Reheating after Inflation, Gauge fields and GWs

Kaloian Lozanov

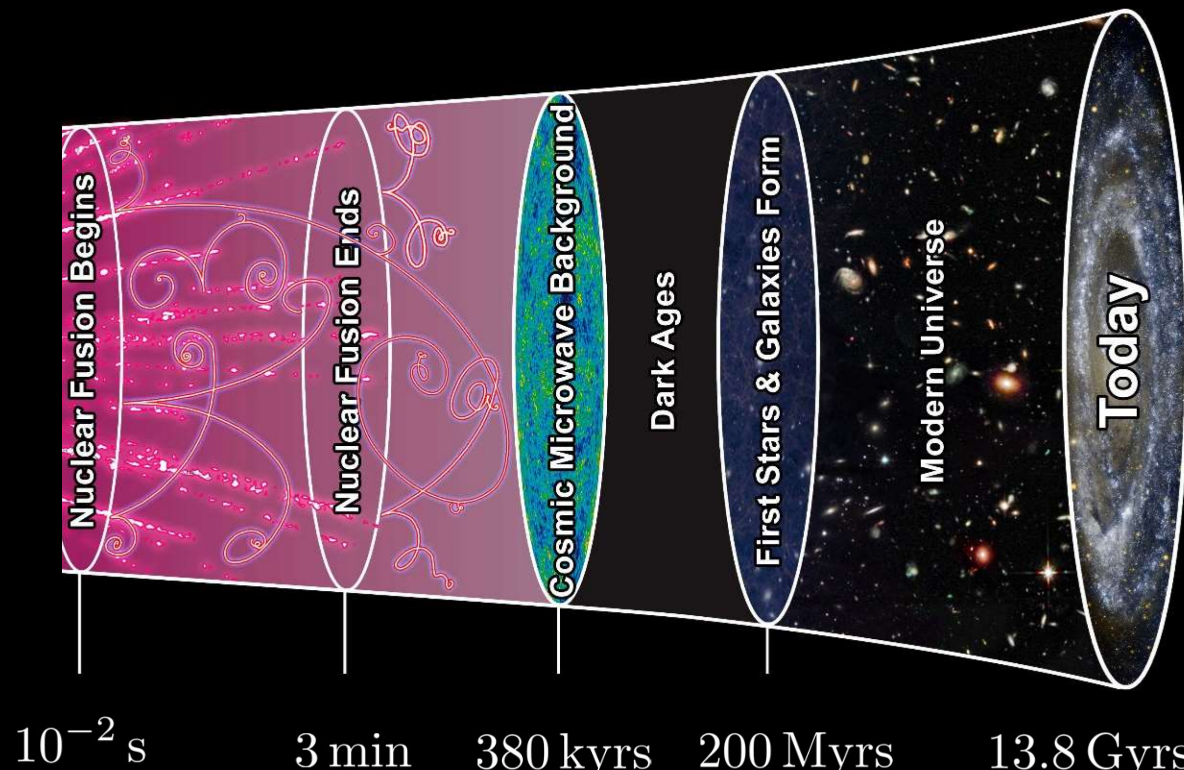
UIUC, Cosmology and HEPheno

Collaborators: Peter Adshead, Zachary Weiner, Mustafa Amin, Eiichiro Komatsu, Matthew Reece, JiJi Fan, Leila Mirzagoli, Yuki Watanabe, Ippei Obata…

