Classical Cosmological Collider Physics

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QMAP Particles/Cosmology Seminar
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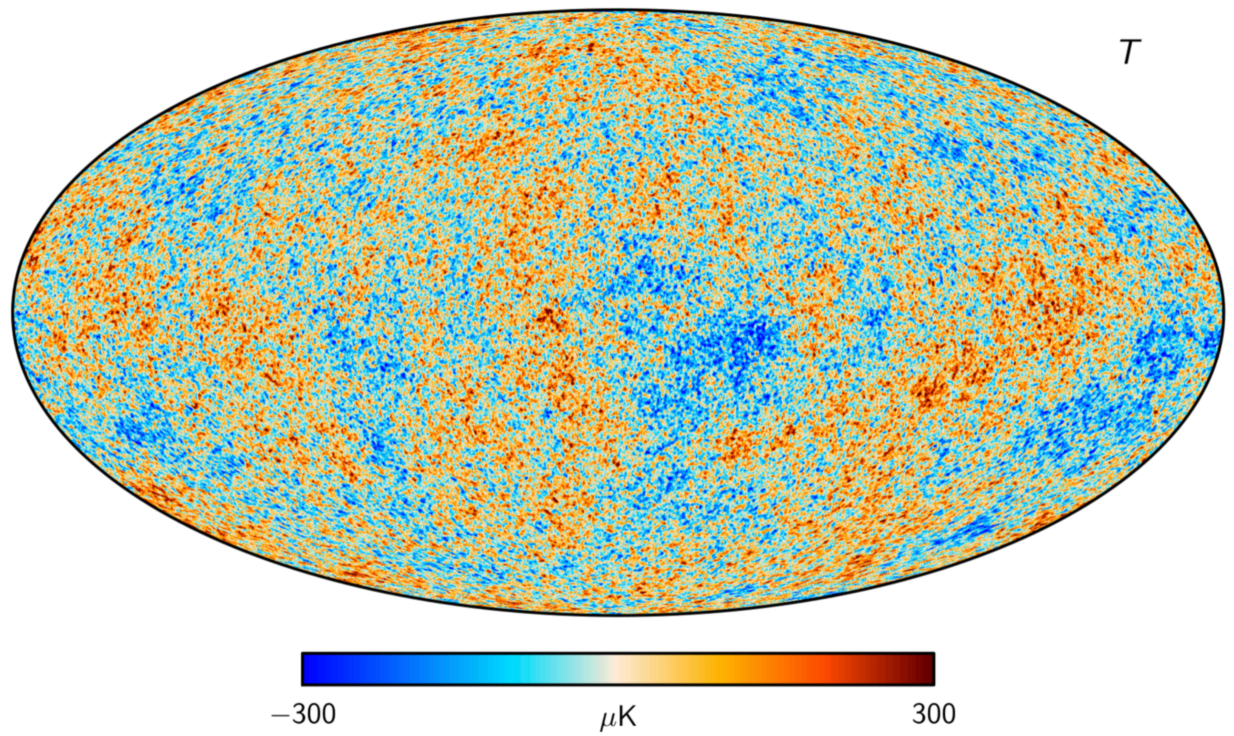
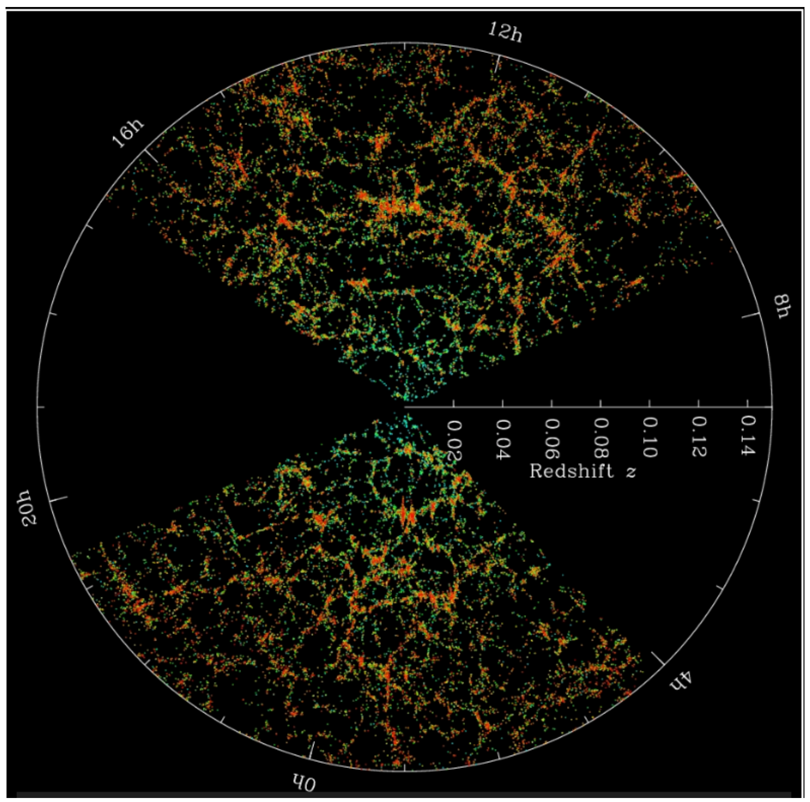
Universe at large and small scales

- Universe is extremely uniform on largest scales, but there are small inhomogeneities as well.



CMB Planck



LSS SDSS

- Origin of these inhomogeneities?

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Cosmic Inflation

- One of the leading paradigms to explain inhomogeneities is cosmic inflation.

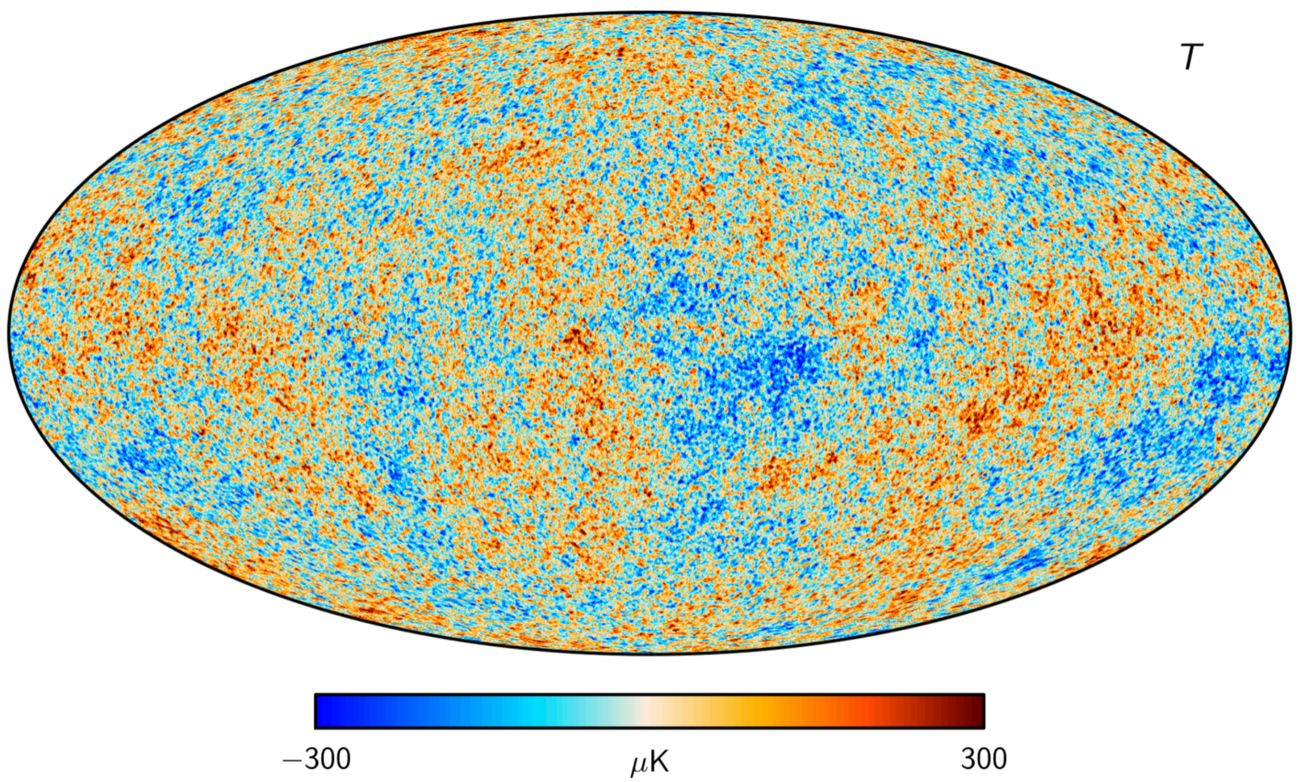
- Rapid expansion of spacetime and creation of classical density perturbations from quantum mechanical fluctuations.

- Excitingly, the Hubble scale during inflation can be as high as 5×10^{13} GeV!

From primordial to the CMB era

$$\delta T_{\text{CMB}} \sim \boxed{\text{Transfer function}} \times \boxed{\mathscr{R}}$$

known (ΛCDM) focus



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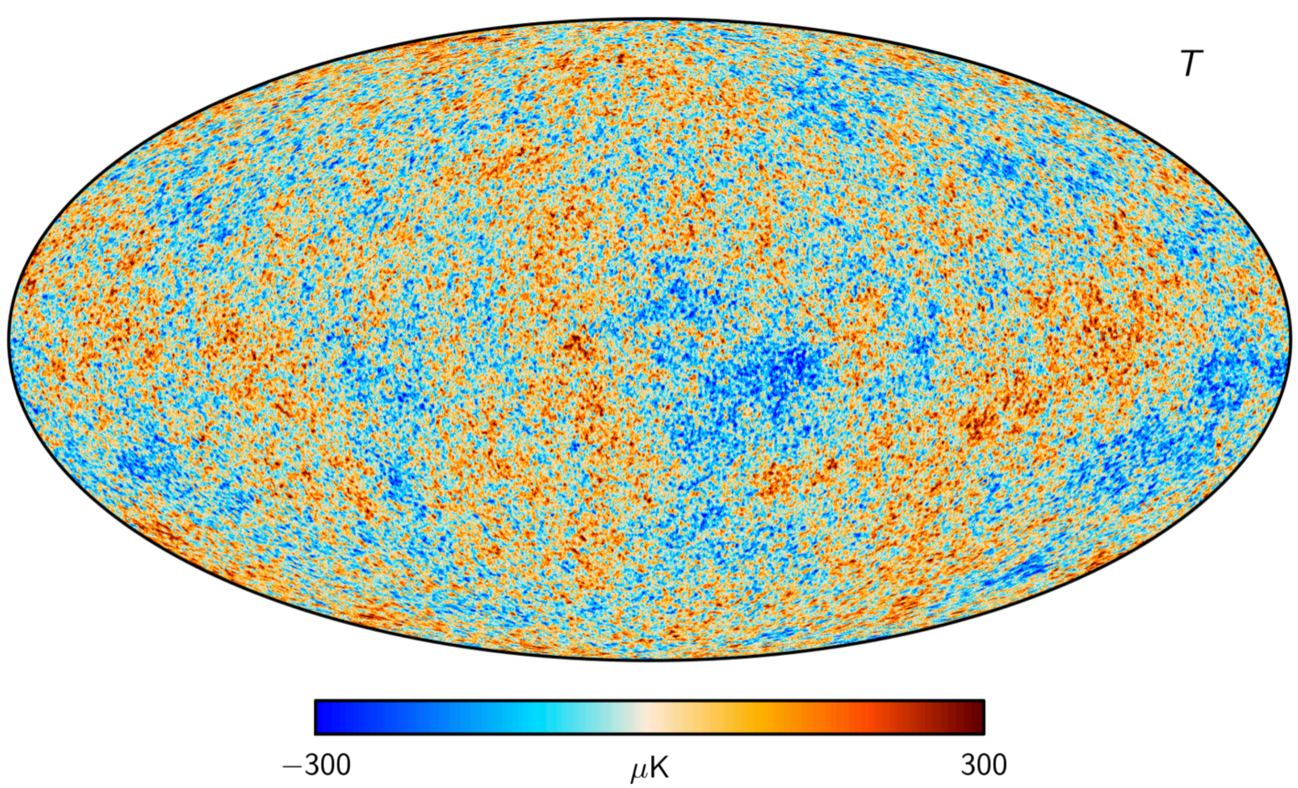
From primordial to the CMB era

