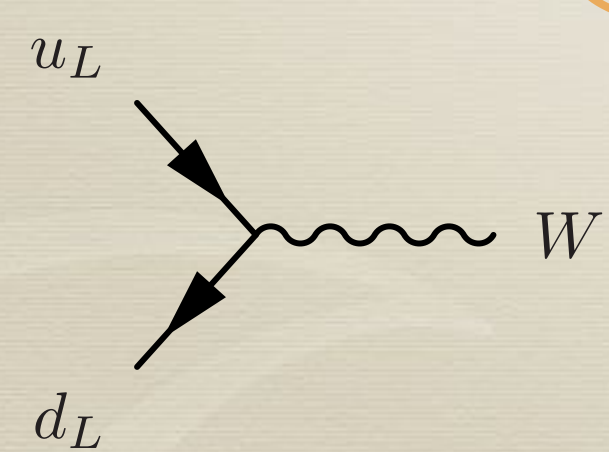


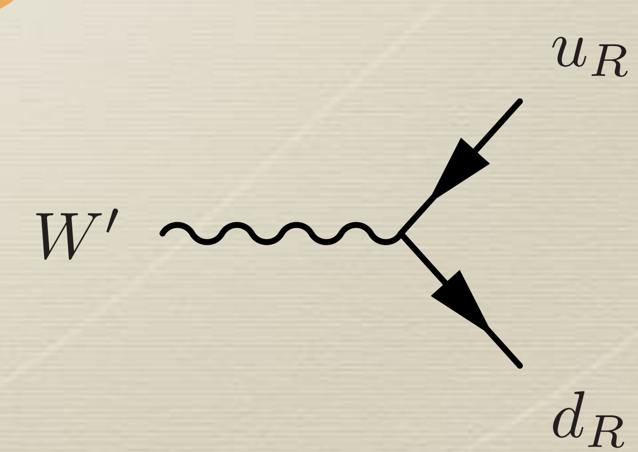
Breaks CP spontaneously to introduce CP phases in quark masses, but without introducing the strong CP phase

Known Solutions

Mohapatra and Senjanovic (1978)
Beg and Tsao (1978)
Babu and Mohapatra (1989)
Barr, Chang and Senjanovic (1991)
D'Agnolo and Hook (2015)
Hall and KH (2018,2019)

- QCD axion
- Spontaneously broken CP
- Spontaneously broken parity





(I will detail more later)

Pros and cons

See also "Solutions of the Strong CP Problem: A Scorecard" by Dine

	Pros	Cons
Axion	Dark matter, interactions are (nearly) predicted by its mass	The shift symmetry is NOT a symmetry (at the best, an accidental one)
CP	Can be a symmetry	Naively large strong CP phase without Nelson-Barr's trick No obvious experimental signatures
Parity	Can be a symmetry, connection with neutrino mass	No guaranteed experimental signatures

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CP	Can be a symmetry	Naively large strong CP phase without Nelson-Barr’s trick No obvious experimental signatures
Parity	Can be a symmetry, connection with neutrino mass Experimental signatures?	~~No guaranteed experimental signatures~~

Outline

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Minimal fermion model

Mohapatra and Senjanovic (1978)
Beg and Tsao (1978)

$SU(2)_L$		$SU(2)_R$
$q = (u, d)$	Parity	$\bar{q} = (\bar{u}, \bar{d})$
$\ell = (\nu, e)$	$\leftrightarrow$	$\bar{\ell} = (N, \bar{e})$

$$q(t, x) \leftrightarrow i\sigma_2 \bar{q}^*(t, -x)$$

Higgses

$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{\mathrm{B-L}}$$

$T_R(1,1,3,1)$ parity $T_L(1,3,1,1)$

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

$\Phi(1,2,2,0)$ parity Φ^*

$$SU(3)_c \times U(1)_{\mathrm{EM}}$$

Yukawa couplings

Let us concentrate on quarks

$(SU(3)_c, SU(2)_L, SU(2)_R, U(1)_{\mathrm{B-L}})$

$$q(3,2,1,1/6) \quad \Phi(1,2,2,0) \quad \bar{q}(\bar{3},1,2,-1/6)$$

$$y_{ij} q_i \Phi \bar{q}_j + y'_{ij} q_i \Phi^* \bar{q}_j$$

Parity

$$+ y^*_{ij} q^*_i \Phi^* \bar{q}^*_j + y'^*_{ij} q^*_i \Phi \bar{q}^*_j$$

$y = y^\dagger$ because of parity symmetry, $\det(y)$, $\det(y')$ are real

The strong CP problem is solved??

Wait, the phases of Higgs vev?

Phase of Higgs VEV

Most of the parameters of the Higgs potential
are real because of Hermiticity and parity

$\Phi \leftrightarrow \Phi^*$

$$|\Phi|^4,\ |\Phi|^2,\ \Phi^2 + \Phi^{*2}$$

However, we must introduce
$SU(2)_R$ symmetry breaking field T_R and its parity partner T_L

$$e^{i\alpha}|T_R|^2\Phi^2 + e^{i\alpha}|T_L|^2\Phi^{*2} + \mathrm{h.c.}$$

$T_R \gg T_L$

$$\rightarrow\ e^{i\alpha}\Phi^2 + e^{-i\alpha}\Phi^{*2}$$

complex Higgs VEV!

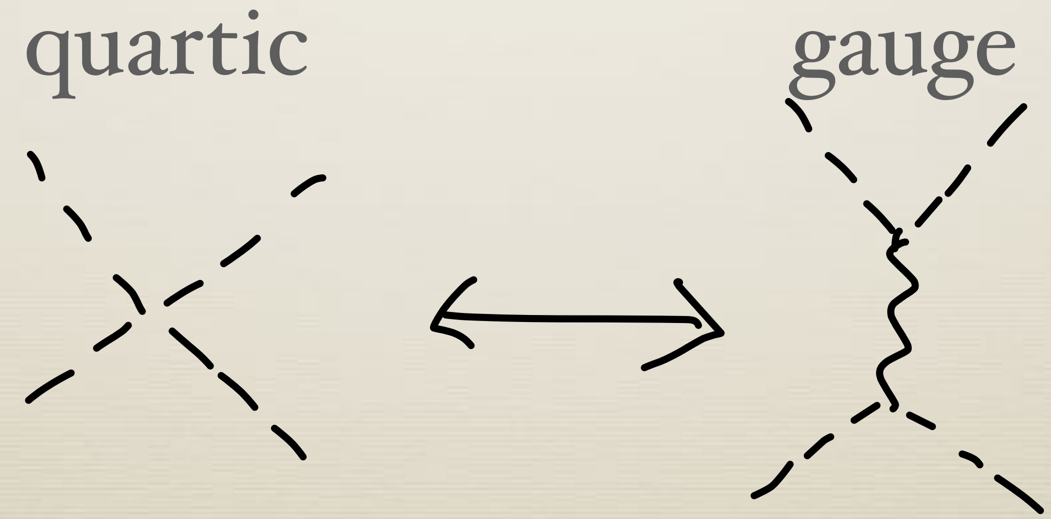
Way out

$$e^{i\alpha}|T_R|^2\Phi^2 + e^{i\alpha}|T_L|^2\Phi^{*2}$$
$$+e^{-i\alpha}|T_R|^2\Phi^{*2} + e^{-i\alpha}|T_L|^2\Phi^2$$

We must forbid these quartic couplings

Ex. supersymmetry

Kuchimanchi (1995), Mohapatra and Rasin (1995)



Minimal Higgs model

Babu and Mohapatra(1989)

$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{\mathrm{B-L}}$$

$$H_R(1,1,2,-1/2)$$

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

parity

$$H_L(1,2,1,1/2)$$

$$SU(3)_c \times U(1)_{\mathrm{EM}}$$

Fermion sector?

$SU(2)_L$

$q = (u, d)$

$(3,2,1,1/6)$

$\ell = (\nu, e)$

$(1,2,1,-1/2)$

Parity

$SU(2)_R$

$\bar{q} = (\bar{u}, \bar{d})$

$(\bar{3},1,2,-1/6)$

$\bar{\ell} = (N, \bar{e})$

$(1,1,2,1/2)$

But yukawa couplings are forbidden

~~$q\bar{q}H_L$~~

Yukawa couplings

Babu and Mohapatra(1989)

Introduce $SU(2)_L \times SU(2)_R$ singlet, Dirac fermions $U, \bar{U}, D, \bar{D}$

$$y_{ij} q_i H_L \bar{U}_j + y_{ij}^* \bar{q}_i H_R U_j + M_{ij} U_i \bar{U}_j$$

$M \gg y v_R$ → $\frac{y^2}{M} q \bar{q} H_L H_R$

right-handed quarks $\simeq \bar{q}$

$M \ll y v_R$ → $y_{ij} q_i \bar{U}_j H_L$

right-handed quarks $\simeq \bar{U}$

Yukawa couplings

Babu and Mohapatra(1989)

$$y_{ij} q_i H_L \bar{U}_j + y_{ij}^* \bar{q}_i H_R U_j + M_{ij} U_i \bar{U}_j$$

$$\begin{pmatrix} q & U \end{pmatrix} \begin{pmatrix} 0 & y v_L \\ y^\dagger v_R & M \end{pmatrix} \begin{pmatrix} \bar{q} \\ \bar{U} \end{pmatrix}$$

$\det(m_u) \propto \det(y y^\dagger)$ is real

Strong CP problem is solved!

(Quantum corrections are found to be small enough)

Hall, KH (2018)

(When embedded into SO(10) GUT, up/down, bottom/tau, and neutrino masses can be explained without complexing GUT scale Higgs)

Hall, KH (2019)

Mirror variant

Barr, Chang and Senjanovic (1991)

SM particles		New particles
q	strong interaction	q'
u	$SU(3)_c$	u'
d		d'
L	parity	L'
e	⟷	e'
H		H'

	electroweak	electroweak'	
W, Z	$SU(2)_L \times U(1)_Y$	$SU(2)'_L \times U(1)'_Y$	W', Z'
	$\downarrow$ H	H' $\downarrow$	
γ	$U(1)_{\rm EM}$	$U(1)'_{\rm EM}$	γ'

Yukawa couplings

$$y_{ij} q_i H_L \bar{U}_j + y^*_{ij} \bar{q}_i H_R U_j + \cancel{M_{ij} U_i \bar{U}_j}$$

$$U(1)_Y \times U(1)'_Y$$

$$\begin{pmatrix} q & U \end{pmatrix} \begin{pmatrix} 0 & y v_L \\ y^\dagger v_R & \cancel{M} \end{pmatrix} \begin{pmatrix} \bar{q} \\ \bar{U} \end{pmatrix}$$

$\det(m_u) \propto \det(y y^\dagger)$ is real

Strong CP problem is solved!

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Minimal Higgs model

$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{\mathrm{B-L}}$$

$\downarrow$ $H_R(1,1,2,-1/2)$

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

$\updownarrow$ parity

$\downarrow$ $H_L(1,2,1,1/2)$

$$SU(3)_c \times U(1)_{\mathrm{EM}}$$

How to obtain $v_R \gg v_L$ despite the parity symmetry?

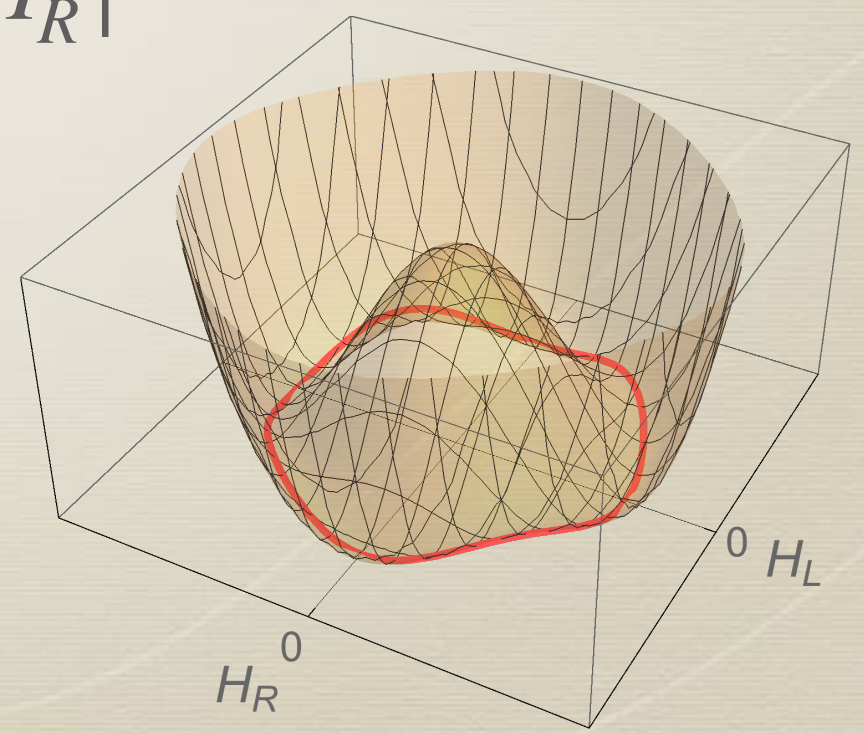
Higgs potential

$$V = \left(\lambda |H_L|^4 - m^2 |H_L|^2\right) + \left(\lambda |H_R|^4 - m^2 |H_R|^2\right) + \tilde{y} |H_L|^2 |H_R|^2$$

$$= \lambda \left(|H_L|^2 + |H_R|^2 - v'^2 \right)^2 + y |H_L|^2 |H_R|^2$$

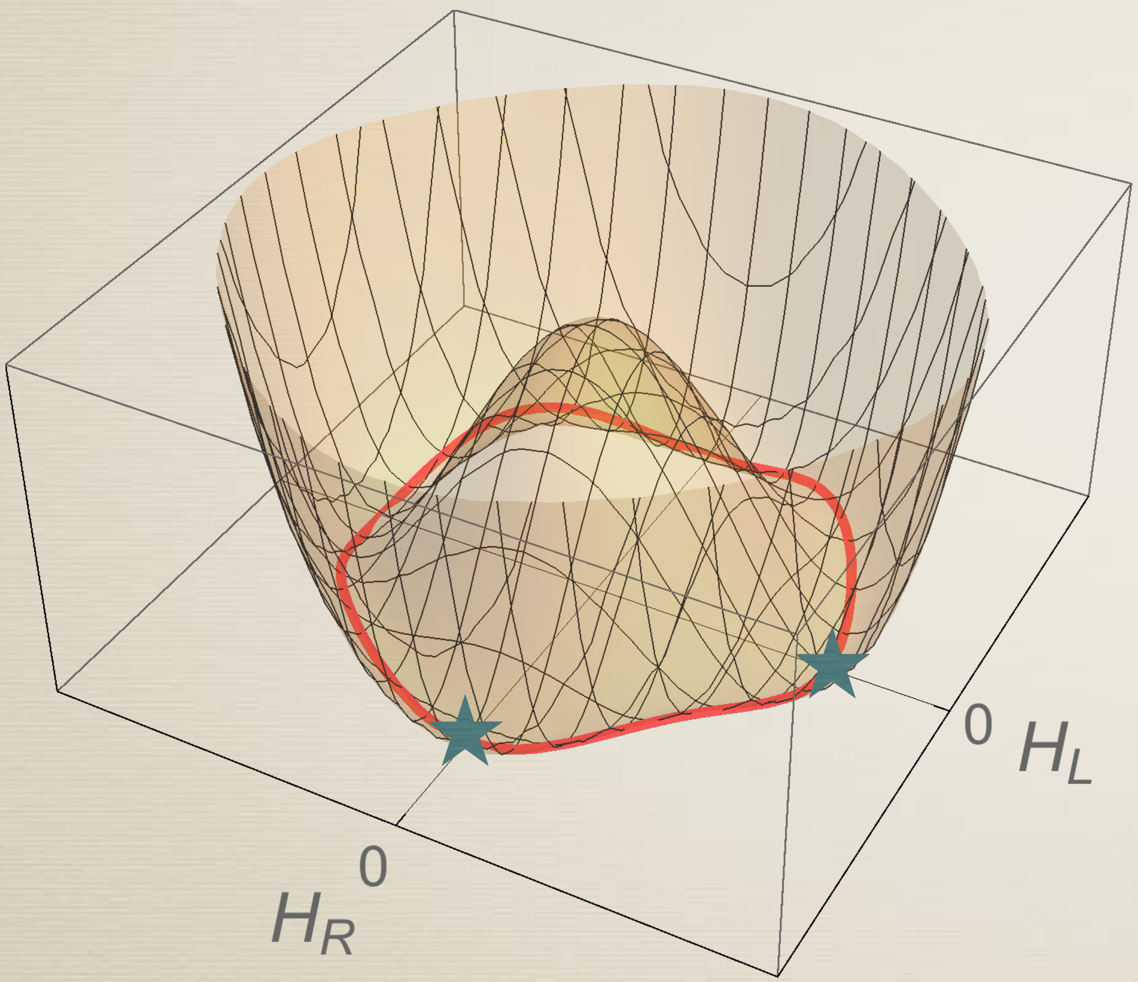
Can we find the minimum with

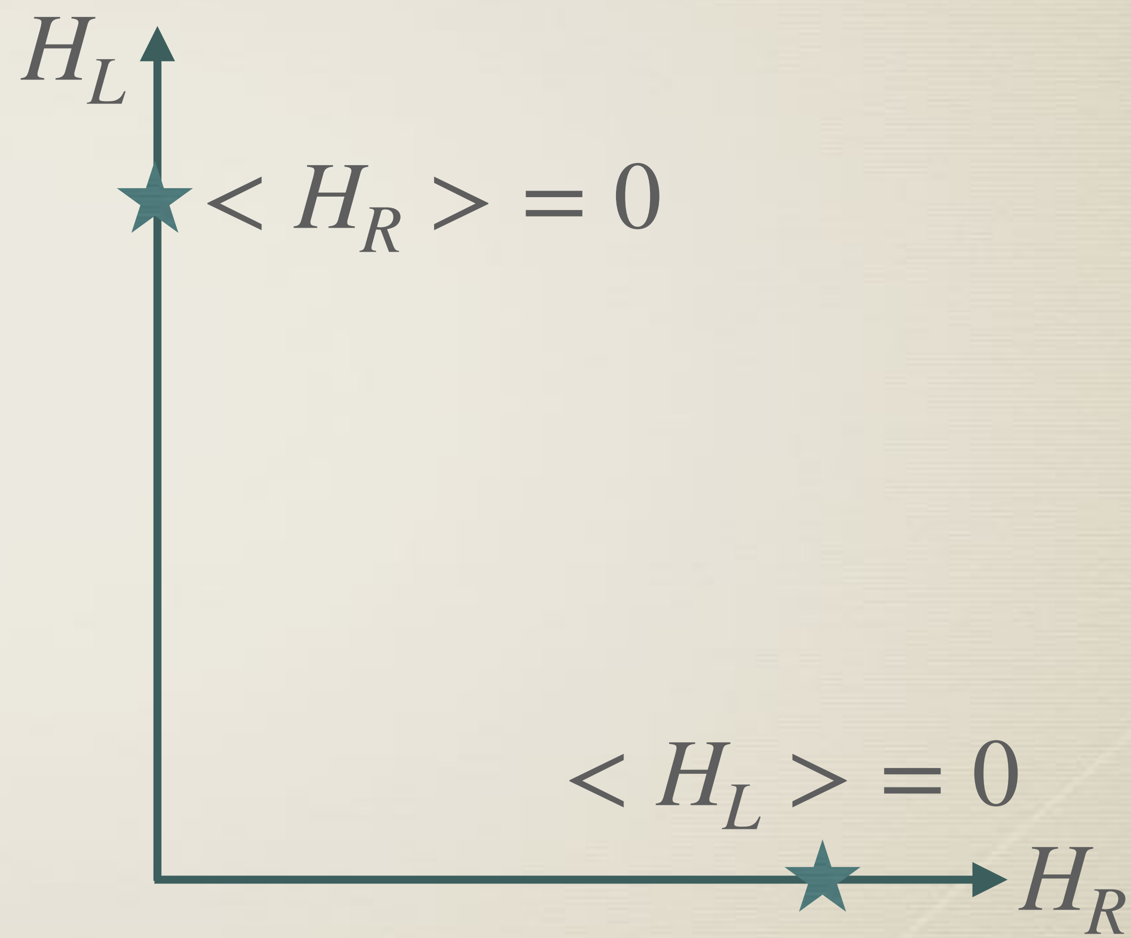
$$\langle H_L \rangle \ll \langle H_R \rangle \quad ?$$



$y > 0$

$$V = \lambda(|H_L|^2 + |H_R|^2 - v'^2)^2 + y|H_L|^2|H_R|^2$$

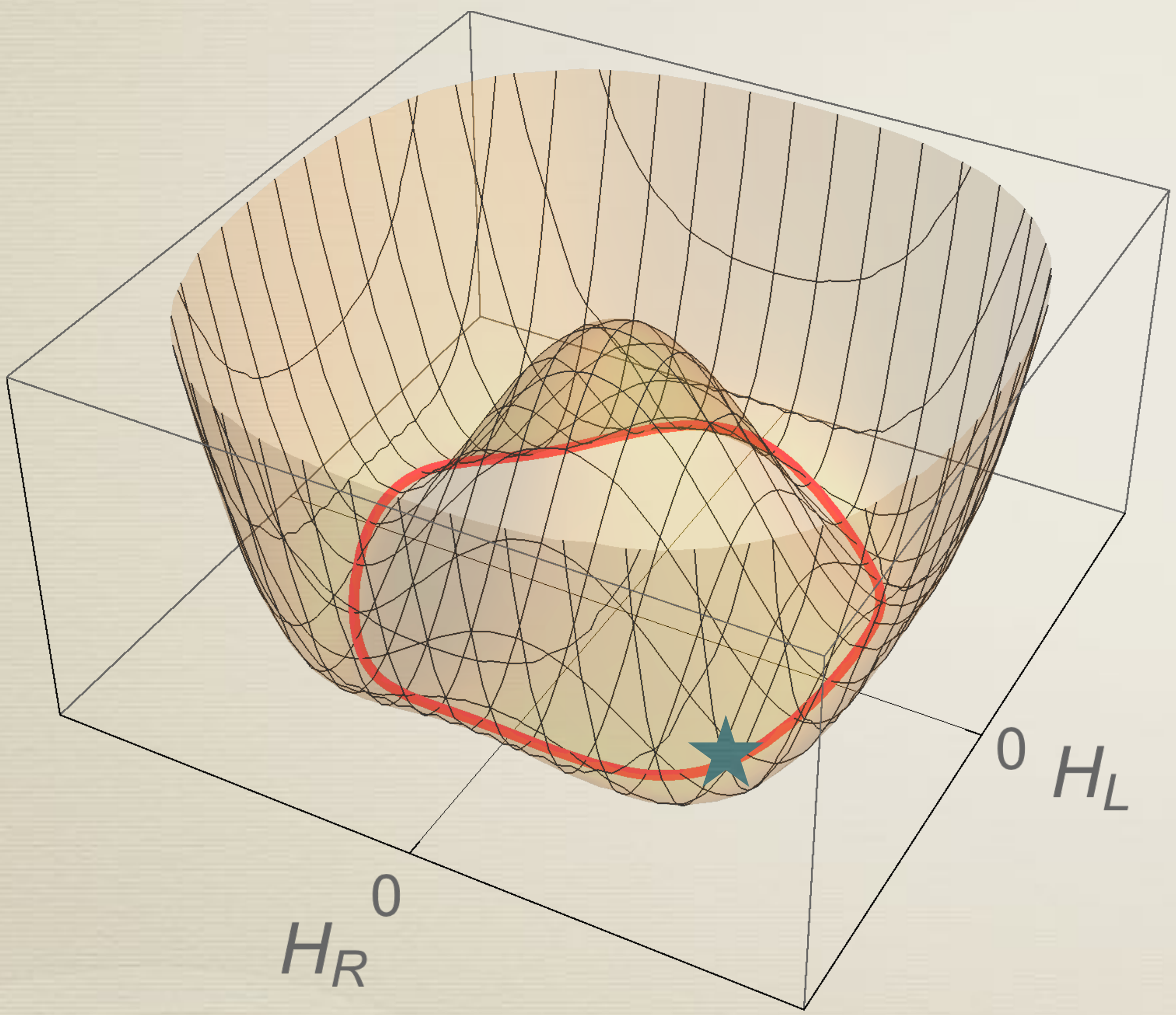


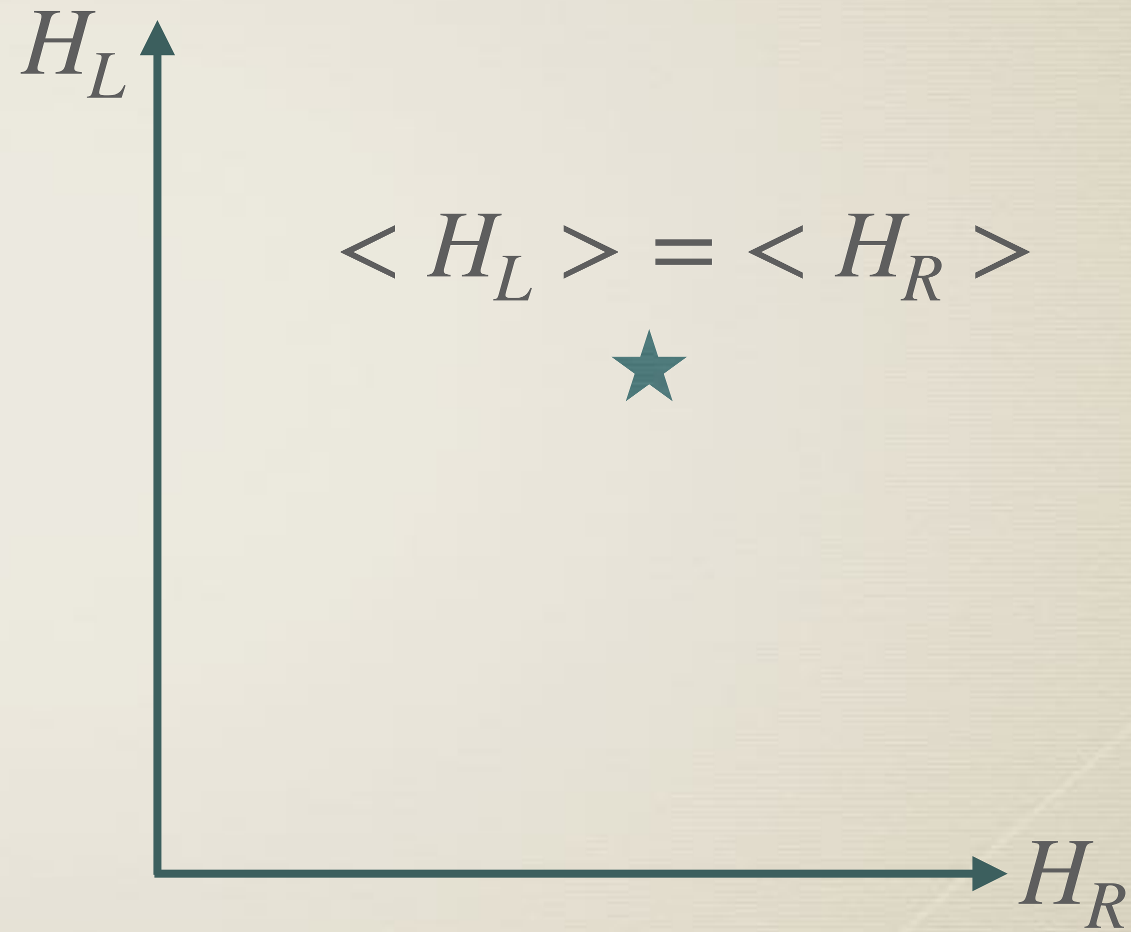


~~$0 \neq \langle H_L \rangle \ll \langle H_R \rangle$~~

$y < 0$

$$V = \lambda(|H_L|^2 + |H_R|^2 - v'^2)^2 + y|H_L|^2|H_R|^2$$





~~$\langle H_L \rangle \lll \langle H_R \rangle$~~

Soft breaking ?