Overview

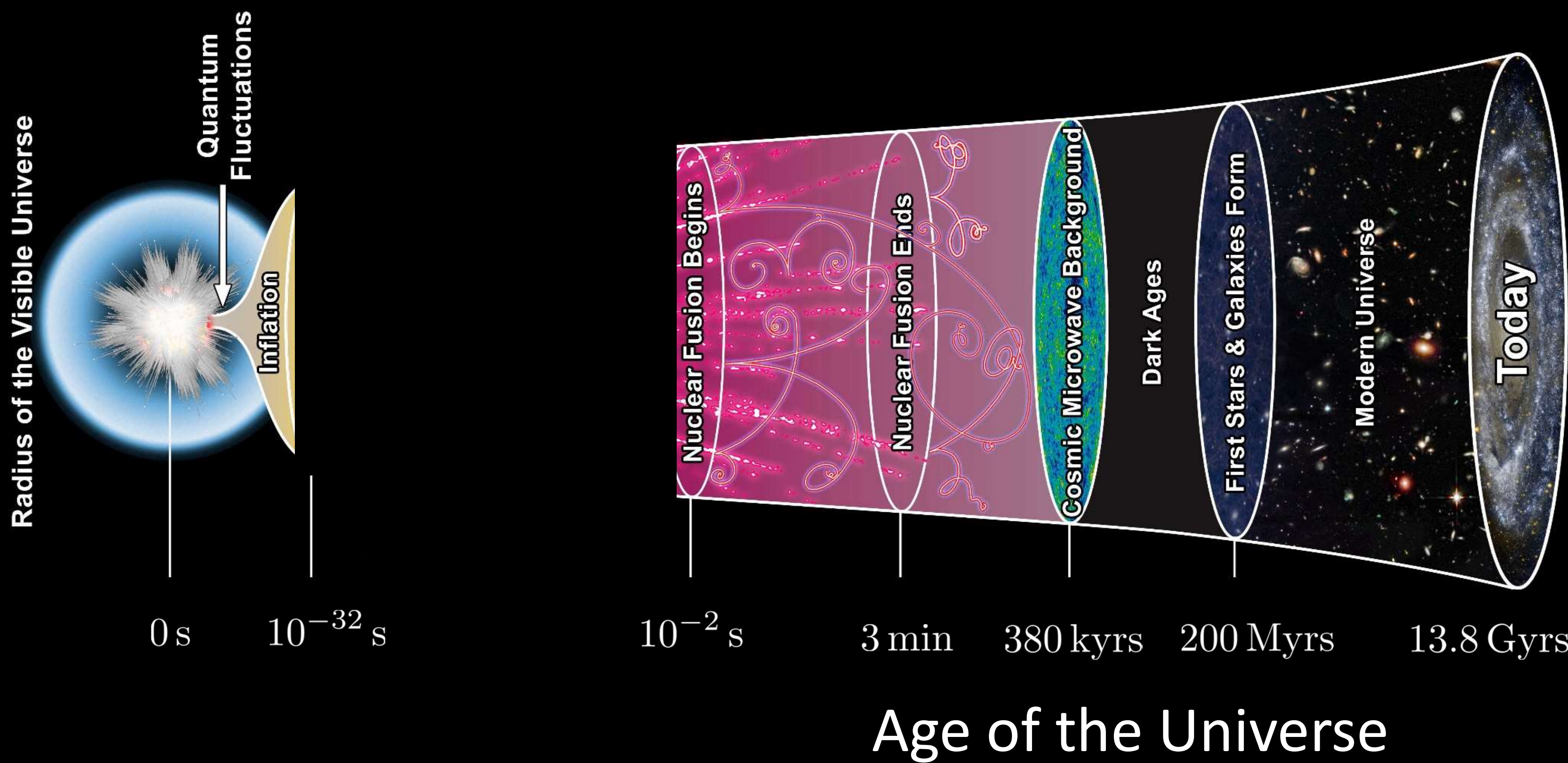
History of the Universe

Radius of the Visible Universe



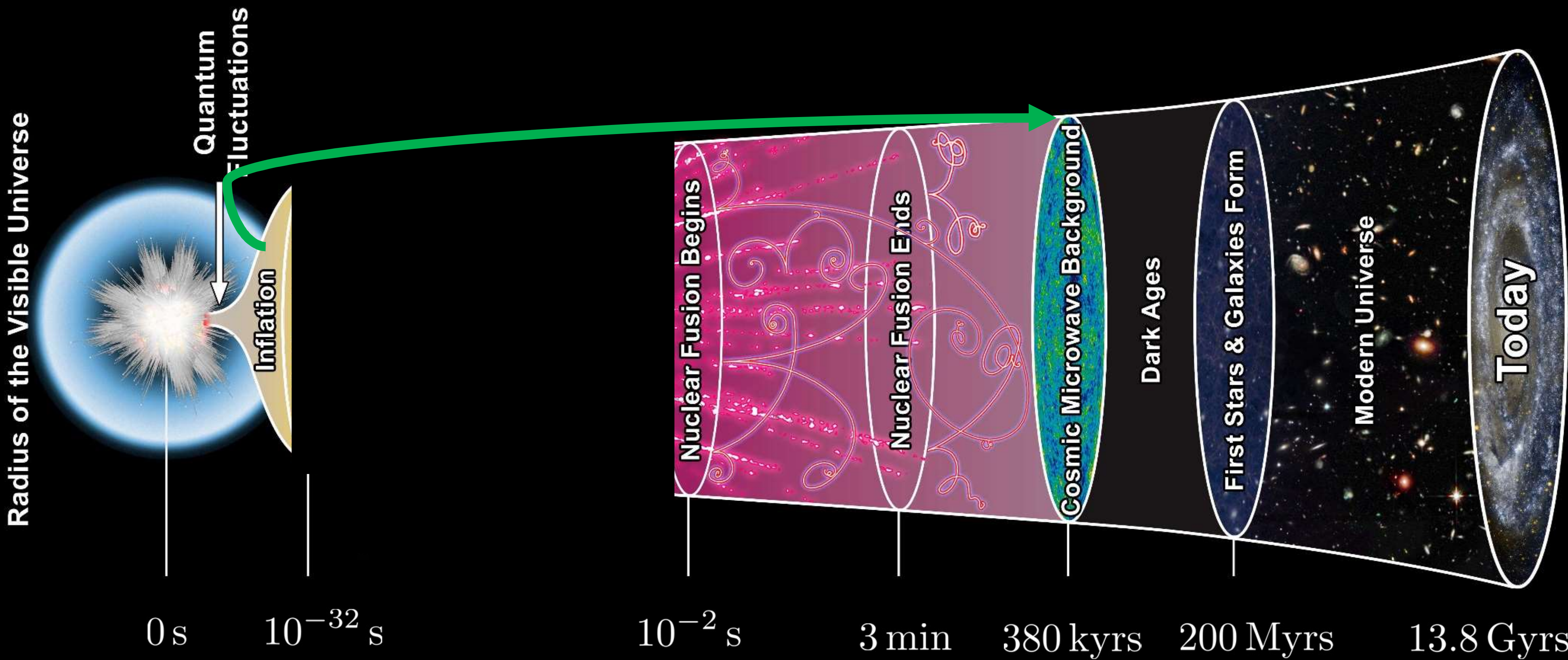