$$\delta T_{\text{CMB}} \sim \boxed{\text{Transfer function}} \times \boxed{\mathscr{R}}$$

known (ΛCDM) focus

- A non-zero correlation $\langle \mathscr{R}(\overrightarrow{k_1})\mathscr{R}(\overrightarrow{k_2})\mathscr{R}(\overrightarrow{k_3})\cdots\rangle$ gives rise to a non-zero $\langle \delta T(\hat{n}_1)\delta T(\hat{n}_2)\delta T(\hat{n}_3)\cdots\rangle$



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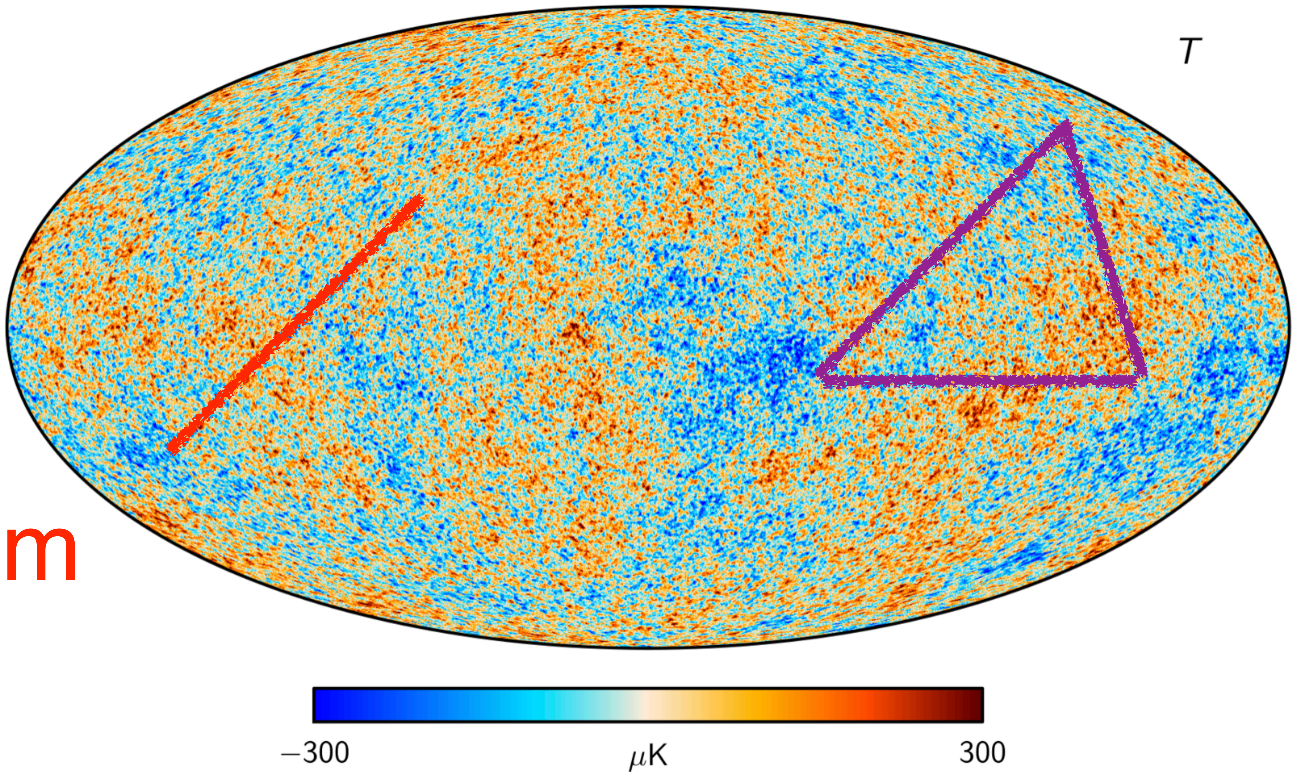
power spectrum



bispectrum

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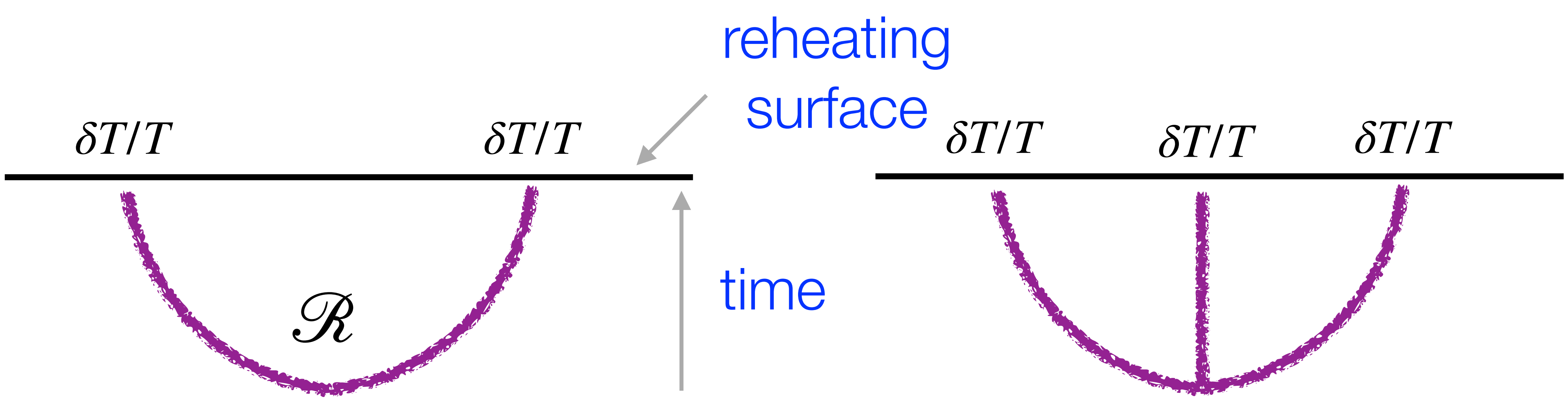
How do we compute $\langle \mathscr{R}(\overrightarrow{k_1})\mathscr{R}(\overrightarrow{k_2})\mathscr{R}(\overrightarrow{k_3})\cdots\rangle$?

- A quantum mechanical 'in-in' expectation value computed at reheating with $Q = \mathscr{R}(\overrightarrow{k_1})\mathscr{R}(\overrightarrow{k_2})\mathscr{R}(\overrightarrow{k_3})\cdots$

$$\langle\Omega|U(t_f,t_i)^\dagger Q U(t_f,t_i)|\Omega\rangle = \langle 0|\bar{T}e^{+i\int_{-\infty(1+i\epsilon)}^{t_f} dt_2 \mathsf{H}_I^{\text{int}}(t_2)} Q_I(t_f) T e^{-i\int_{-\infty(1-i\epsilon)}^{t_f} dt_1 \mathsf{H}_I^{\text{int}}(t_1)}|0\rangle.$$

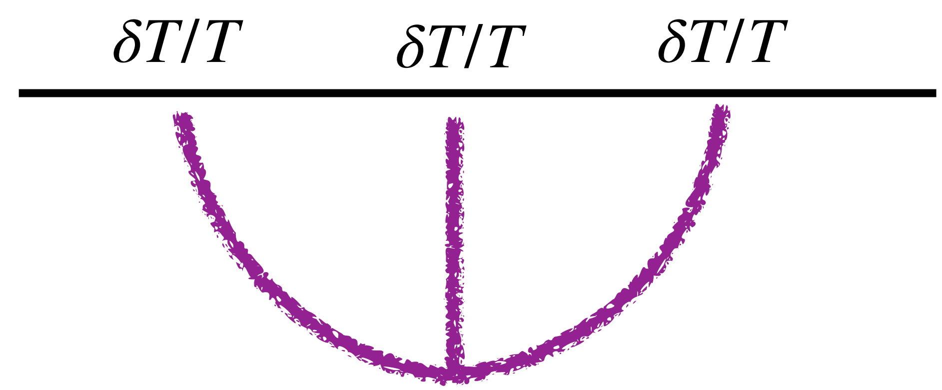
Weinberg, '05

- Diagrammatically,



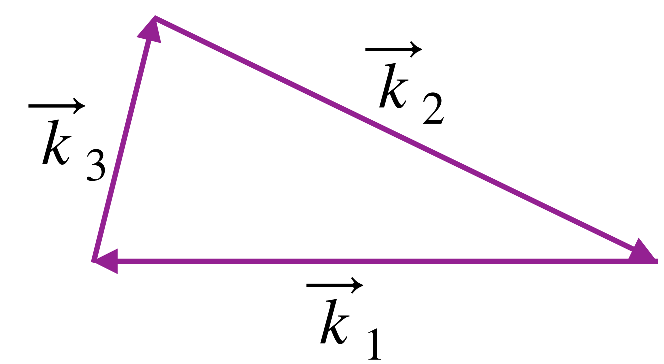
Bispectrum: definitions and notations

- Dimensionless measure of non-gaussianity (NG):



$$F(k_1, k_2, k_3) \equiv \frac{\langle \mathscr{R}(\vec{k}_1)\mathscr{R}(\vec{k}_2)\mathscr{R}(\vec{k}_3)\rangle'}{\langle \mathscr{R}(\vec{k}_1)\mathscr{R}(-\vec{k}_1)\rangle'\langle \mathscr{R}(\vec{k}_3)\mathscr{R}(-\vec{k}_3)\rangle' + \text{perms}}\,.$$