Babu and Mohapatra(1989)

$$V = \lambda(|H_L|^2 + |H_R|^2 - v'^2)^2 + y|H_L|^2|H_R|^2$$

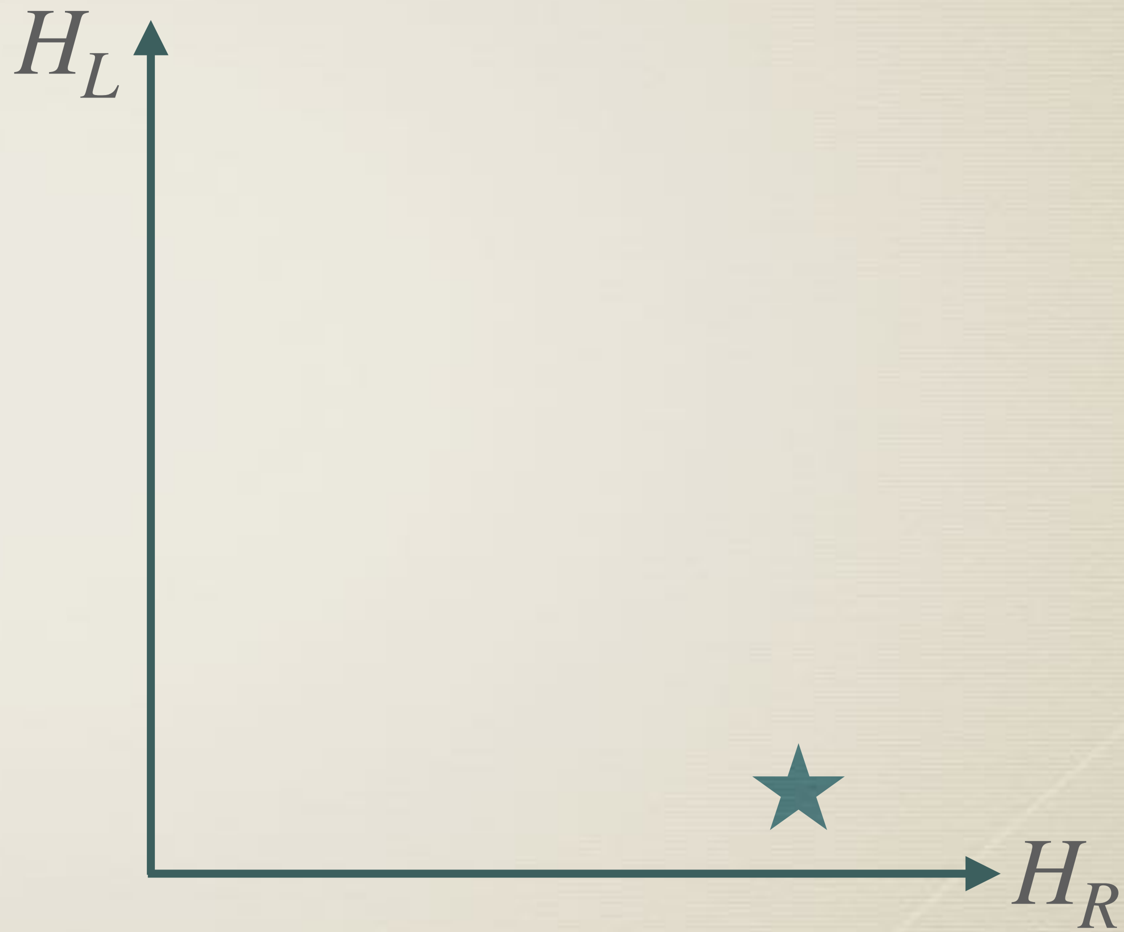
$$+\Delta m^2(|H_L|^2 - |H_R|^2)$$

$$< \mathcal{O} > \neq 0$$

$$\mathcal{O}(|H_L|^2 - |H_R|^2)$$

Parity

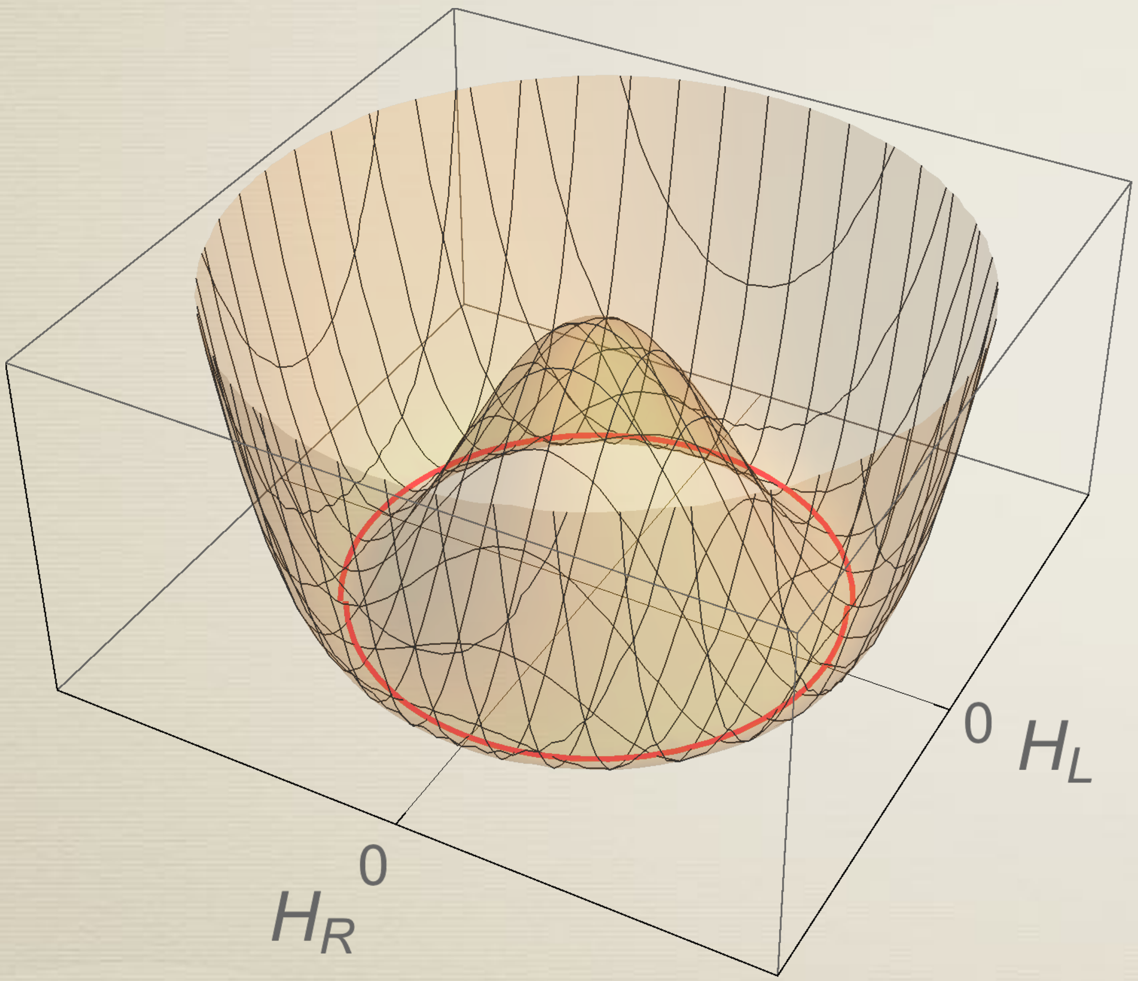
$$\mathcal{O} \longleftrightarrow -\mathcal{O}$$

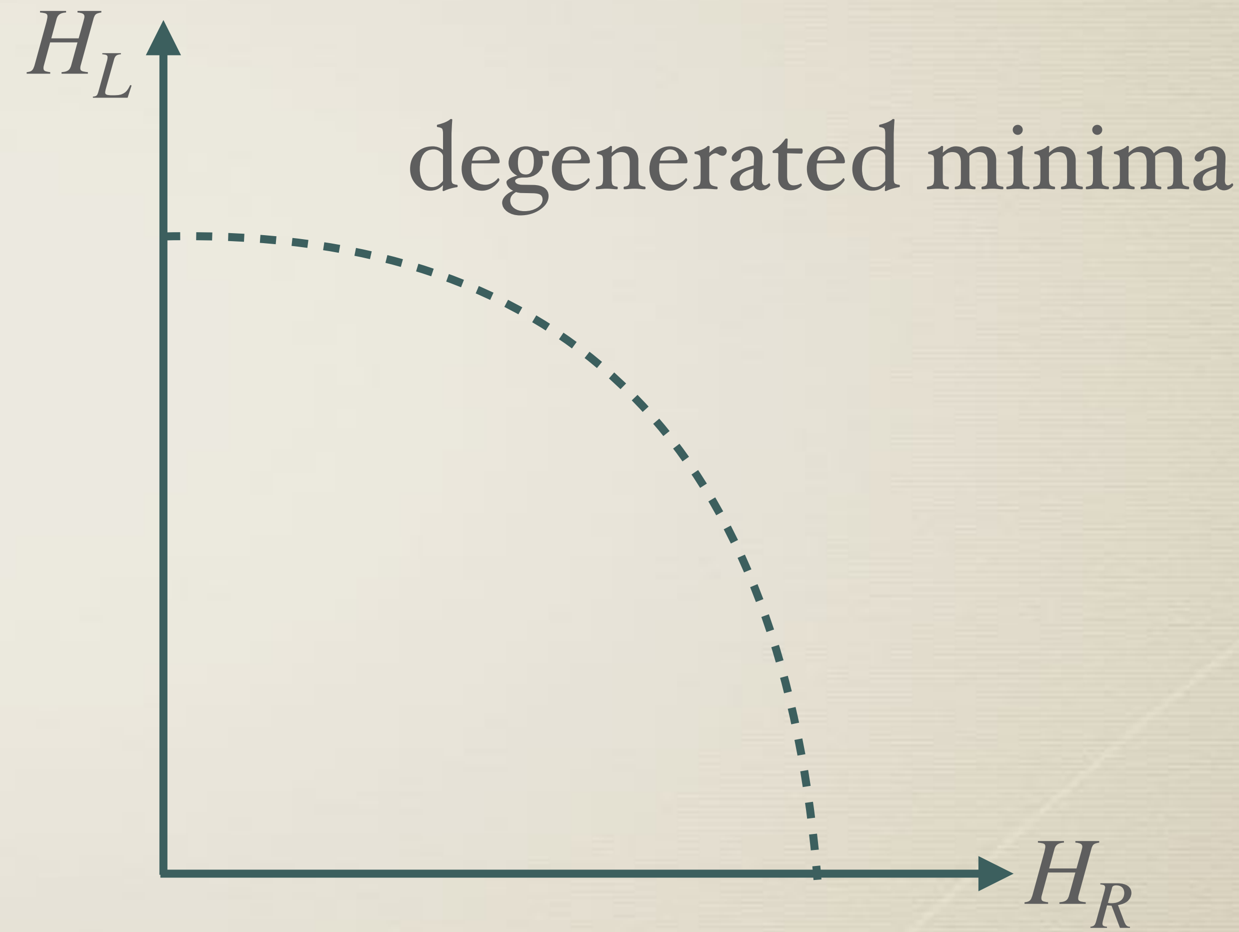


I will introduce more minimal scenario

$y \simeq 0$

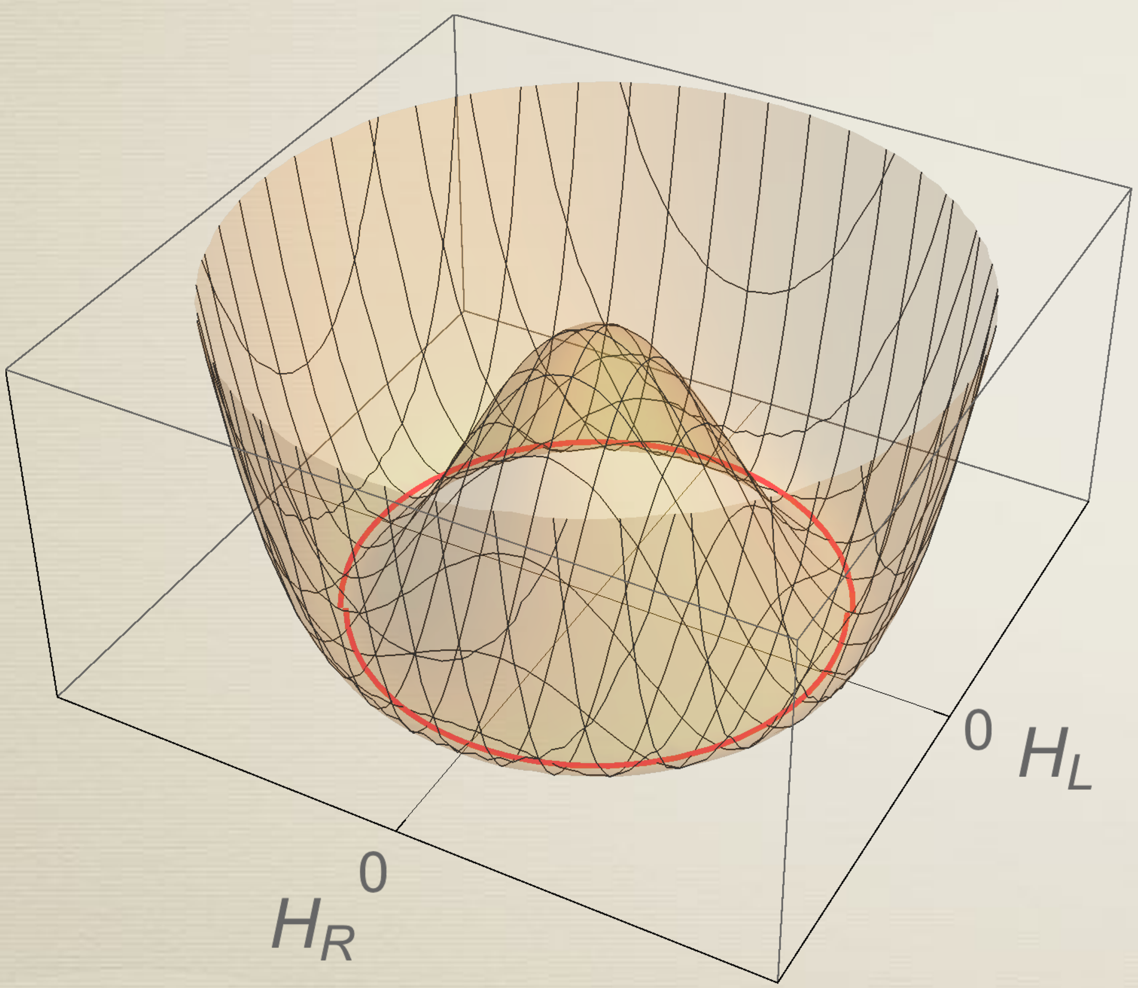
$$V = \lambda(|H_L|^2 + |H_R|^2 - v'^2)^2 + y|H_L|^2|H_R|^2$$

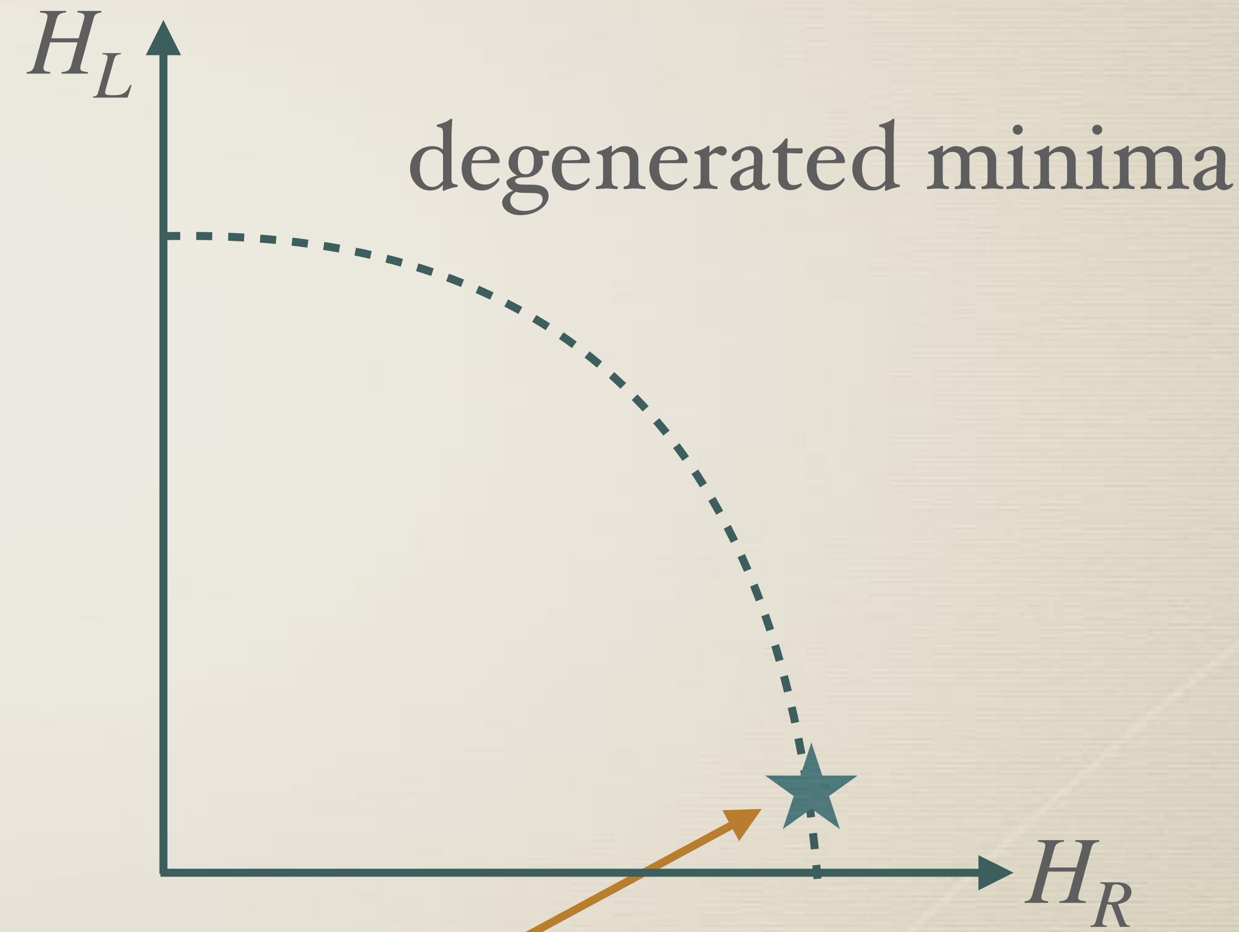




$y \simeq 0$

$$V = \lambda(|H_L|^2 + |H_R|^2 - v'^2)^2 + \sout{y|H_L|^2|H_R|^2}$$



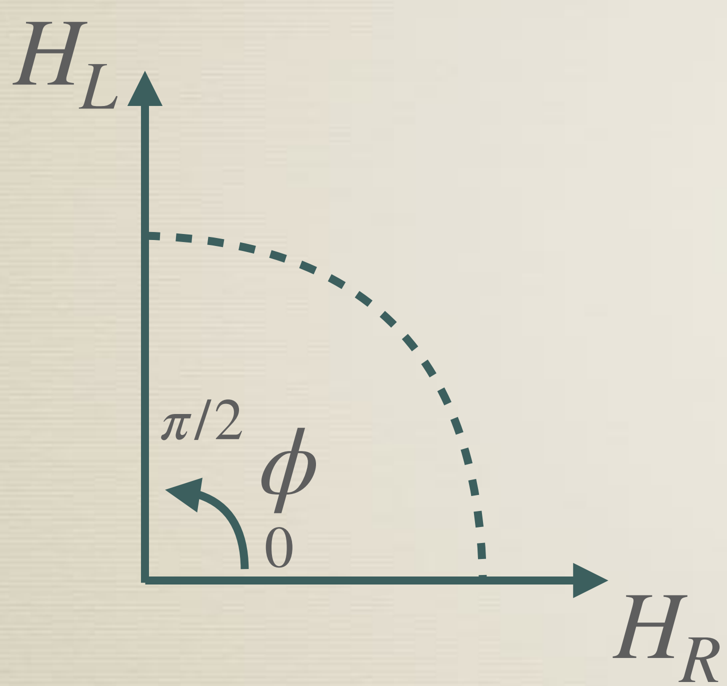


$$\langle H_L \rangle \ll \langle H_R \rangle ?$$

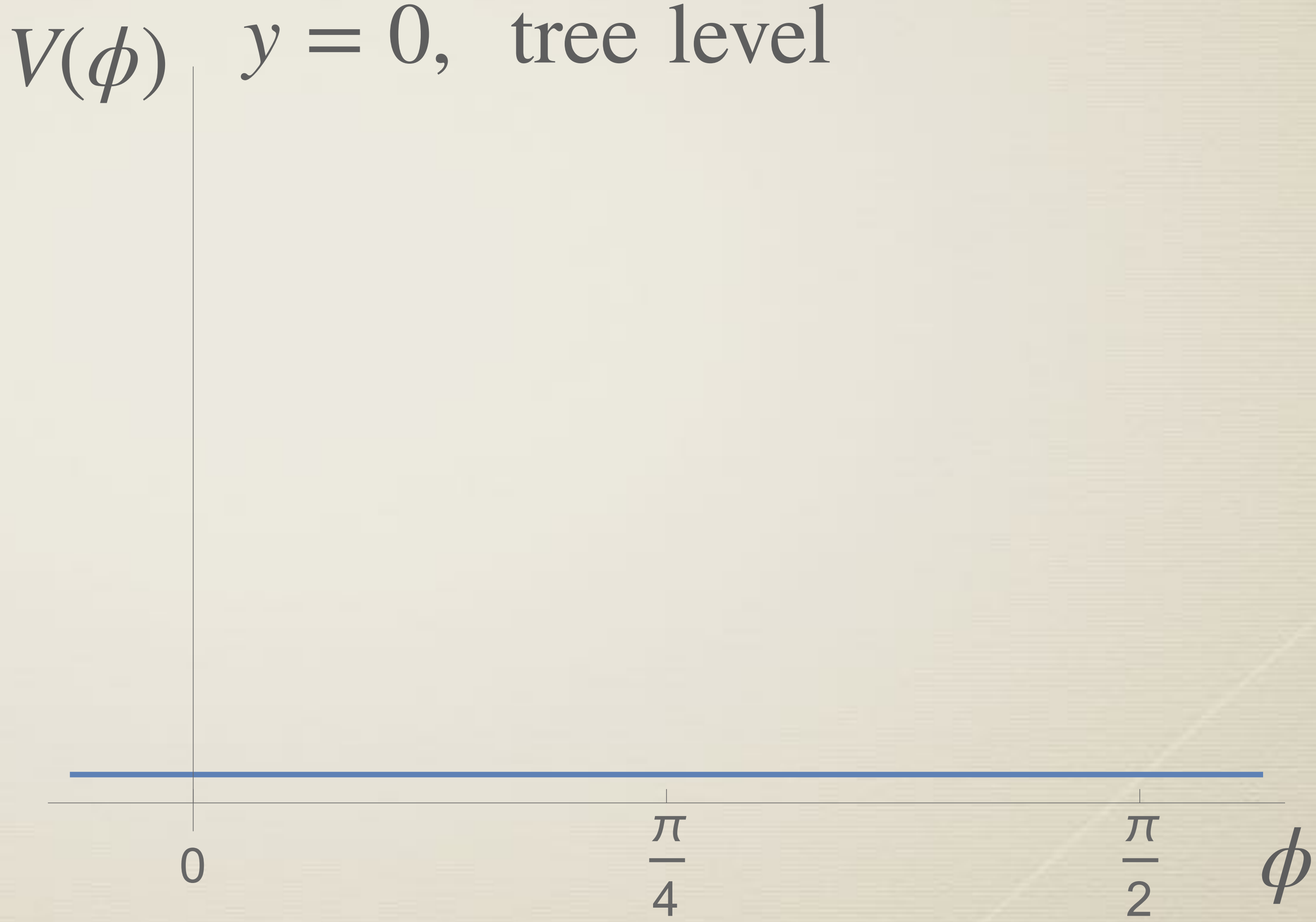
Degeneracy is resolved by quantum corrections