Age of the Universe

History of the Universe



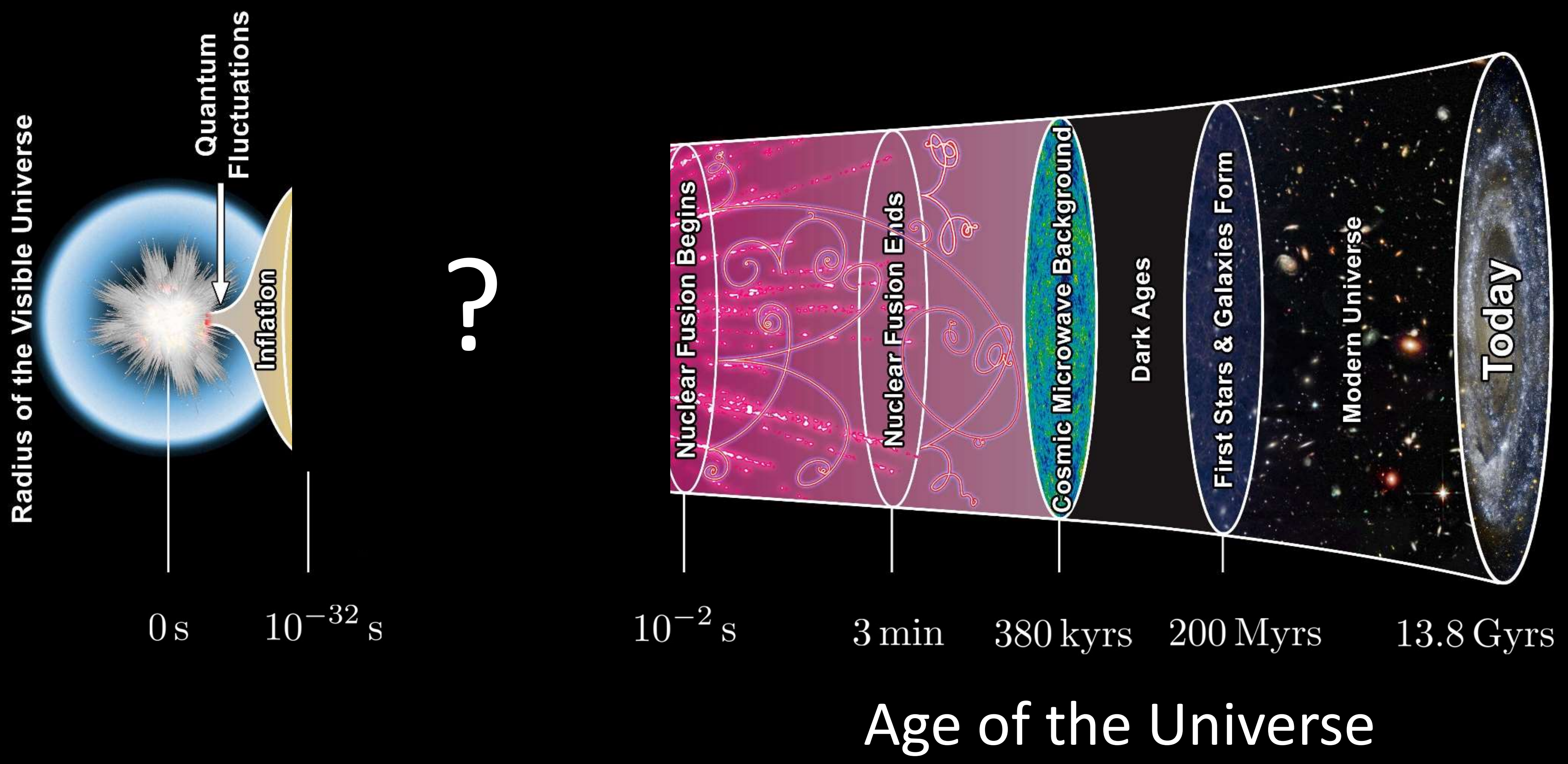
History of the Universe

Mukhanov, Chibisov (1981)