$$f_{\text{NL}} = \frac{5}{18}F(k,k,k)$$



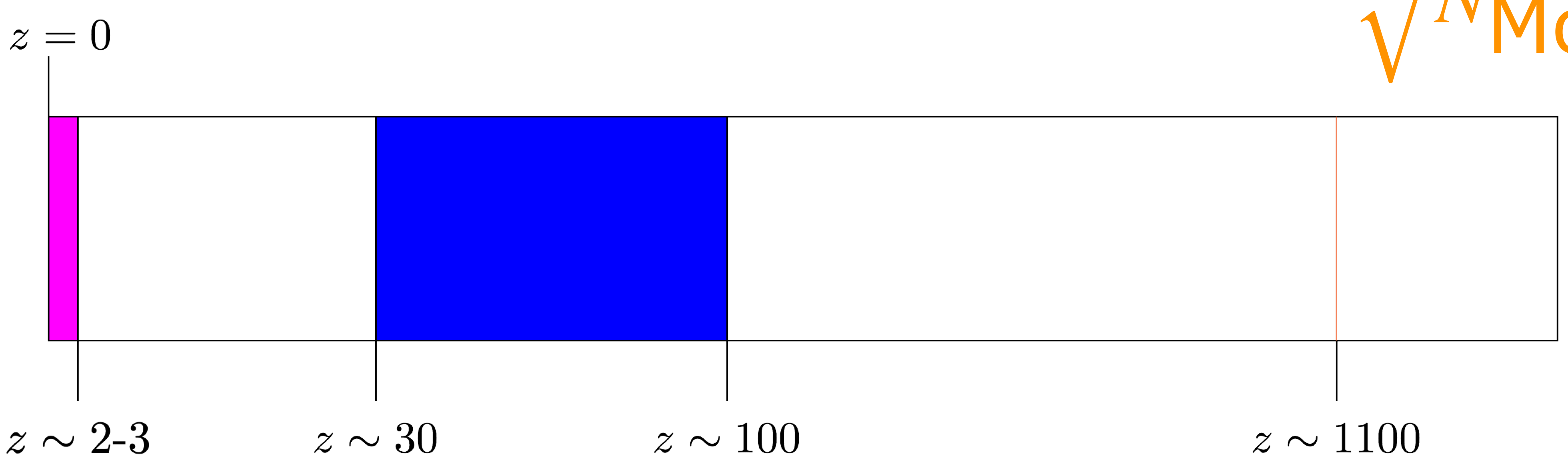
- $g\mathscr{R}^3$ interaction with $g \sim H$ i.e. maximally strong coupling implies $f_{\text{NL}} \sim 10^4$

Bispectrum: current and future

- Current bound from CMB roughly $|f_{\rm NL}| \lesssim \mathcal{O}(10)$

Planck '18

- Number of modes matter : $\delta f_{\rm NL} \sim \dfrac{10^4}{\sqrt{N_{\rm Modes}}}$



Alvarez et. al. '14

Loeb, Zaldarriaga '03

Planck: $N_{\rm modes} \sim 10^7 \Rightarrow \delta f_{\rm NL} \sim 10$ **Current bound**

LSS: $\delta f_{\rm NL} \sim 0.2$ **(SPHEREx)** 2025; ~ 1 **(Euclid, Vera Rubin Obs.)** 2023/24

21-cm Cosmology: $\delta f_{\rm NL} \sim 10^{-4} - 10^{-3}$ **20??**

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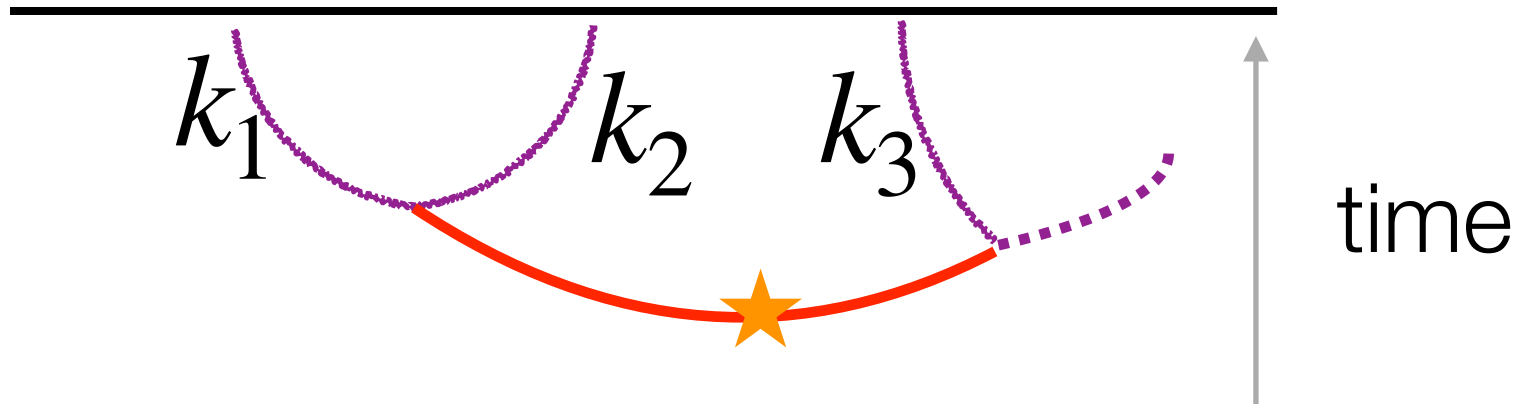
Cosmological collider physics

- H scale particles can get cosmologically produced during inflation and decay into inflaton fluctuations



$$\phi(t, \vec{x}) = \phi_0(t) + \delta\phi(t, \vec{x})$$

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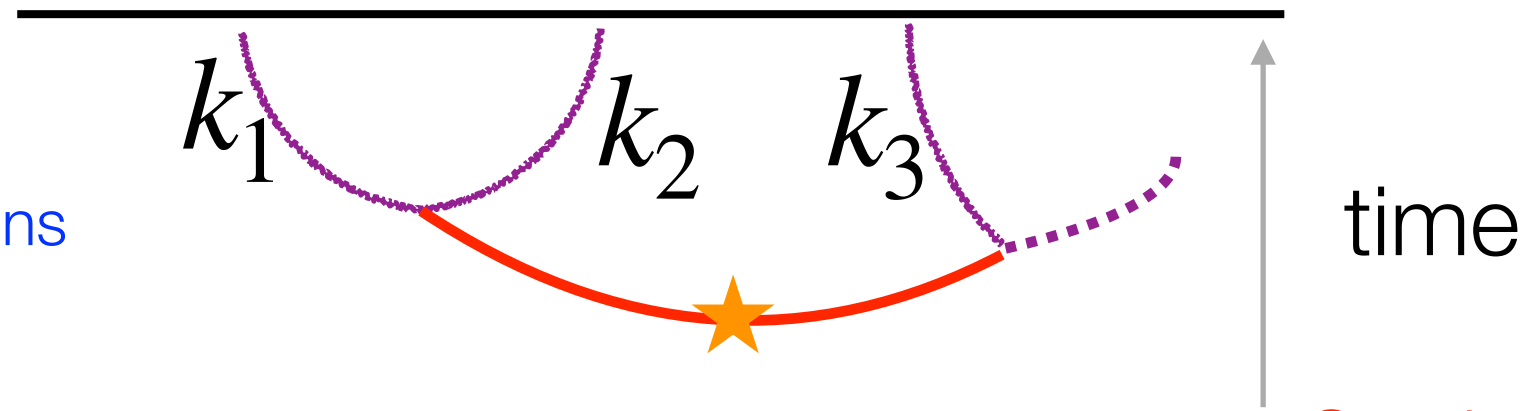
Cosmological collider physics



oscillations
e^{imt}

On-shell info!

squeezed limit
$k_3 \ll k_1$

$$F(k_1, k_2, k_3) \sim \left(\frac{k_3}{k_1}\right)^{\frac{3}{2}+i\mu} f_s(\cos\theta) \Leftrightarrow \frac{i}{s - m^2 + im\Gamma} P(\cos\theta)$$

mass

spin

$$\mu = \sqrt{\frac{m^2}{H^2} - \frac{9}{4}}$$

$$\theta = \hat{k}_1 \cdot \hat{k}_3$$

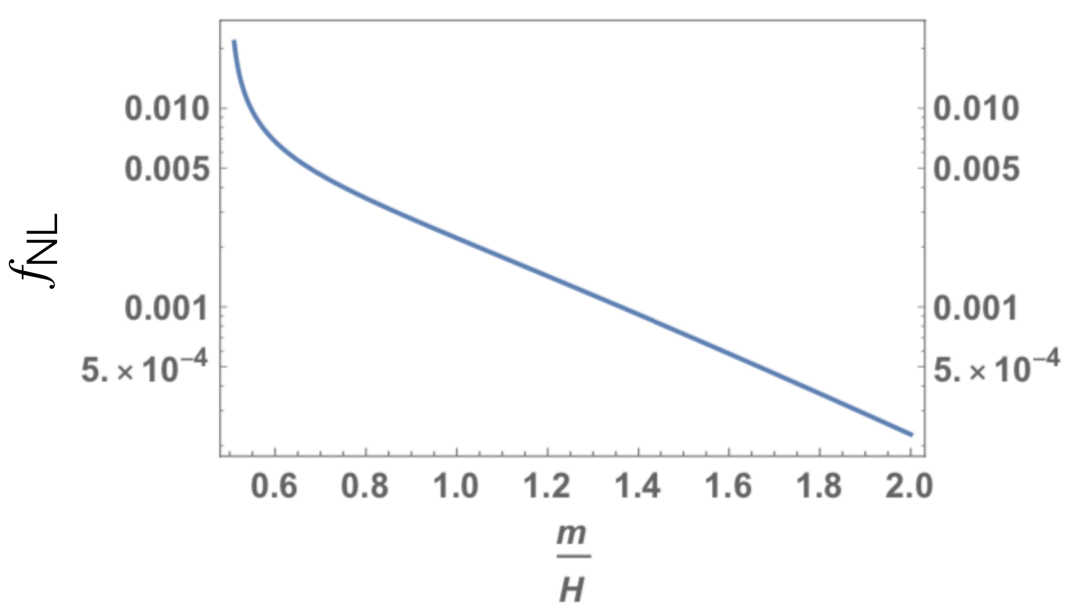
Chen, Wang ’09;
Arkani-Hamed, Maldacena ’15
+ many more