$y \simeq 0$

$$V = \lambda(|H_L|^2 + |H_R|^2 - v'^2)^2$$



angular direction ϕ

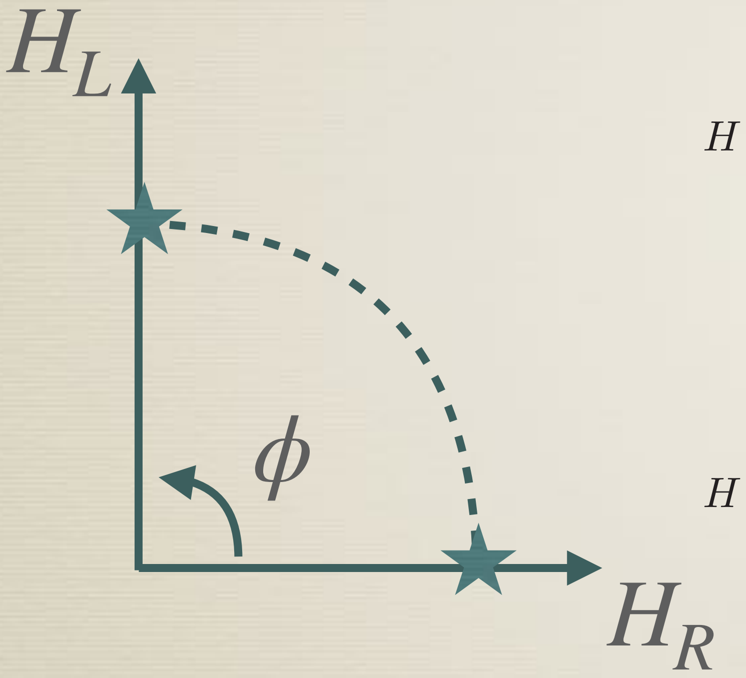


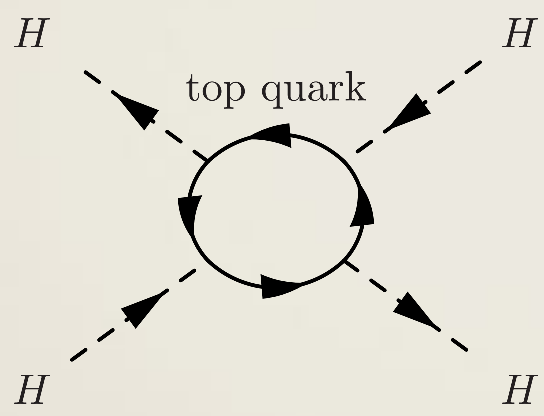
$$y \simeq 0$$

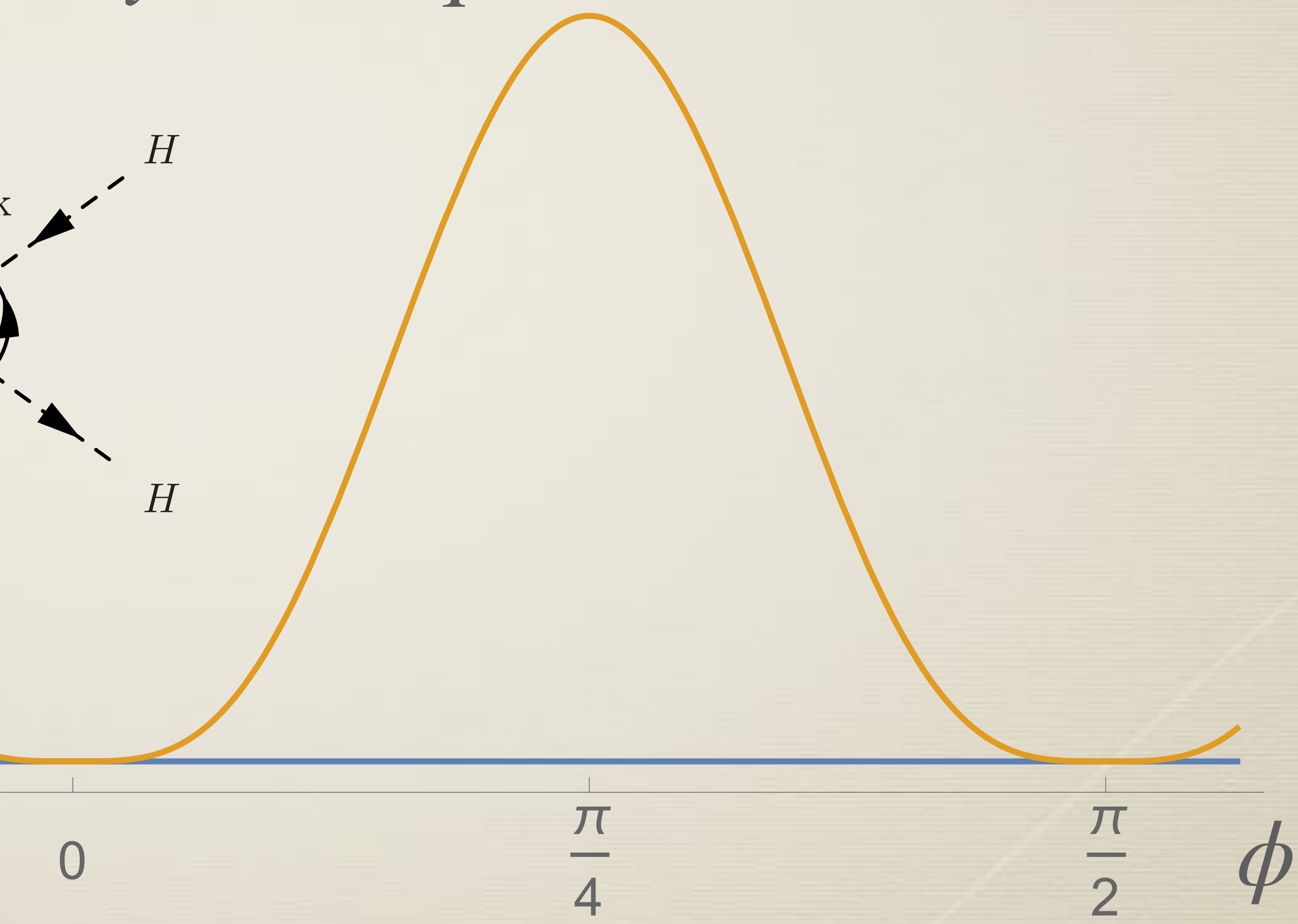
Colemann-Weinberg potential

$$V = \lambda(|H_L|^2 + |H_R|^2 - v'^2)^2 + V_{\text{quantum}}(H_L, H_R)$$

$y = 0,\ \text{quantum correction}$





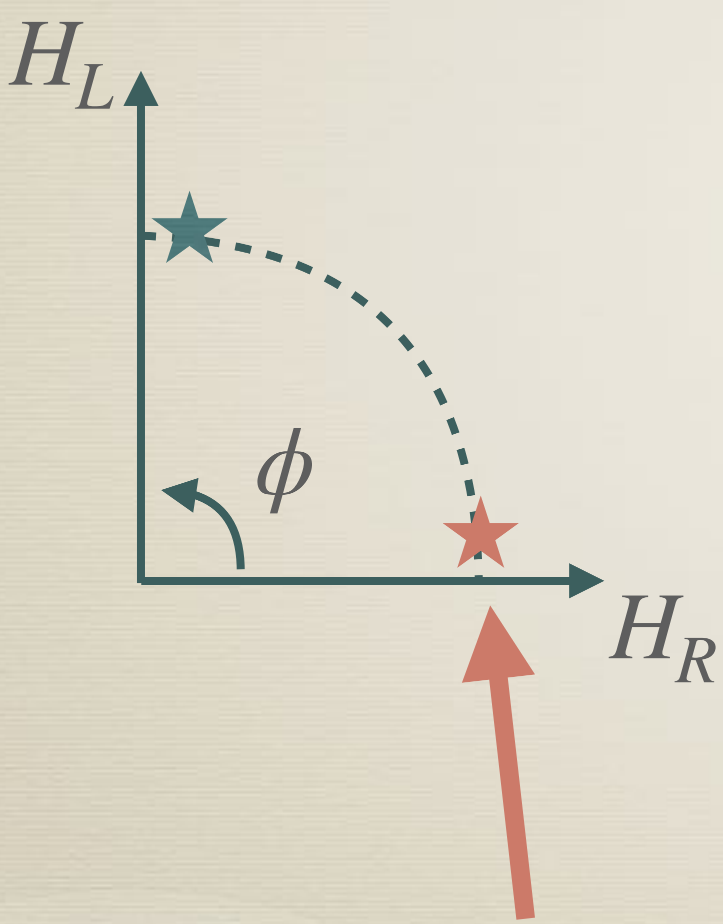


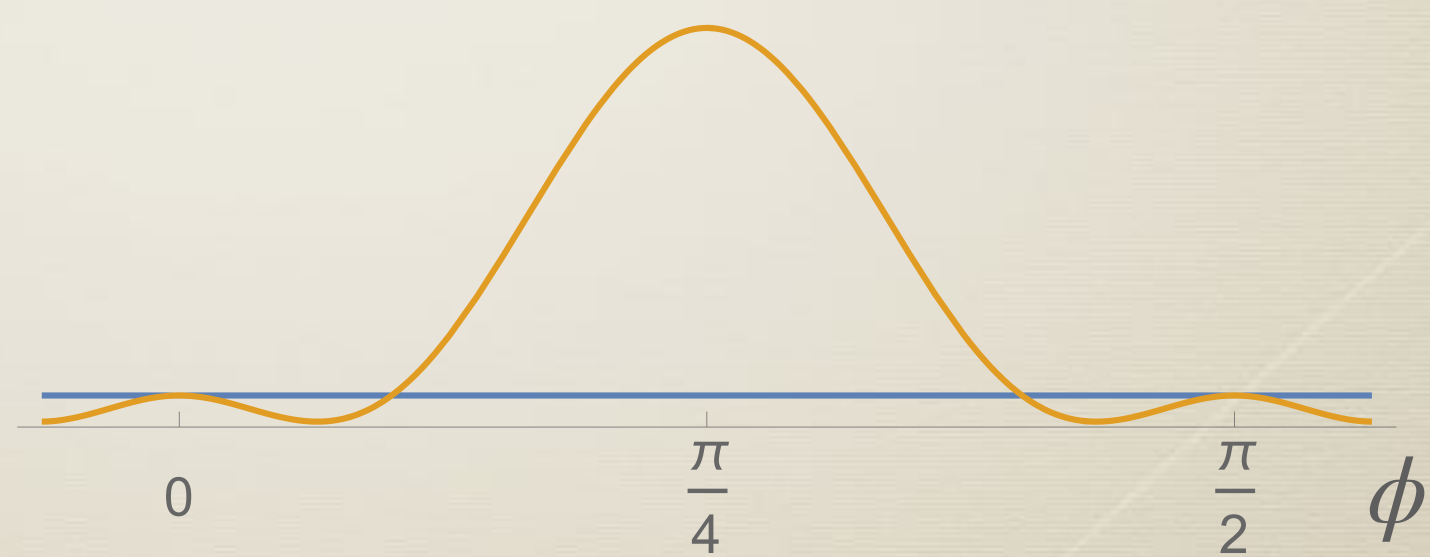
$y \simeq 0$

$$V = \lambda(|H_L|^2 + |H_R|^2 - v'^2)^2 + V_{\text{quantum}}(H_L, H_R) + y|H_L|^2|H_R|^2$$

$$y \simeq -\frac{v^2}{v'^2}, \quad \text{quantum correction}$$

(fine-tuned Higgs mass)





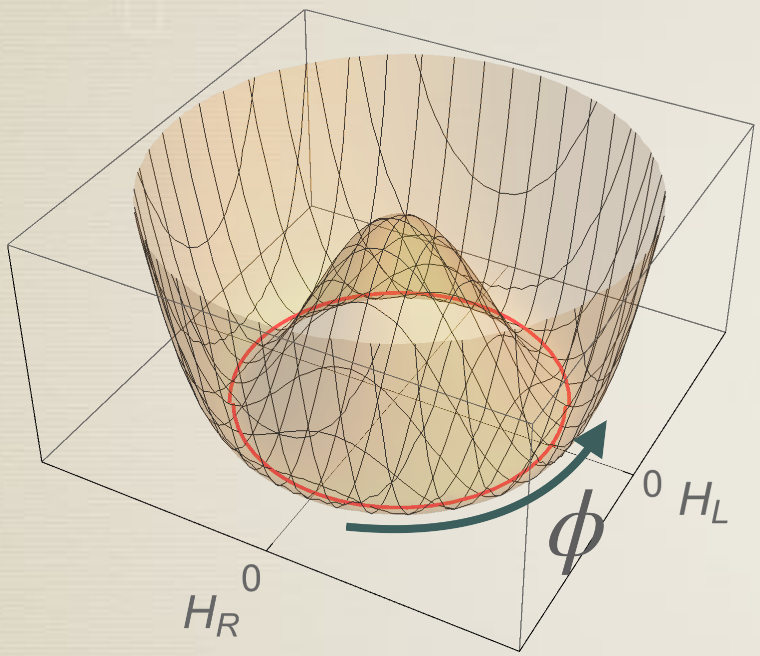
$\langle H_L \rangle \ll \langle H_R \rangle$ is achieved !

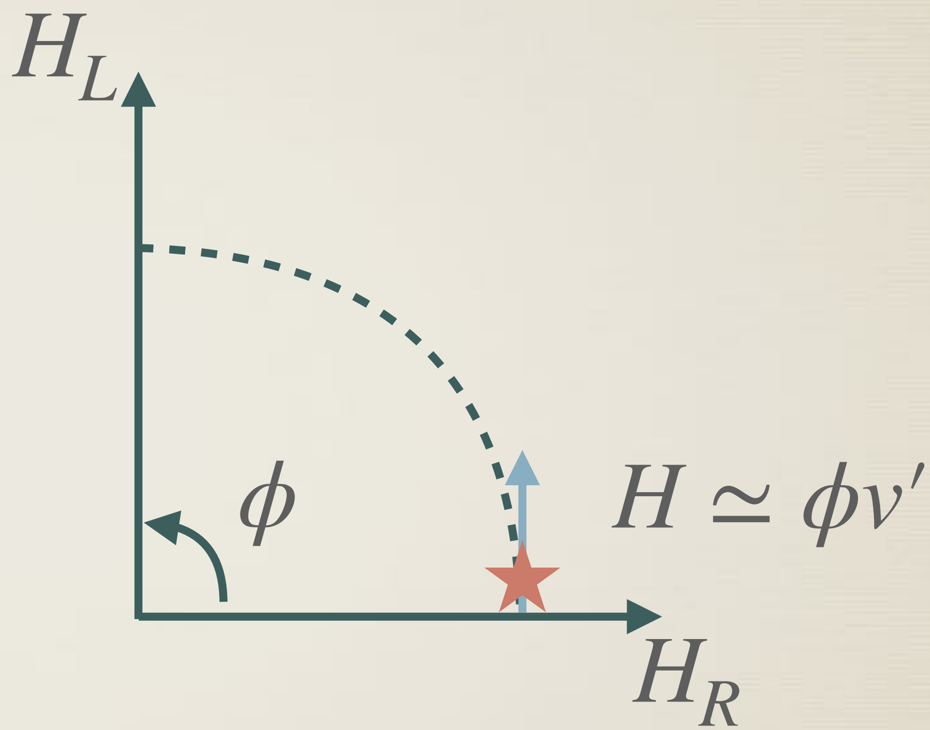
Hall, KH (2018)

Prediction on the quartic coupling

Hall, KH (2018)

$V \simeq \lambda(|H_L|^2 + |H_R|^2 - v'^2)^2$ + small corrections





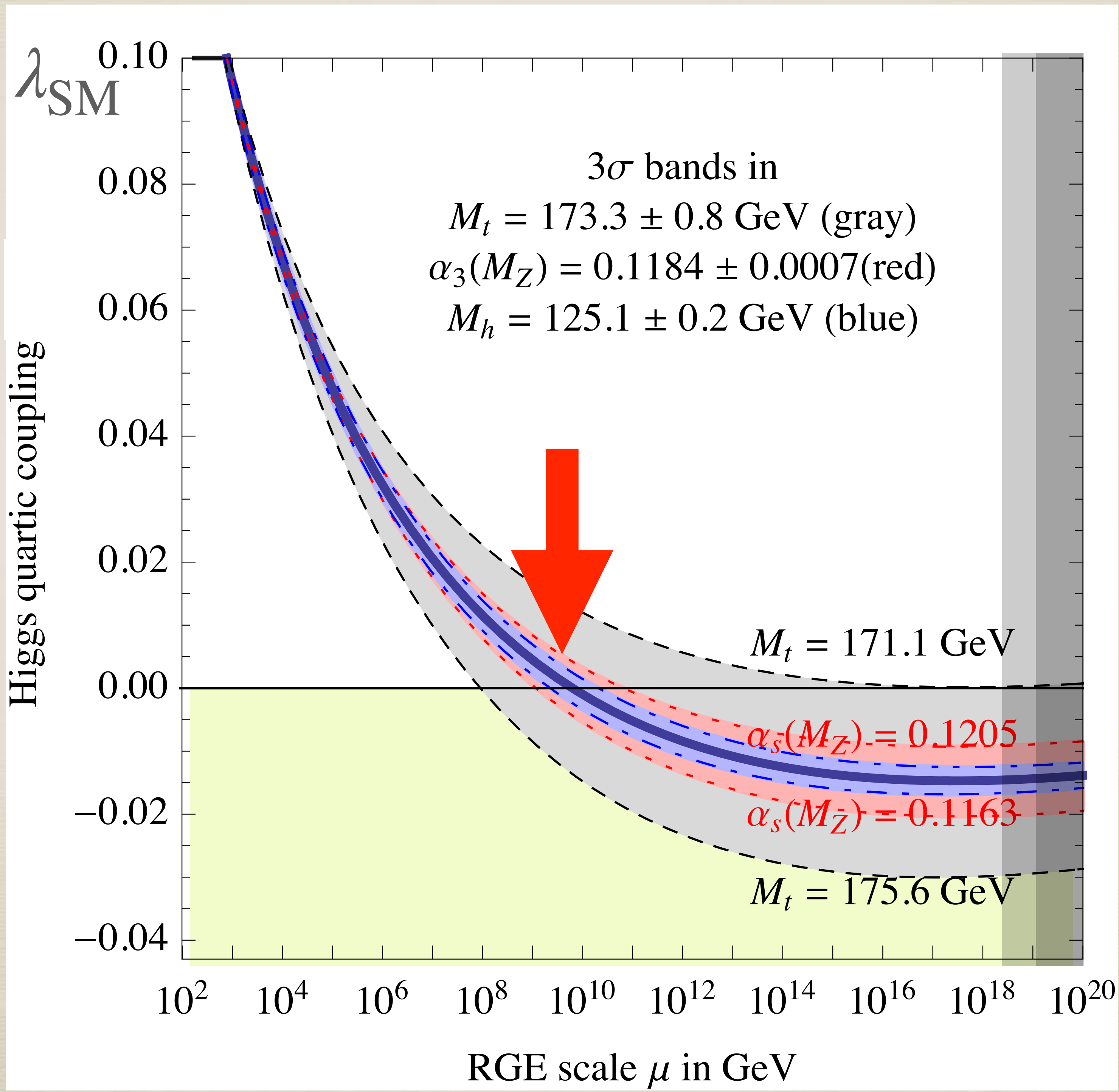
symmetry rotating the vector (H_L, H_R)

Standard Model Higgs is a (pseudo) Nambu-Goldstone boson associated with symmetry breaking by $\langle H_R \rangle = v' = v_R$

$$\lambda_{SM}(v_R) \simeq 0$$

(up to calculable threshold correction)

Parity breaking scale



Buttazzo et.al (2013)

Fine-tuning

$$V = \lambda \left(|H_L|^2 + |H_R|^2 - {v_R}^2 \right)^2 + y |H_L|^2 |H_R|^2$$

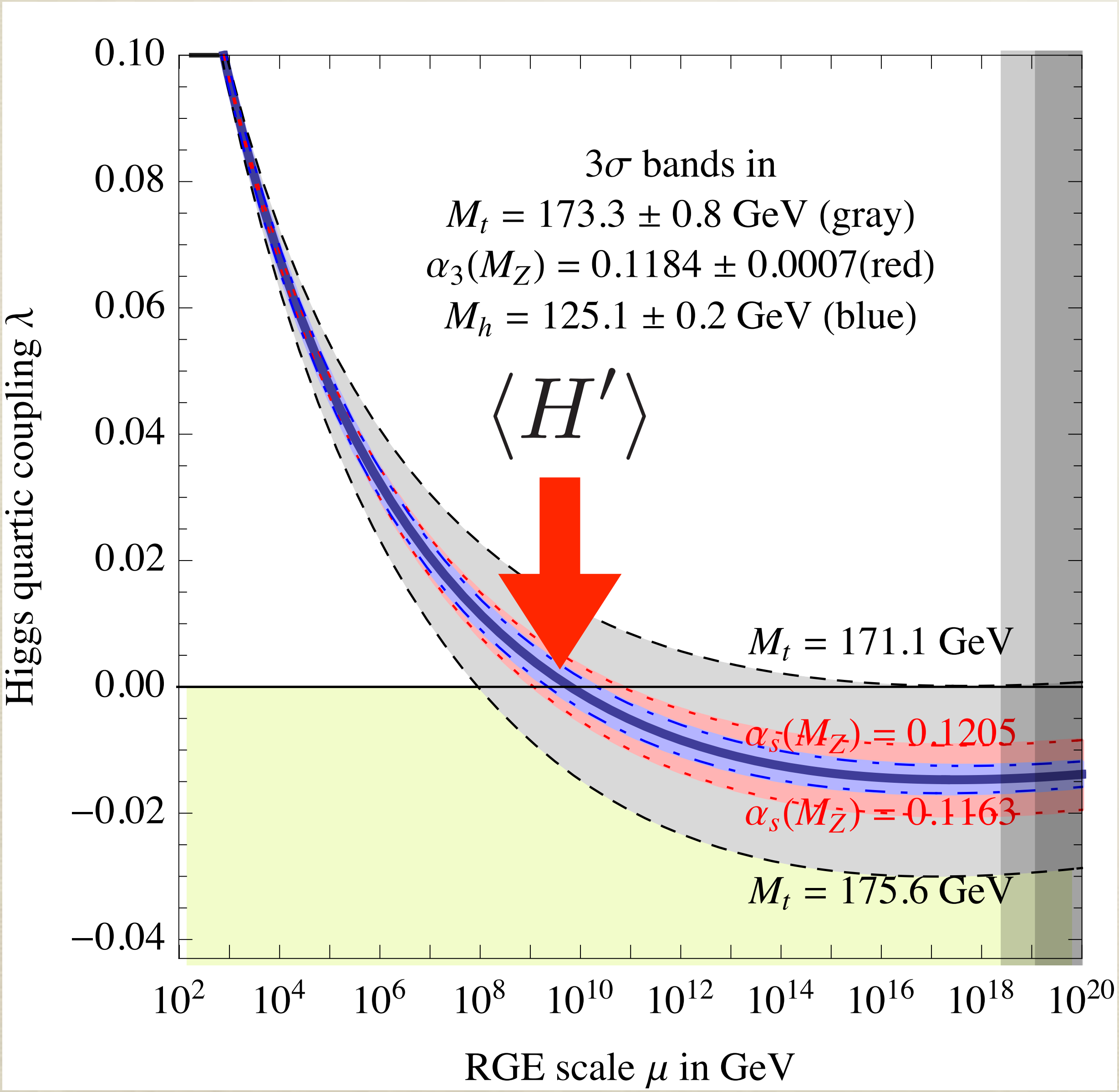
$$\frac{{v_R}^2}{\Lambda^2_{\mathrm{cut}}} \times \frac{v^2}{{v_R}^2} = \frac{v^2}{\Lambda^2_{\mathrm{cut}}}$$

Despite the intermediate scale v_R,

same as that of standard model

Generalization

The scheme is applicable to generic models with the SM Higgs H and its Z2 partner H'



Higgs Parity

Left-Right symmetry in SO(10) or Pati-Salam, Mirror Parity, ...

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Left-Right symmetry and Grand Unified Theory

Grand Unification

$SO(10)$ Fritzsch and Minkowski (1975), Georgi (1975)

Left-Right Symmetry

$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)$

$\langle H_R \rangle$

Standard Model

$SU(3)_c \times SU(2)_L \times U(1)_Y$

ψ_{16}

Single gauge group,
single type of chiral fermions

(+ vector-like multiplets)

$H_L, H_R \subset \mathbf{16}$

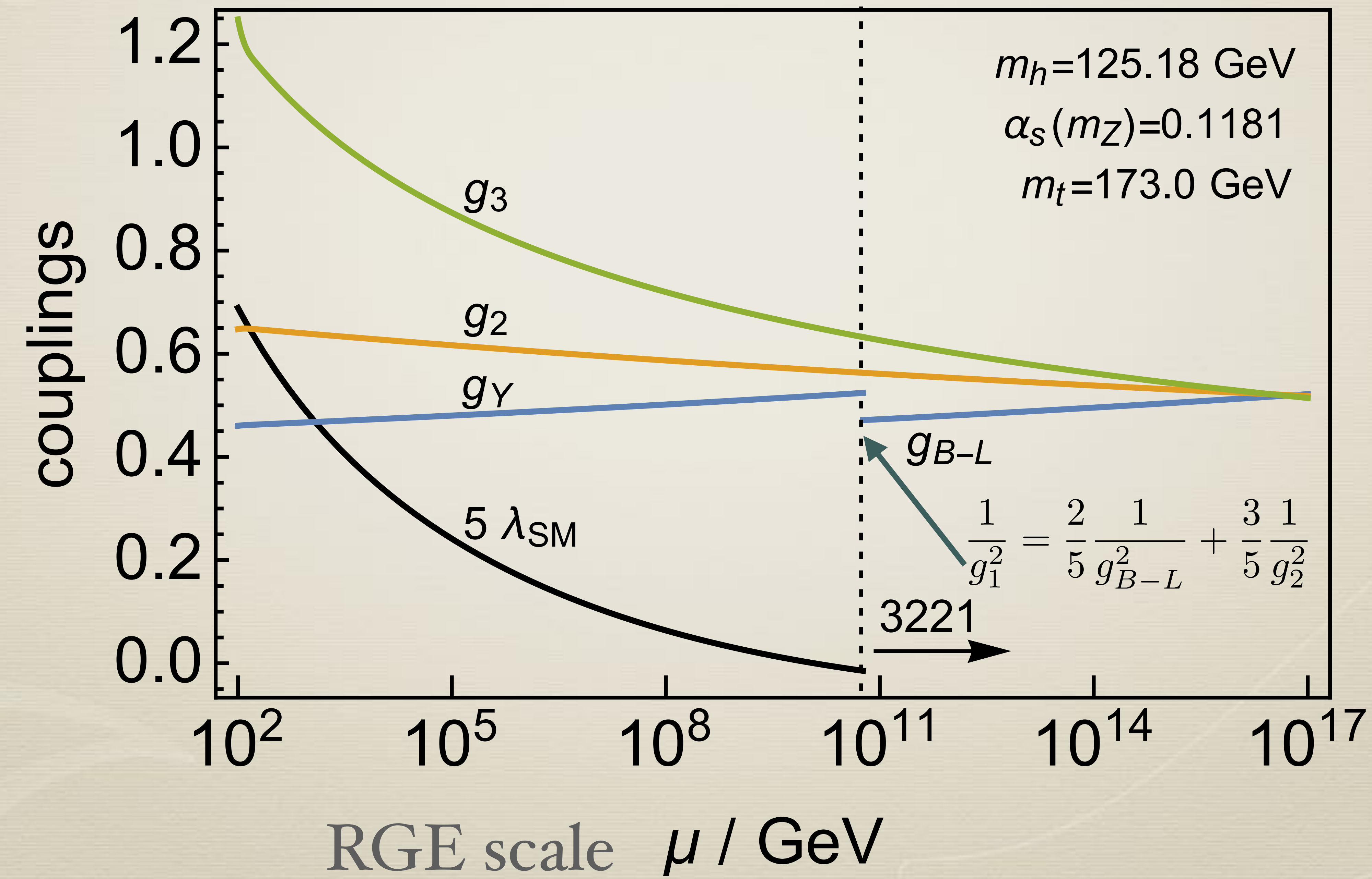
$q_L, \;\; L_L, \;\; q_R, \;\; L_R$ (+ vector-like multiplets)

$q_L = (u_L, d_L), \;\; L_L = (\nu_L, e_L), \;\; u_R, \;\; d_R, \;\; e_R$

Coupling unification

Hall, KH (2018, 2019)

energy-dependent couplings

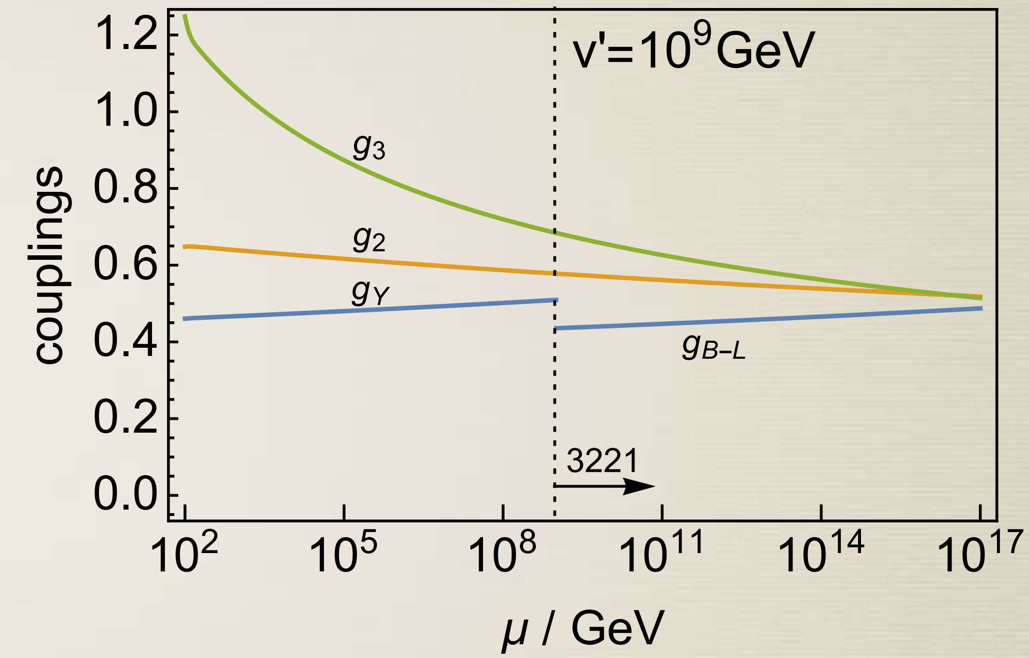


RGE scale

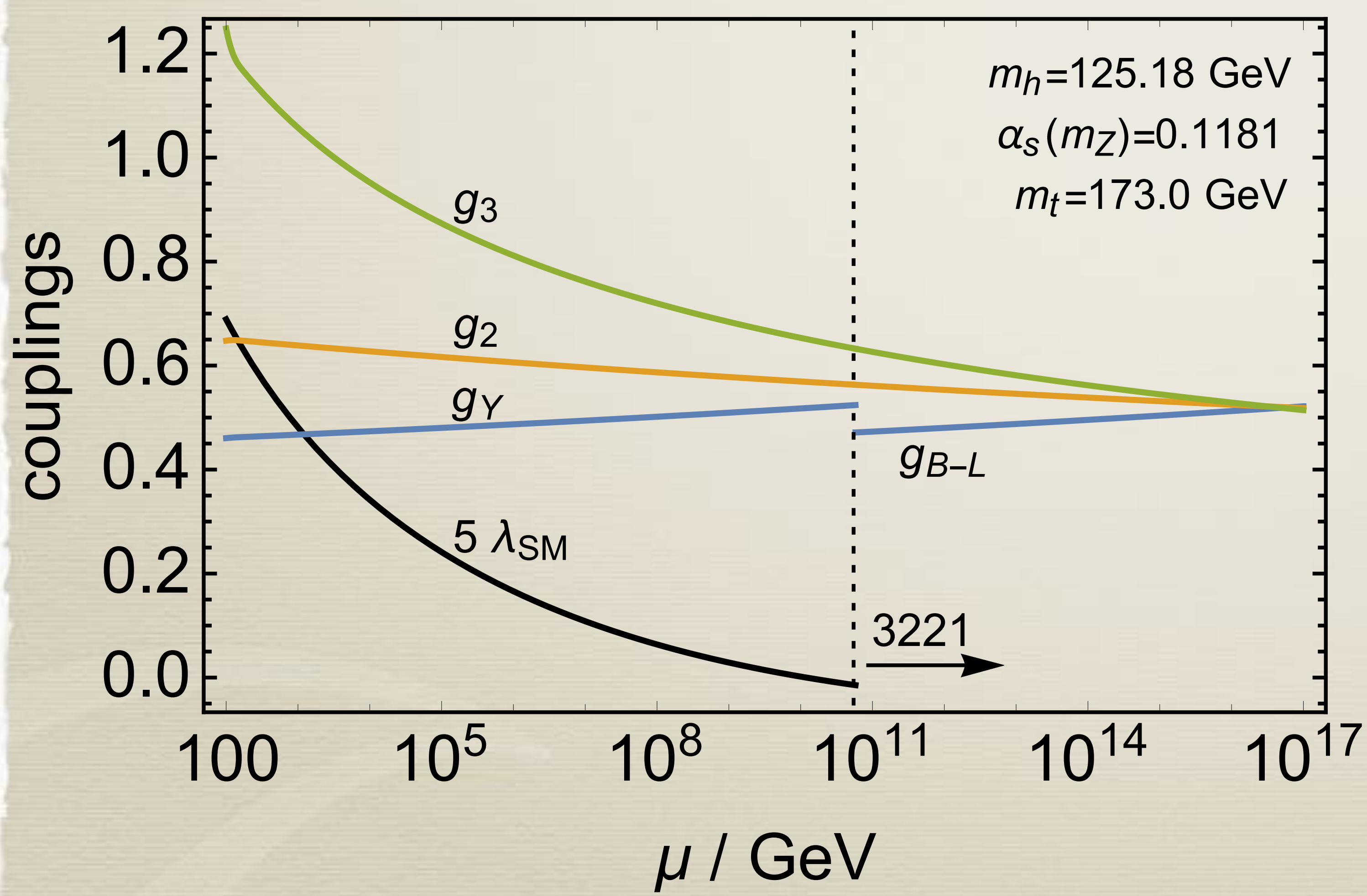
Coupling unification

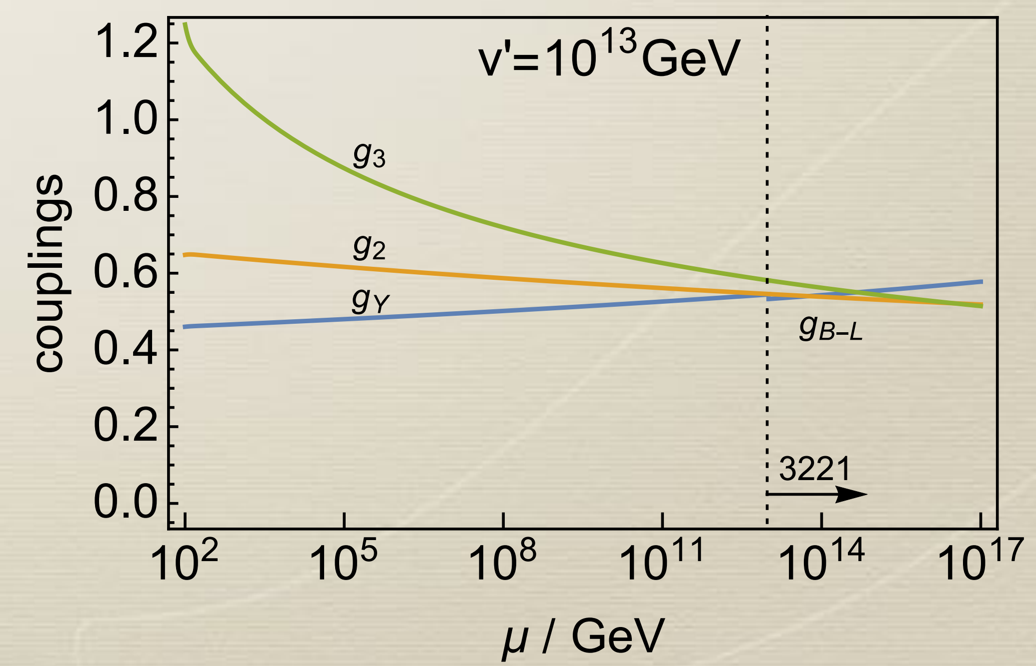
Hall, KH (2018, 2019)