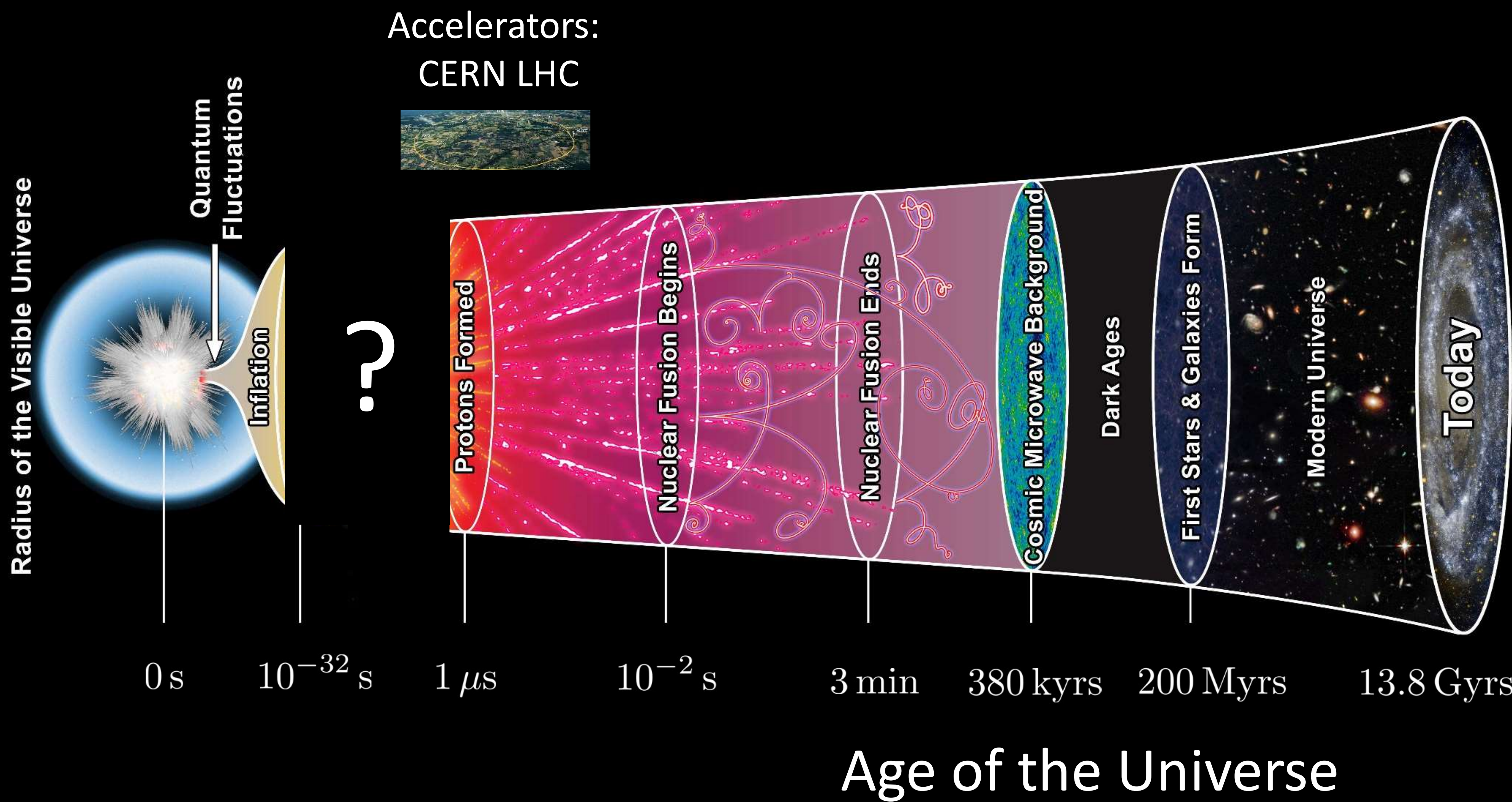


Age of the Universe

History of the Universe

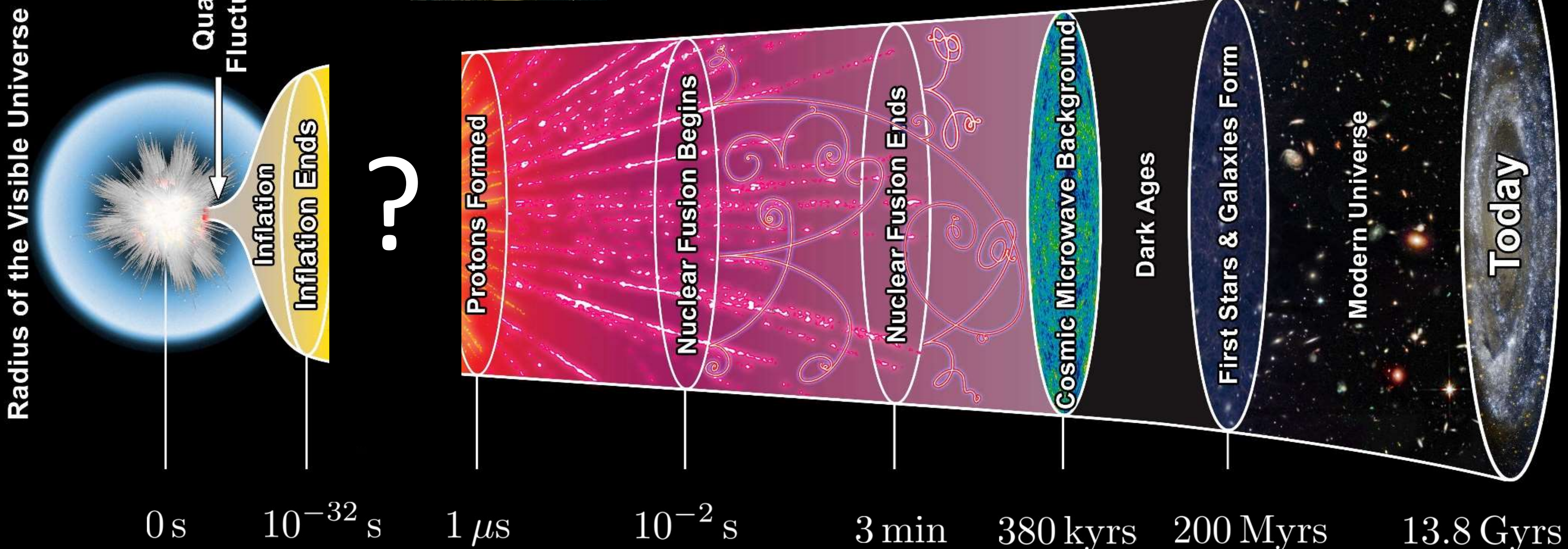


History of the Universe



History of the Universe

Accelerators:
CERN LHC



10^{16} GeV ⟷ 10^{3} GeV

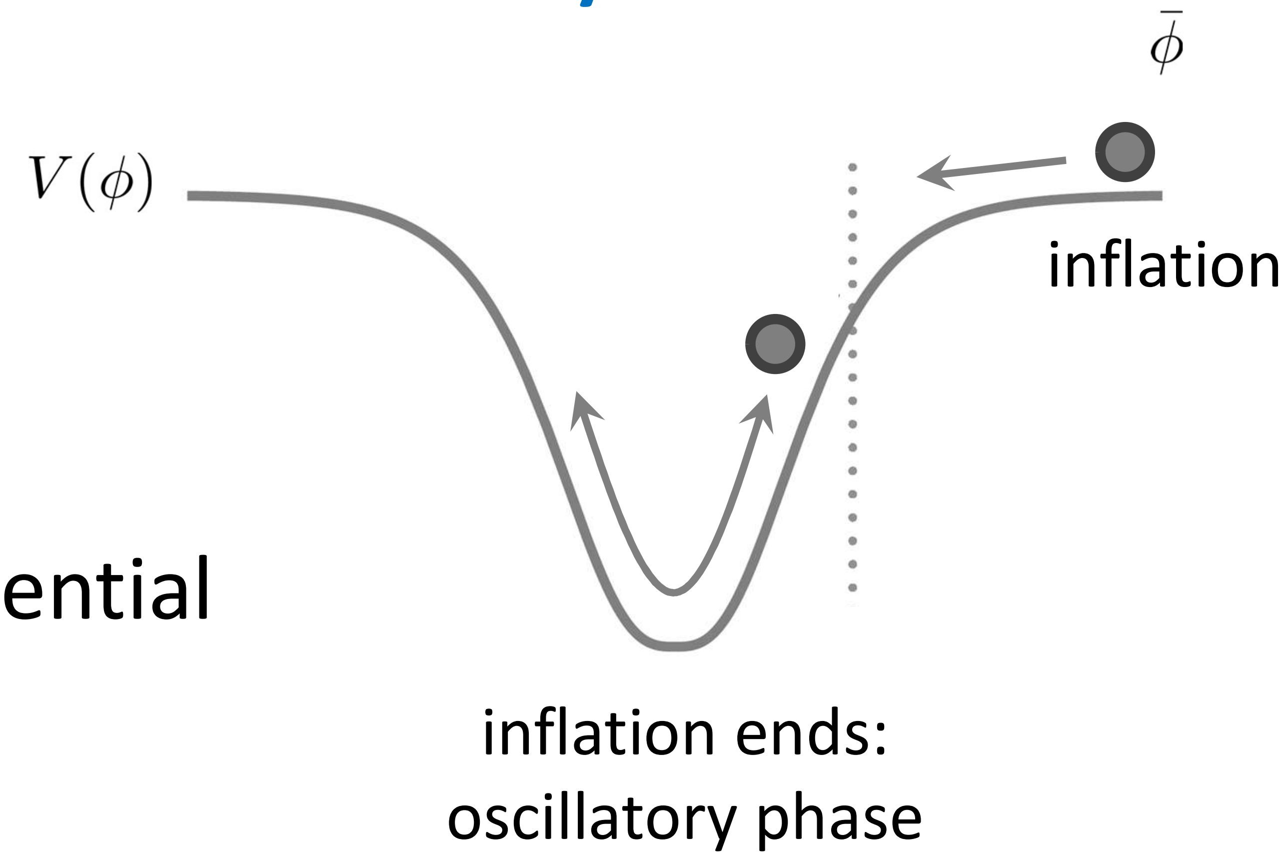
Age of the Universe

The Universe at the end of inflation

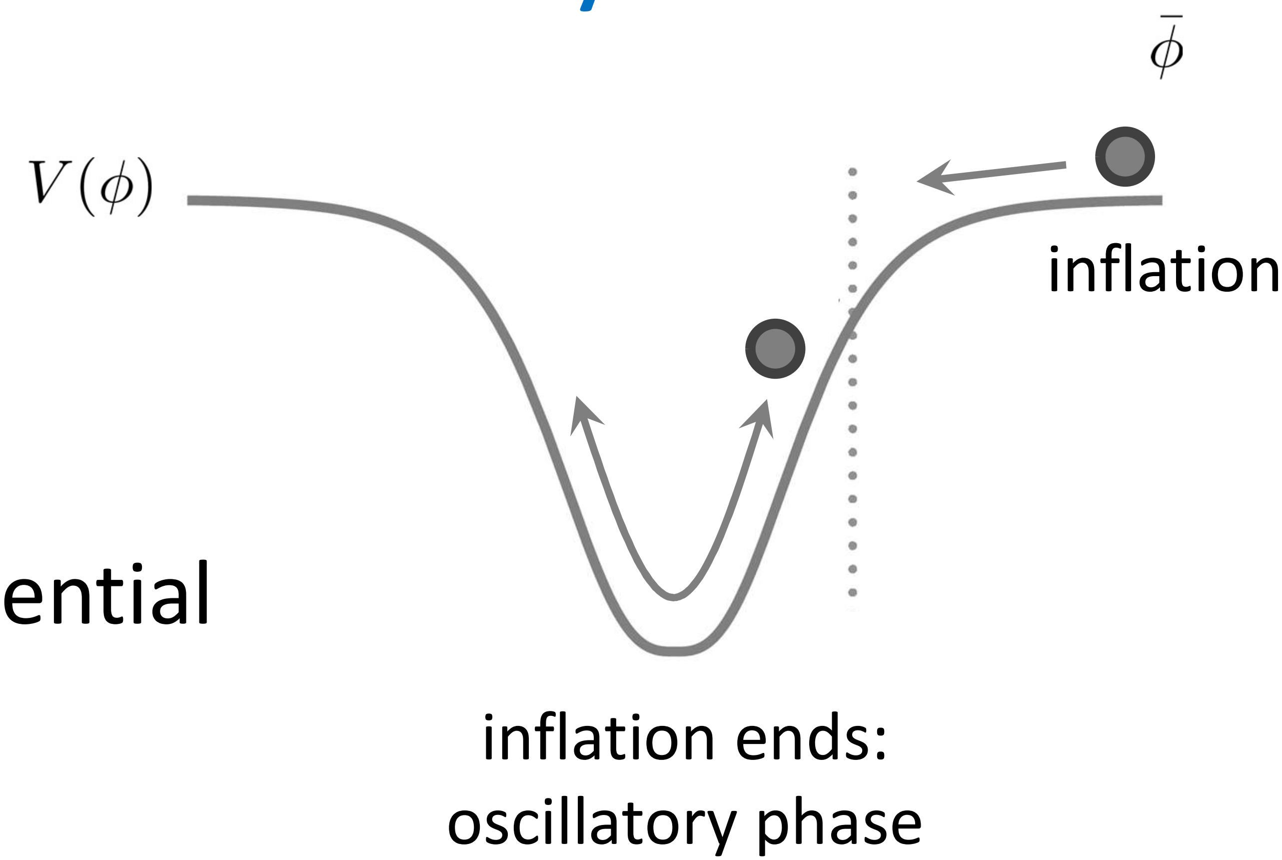