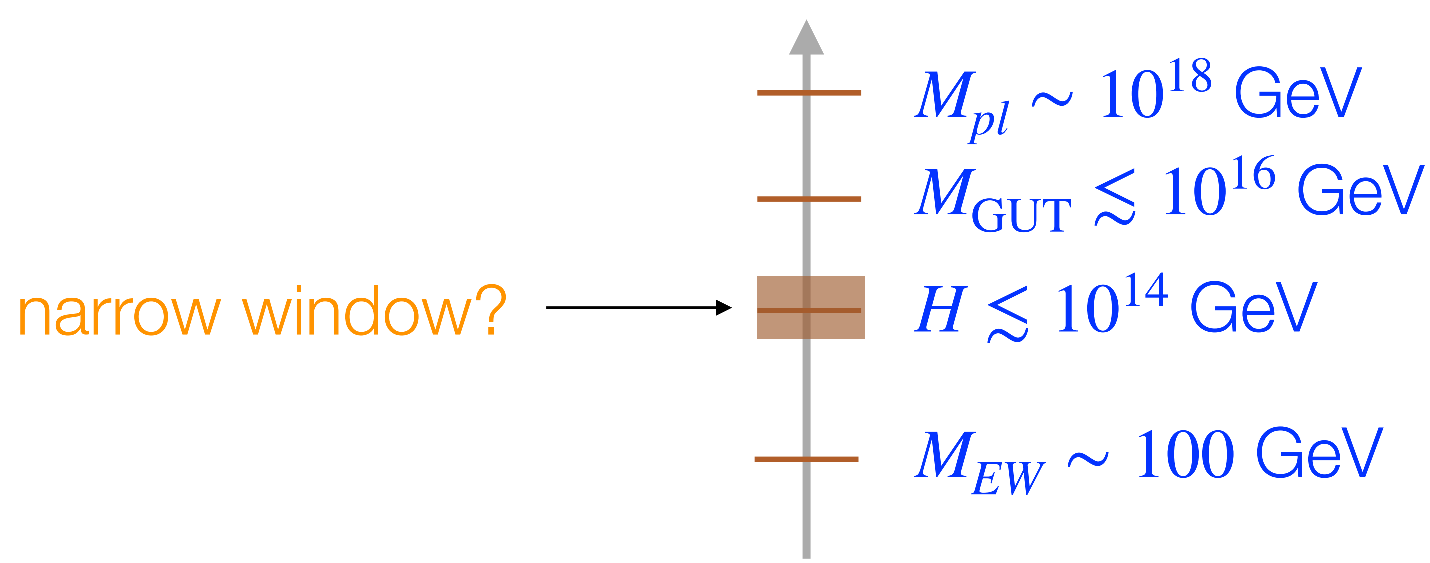
Window of opportunity

- Loss of non-analyticity for $m \ll H$
- "Boltzmann suppression" $e^{-\pi m/H}$ for $m \gg H$

$$F \sim \left(\frac{k_3}{k_1}\right)^{\frac{3}{2}+i\sqrt{\frac{m^2}{H^2}-\frac{9}{4}}}$$

≈ 0 for $m \ll H$





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BSM targets of the cosmological collider

- Rich particle physics candidates can lie around or above 10^{14} GeV, e.g., GUTs, right-handed neutrinos in see-saw …

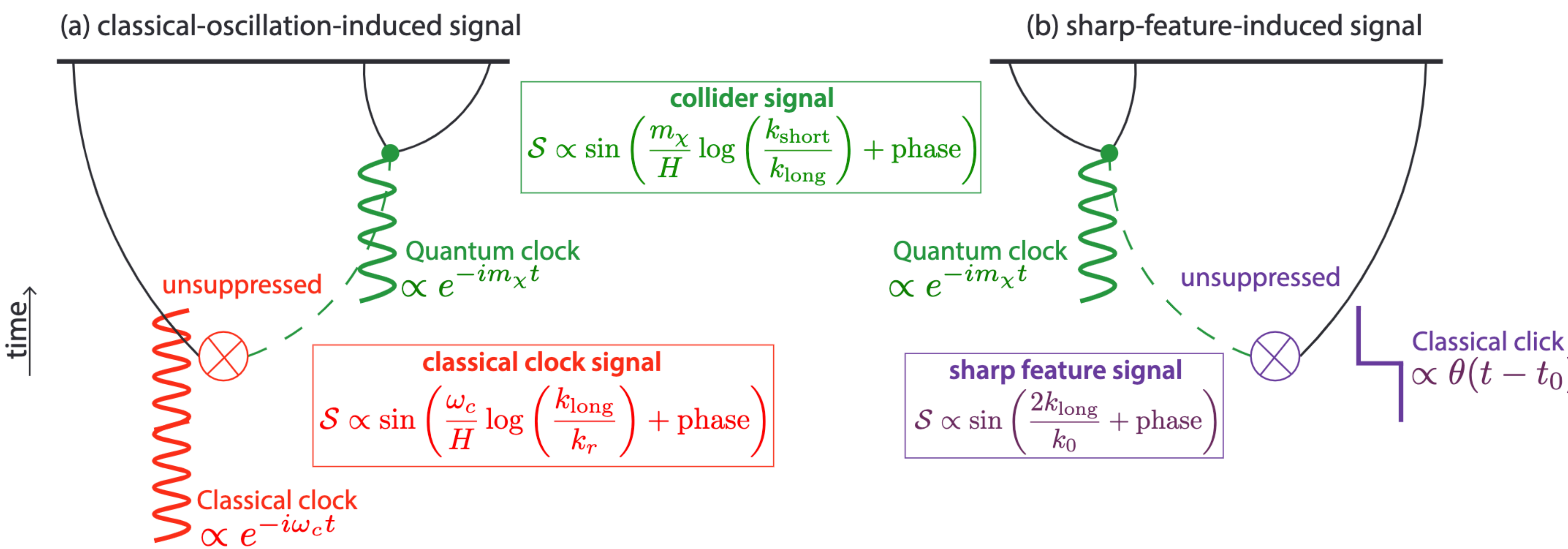
- But they need not be exactly around 10^{14} GeV, can easily be one of two orders of magnitude higher!

- Chemical potential for charged spin-0, spin-1/2, 1, 2. But real scalars? More generally what other mechanisms?

Chen, Wang, Xianyu '18;
Wang, Xianyu '20;
Bodas, **SK**, Sundrum '20; Tong, Xianyu '22; …

- Goal: construct plausible mechanisms in which such signatures are observable in the near future.

Big picture



(a) classical-oscillation-induced signal
(b) sharp-feature-induced signal
collider signal
$\mathcal{S} \propto \sin\left(\frac{m_\chi}{H}\log\left(\frac{k_{\rm short}}{k_{\rm long}}\right)+\text{phase}\right)$
Quantum clock
$\propto e^{-im_\chi t}$
unsuppressed
time
classical clock signal
$\mathcal{S} \propto \sin\left(\frac{\omega_c}{H}\log\left(\frac{k_{\rm long}}{k_r}\right)+\text{phase}\right)$
Classical clock
$\propto e^{-i\omega_c t}$
Quantum clock
$\propto e^{-im_\chi t}$
unsuppressed
sharp feature signal
$\mathcal{S} \propto \sin\left(\frac{2k_{\rm long}}{k_0}+\text{phase}\right)$
Classical click
$\propto \theta(t-t_0)$

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- Bispectrum with oscillatory features
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- Example implementations
    - Oscillatory inflaton potential
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Interaction

$$\mathscr{L} = \mathscr{L}_\phi + \mathscr{L}_\chi + \mathscr{L}_{\text{int}}(\partial\phi, \chi)$$

consistent with ϕ shift symmetry

- $\mathscr{L}_\phi$ satisfies slow roll at leading order; χ massive field of interest

- Simplified parametrization:

$$\mathscr{L}_{\text{int}} \supset \rho(1+B(t))\delta\dot{\phi}\delta\chi + \frac{\lambda}{\Lambda}(1+C(t))\left[(\delta\dot{\phi})^2 - \frac{1}{a^2}(\partial_i\delta\phi)^2\right]\delta\chi + \cdots$$

main focus

alone insufficient

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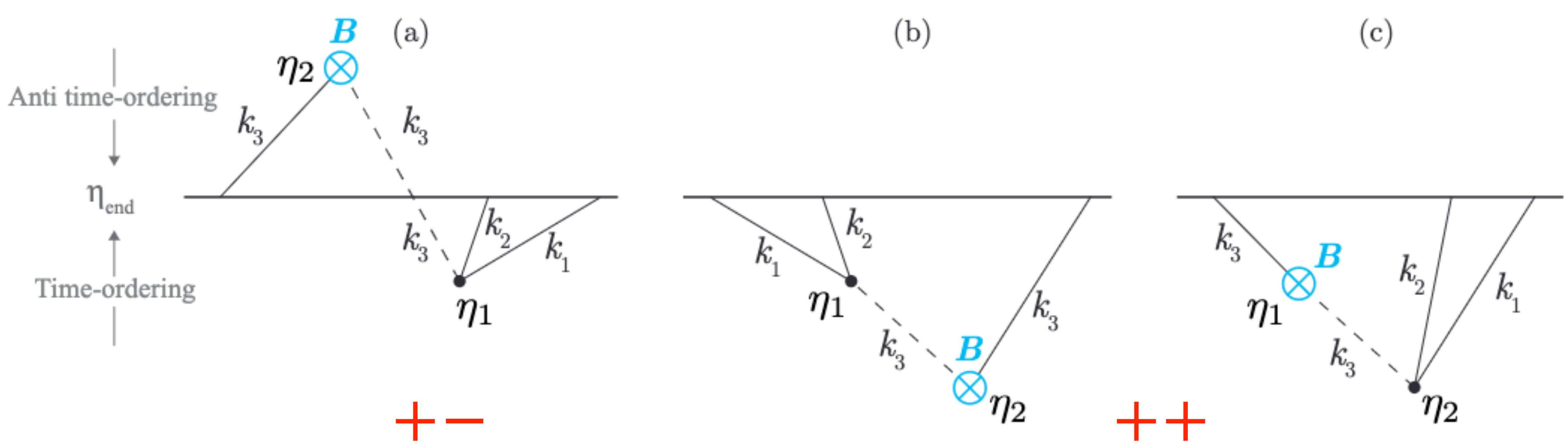
In-in diagrams

$$\mathscr{L}_{\rm int} \supset \rho(1+B(t))\delta\dot{\phi}\delta\chi + \frac{\lambda}{\Lambda}(1+C(t))\left[(\delta\dot{\phi})^2 - \frac{1}{a^2}(\partial_i\delta\phi)^2\right]\delta\chi$$

$$\langle\delta\phi^3\rangle = \langle 0|\underbrace{\left[\bar{T}e^{i\int_{-\infty}^{t_{\rm end}}dt' H_{\rm int}(t')}\right]}_{-}\,\delta\phi^3(t_{\rm end})\,\underbrace{\left[Te^{-i\int_{-\infty}^{t_{\rm end}}dt' H_{\rm int}(t')}\right]}_{+}|0\rangle$$



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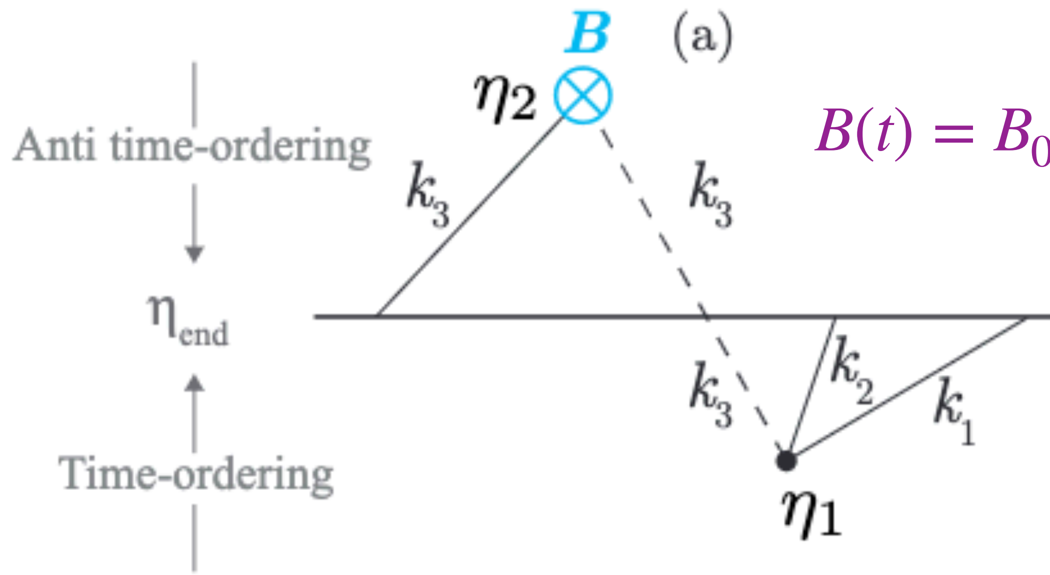
$+-$ contribution