Other v_R



v_R determined by Higgs Parity

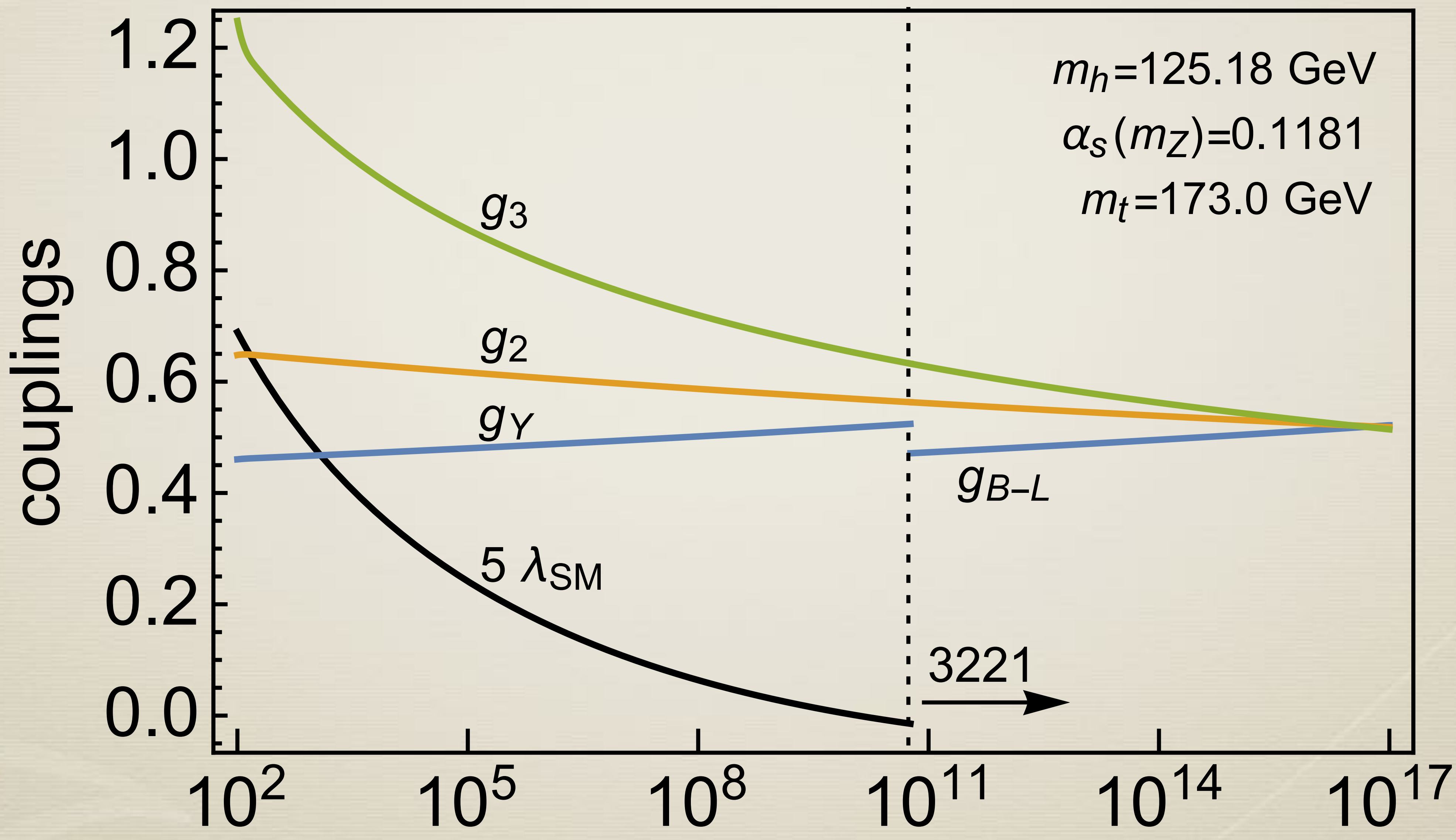




Higgs Parity GUT

Hall, KH (2018, 2019)

energy-dependent couplings



RGE scale μ / GeV

Higgs Parity GUT

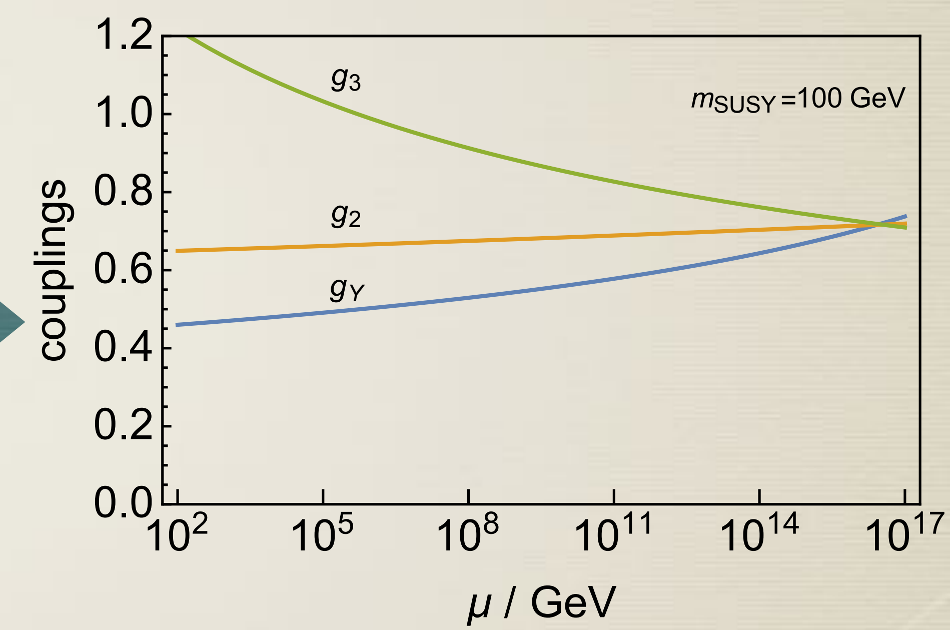
Hall, KH (2018, 2019)

weak scale 10^2 GeV

GUT scale 10^{16} GeV

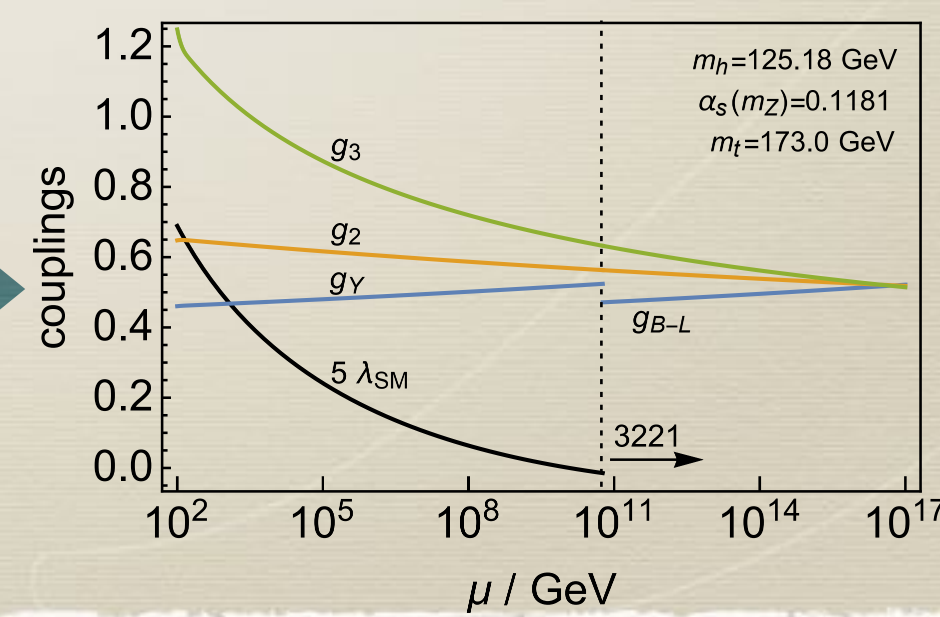
gauge coupling constants
(LEP, ...)

SUSY GUT

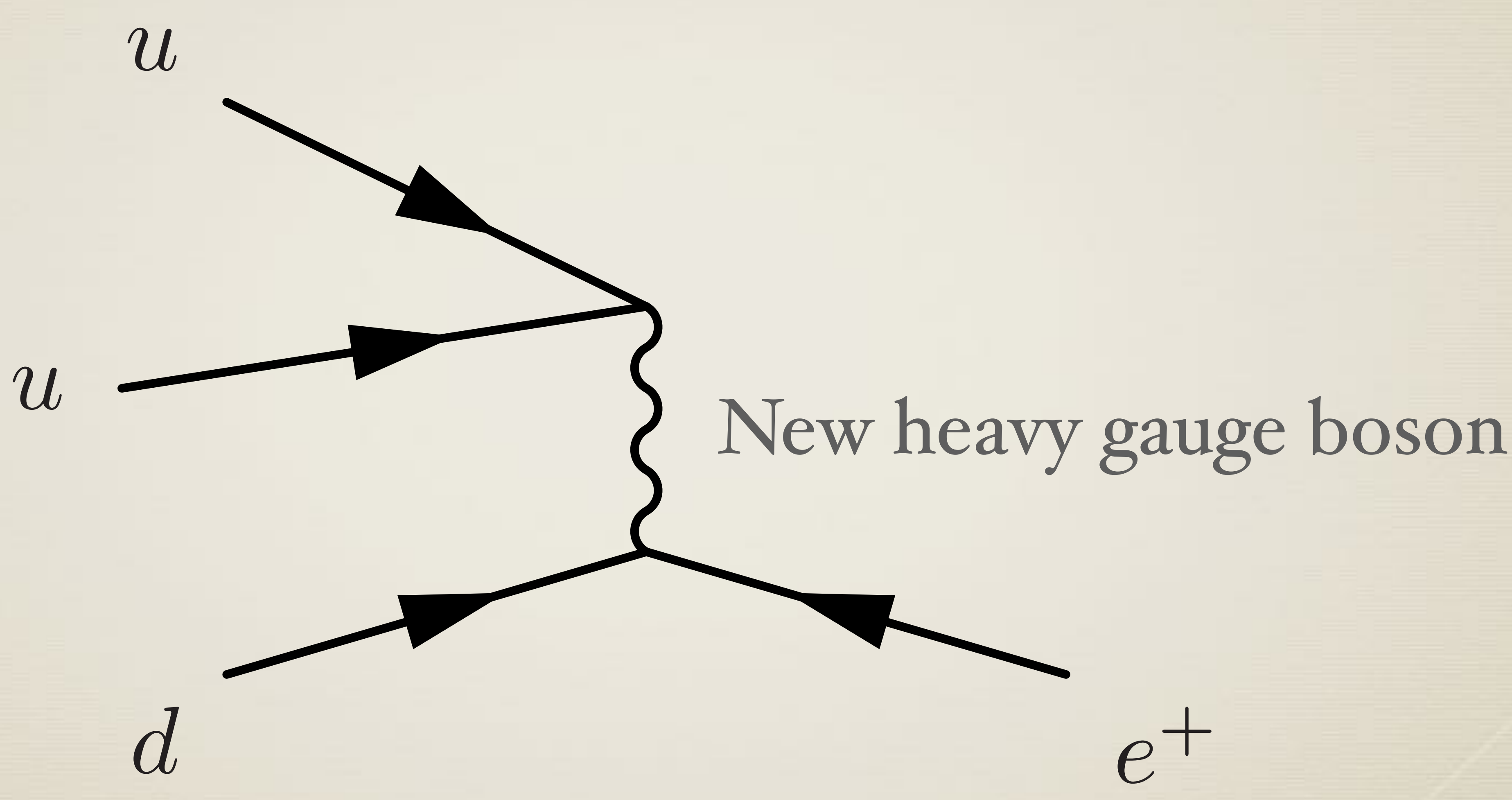


gauge coupling constants
top quark mass
Higgs mass
(LHC, lattice, future colliders, ...)

HP GUT



Proton decay

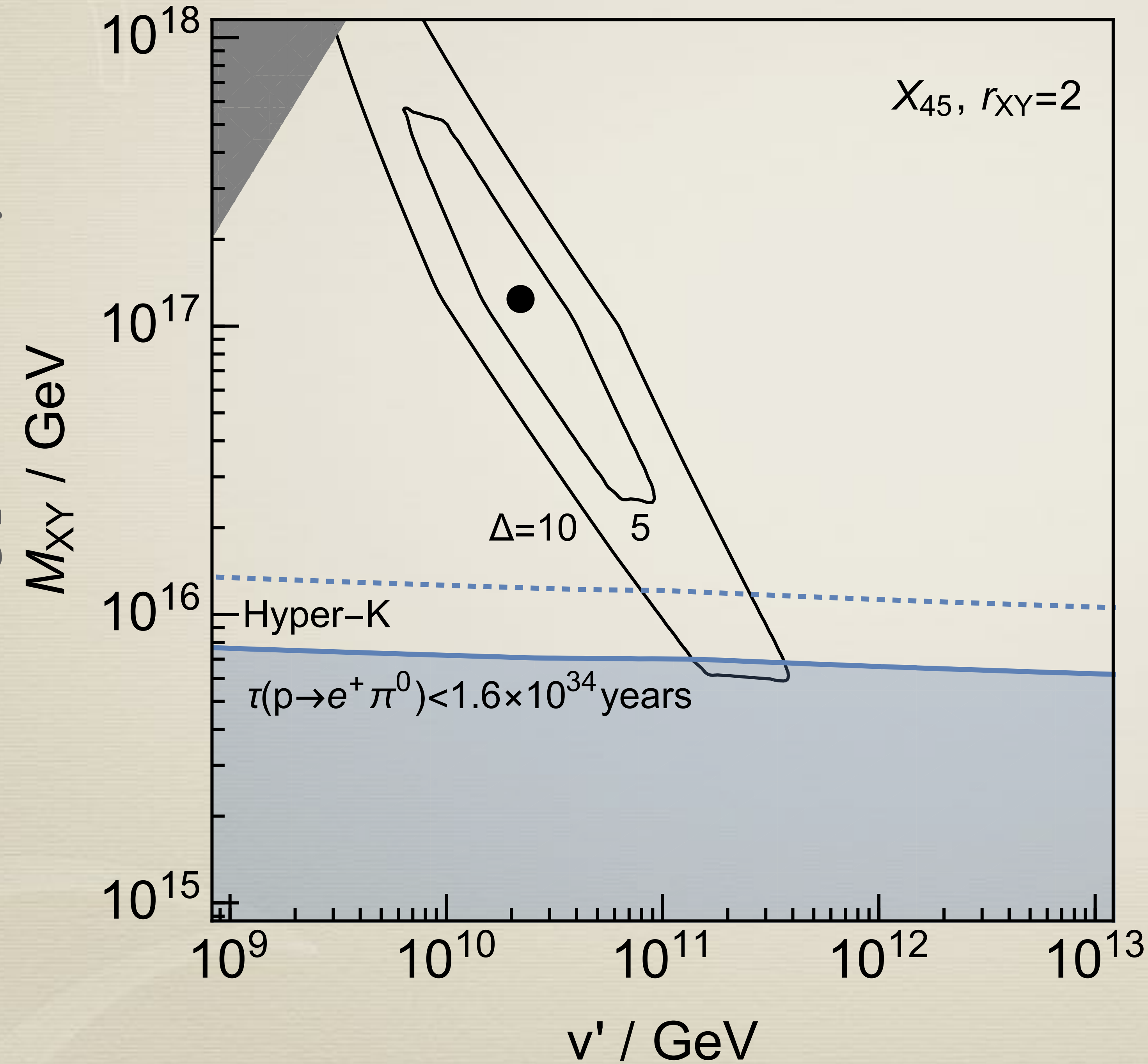


$$p \to e^+\pi^0$$

Quantify unification

Hall, KH (2019)

mass of new gauge boson mediating proton decay



There can be quantum corrections from heavy particles around the GUT scale

$$\Delta = \max_{i,j}\left|\frac{8\pi^2}{g_i^2}-\frac{8\pi^2}{g_j^2}\right|$$

typically

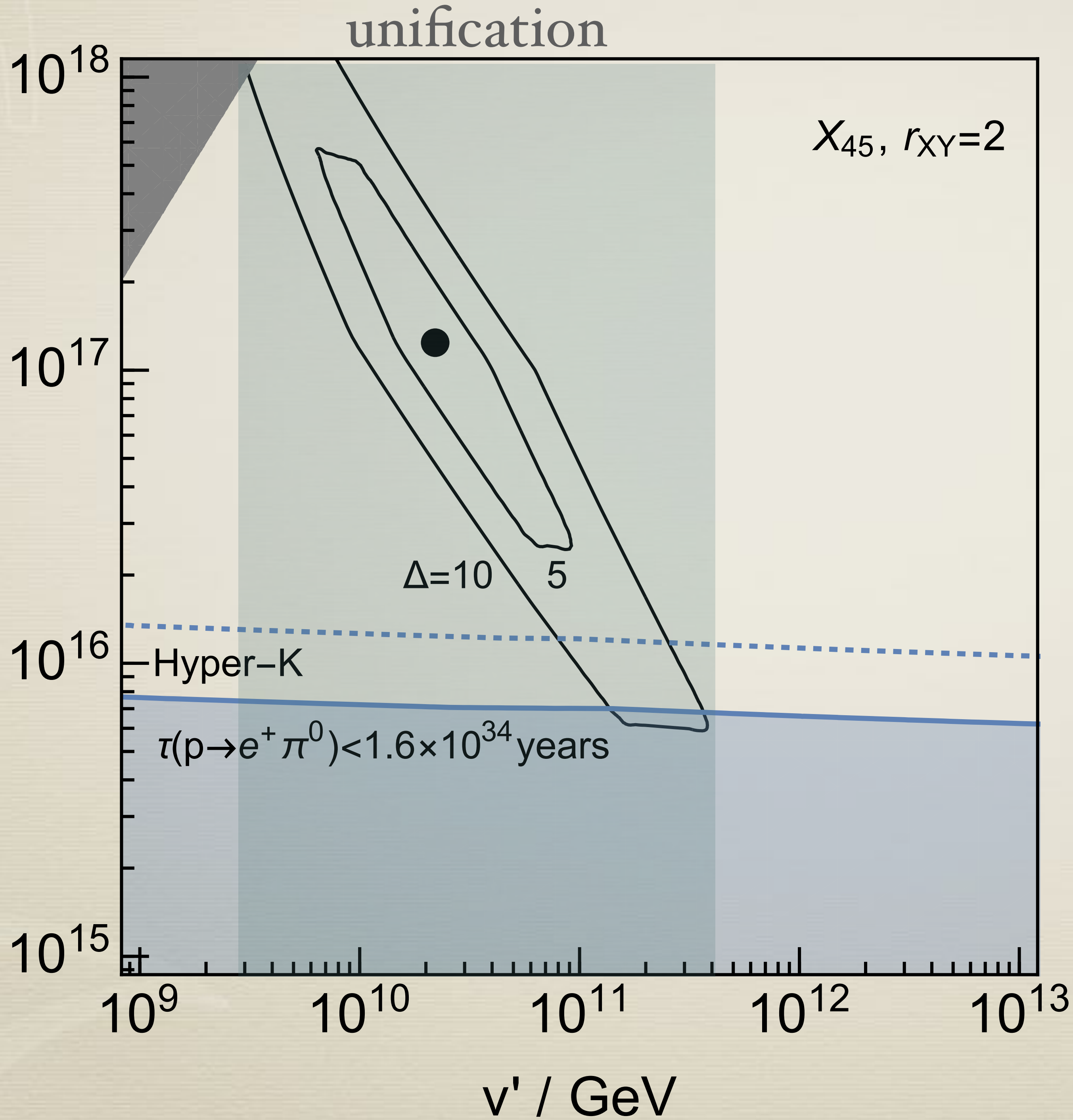
$$\Delta = \text{few} - 10$$

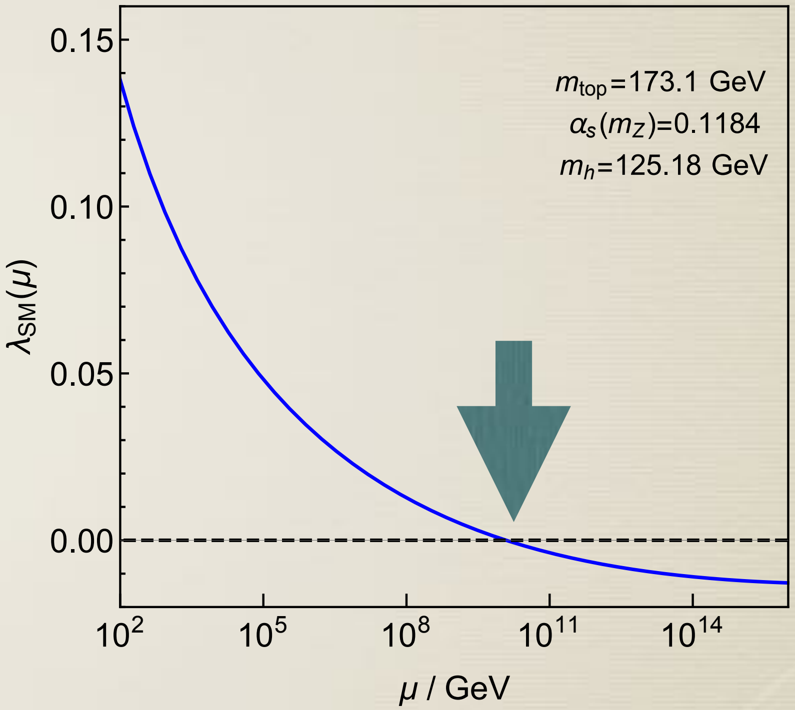
(smaller than SUSY GUT)

Quantify unification

Hall, KH (2019)

mass of new gauge boson
mediating proton decay





typically

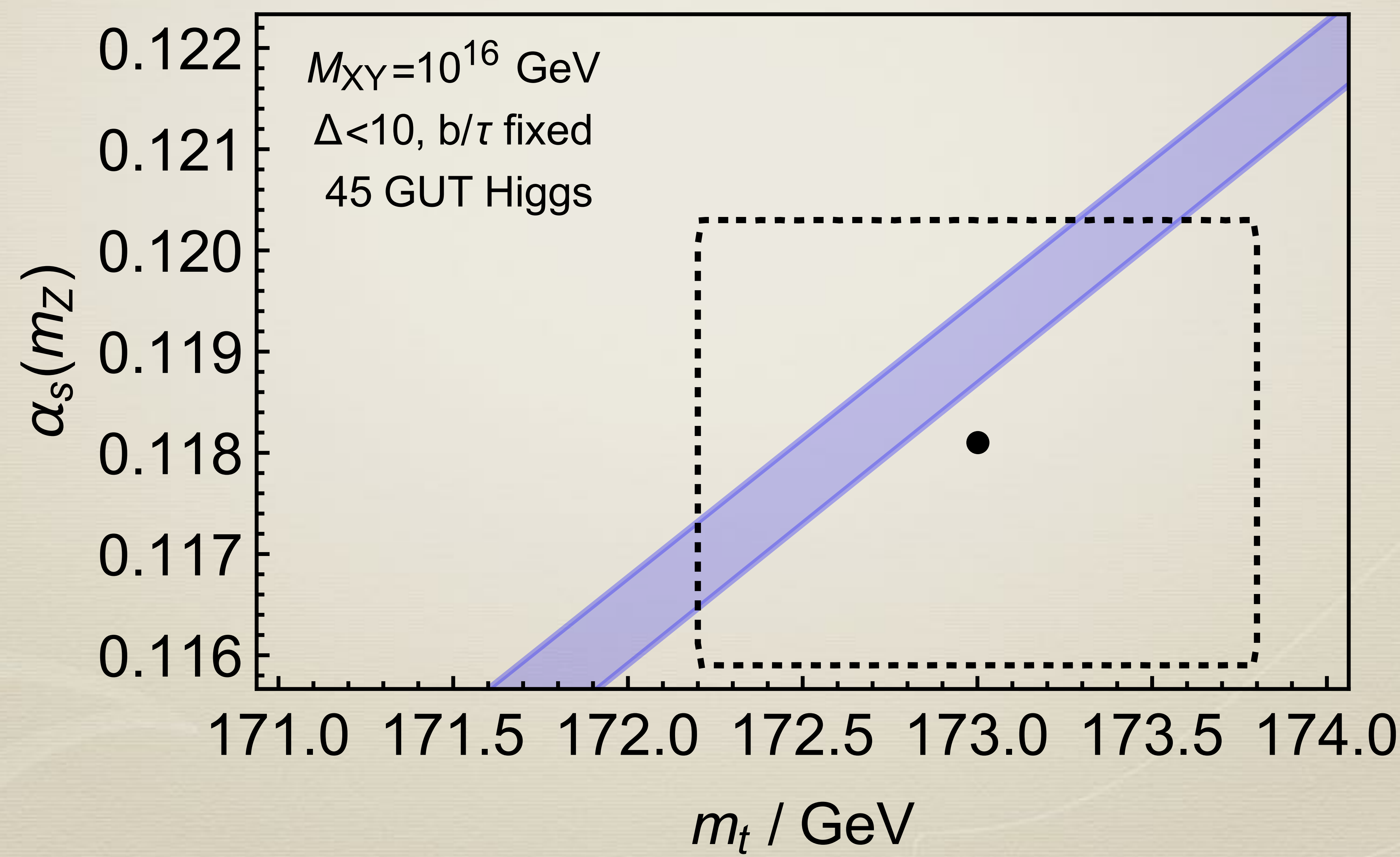
Δ = few − 10

(smaller than SUSY GUT)

Proton decay

Hall, KH (2019)

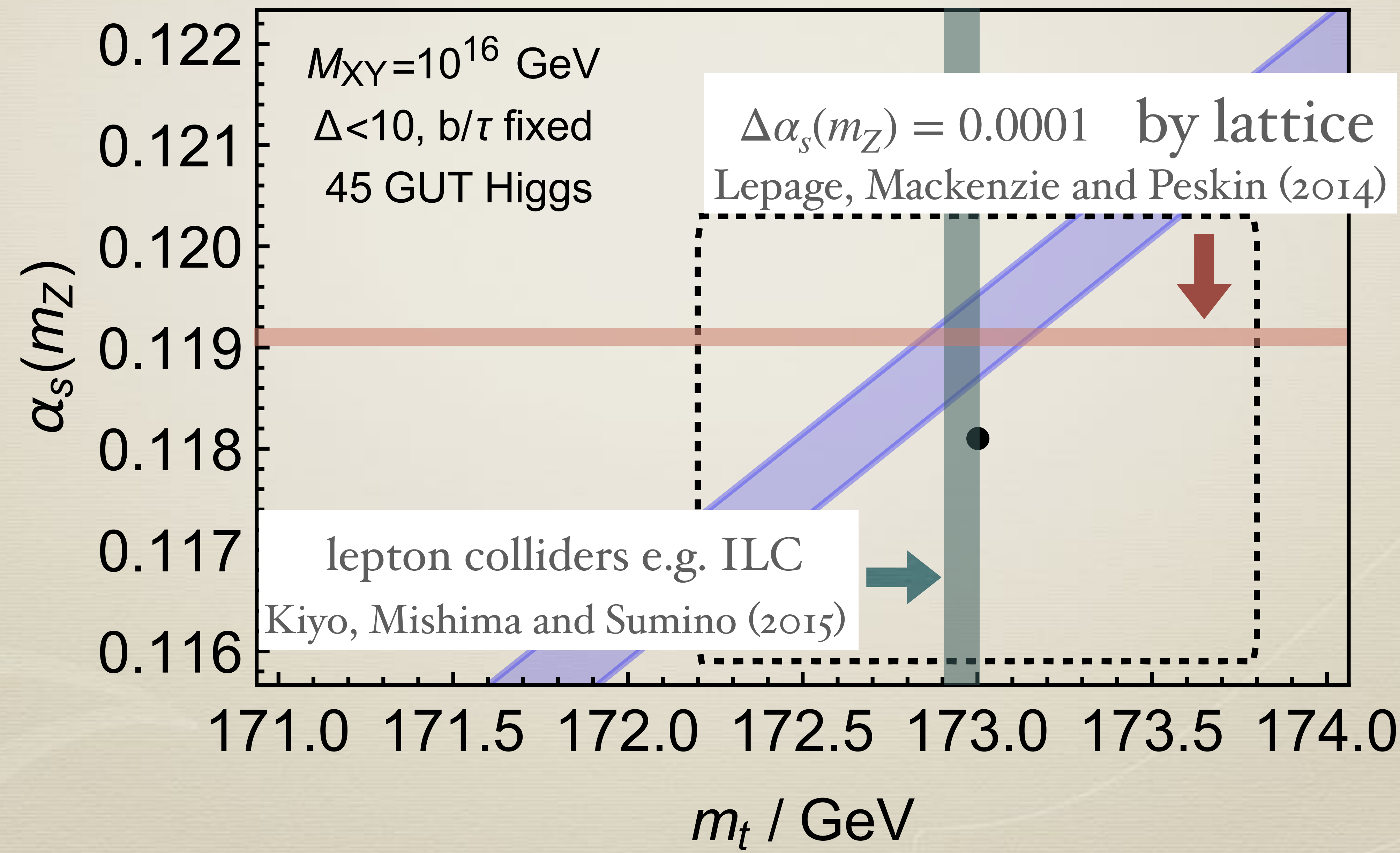
Suppose proton decay is observed at Hyper-K (2027-)