- Cold, dominated by inflaton $\phi(t)$
- $\phi(t)$ should decay to matter and radiation to recover BBN - *REHEATING*

Inflaton decay

- Form of the potential



Inflaton decay



- Form of the potential

- Couplings to other fields

Inflaton decay

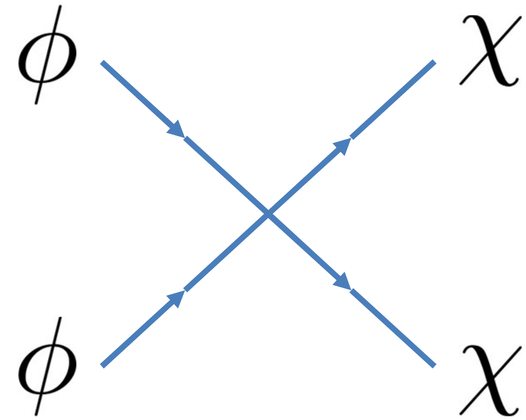
Allahverdi, Brandenberger, et al. (2010), Amin, et al. (2015), KL (2019) (reviews)

- Perturbative

Inflaton decay

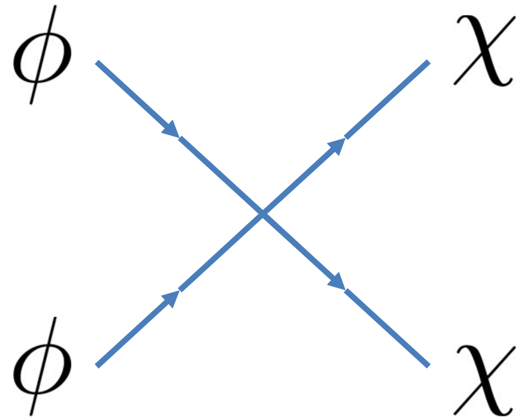
Allahverdi, Brandenberger, et al. (2010), Amin, et al. (2015), KL (2019) (reviews)

- Perturbative
  -Rate and x-section



Inflaton decay

Allahverdi, Brandenberger, et al. (2010), Amin, et al. (2015), KL (2019) (reviews)



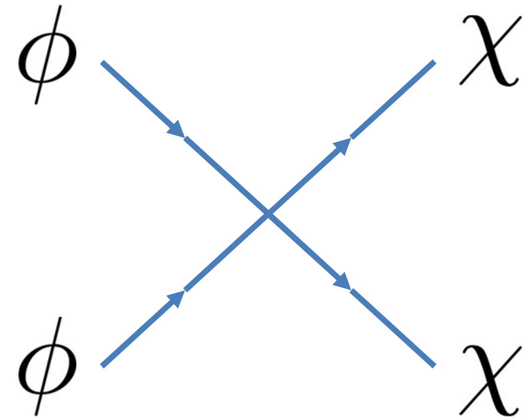
- Perturbative
  -Rate and x-section

- Non-perturbative (explosive)