$B(t) = B_0(a/a_0)^{-n}\sin(\omega_c(t-t_0))\theta(t-t_0)$

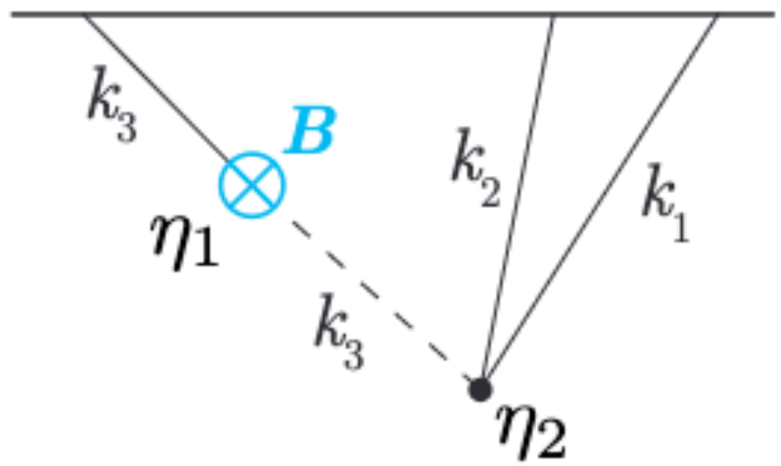
$+-$

- Energy injection at η_2 vertex, but none at η_1 $\Rightarrow$ always exponentially $e^{-\pi m/H}$ suppressed

$$\langle\delta\phi^3\rangle'_{+-} = -\frac{\rho\lambda}{\Lambda}u_{k_1}u_{k_2}u^*_{k_3}(\eta_{\rm end})\int_{-\infty}^{0}\frac{{\rm d}\eta_1}{(H\eta_1)^4}\dot{u}^*_{k_1}\dot{u}^*_{k_2}v^*_{k_3}(\eta_1)\int_{-\infty}^{0}\frac{{\rm d}\eta_2}{(H\eta_2)^4}\dot{u}_{k_3}v_{k_3}B_c(\eta_2)$$

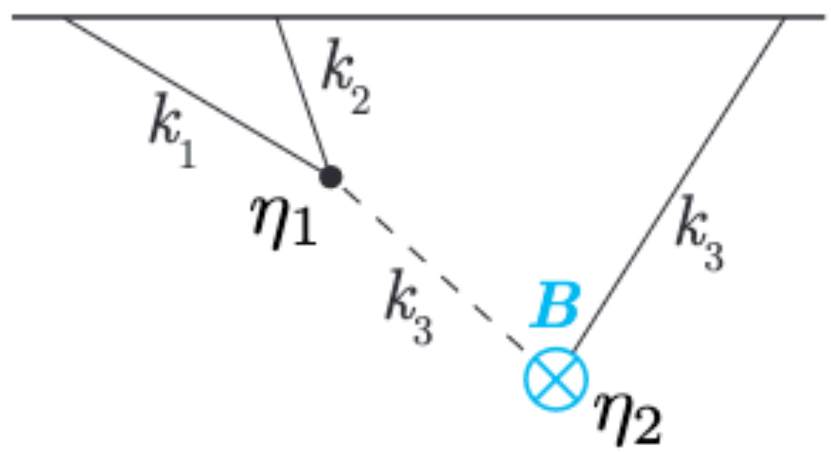
$$\propto \underbrace{e^{\pi\mu/2}\int_0^\infty {\rm d}z_1 z_1^{3/2}H^{(2)}_{i\mu}(z_1)\,e^{-ipz_1}}_{\mathcal{I}_3^+}\ \underbrace{e^{-\pi\mu/2}\int_0^\infty\frac{{\rm d}z_2}{\sqrt{z_2}}H^{(1)}_{i\mu}(z_2)\,e^{iz_2}\,z_2^{n-i\mu_c}}_{\mathcal{I}_2^-}-\{\mu_c\to-\mu_c\}$$

++ contribution: non-spectroscopic



- Three point takes place first $\Rightarrow$ again no energy injection to produce on-shell χ
- But important and un-suppressed EFT contribution $\sim H^2/M^2$ where χ integrated out

++ contribution: spectroscopic!

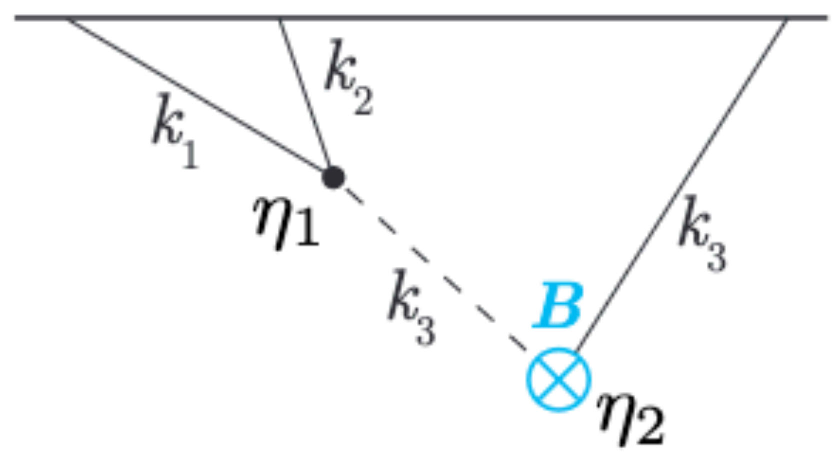


- Two point takes place first $\Rightarrow$ energy injection to produce on-shell χ at η_2!

- Then χ just decays into inflaton quanta at η_1

- χ oscillates to imprint mass information: spectroscopy!

Stationary phase estimate



$\mu_c = \omega_c / H$

- Resonance at η_2: $\mu_c = |k_3\eta_2^{\text{res}}| + \sqrt{(k_3\eta_2^{\text{res}})^2 + \mu^2}$

$$z_2^{\text{res}} \equiv |k_3\eta_2^{\text{res}}| \simeq \frac{\mu_c^2 - \mu^2}{2\mu_c}$$

- Resonance at η_1: $\sqrt{(k_3\eta_1^{\text{res}})^2 + \mu^2} = |k_{12}\eta_1^{\text{res}}|$

For most $\mu_c \gtrsim \mu$ 2pt indeed happens earlier

$p \equiv k_{12}/k_3$

$$z_1^{\text{res}} \equiv |k_3\eta_1^{\text{res}}| \simeq \frac{\mu}{p}$$

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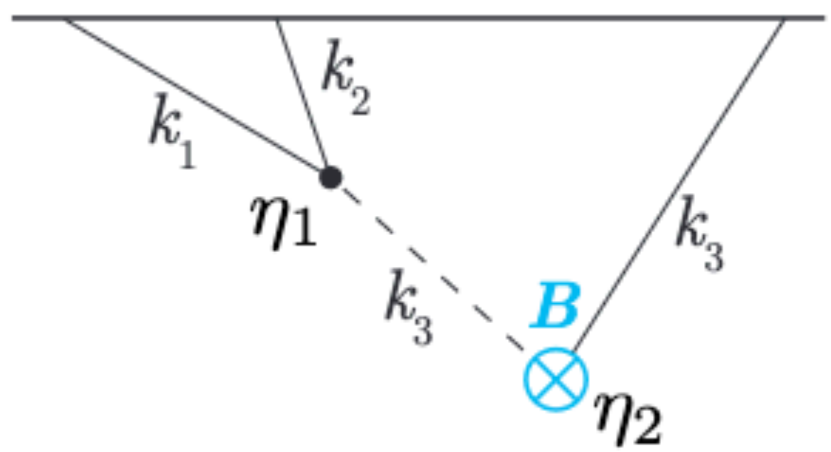
Scale and shape dependence



- Oscillation of source from η_0 to η_2: $\left(\frac{\eta_0}{\eta_2}\right)^{\pm i\mu_c} \to \left(\frac{k_3}{k_0\mu_c}\right)^{\pm i\mu_c}$
- Oscillation of χ from η_2 to η_1: $\left(\frac{\eta_2}{\eta_1}\right)^{\pm i\mu} \to \left(\frac{k_3}{k_1}\right)^{\pm i\mu}$

$$\mathcal{S}(k_1, k_2, k_3) \xrightarrow{k_r \ll k_3 \ll k_1} \left(\frac{k_3}{k_0\mu_c}\right)^{-n-i\mu_c} \left(\frac{k_1}{k_3}\right)^{-1/2-i\mu} + \mathrm{c.c.}$$

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Analytical Computation

- Computation involving ++ diagram: simplifies in the squeezed limit with factorization

$$
\begin{aligned}
\langle\delta\phi^3\rangle'_{++} &\supset \frac{\rho\lambda}{\Lambda} u_{k_1}u_{k_2}u_{k_3}(\eta_{\rm end}) \int_{-\infty}^{0} \frac{{\rm d}\eta_1}{(H\eta_1)^4}\, \dot{u}^*_{k_1}\dot{u}^*_{k_2}v_{k_3}(\eta_1) \int_{-\infty}^{\eta_1} \frac{{\rm d}\eta_2}{(H\eta_2)^4}\, \dot{u}^*_{k_3}v^*_{k_3}B_{\rm c}(\eta_2) \\
&= \frac{\rho\lambda}{\Lambda}\frac{-\pi H^3}{32k_1k_2k_3^4}\frac{i}{2}B_0\, z_0^{-n-i\mu_{\rm c}} \\
&\quad\times e^{-\pi\mu/2}\int_0^\infty {\rm d}z_1 z_1^{3/2}\, H^{(1)}_{i\mu}(z_1)\, e^{-ipz_1}\, \underbrace{e^{\pi\mu/2}\int_{z_1}^\infty \frac{{\rm d}z_2}{\sqrt{z_2}}\, H^{(2)}_{i\mu}(z_2)\, e^{-iz_2}\, z_2^{n+i\mu_{\rm c}}\theta(z_0-z_2)}_{\mathcal{I}_2^+(z_1)} \\
&\quad - \{\mu_c \to -\mu_c\}
\end{aligned}
$$