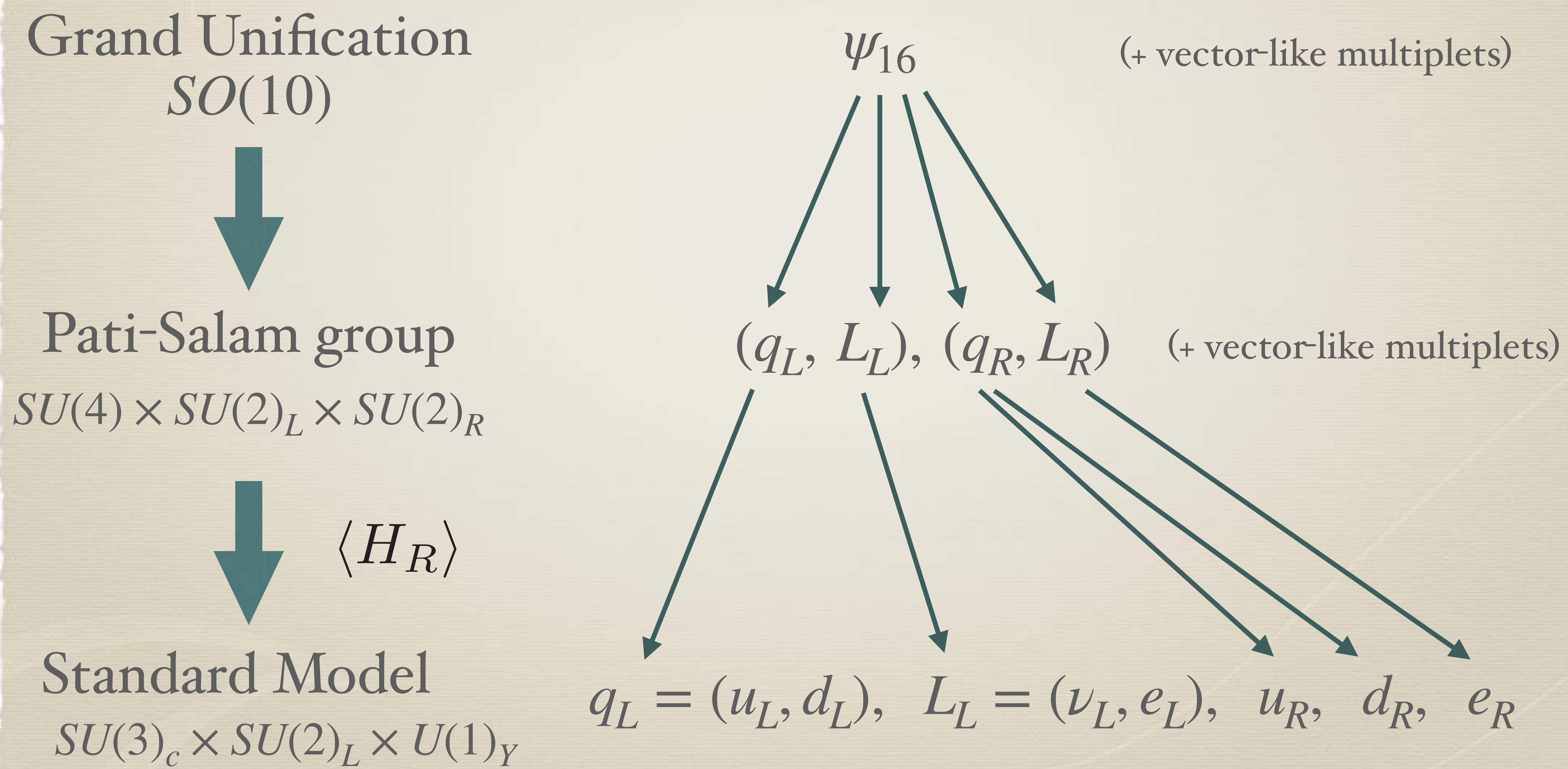
Proton decay

Hall, KH (2019)

Suppose proton decay is observed at Hyper-K (2027-)

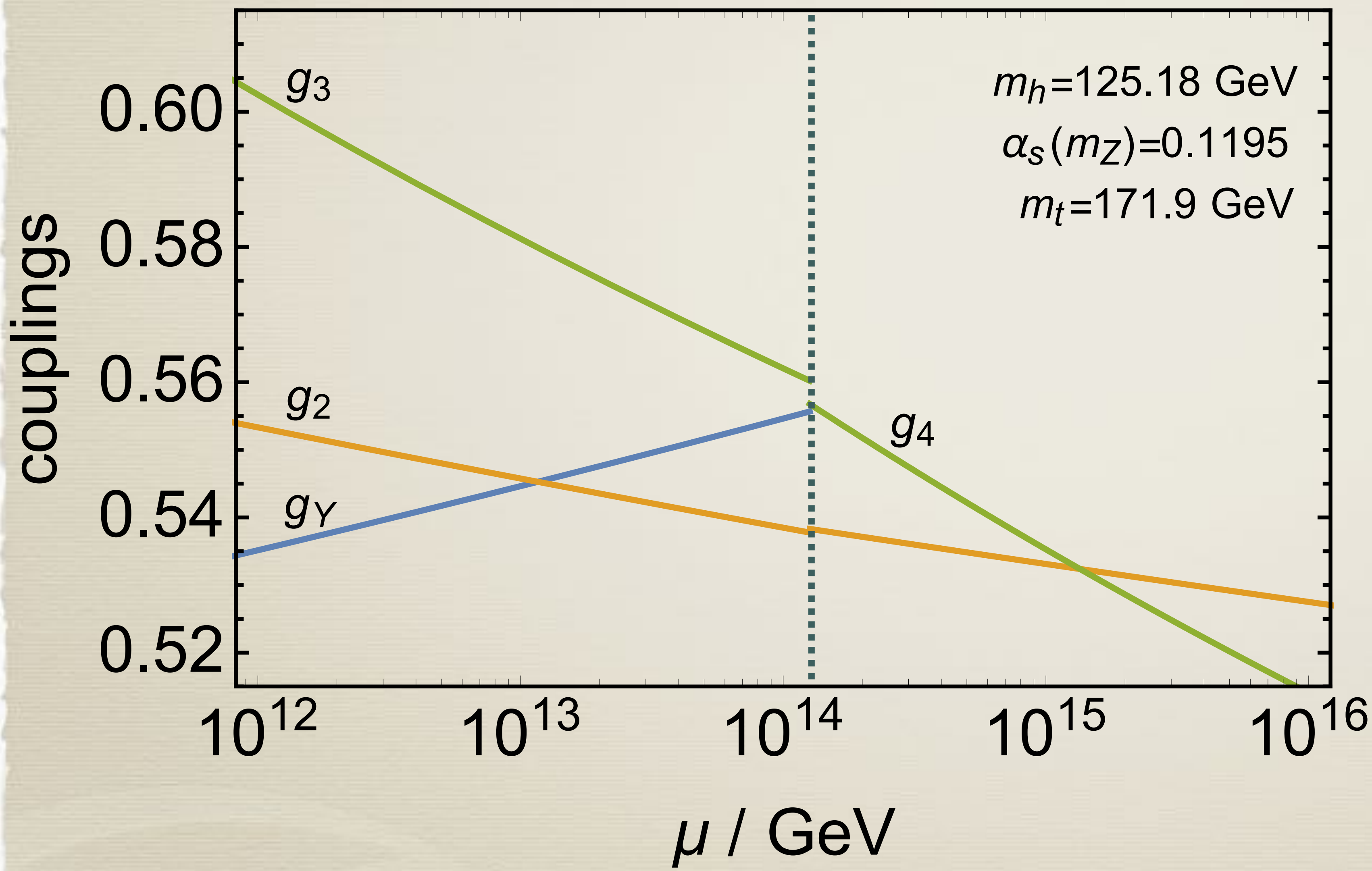


Intermediate Pati-Salam



Intermediate Pati-Salam

Hall, KH (2018, 2019)



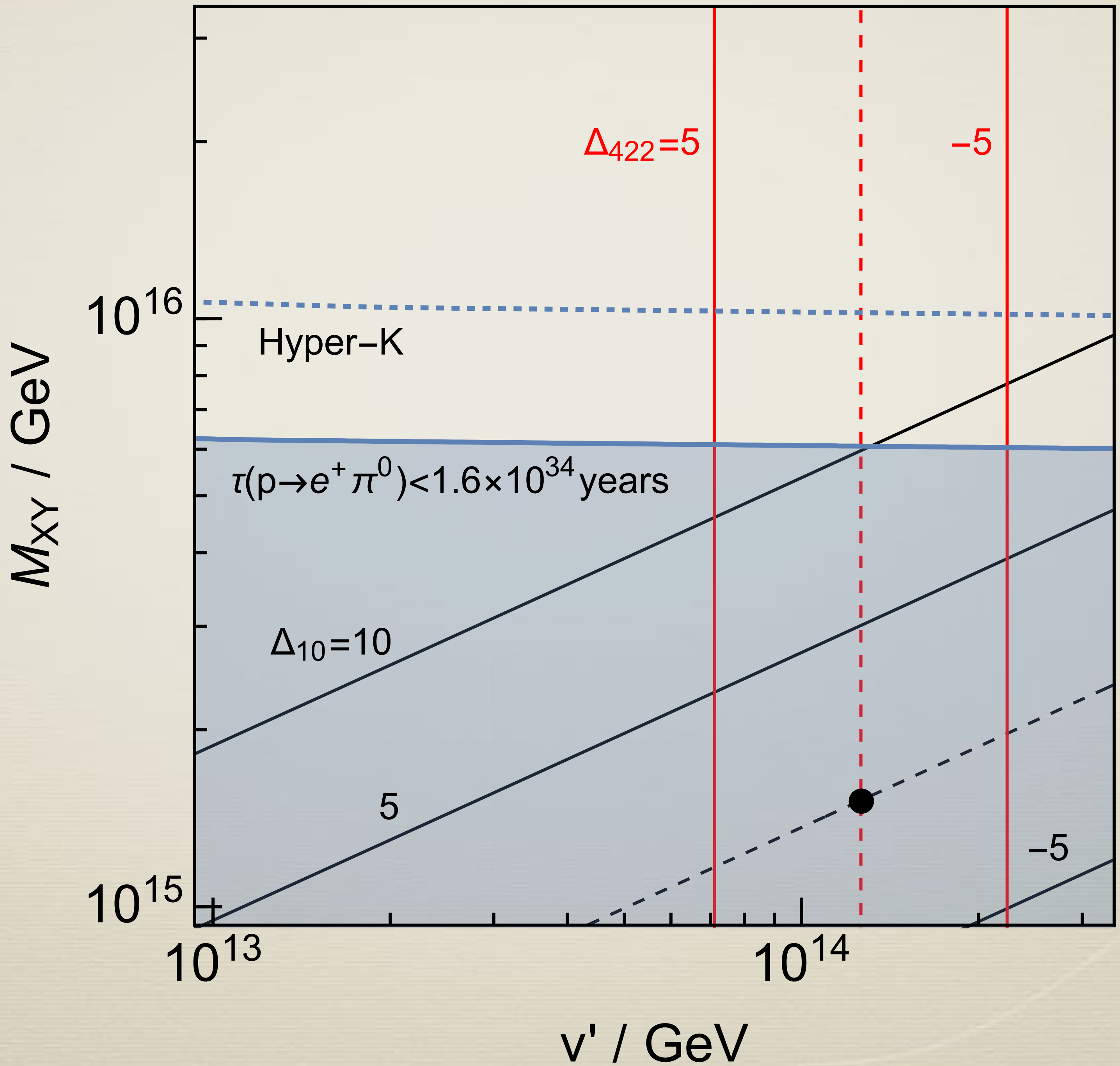
$SU(4) \times SU(2)_L \times SU(2)_R$

2 couplings

Pati-Salam scale is fixed by coupling unification

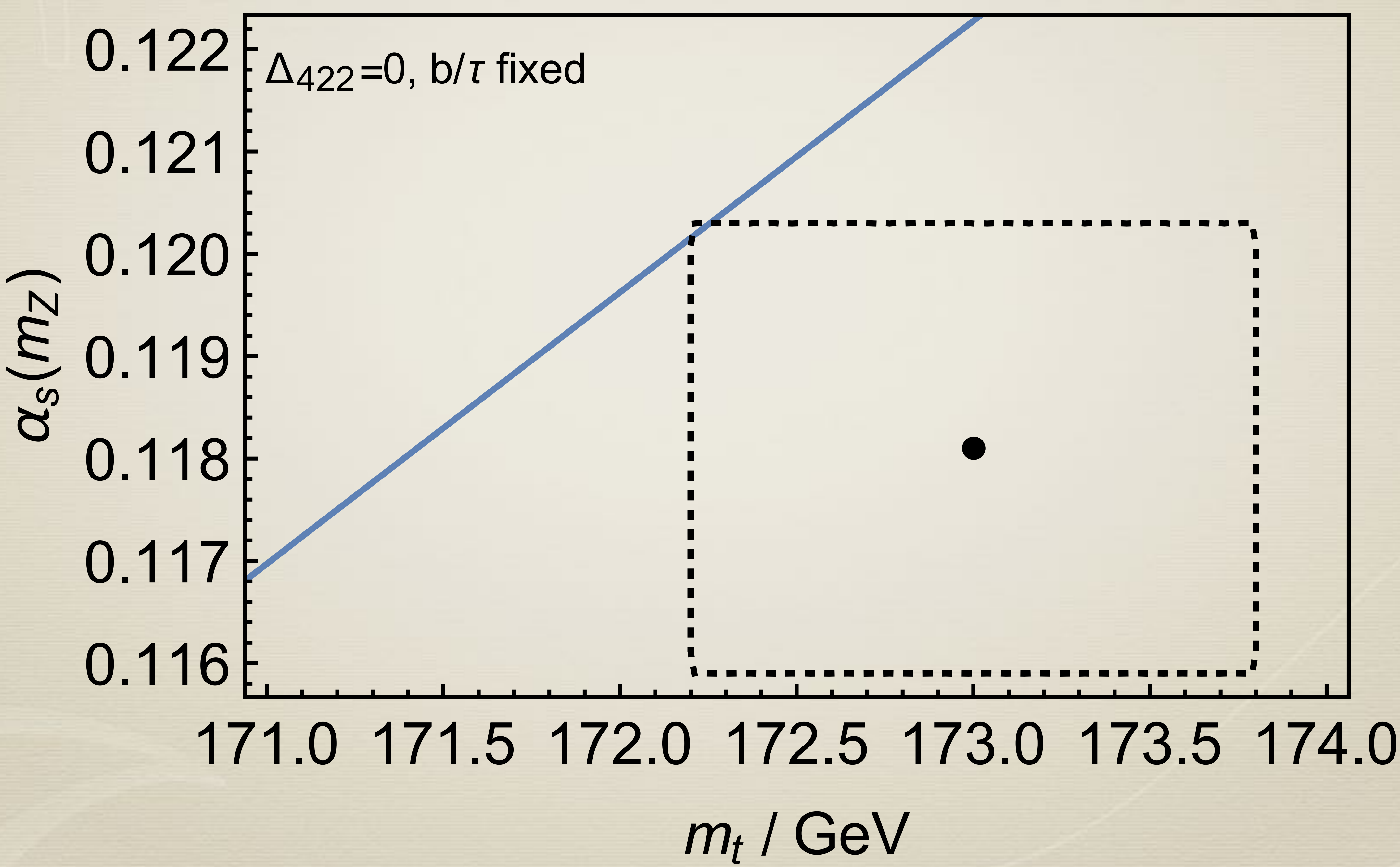
$$\frac{1}{g_1^2} \simeq \frac{2}{5}\frac{1}{g_4^2} + \frac{3}{5}\frac{1}{g_2^2}$$

Pati-Salam



Δ_{422}=5
−5
10^{16}
Hyper−K
$\tau(p\to e^+\pi^0)<1.6\times10^{34}$years
M_{XY} / GeV
Δ_{10}=10
5
−5
10^{15}
10^{13}
10^{14}
v' / GeV

Pati-Salam



Δ_{422}=0, b/τ fixed
0.122
0.121
0.120
0.119
0.118
0.117
0.116
$\alpha_s(m_Z)$
171.0
171.5
172.0
172.5
173.0
173.5
174.0
m_t / GeV

Outline

* Strong CP problem and solutions to it
* Parity solutions with minimal fermions or Higgses
* Parity symmetry breaking scale
* Unification
* Dark matter

Mirror variant

Barr, Chang and Senjanovic (1991)

SM particles		New particles
q	strong interaction	q’
u	$SU(3)_c$	u’
d		d'
L	parity	L’
e	⟷	e’
H		H’

electroweak

$W, Z \quad SU(2)_L \times U(1)_Y$

$\downarrow$

$\gamma \quad U(1)_{\text{EM}}$

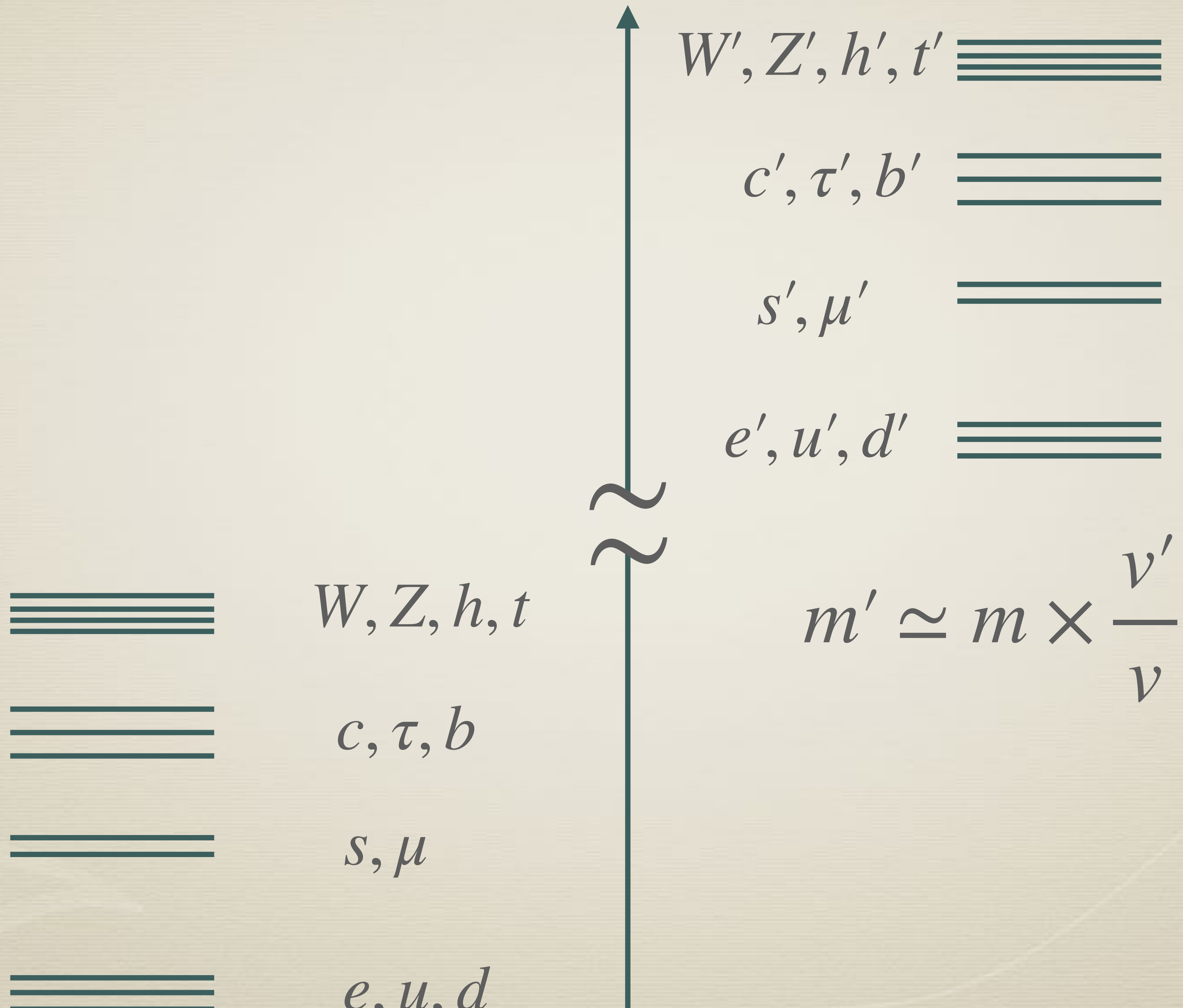
electroweak’

$SU(2)'_L \times U(1)'_Y \quad W', Z'$

$\downarrow$

$U(1)'_{\text{EM}} \quad \gamma'$

Mass spectrum



$$m' \simeq m \times \frac{v'}{v}$$

Mirror dark matter

Dunsky, Hall, KH (2019)

New particles

q', u', d'
L', e'
H'

electroweak'

$$SU(2)'_L \times U(1)'_Y \quad W', Z'$$

$$\downarrow$$

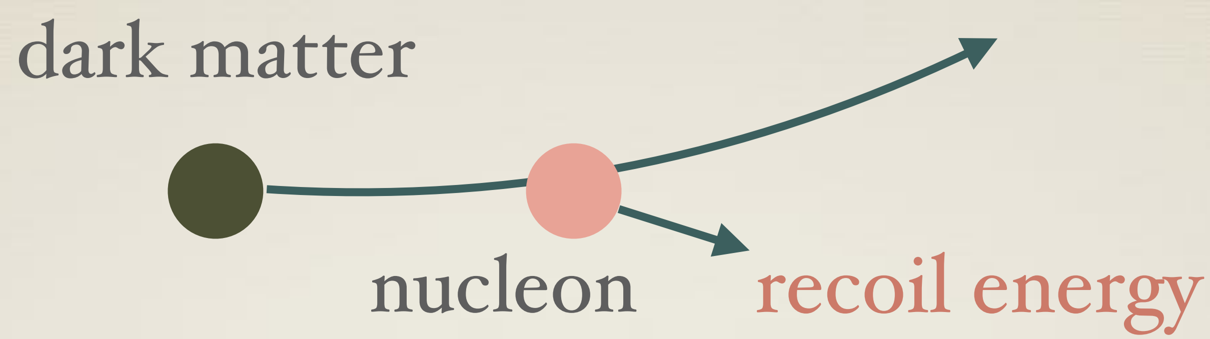
$$U(1)'_{\rm EM} \quad \gamma'$$

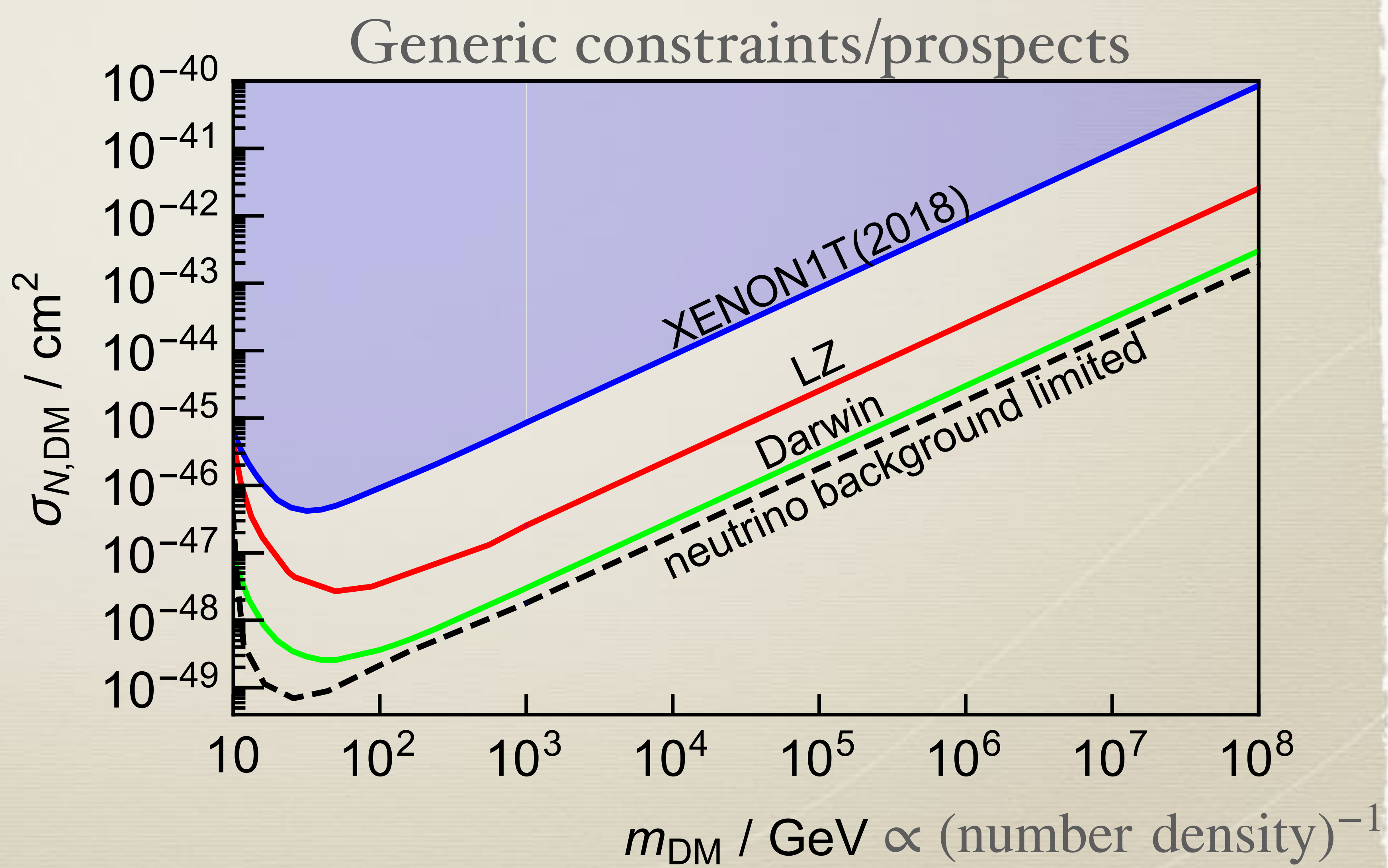
The mirror electron is absolutely stable

$$e'$$

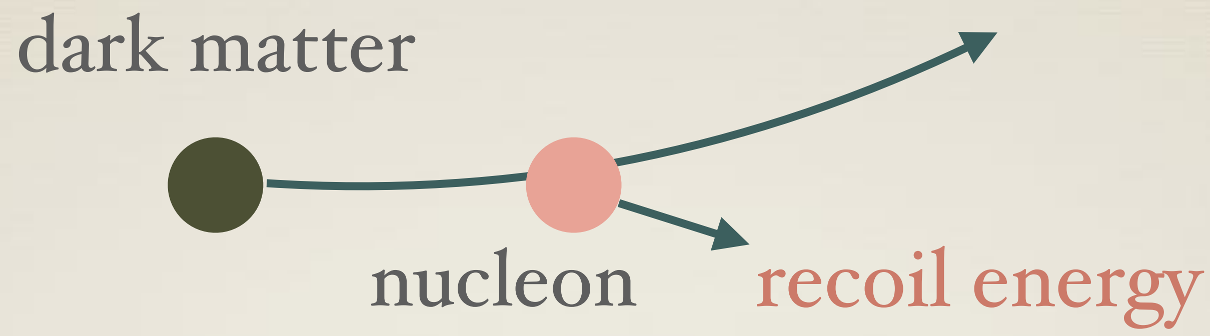
I assume that it comprises all of dark matter