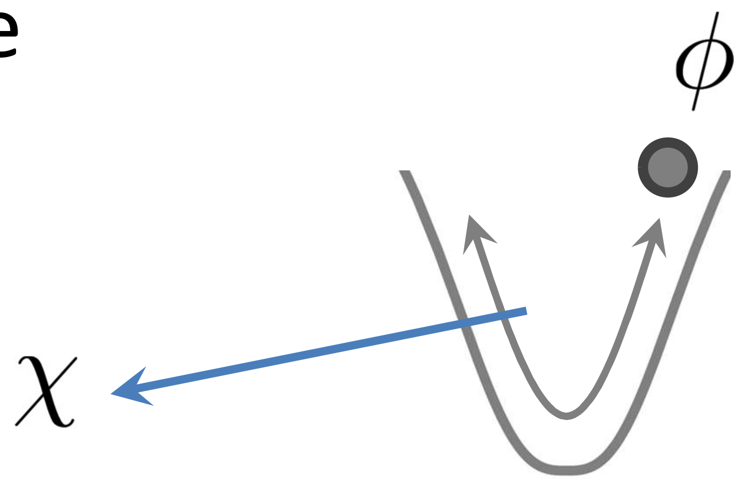
Inflaton decay

Allahverdi, Brandenberger, et al. (2010), Amin, et al. (2015), KL (2019) (reviews)

- Perturbative
  -Rate and x-section



- Non-perturbative (explosive)
  -parametric resonance



Non-perturbative decay (parametric resonance)

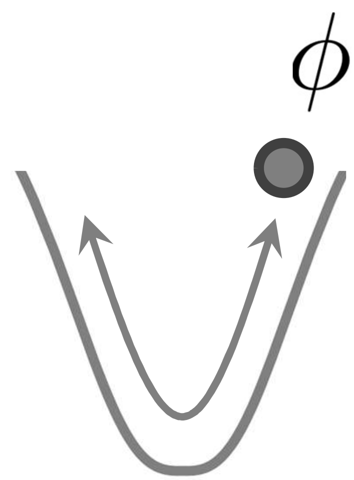
$$V(\phi,\chi) \sim \frac{1}{2}m^2\phi^2 + g^2\phi^2\chi^2 + ...$$

$$\ddot{\chi} - \Delta\chi + g^2\phi^2\chi = 0$$

Non-perturbative decay (parametric resonance)

$$V(\phi, \chi) \sim \frac{1}{2} m^2 \phi^2 + g^2 \phi^2 \chi^2 + \ldots$$

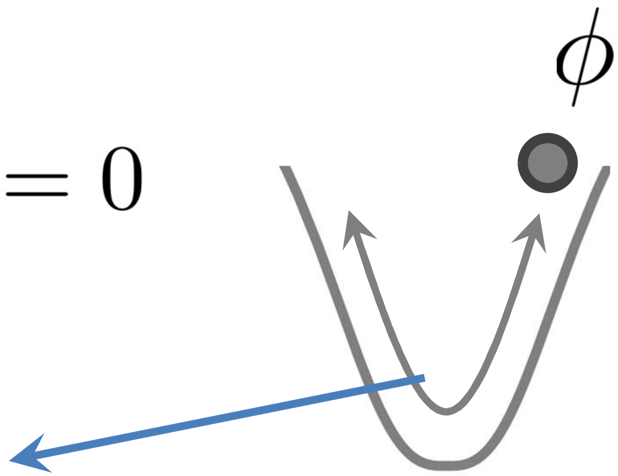
$$\ddot{\chi} - \Delta \chi + g^2 \phi^2 \chi = 0$$



Non-perturbative decay (parametric resonance)

$$V(\phi, \chi) \sim \frac{1}{2} m^2 \phi^2 + g^2 \phi^2 \chi^2 + \ldots$$

$$\ddot{\chi} - \Delta \chi + g^2 \phi^2 \chi = 0$$

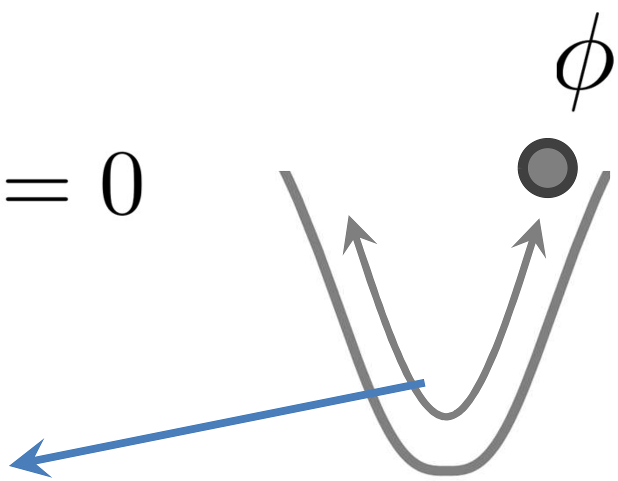


$$\chi_k \propto \exp(\pm \mu_k t)$$

Non-perturbative decay (parametric resonance)