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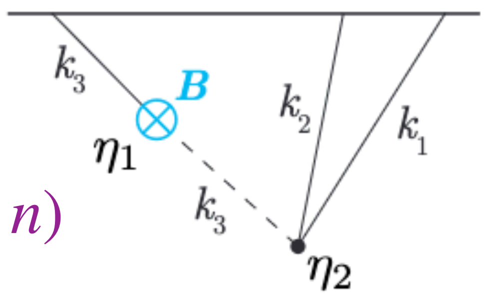
Analytical Computation

$$\langle \delta\phi^3 \rangle'_{++} \approx \frac{H^3}{k_1^3 k_3^3} \left(\frac{k_3}{k_0 \mu_c} \right)^{-n-i\mu_c} \left(\frac{k_1}{k_3} \right)^{-3/2-i\mu} \mathscr{F}(\mu, \mu_c, n)$$



$$\mathscr{F}(\mu, \mu_c, n) = \frac{1}{16} \frac{\rho\lambda}{\Lambda} \frac{B_0}{2} e^{\pi\mu} \left(\frac{-i\mu_c}{\mu_c^2 - \mu^2} \right)^{n+i\mu_c} \left(\frac{3}{2} + i\mu \right) \left(\frac{5}{2} + i\mu \right)$$

$$\times \frac{\Gamma(n + \frac{1}{2} + i\mu_c + i\mu)\Gamma(n + \frac{1}{2} + i\mu_c - i\mu)}{\Gamma(n + 1 + i\mu_c)} \frac{\Gamma(-2i\mu)\Gamma(\frac{1}{2} + i\mu)}{\Gamma(\frac{1}{2} - i\mu)} + \cdots$$

- Asymptotically, $\mathscr{F}(\mu, \mu_c, n) \sim e^{-\pi(\mu - \mu_c)}$

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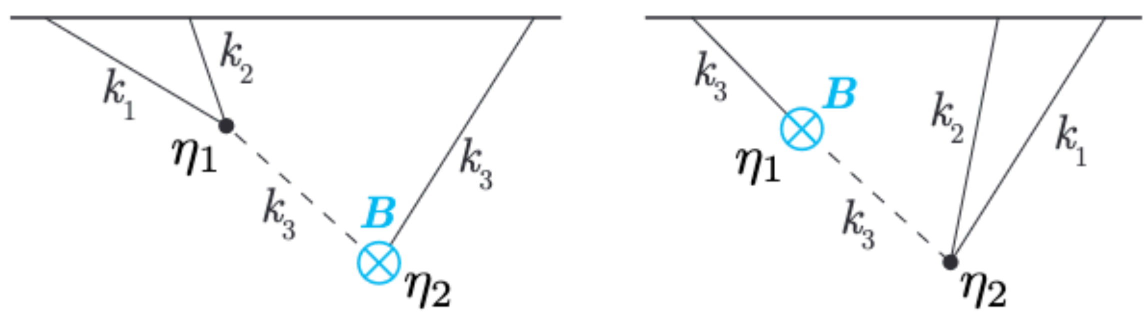
Numerical computation

- Keep both the diagrams



$$T_1(z) = z^{3/2}\left(2 - \frac{2}{p^2}\right)$$

$$T_2(z) = -z^{-1/2}\frac{4}{p^2}\left(1 - \frac{2}{p^2}\right)$$

$$T_3(z) = -z^{1/2}\frac{4i}{p}\left(1 - \frac{2}{p^2}\right)$$

$$\langle\delta\phi^3\rangle'_{++}\Big|_{\text{left}} = \frac{\rho\lambda}{\Lambda}\frac{-\pi H^3}{32k_1k_2k_3^4}\frac{i}{2}B_0\, z_0^{-n-i\mu_c}$$

$$\times\int_0^\infty dz_1\left(T_1(z_1)+T_2(z_1)+T_3(z_1)\right)H_{i\mu}^{(1)}(z_1)e^{-ipz_1}\int_{z_1}^\infty\frac{dz_2}{\sqrt{z_2}}H_{i\mu}^{(2)}(z_2)e^{-iz_2}z_2^{n+i\mu_c}\theta(z_0-z_2)-\{\mu_c\to-\mu_c\}$$

$$\langle\delta\phi^3\rangle'_{++}\Big|_{\text{right}} = \frac{\rho\lambda}{\Lambda}\frac{-\pi H^3}{32k_1k_2k_3^4}\frac{i}{2}B_0 z_0^{-n-i\mu_c}$$

$$\times\int_0^\infty\frac{dz_1}{\sqrt{z_1}}H_{i\mu}^{(2)}(z_1)e^{-iz_1}z_1^{n+i\mu_c}\theta(z_0-z_1)\int_{z_1}^\infty dz_2\left(T_1(z_2)+T_2(z_2)+T_3(z_2)\right)H_{i\mu}^{(1)}(z_2)e^{-ipz_2}-\{\mu_c\to-\mu_c\}$$

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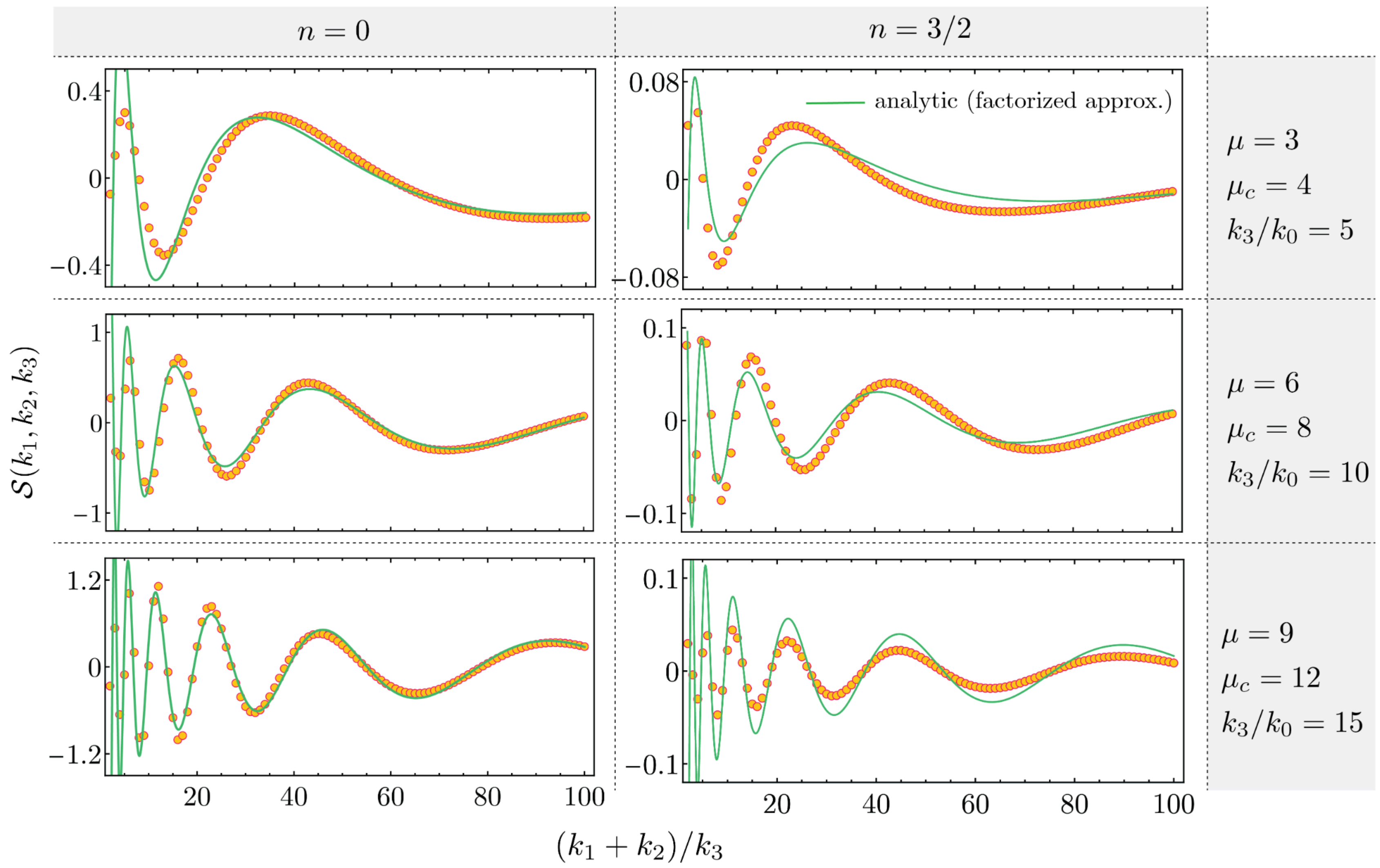
Shape dependence



$n = 0$
no volume dilution of the source
e.g., oscillation on the potential

$n = 3/2$
volume dilution of the source
e.g., classical oscillation of a 2nd field

We see $\left((k_1 + k_2)/k_3\right)^{-3/2 - i\mu}$ + c.c. behavior

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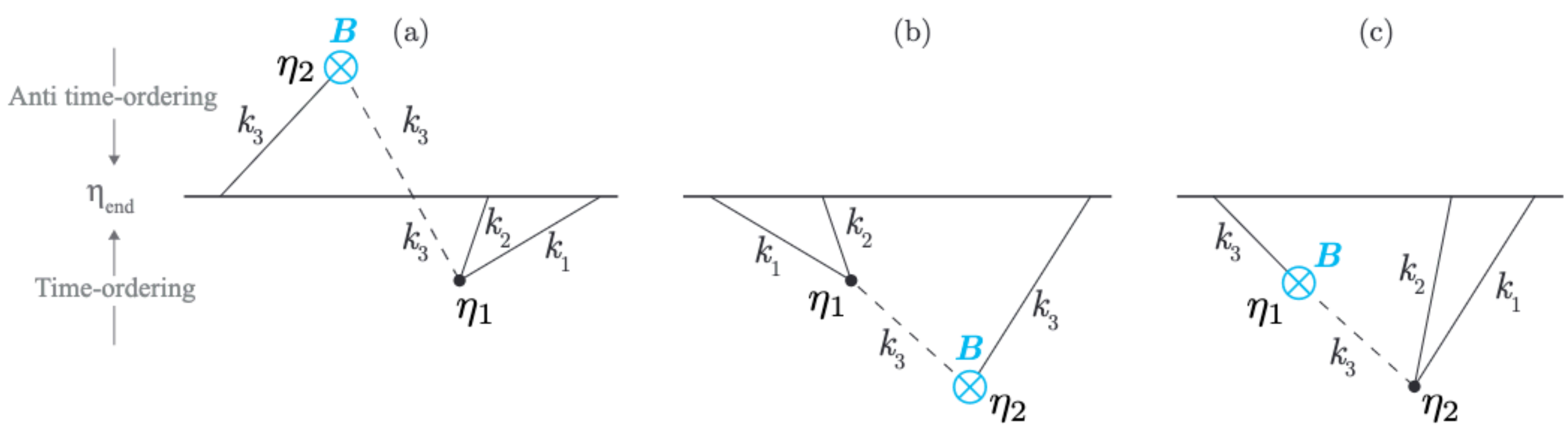
Sharp feature

$$\mathscr{L}_{\text{int}} \supset \rho(1+B(t))\delta\dot{\phi}\delta\chi + \frac{\lambda}{\Lambda}(1+C(t))\left[(\delta\dot{\phi})^2 - \frac{1}{a^2}(\partial_i\delta\phi)^2\right]\delta\chi$$

$$B(t) = B_0\theta(t-t_0)$$

- Sharp feature contains infinitely many frequencies: $\theta(t) \to \frac{1}{2}\left(\delta(\omega) - \frac{i}{\pi\omega}\right)$