Direct detection of dark matter

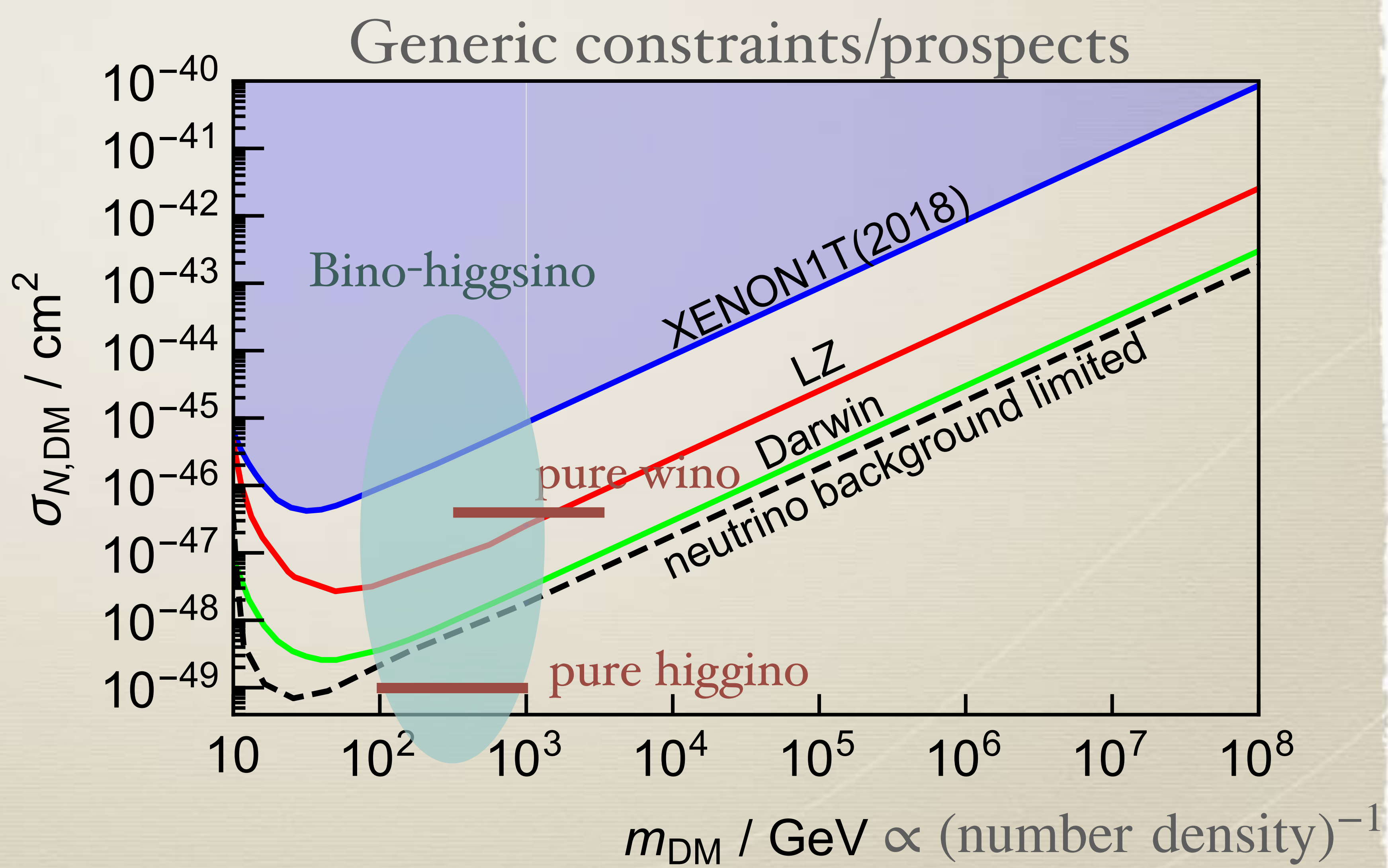




Direct detection of dark matter

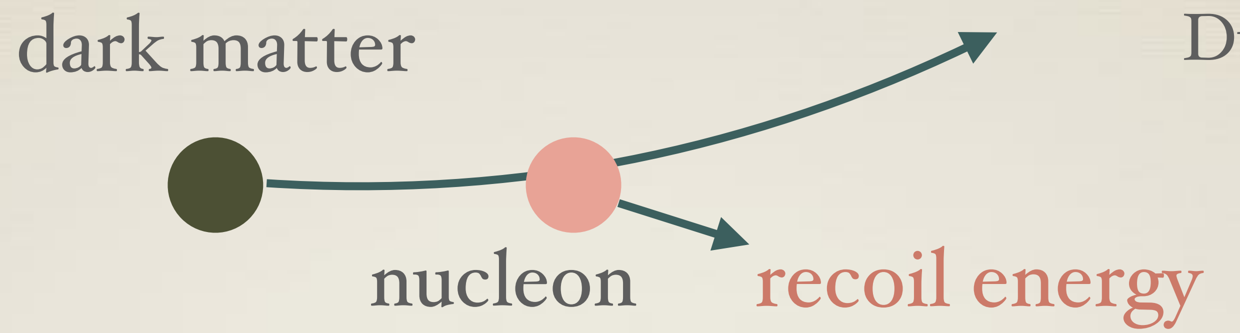


ex.
Dark matter in supersymmetric theories



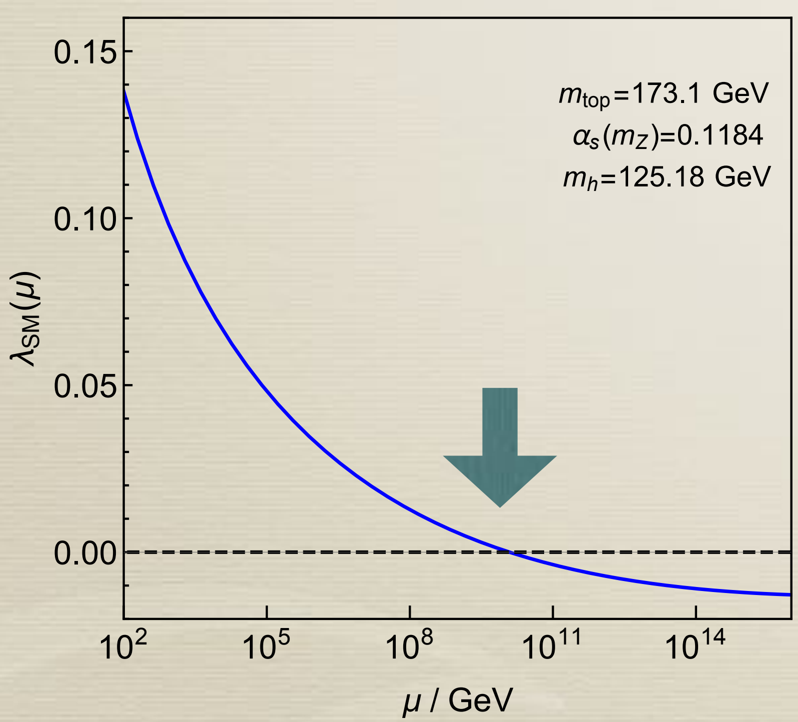
Direct detection of dark matter

Dunsky, Hall, KH (2019)



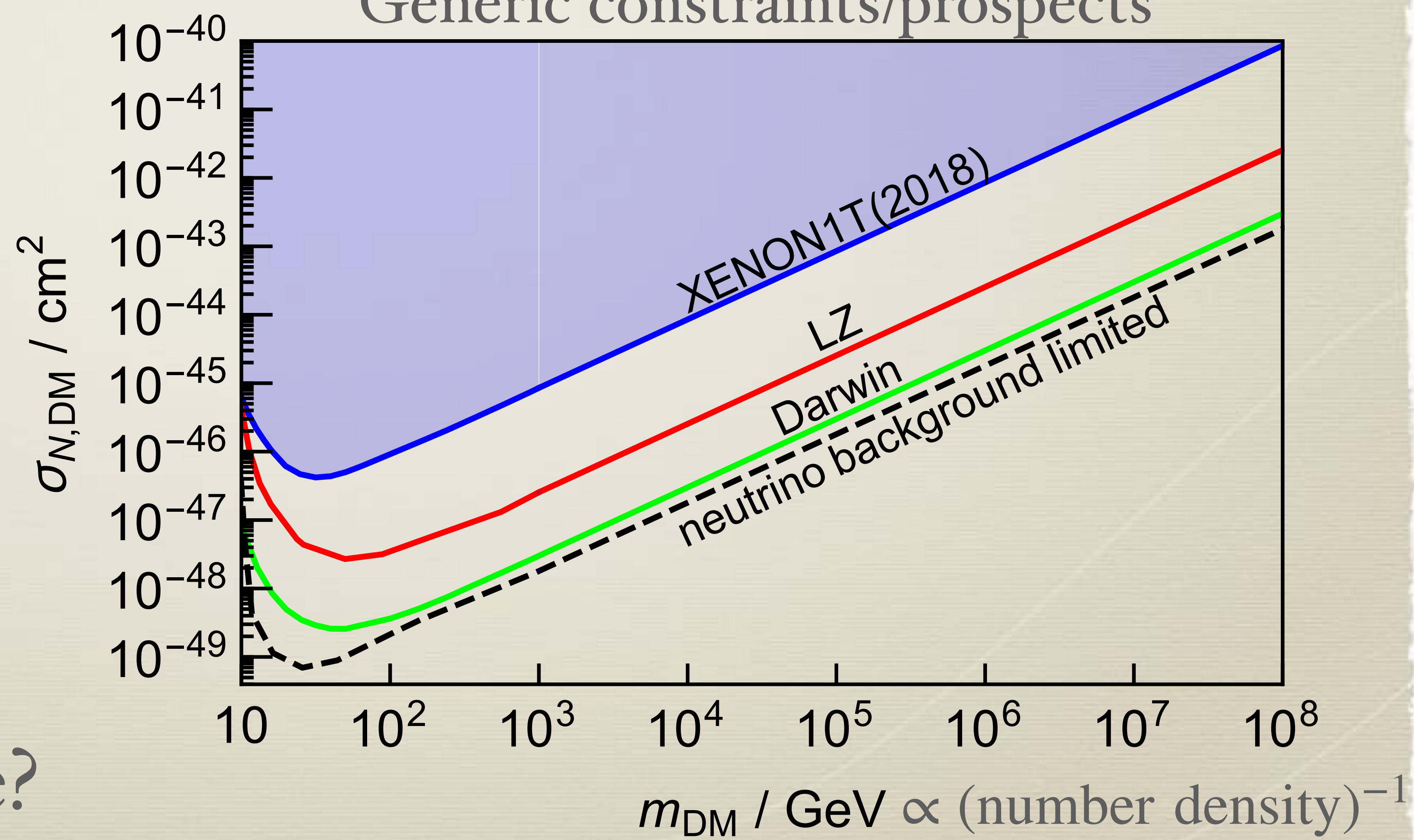
Higgs Parity

$$m_{e'} = y_e v'$$



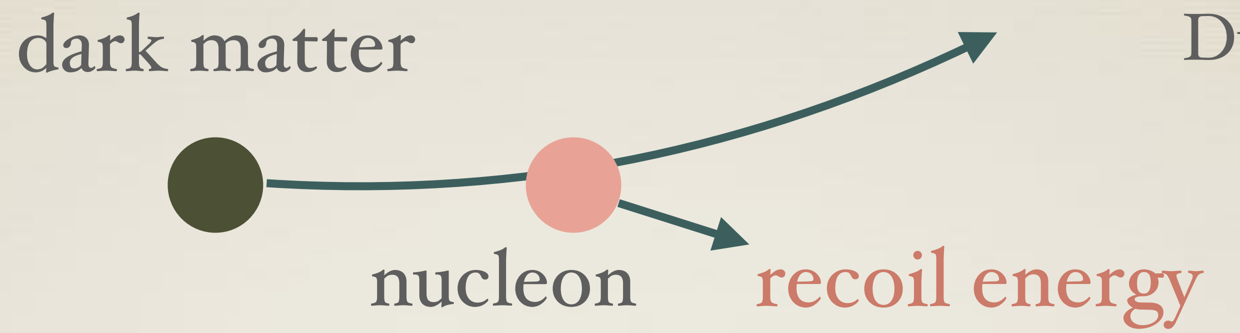
Interaction rate?

Generic constraints/prospects



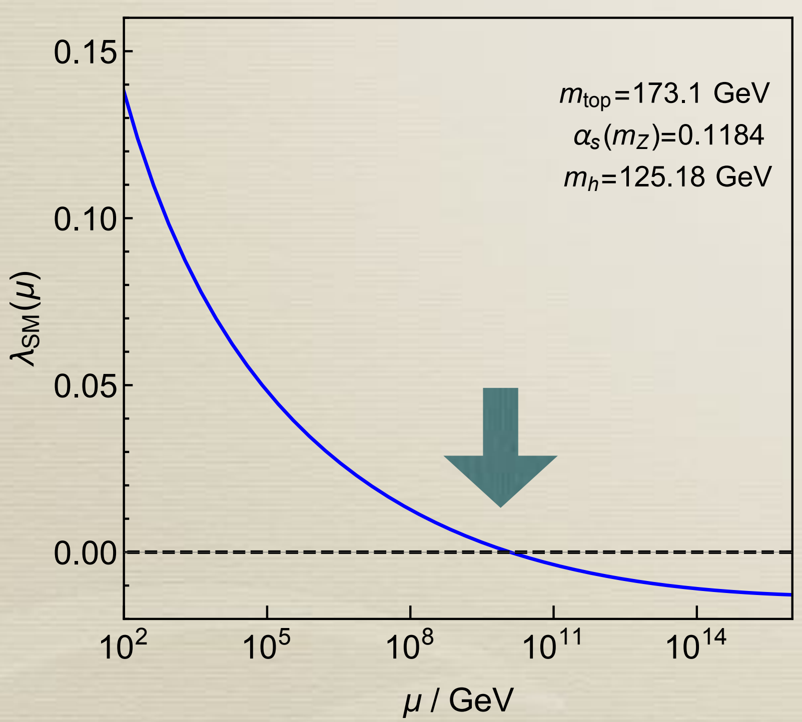
Direct detection of dark matter

Dunsky, Hall, KH (2019)

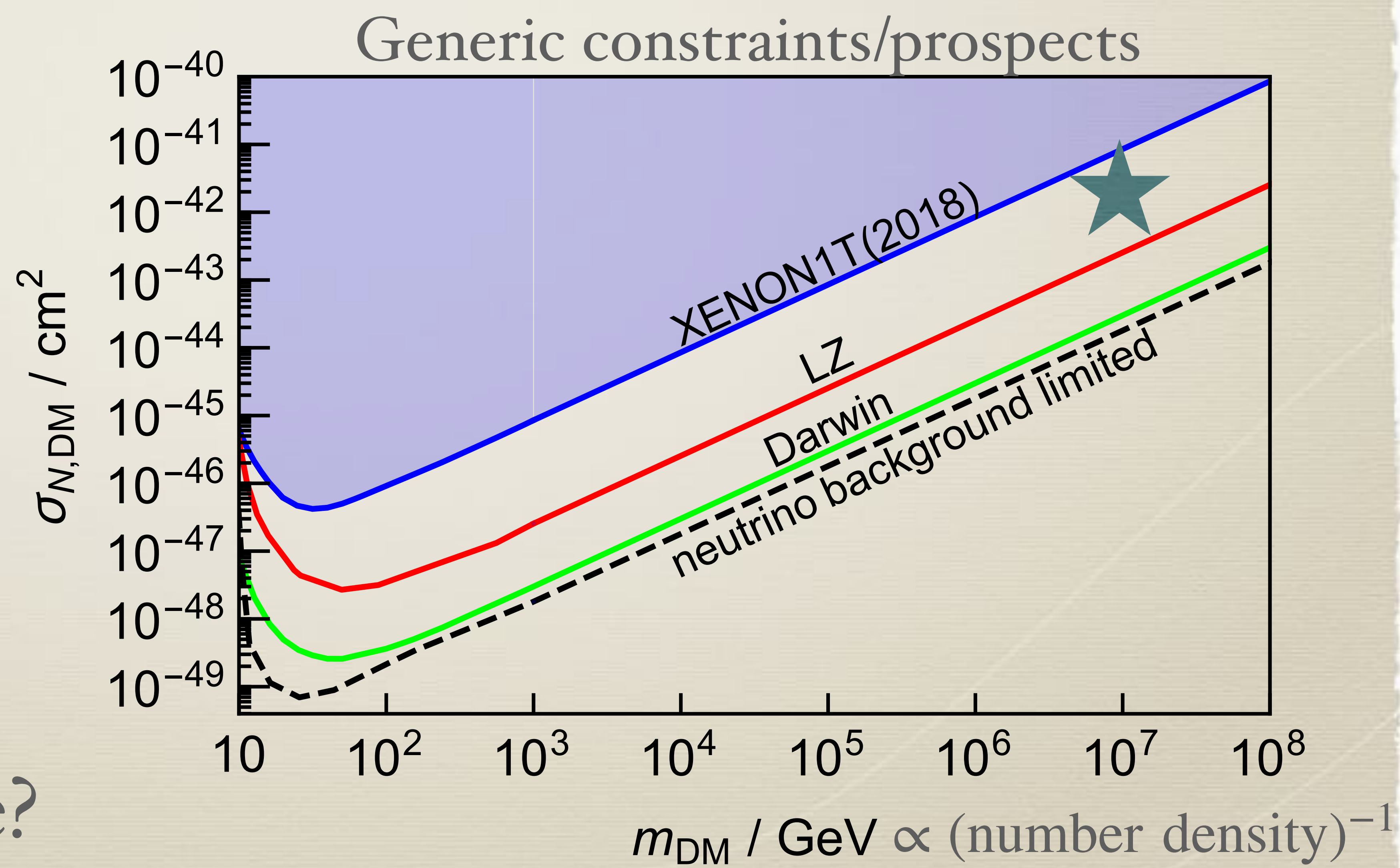


Higgs Parity

$$m_{e'} = y_e \underline{v'}$$

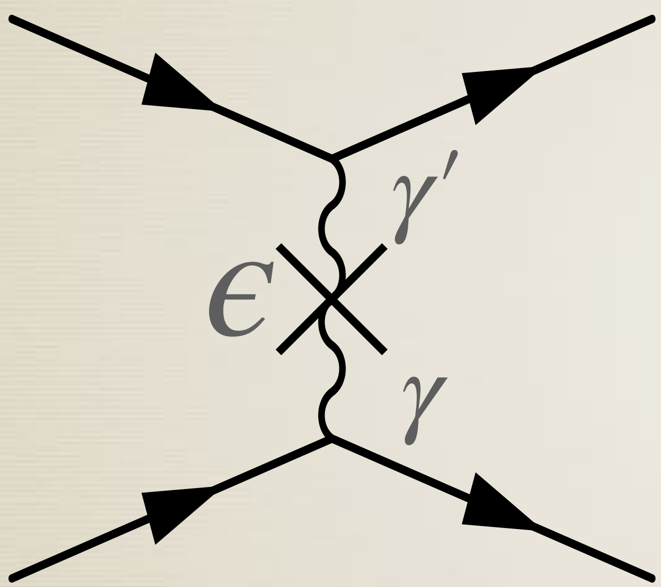


Interaction rate?



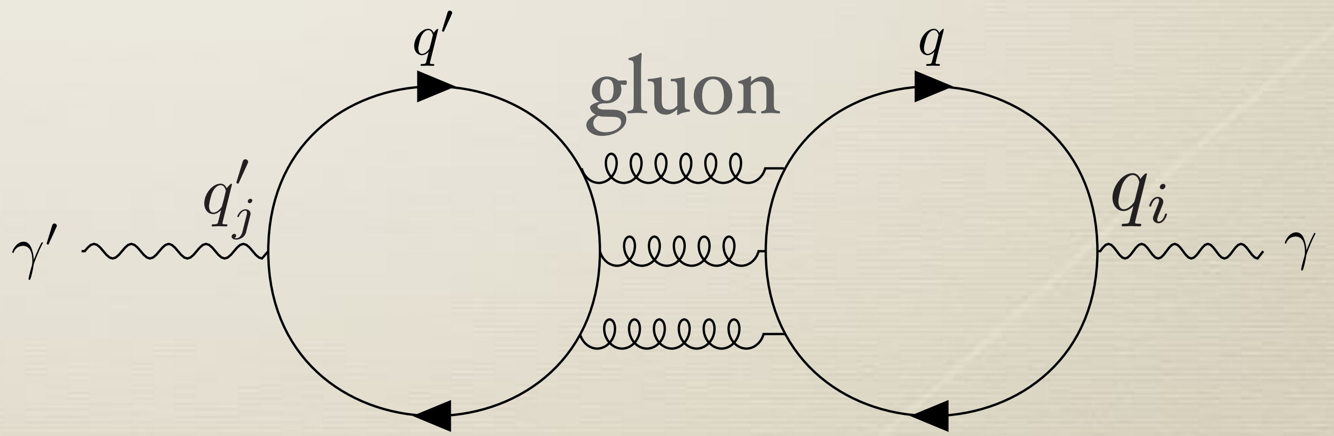
Prediction on interaction

Dunsky, Hall, KH (2019)



photon-mirror photon mixing

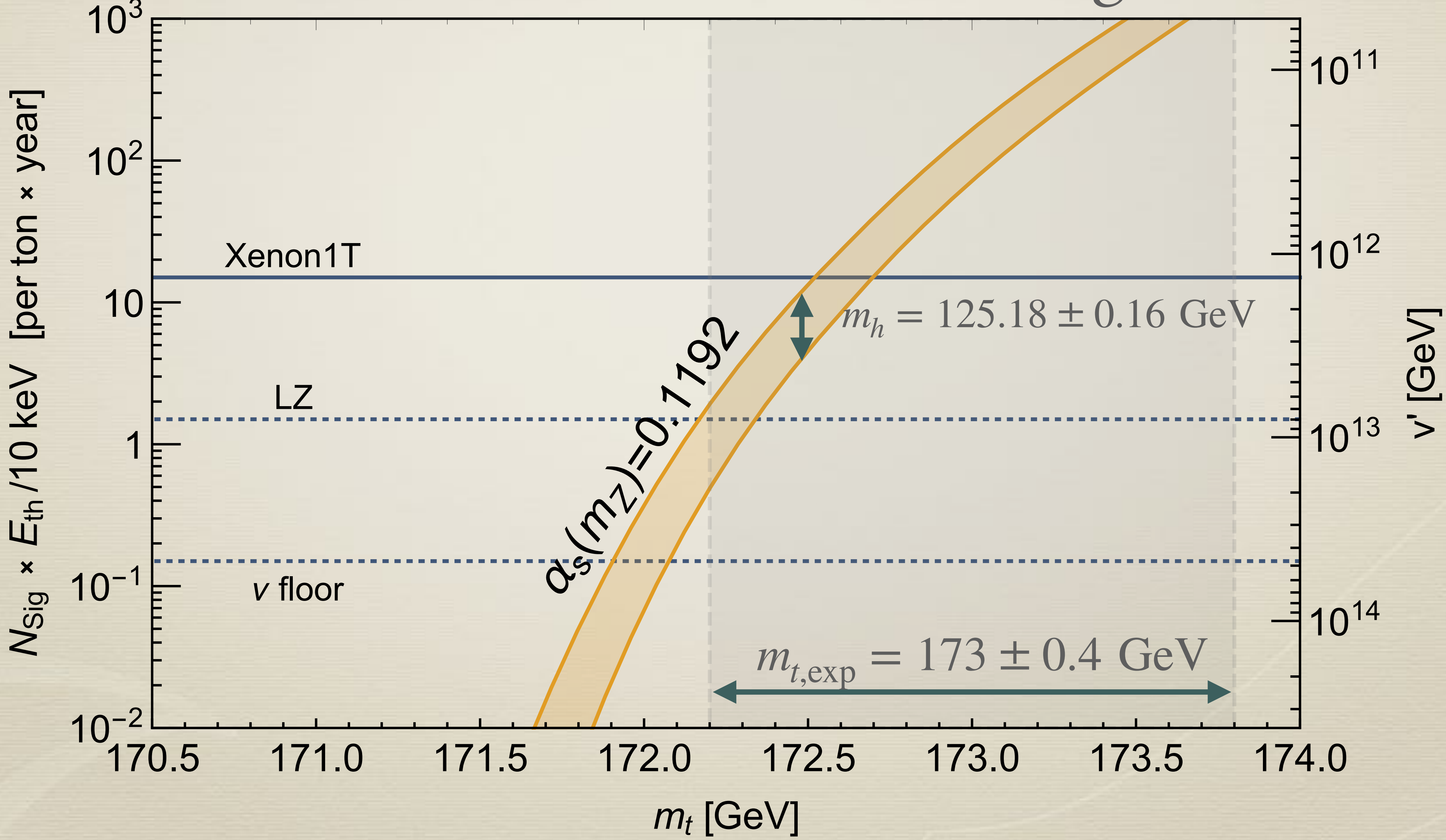
$$\epsilon = \epsilon_{\text{tree}} + \epsilon_{\text{quantum correction}}$$



SM parameters and DM

Dunsky, Hall, KH (2019)

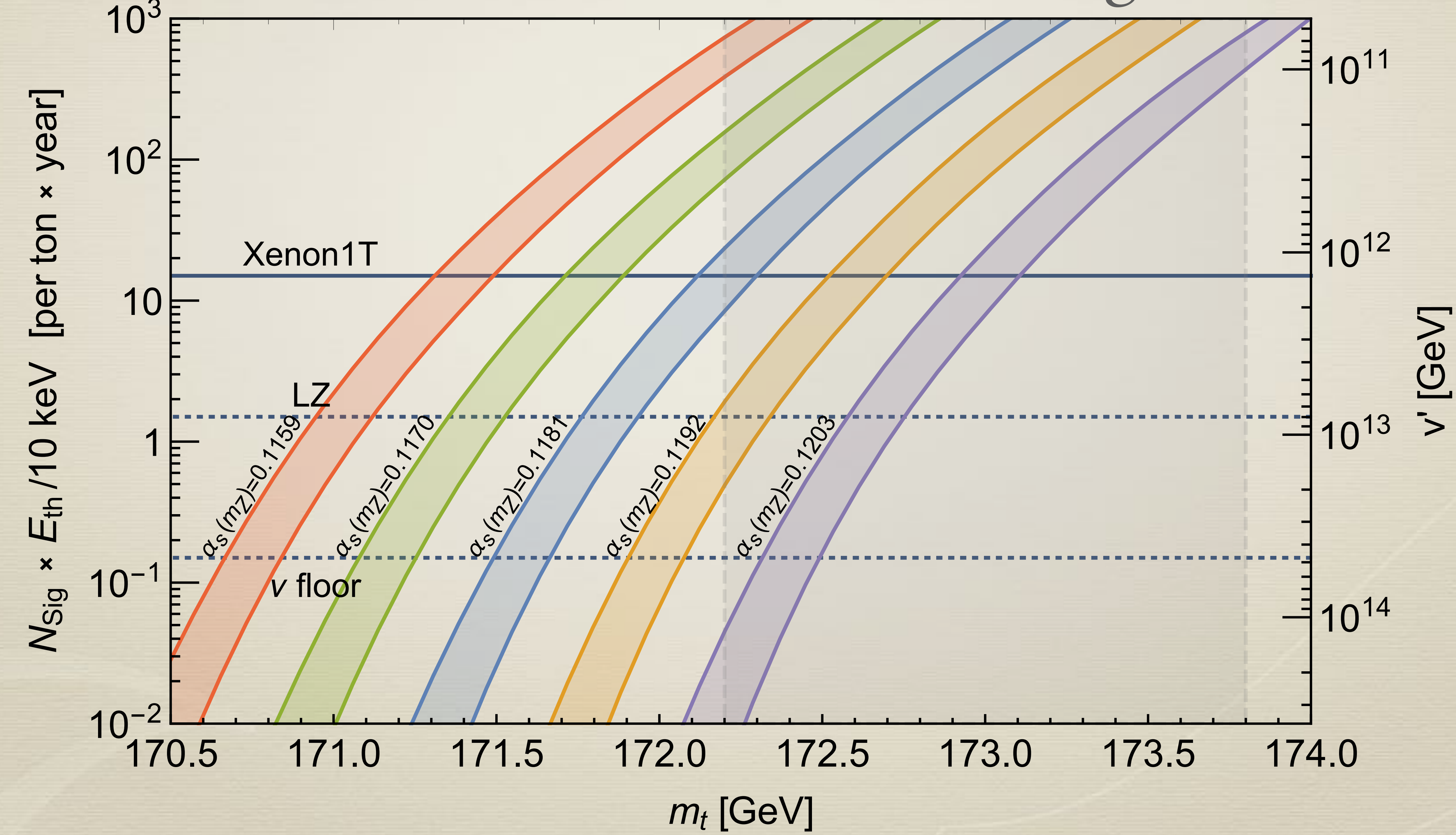
Direct detection rate for Xe targets



SM parameters and DM

Dunsky, Hall, KH (2019)