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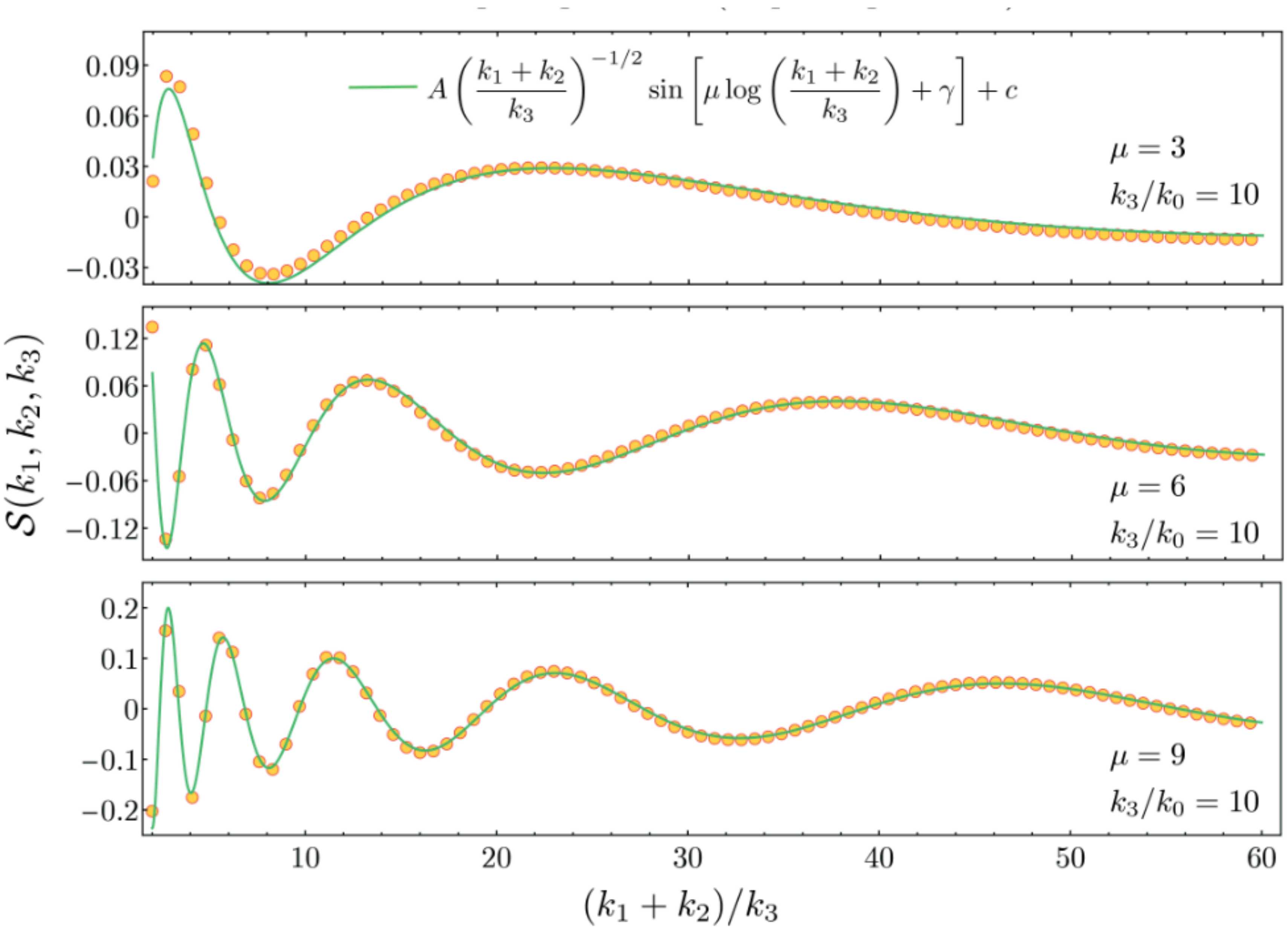
Shape dependence



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EFT Parametrization

- Imposing shift symmetry on ϕ

$$\frac{c_1}{\Lambda_\chi}(\partial\phi)^2\chi + \frac{c_2}{\Lambda_\sigma}(\partial\phi)^2\sigma + \frac{(\partial\phi)^2\chi\sigma}{\Lambda_\chi\Lambda_\sigma} + \frac{c_3}{\Lambda_\chi^2}(\partial\phi)^2\chi^2 + \frac{c_4}{\Lambda_\sigma^2}(\partial\phi)^2\sigma^2 + \cdots .$$

- Constraints from power spectrum

$$\Lambda_\chi \gtrsim \dot{\phi}_0/m_\chi \qquad \Lambda_\sigma \gtrsim \dot{\phi}_0/m_\sigma$$

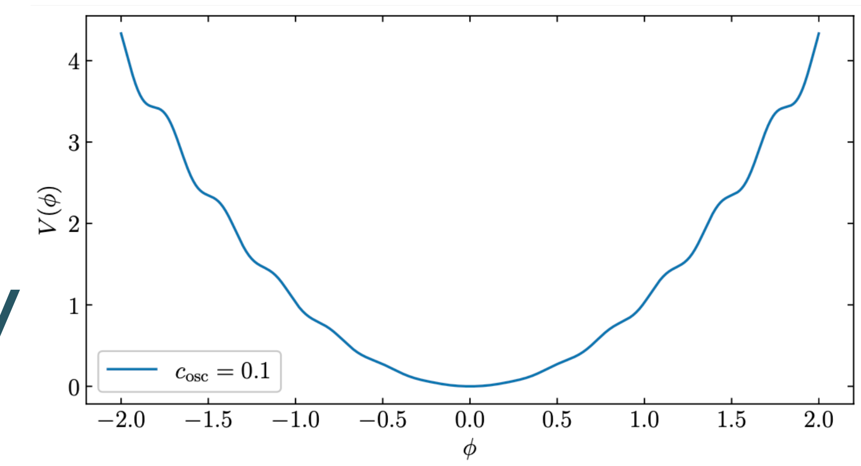
- Constraints from derivative expansion $\left[(\partial\phi)^2/\Lambda_\chi^4\right]^n$

$$\Lambda_\chi > \sqrt{\dot{\phi}_0} \qquad \Lambda_\sigma > \sqrt{\dot{\phi}_0}$$

Creminelli '03

Oscillations on inflaton potential

$$V_{\text{resonant model}} = V_{\text{sr}}(\phi)\left[1 + c_{\text{osc}} \sin\left(\frac{\phi}{\phi_r} + \beta\right)\right]$$



- Subdominant but high frequency oscillatory motion of the inflaton

$$\phi_{\text{background}} = \phi_0(t) + \phi_1(t)$$

$$\phi_1 \sim \frac{c_{\text{osc}} V_{\text{sr}}}{\phi_r \omega_c^2} \cos(\omega_c t)$$

- Induces the oscillatory energy source

$$\frac{c_1}{\Lambda_\chi}(\partial\phi)^2 \delta\chi \supset -2\frac{c_1(\dot{\phi}_0 + \dot{\phi}_1)}{\Lambda_\chi}\dot{\delta\phi}\delta\chi - \frac{c_1}{\Lambda_\chi}\left[(\dot{\delta\phi})^2 - \frac{1}{a^2}(\partial_i \delta\phi)^2\right]\delta\chi + \cdots$$

$$-2\frac{c_1 \dot{\phi}_1}{\Lambda_\chi} \equiv \rho B(t)$$

Oscillatory classical field

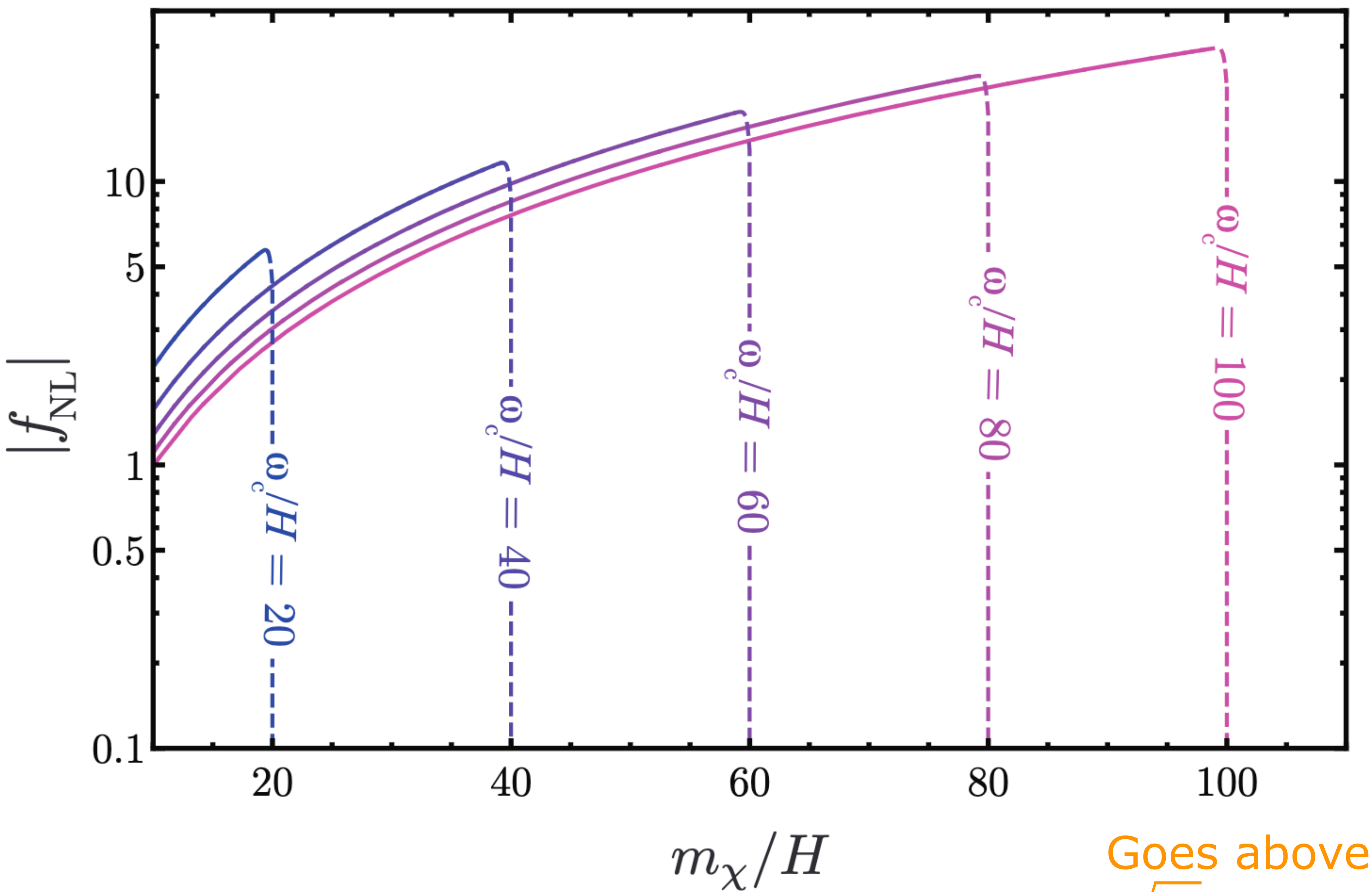
- Imagine having a second oscillatory classical field excited due to features in the landscape: `classical clock field'

$$\sigma_c(t) = \sigma_s(a/a_0)^{-3/2}\sin(m_\sigma(t-t_\sigma))\ \theta(t-t_0)$$