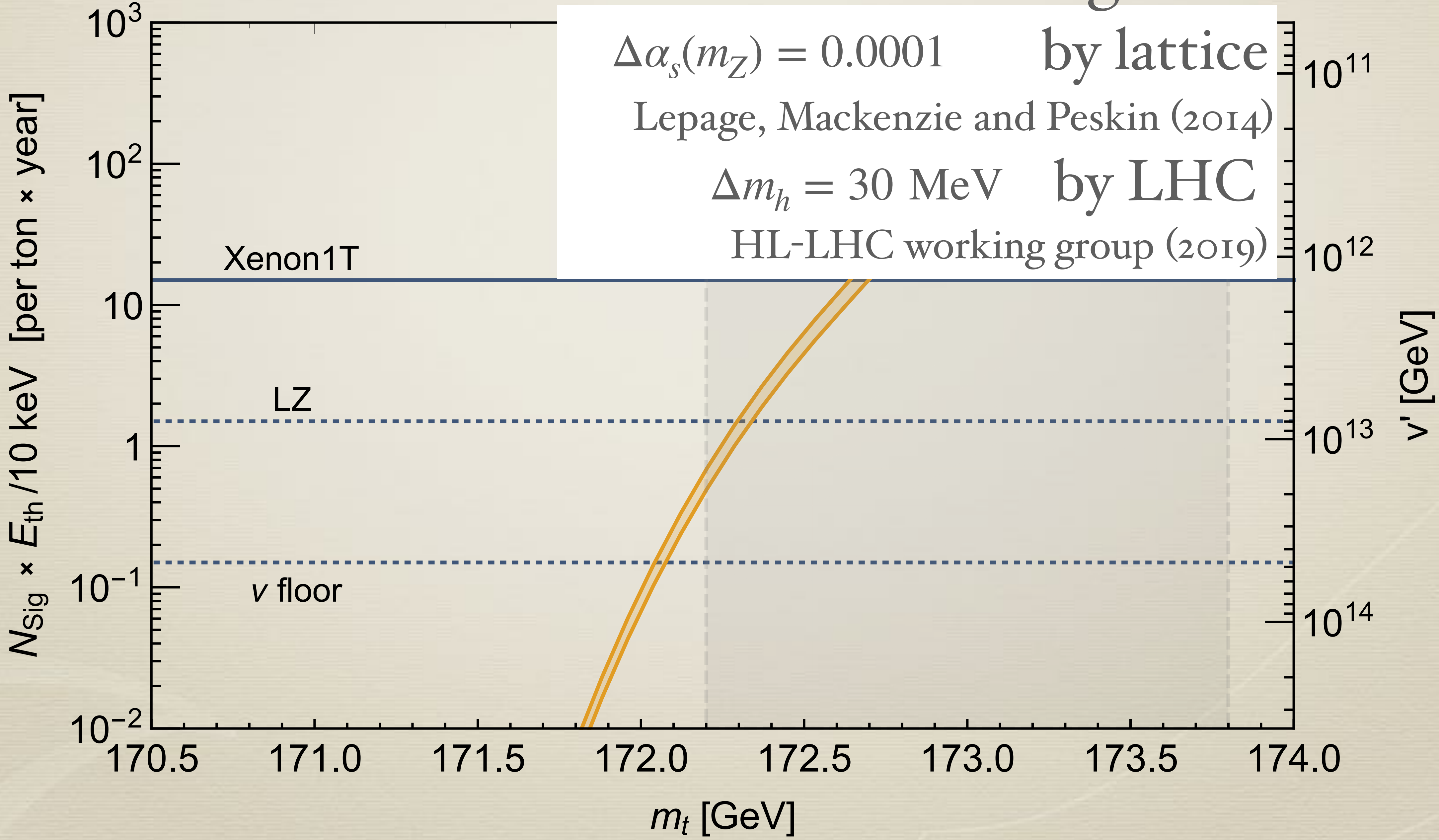
Direct detection rate for Xe targets



SM parameters and DM

Dunsky, Hall, KH (2019)

Direct detection rate for Xe targets



$\Delta\alpha_s(m_Z) = 0.0001$ by lattice

Lepage, Mackenzie and Peskin (2014)

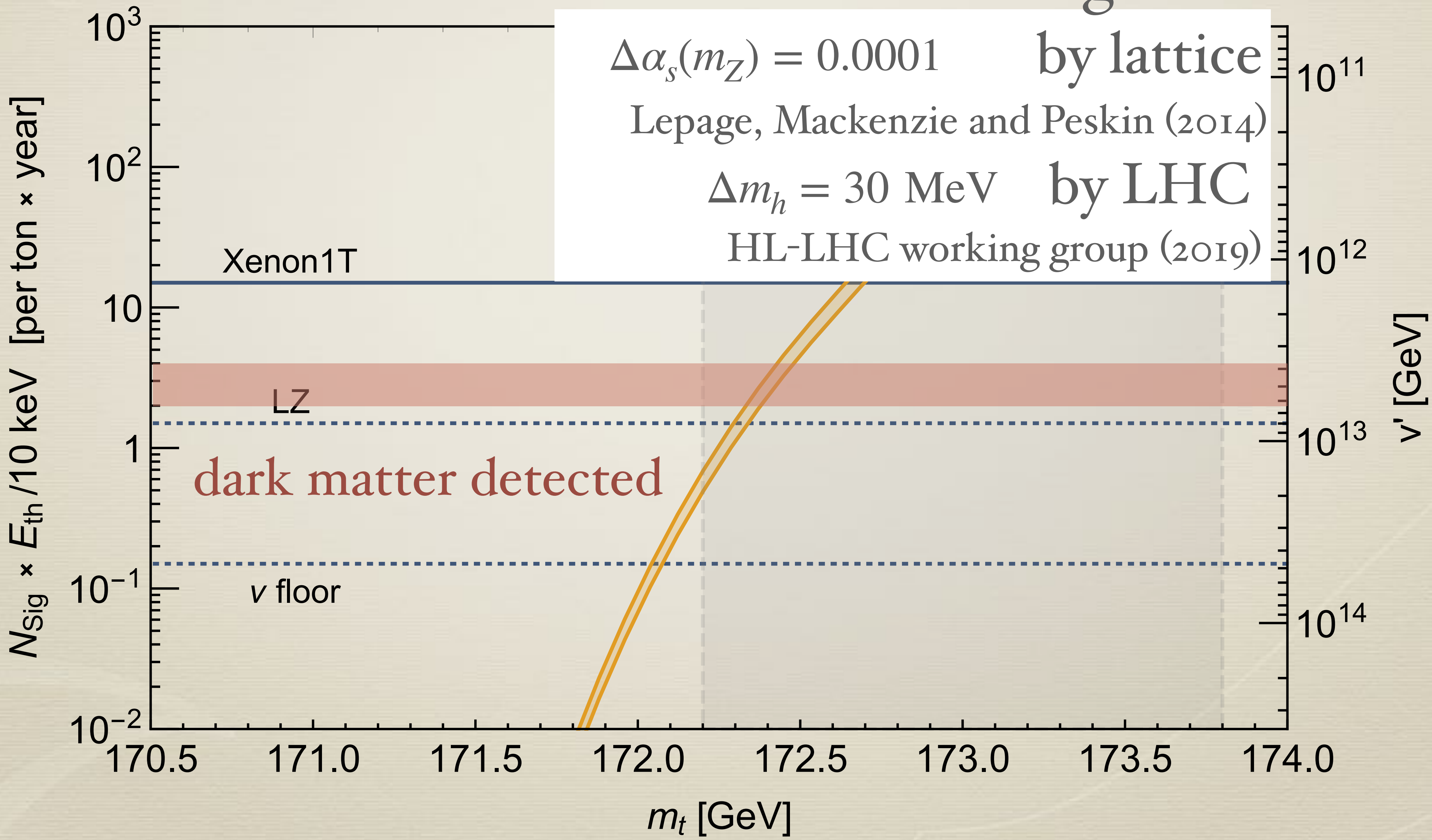
$\Delta m_h = 30$ MeV by LHC

HL-LHC working group (2019)

SM parameters and DM

Dunsky, Hall, KH (2019)

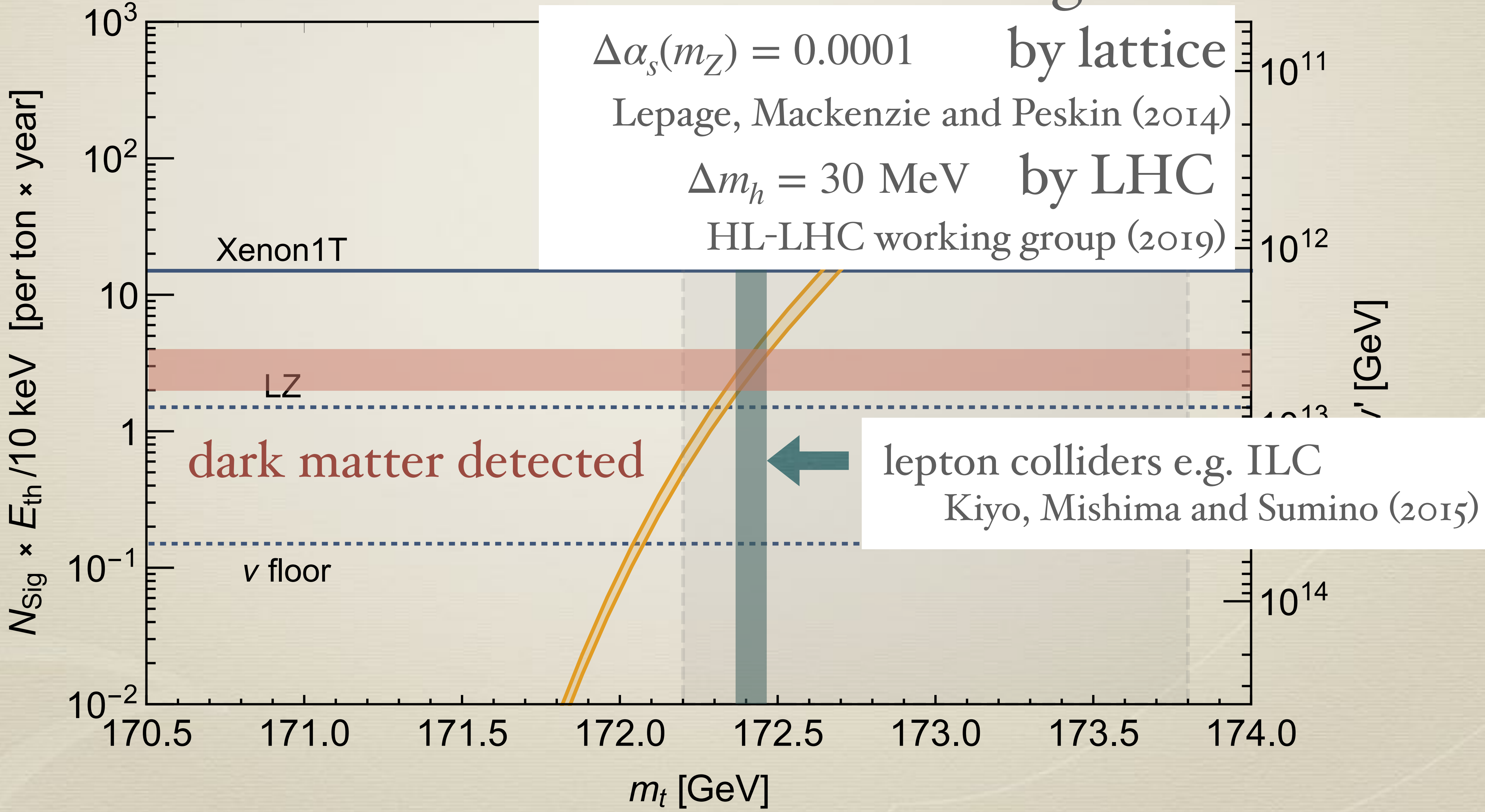
Direct detection rate for Xe targets



SM parameters and DM

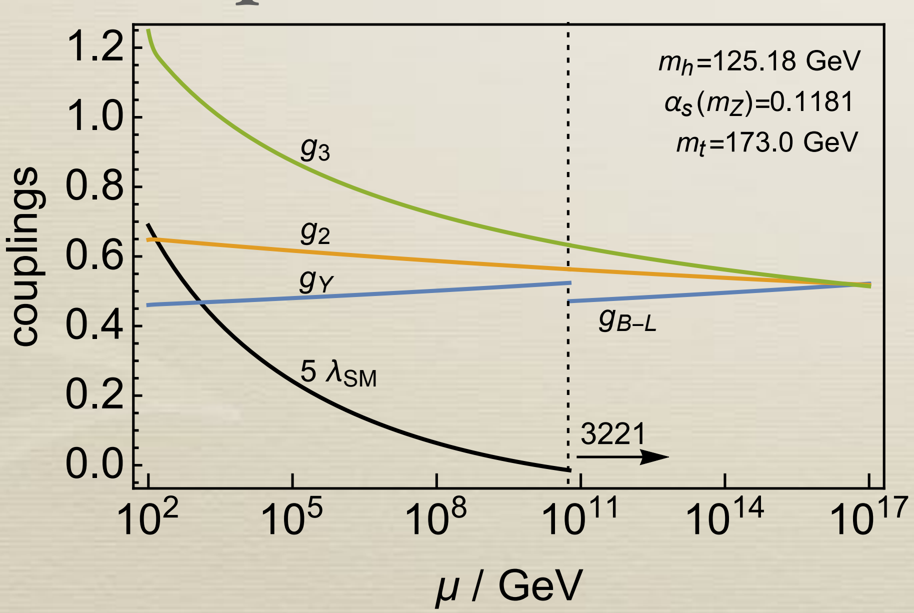
Dunsky, Hall, KH (2019)

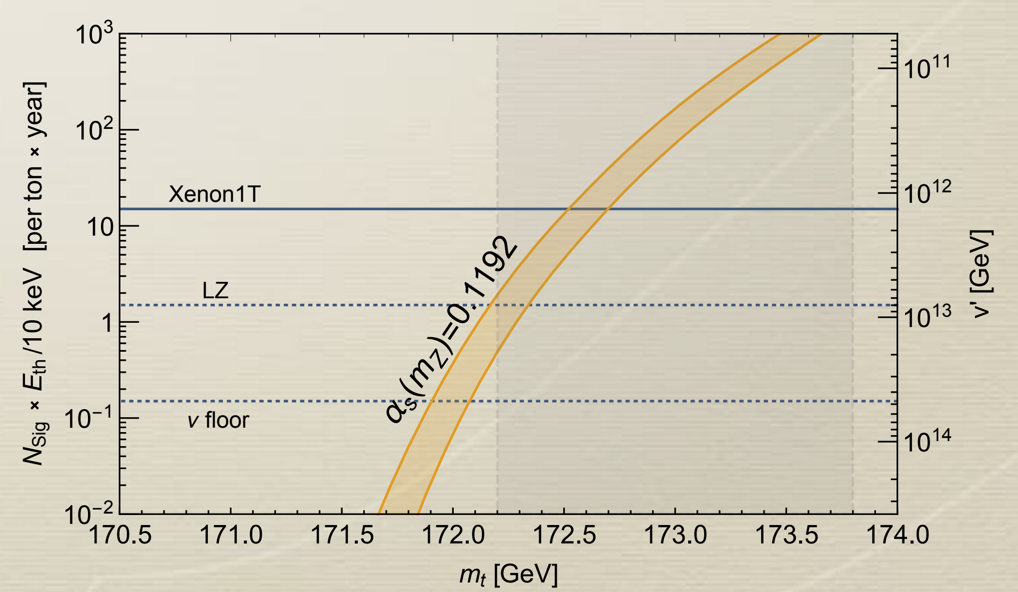
Direct detection rate for Xe targets



Summary

- Parity can solve the strong CP problem
- In models with minimal Higgs content, the SM Higgs quartic coupling vanishes at the scale where exact parity is spontaneously broken
- The small SM Higgs quartic coupling at UV is explained. Possible experimental signals are correlated with SM parameters





Future of colliders

We should maximize the impact of future colliders

* Searches for new particles
* Searches for deviation from the standard model predictions

$$N_{\text{events}} = N_{\text{SM prediction}} \;?$$

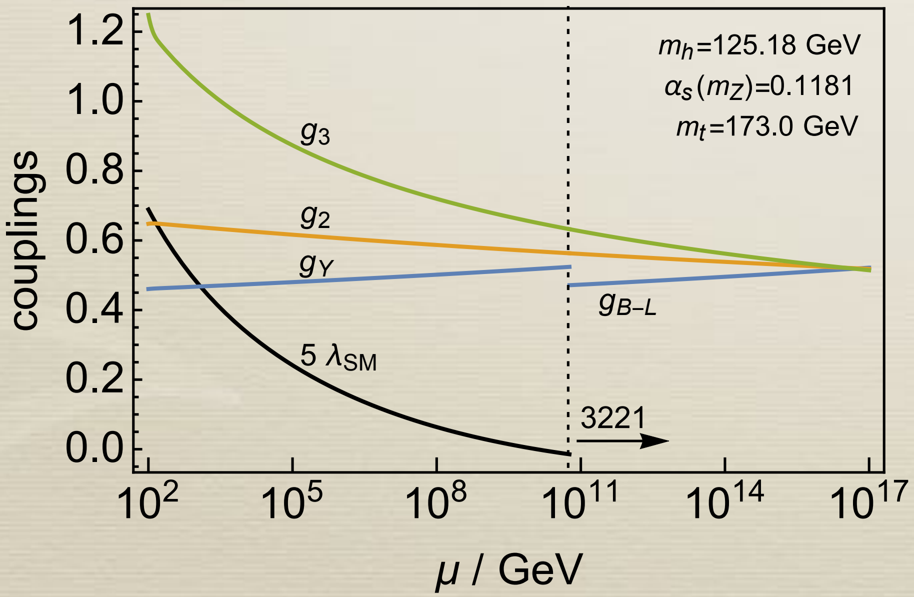
* Precise measurements of standard model parameters

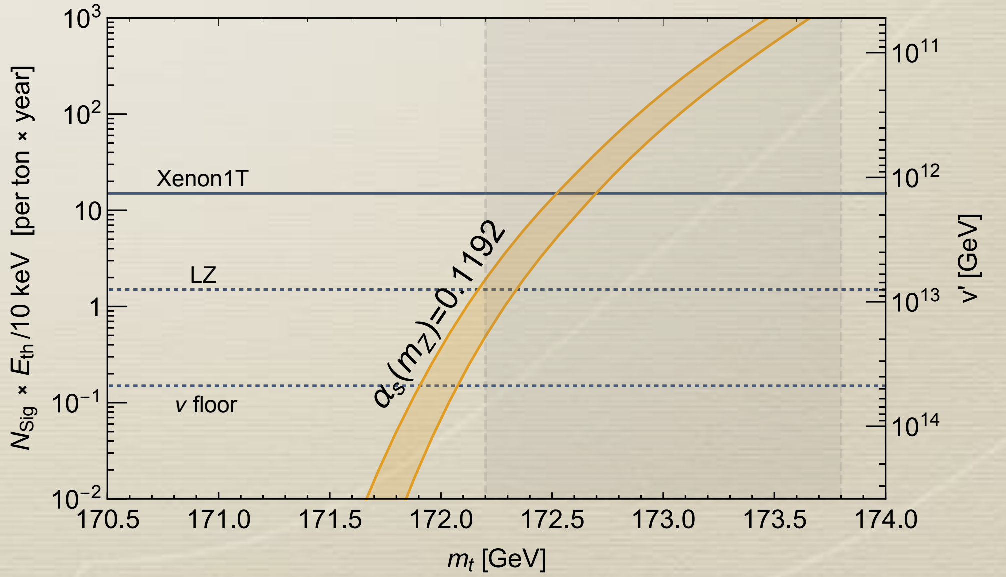
top quark mass,
strong coupling constant,
Higgs mass, etc.

Any other new physics models
impacted by precise measurements of parameters?

Summary

- Parity can solve the strong CP problem
- In models with minimal Higgs content, the SM Higgs quartic coupling vanishes at the scale where exact parity is spontaneously broken
- The small SM Higgs quartic coupling at UV is explained. Possible experimental signals are correlated with SM parameters





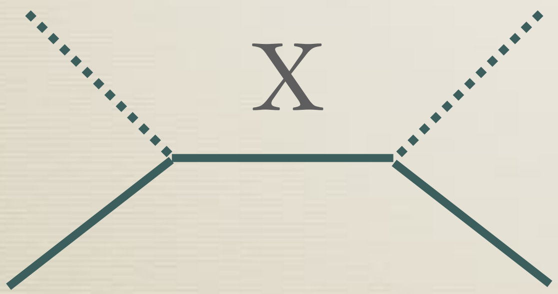
Back up

GUT, yukawa

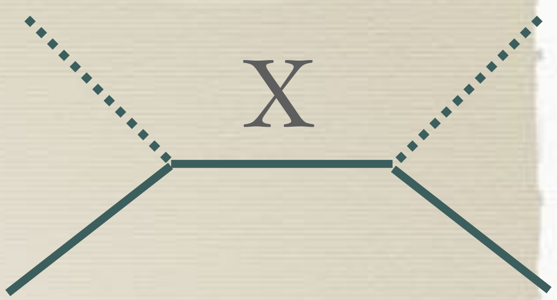
Yukawa interaction

No gauge invariant renormalizable coupling

$$\frac{c_{ij}}{M} H H' q_L \bar{q}_R$$



Yukawa couplings



Small enough not to blow up the gauge coupling

	$SU(3)_c$	$SU(2)_L$	$SU(2)_R$	$U(1)$	$SU(4)$	$SO(10)$	coupling
up	**3**	**1**	**1**	2/3	**15**	**45**	$\bar{X}qH^\dagger + Xq'H'^\dagger$
	3	**2**	**2**	−1/3	**6/10**	**45, 54, 210/210**	$\bar{X}qH'^\dagger + Xq'H^\dagger$
down	**3**	**1**	**1**	−1/3	**6/10**	**10, 126/120**	$\bar{X}qH + Xq'H'$
	3	**2**	**2**	2/3	**15**	**120, 126**	$\bar{X}qH' + Xq'H$
electron	**1**	**1**	**1**	−1	**10**	**120**	$\bar{X}\ell H + X\ell' H'$
	1	**2**	**2**	0	**1/15**	**10, 120/120, 126**	$X\ell H' + X\ell' H$
neutrino	**1**	**1**	**1**	0	**1/15**	**1, 54, 210/45, 210**	$X(\ell H^\dagger + \ell' H'^\dagger)$
	1	**2**	**2**	−1	**10**	**210**	$\bar{X}\ell H'^\dagger + X\ell' H^\dagger$
	1	**3**	**1**	0	**1**	**45**	$X\ell H^\dagger$
	1	**1**	**3**	0	**1**	**45**	$X\ell' H'^\dagger$

SO(10) embedding

Hall, KH (2018)

$$q, \ell, q', \ell' = 16$$

$$H, H' \subset 16_H$$

$$q(t,x) \leftrightarrow \bar{q}'(t,x)$$

C: Part of SO(10)

$$q(t,x) \leftrightarrow i\sigma_2 q^*(t,-x)$$

CP

$$q(t,x) \leftrightarrow i\sigma_2 \bar{q}'^*(t,-x)$$

$$SO(10) \times CP \xrightarrow{\phi_{45}^-} SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P_{LR}$$

CKM phase

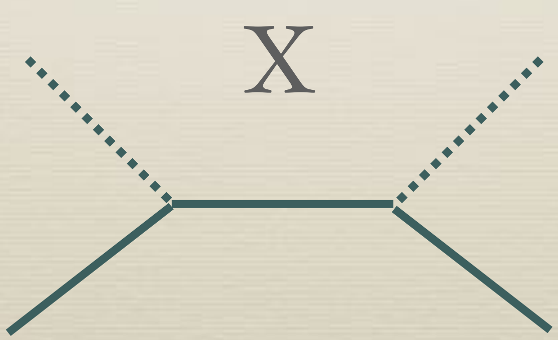
$$SO(10) \times CP \xrightarrow{\phi_{45}^-} SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P_{LR}$$

Real yukawas without CP symmetry breaking...

A simple renormalizable example to obtain CP phases

$$\mathcal{L} = \left(M^{ij} + i\lambda^{ij}\phi_{45}\right) X_{10,i} X_{10,j}$$

$$\frac{c_{ij}}{M} H H' q_i q'_j$$



up-down non-unification

Hall, KH (2018)

down-type

$$x_d 16_f 16_H 10_f + M_d 10_f 10_f$$
$$= x\left(q\bar{D}H_L + \bar{q}DH_R\right) + MD\bar{D}$$

up-type

$$x_u 16_f 45_f 16_H^\dagger + \frac{1}{2} M_u 45_f^2$$
$$\supset x_u\left(q\bar{Q}H_R^\dagger + \bar{q}QH_L^\dagger\right) + M_u Q\bar{Q}$$

→ No unification

bottom-tau unification

Hall, KH (2018)

$$x_d 16_f 16_H 10_f + M_d 10_f 10_f$$

$\Delta(1,2,2,0)$

$$= x_d \left(q\bar{D}H_L + \bar{q}DH_R + \ell\Delta H_L + \bar{\ell}\Delta H_R \right) + M_d \left(D\bar{D} + \frac{1}{2}\Delta^2 \right)$$

$y_b = y_\tau$ at high energy scales?

Left-handed quark is in general linear combination of

$q(3,2,1,1/6) \subset 16$ $\qquad$ $Q(3,2,2,-1/3) \subset 45,54$

$$x_u 16_f 45_f 16_H^\dagger + \frac{1}{2} M_u 45_f^2$$

$$\supset x_u \left(q\bar{Q}H_R^\dagger + \bar{q}QH_L^\dagger \right) + M_u Q\bar{Q}$$

→ change bottom/tau by an O(1) factor

up and neutrino

Hall, KH (2018)

$$x_u 16_f 45_f 16_H^\dagger + \frac{1}{2} M_u 45_f^2$$

$$\supset x_u \left(q\bar{Q}H_R^\dagger + \bar{q}QH_L^\dagger + \ell T_L H_L^\dagger + \bar{\ell} T_R H_R^\dagger \right) + M_u \left(Q\bar{Q} + \frac{1}{2}T_L^2 + \frac{1}{2}T_R^2 \right)$$

Why aren't neutrinos also as hierarchical as up-quarks?

$$x_d 16_f 16_H 10_f + M_d 10_f 10_f$$

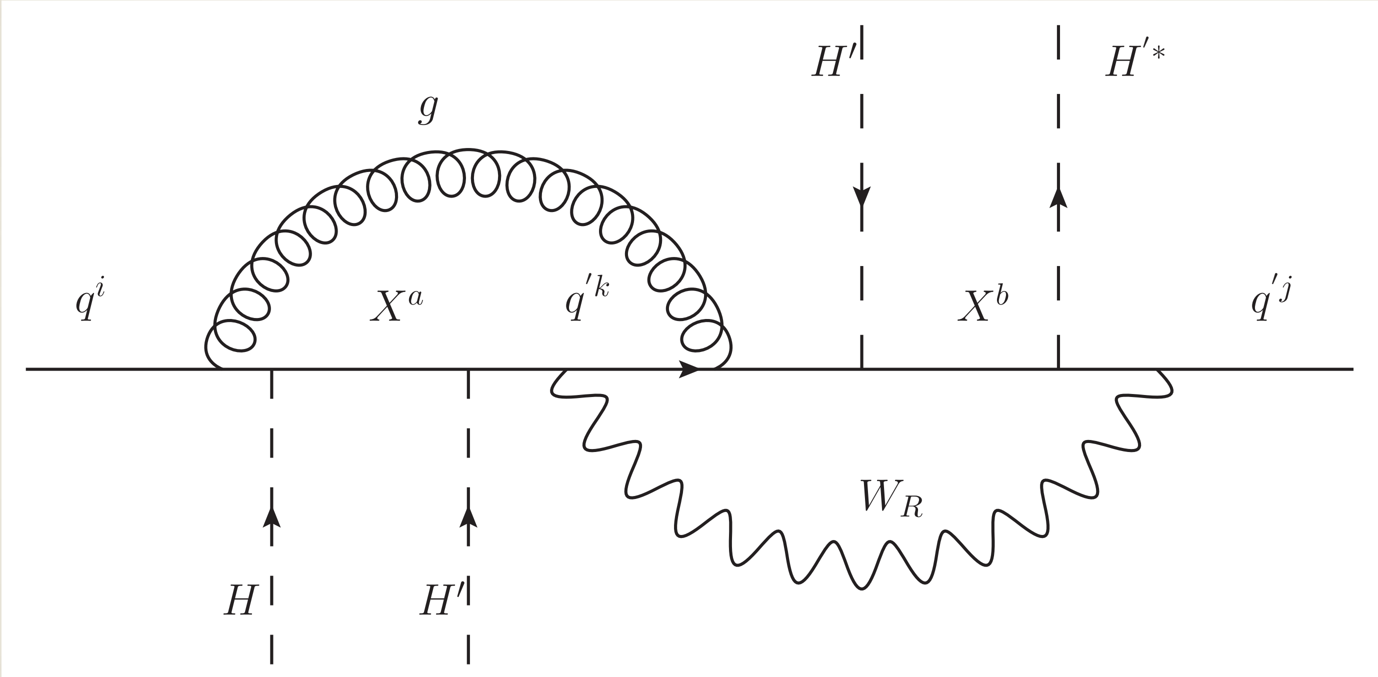
$\Delta(1,2,2,0)$

$$= x \left(\ell \Delta H_R + \bar{\ell} \Delta H_L \right) + \frac{1}{2} M_d \Delta^2$$

If $M_d \ll x_d v_R$, the left-handed neutrino is mainly from Δ

Non-zero CPV

Hall, KH (2018)