- Subdominant inflaton oscillation through $\frac{c_2}{\Lambda_\sigma}(\partial\phi)^2\sigma$ coupling:

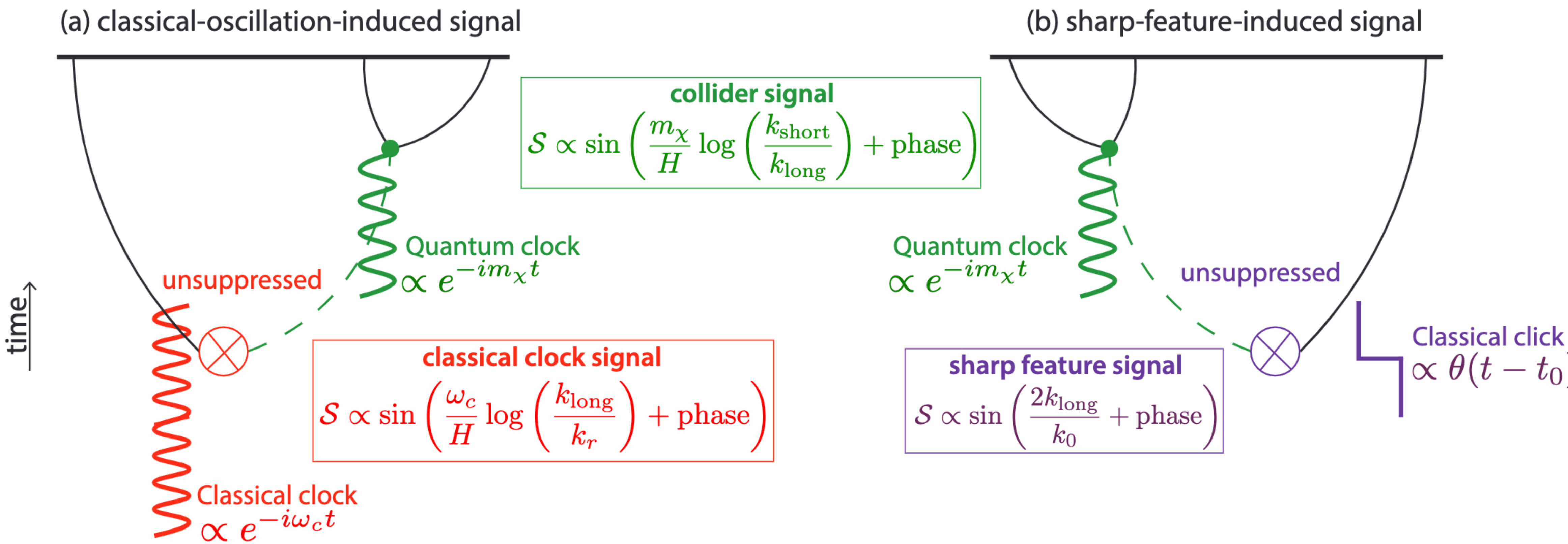
$$B(t) = \frac{\dot{\phi}_1(t)}{\dot{\phi}_0(t)} \approx -\frac{\sigma_c}{\Lambda_\sigma}$$

Strength of non-Gaussianity



Goes above the $\sqrt{\dot{\phi}_0} \approx 60H$ limit

Summary



(a) classical-oscillation-induced signal
(b) sharp-feature-induced signal
collider signal
$\mathcal{S} \propto \sin\left(\frac{m_\chi}{H}\log\left(\frac{k_{\rm short}}{k_{\rm long}}\right) + {\rm phase}\right)$
Quantum clock
$\propto e^{-im_\chi t}$
unsuppressed
time
classical clock signal
$\mathcal{S} \propto \sin\left(\frac{\omega_c}{H}\log\left(\frac{k_{\rm long}}{k_r}\right) + {\rm phase}\right)$
Classical clock
$\propto e^{-i\omega_c t}$
Quantum clock
$\propto e^{-im_\chi t}$
unsuppressed
sharp feature signal
$\mathcal{S} \propto \sin\left(\frac{2k_{\rm long}}{k_0} + {\rm phase}\right)$
Classical click
$\propto \theta(t - t_0)$

Conclusions

- Rich particle physics candidates can lie around or above 10^{14} GeV, e.g., GUTs, right-handed neutrinos in see-saw …
- Typical models suffer from Boltzmann suppression, especially real scalar fields.
- Primordial features on the landscape can `classically' inject energy and excite very heavy fields with $m \sim \mathcal{O}(10-100)H$

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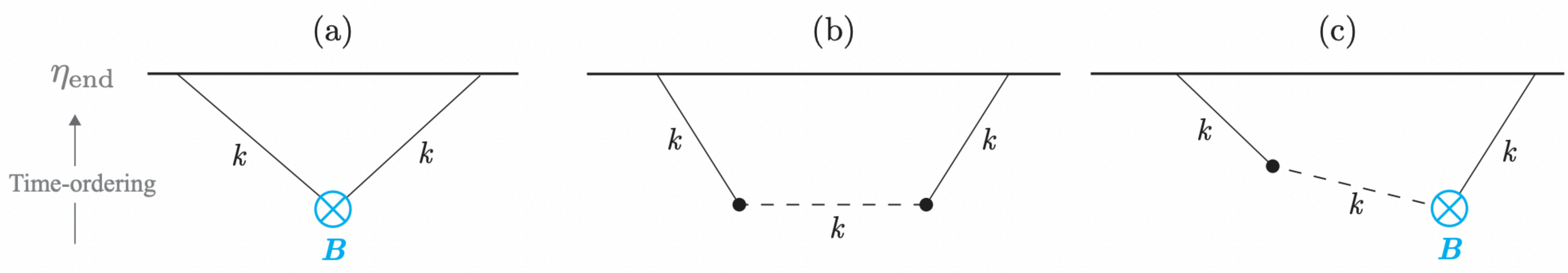
Power spectrum

- Three classes of diagrams:

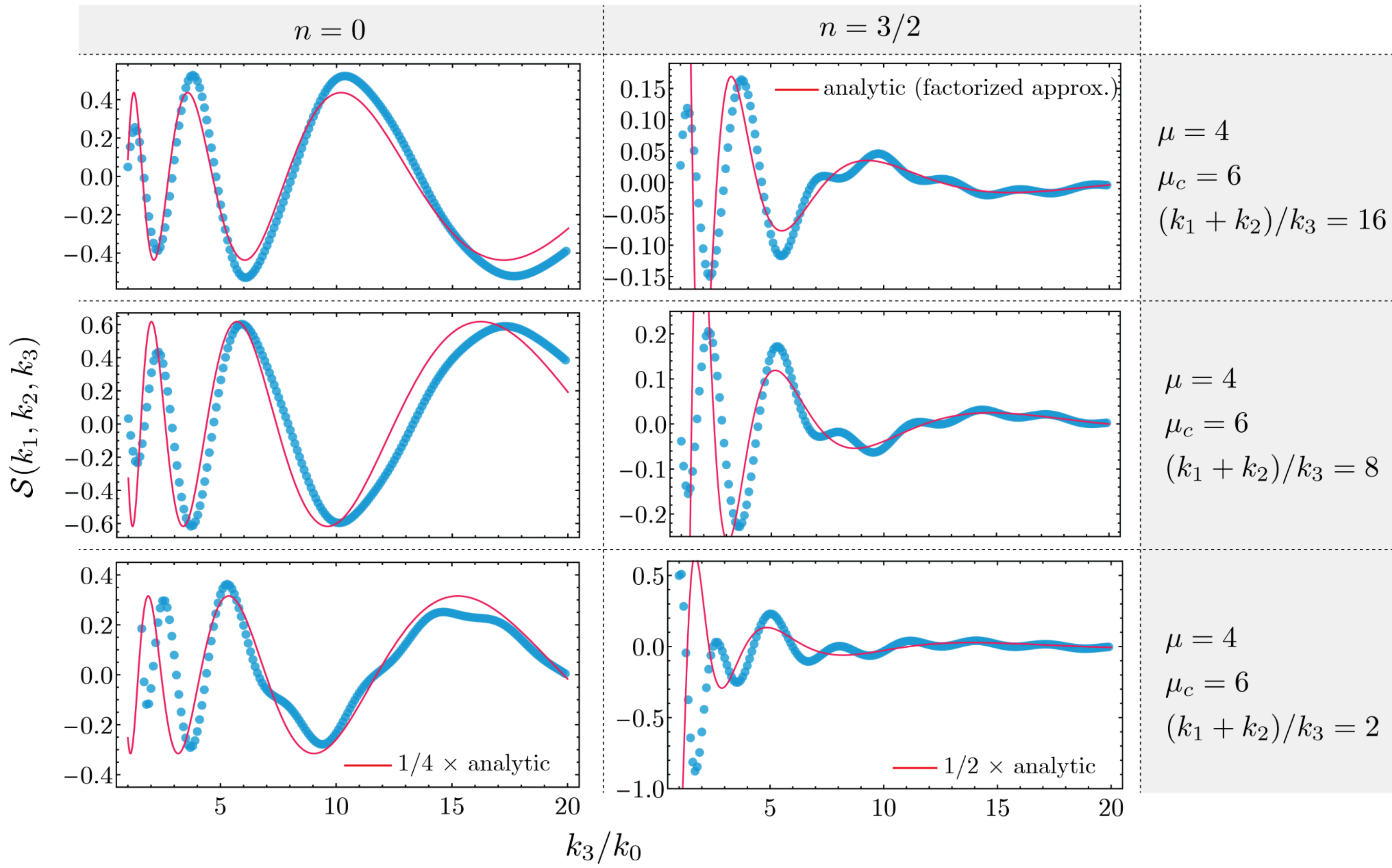


- Diagram (a) is typically the dominant one

$$\frac{\Delta P_\zeta}{P_\zeta} = \sqrt{2\pi}\frac{B_0}{2}\mu_c^{1/2}\left(\frac{2k}{k_r}\right)^{-n-i\mu_c+i\alpha_1} + \text{c.c.}$$

- We choose $B_0 \lesssim 5\times 10^{-3}$ consistent with current bounds

Scale dependence



We see $(k_3/k_0)^{-n-i\mu_c} +$ c.c. behavior

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Scale dependence