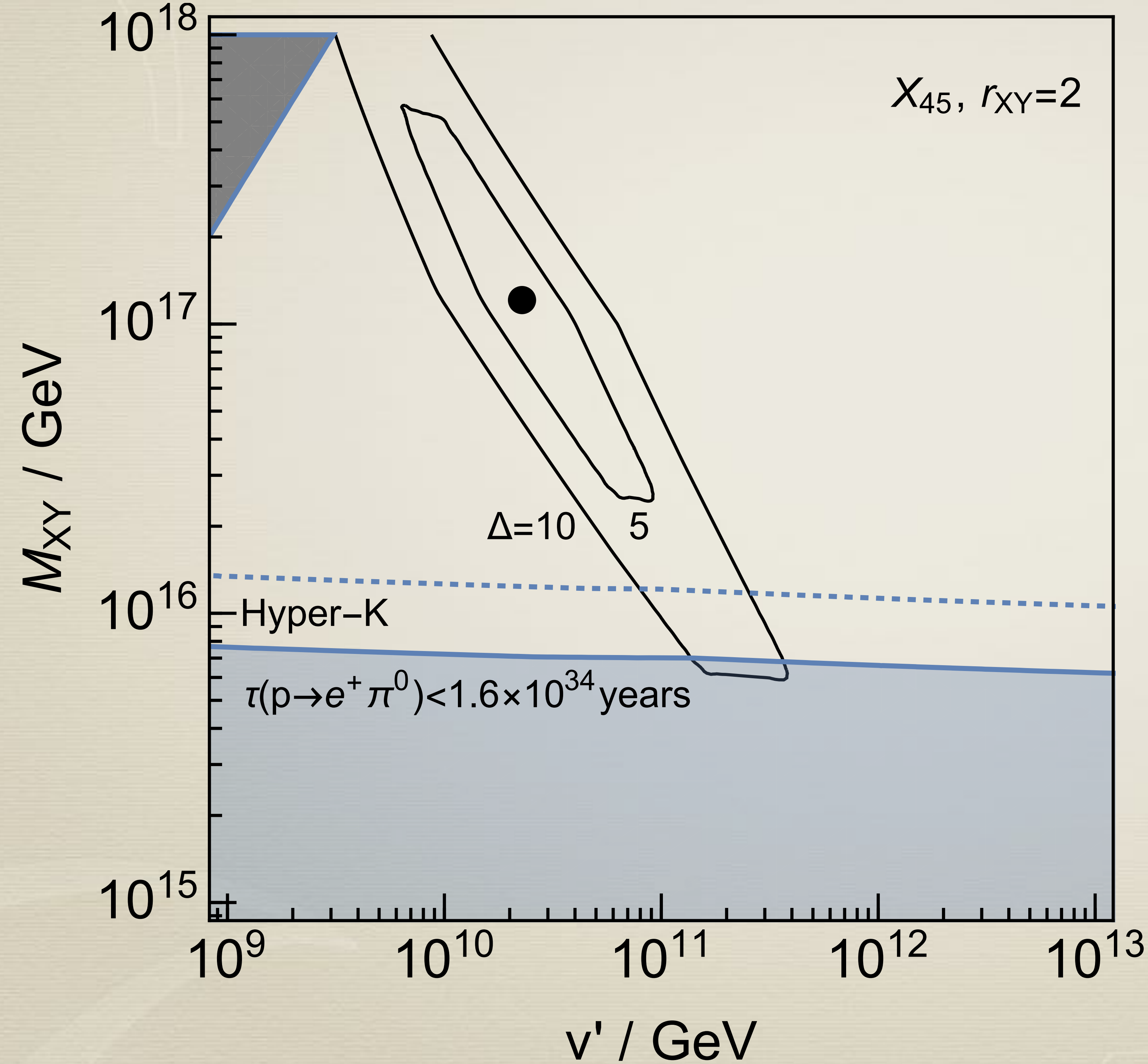


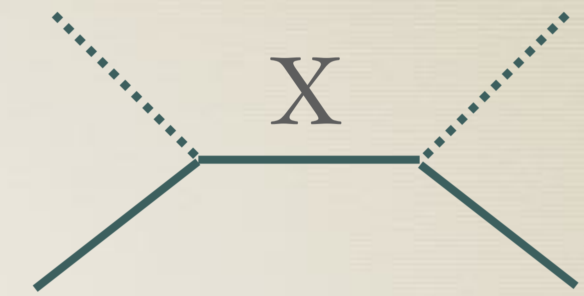
$$\delta\theta \sim 10^{-11} \frac{\theta^u_{23}\theta^d_{23}}{V^2_{cb}}$$

Suppressed by loop factors, flavor mixing

GUT, threshold corrections

Coupling unification





$$r_{XY} = \frac{M_{(3,1,1)}}{M_{XY}}$$

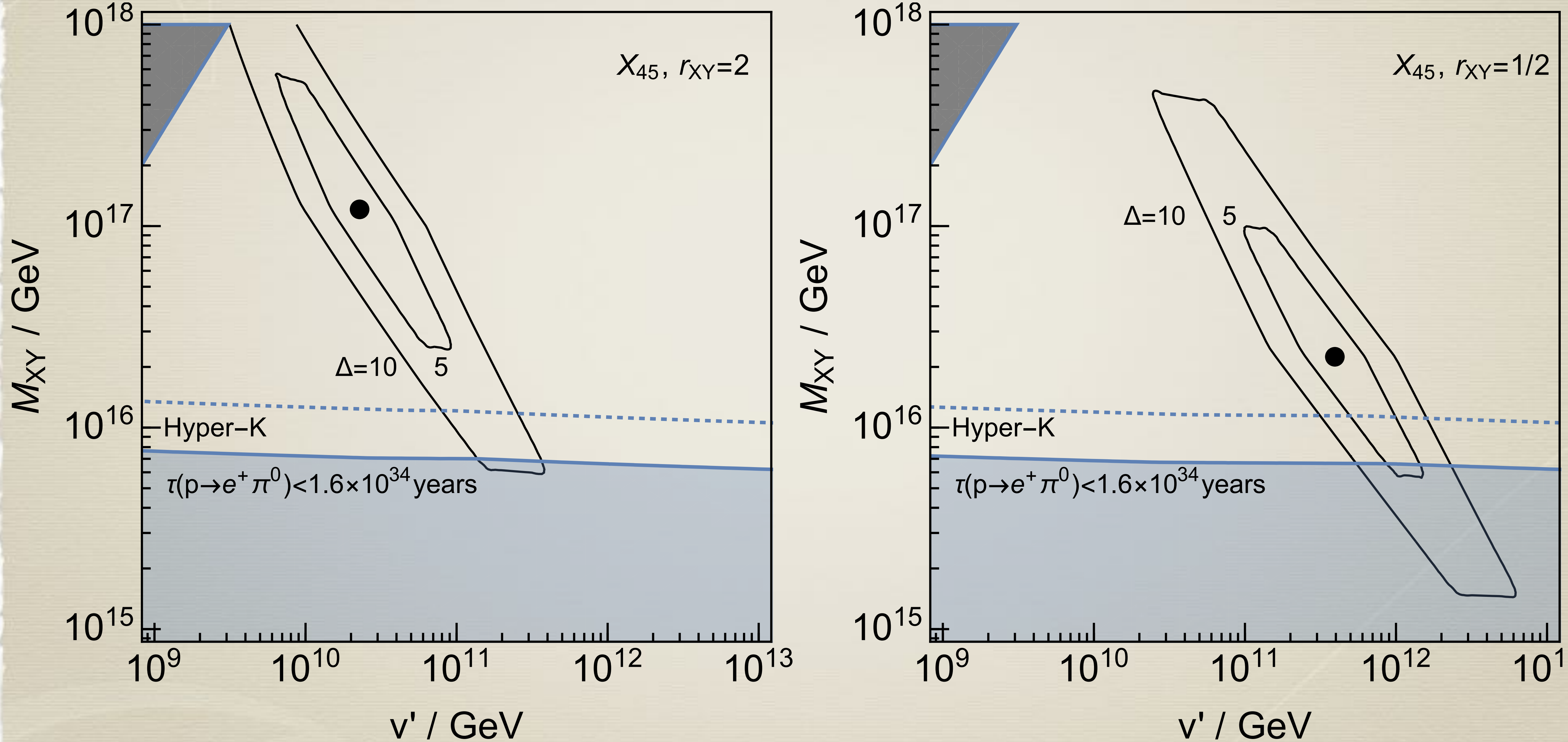
(=2 in the minimal model)

$$\Delta \sim \delta \left(\frac{2\pi}{\alpha} \right)$$

~ Casimir operator ×
Log(mass splitting)

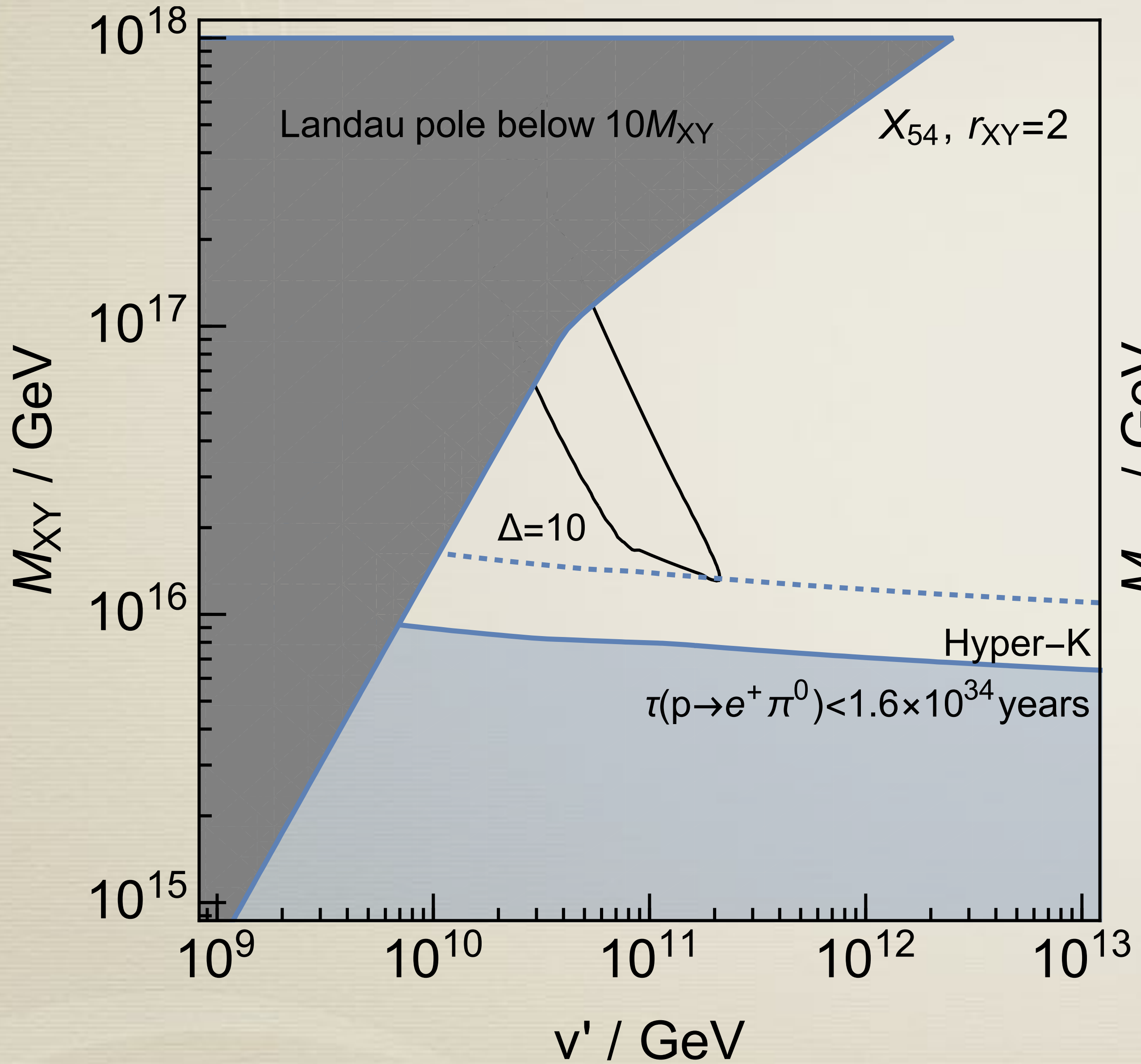
Hall, KH (2019)

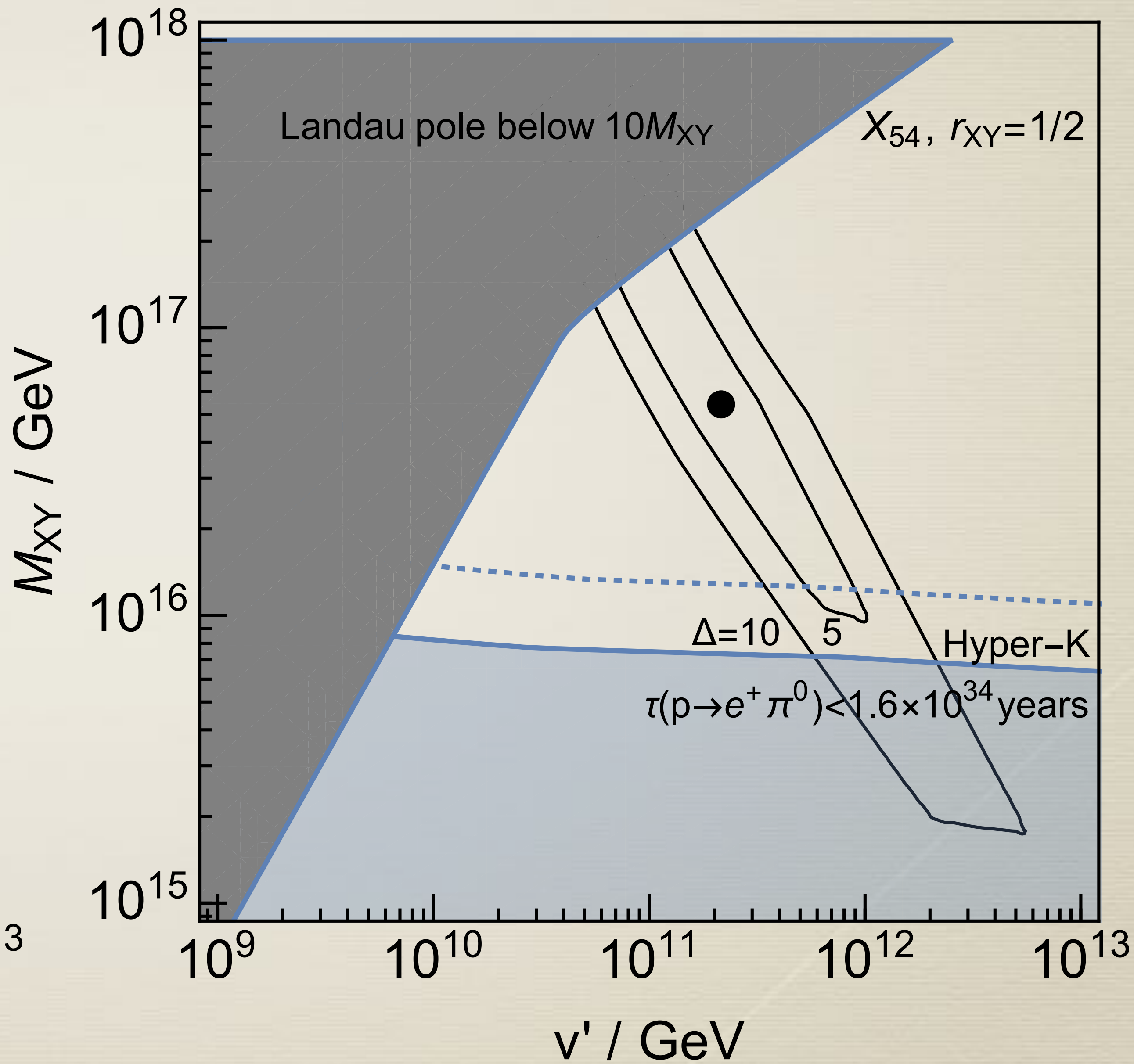
Coupling unification



Hall, KH (2019)

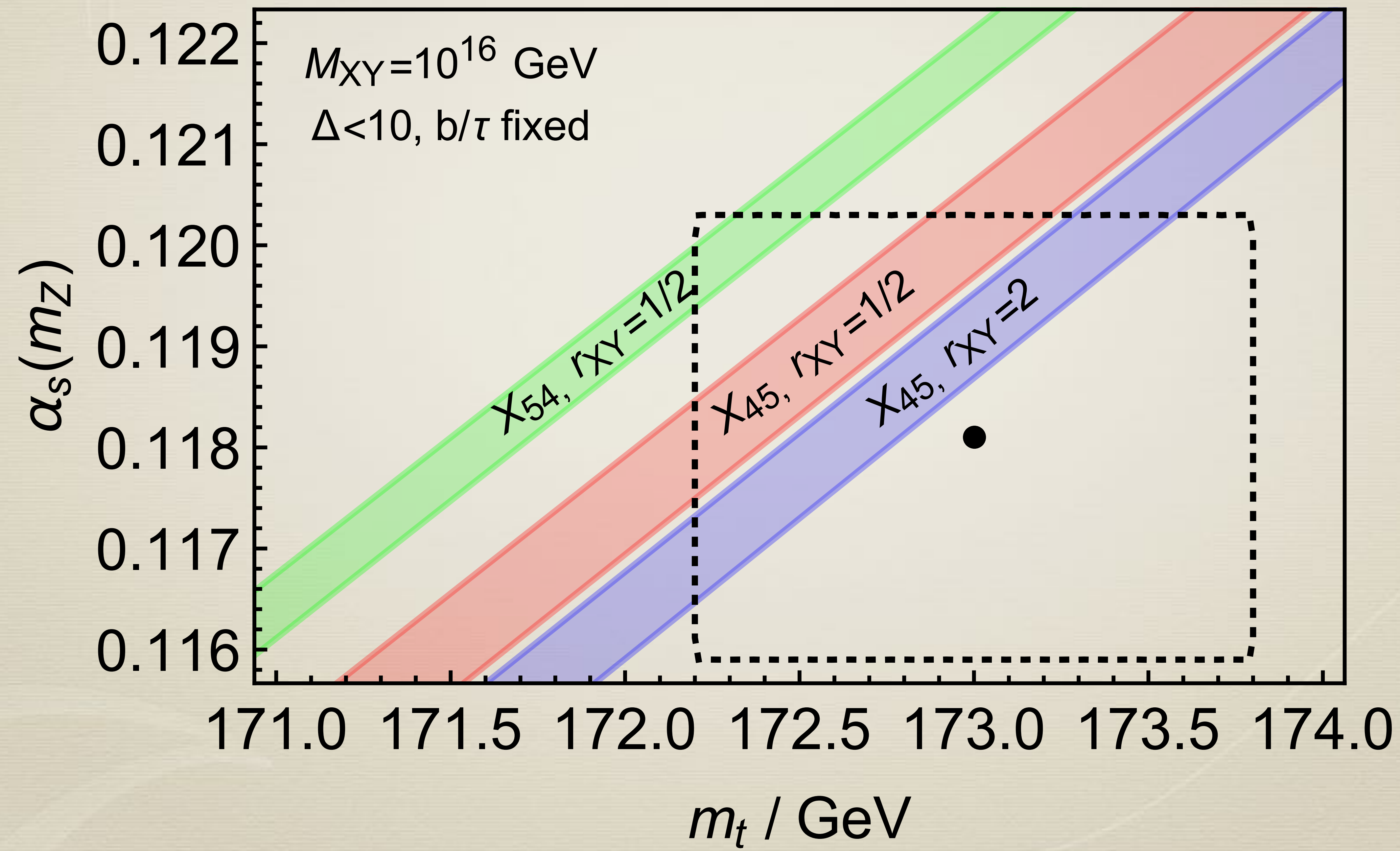
Coupling unification



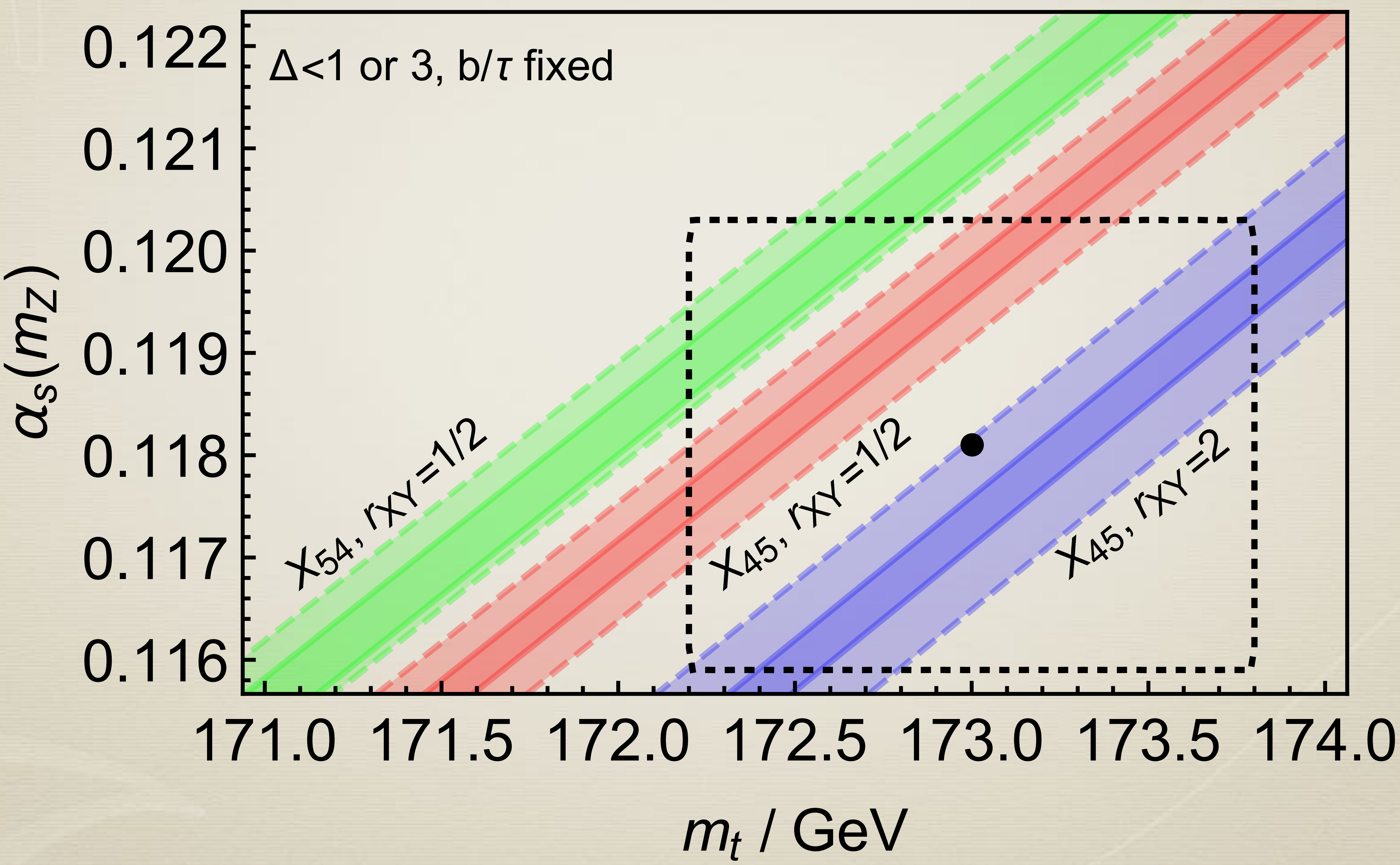


Hall, KH (2019)

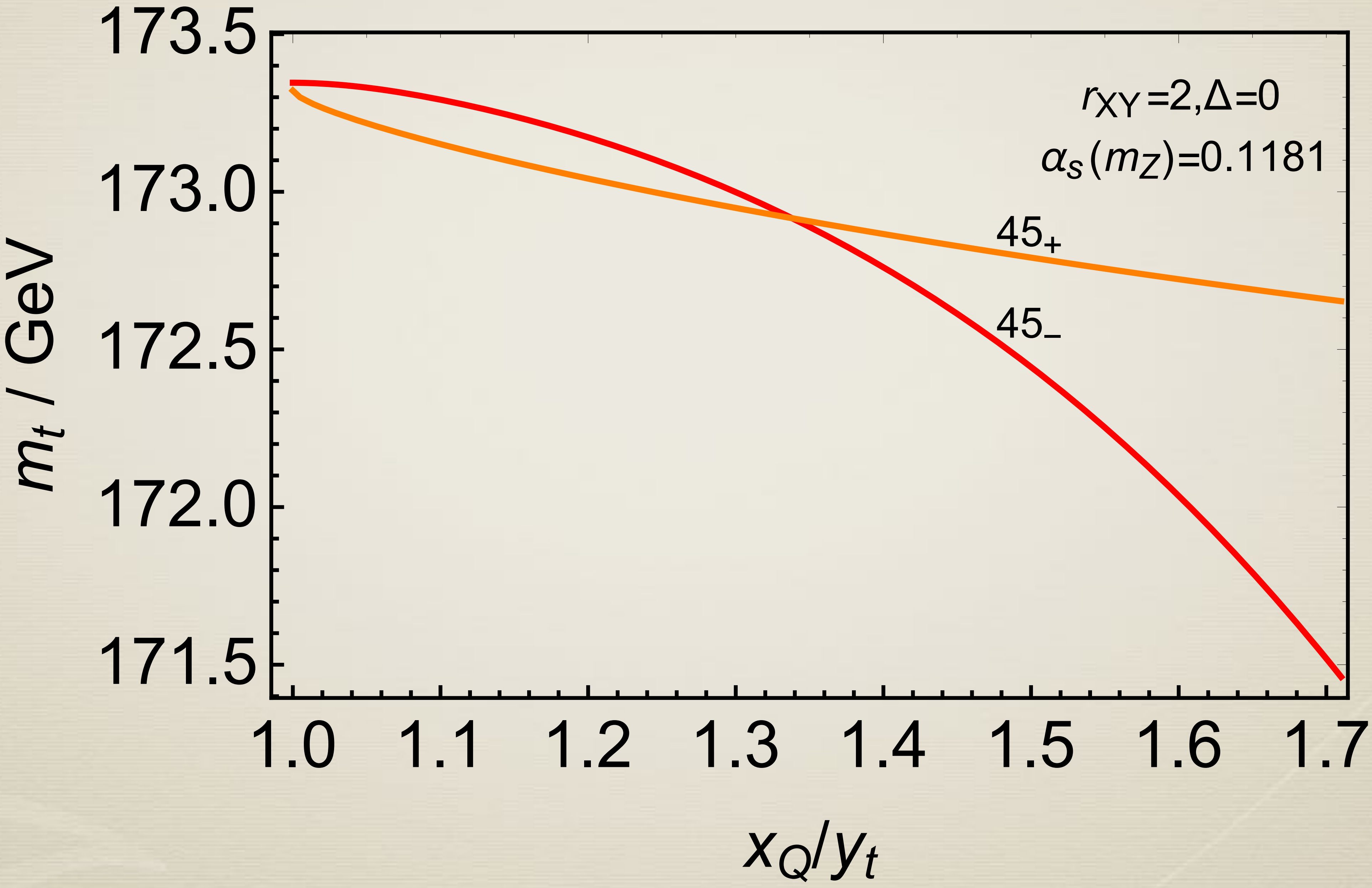
Proton decay observed



Small threshold corrections



W/o b/τ unification



Correction to the gauge coupling unification by high dimensional operator

$$SO(10) \xrightarrow{\phi_{210}} SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times C_{LR}$$

$$\frac{210^{abcd}}{M_*} F_{10}^{ab} F_{10}^{cd} \qquad \Delta\left(\frac{2\pi}{\alpha}\right) \lesssim 10$$

$$SO(10) \times CP \xrightarrow{\phi_{45}} SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P_{LR}$$

$$\frac{45^{ac}}{M_*} \frac{45^{bd}}{M_*} F_{10}^{ab} F_{10}^{cd} \qquad \Delta\left(\frac{2\pi}{\alpha}\right) \lesssim 1$$

Correction to the gauge coupling unification by high dimensional operator

$$SO(10) \xrightarrow{\phi_{54}} SU(4) \times SU(2)_L \times SU(2)_R \times C_{LR}$$

$$\frac{54^{ab}}{M_*} F_{10}^{ac} F_{10}^{bc} \qquad \Delta\left(\frac{2\pi}{\alpha}\right) \lesssim 1$$

$$SO(10) \times CP \xrightarrow{\phi_{210}} SU(4) \times SU(2)_L \times SU(2)_R \times P_{LR}$$

$$\frac{210}{M_*}\frac{210}{M_*} F_{10} F_{10} \qquad \Delta\left(\frac{2\pi}{\alpha}\right) \ll 1$$

DM

e' without u'

Dunsky, Hall, KH (2019)

$T \ll m_{u'}$ and

Inflaton → e'ebar', ~~u'ubar'~~

e.g. by

$$2m_{e'} < \phi < 2m_{u'}$$

Other possibilities?
Baryogenesis for low temperature?

Misc.

Nelson-Barr mechanism

Nelson (1984), Barr (1984)

$$\begin{pmatrix} Q & u \end{pmatrix} \begin{pmatrix} 0 & yH \\ M_1 & i\phi \end{pmatrix} \begin{pmatrix} \bar{u}_1 \\ \bar{u}_2 \end{pmatrix}$$

SM right-handed quarks are linear combination of $\bar{u}_1$ and $\bar{u}_2$