$$V(\phi,\chi) \sim \frac{1}{2}m^2\phi^2 + g^2\phi^2\chi^2 + \ldots$$

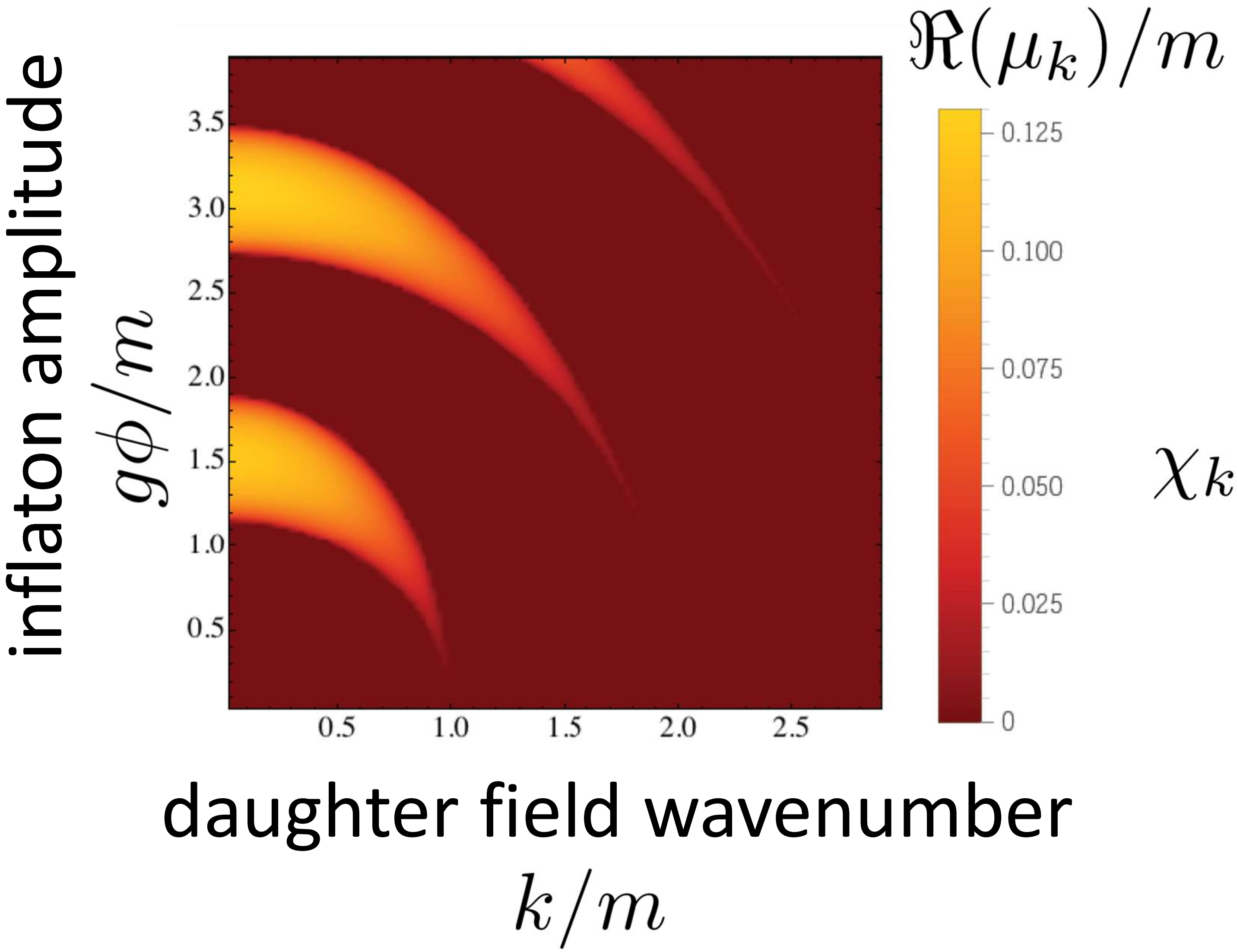
$$\ddot{\chi} - \Delta\chi + g^2\phi^2\chi = 0$$



$$\chi_k \propto \exp(\pm\mu_k t)$$

if $\Re(\mu_k) \neq 0 \implies$ parametric resonance

Non-perturbative decay (parametric resonance)



$$\chi_k \propto \exp(\pm\mu_k t)$$

Time-line of Reheating

Allahverdi, Brandenberger, et al. (2010), Amin, et al. (2015), KL (2019) (reviews)

time

Time-line of Reheating

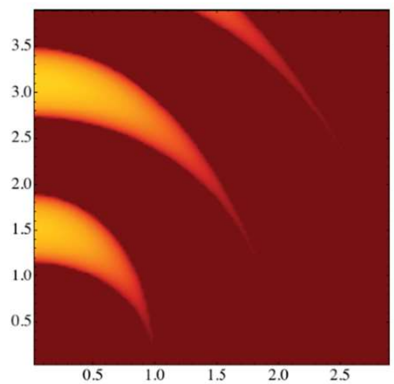
Allahverdi, Brandenberger, et al. (2010), Amin, et al. (2015), KL (2019) (reviews)

time

inflation

Time-line of Reheating

Allahverdi, Brandenberger, et al. (2010), Amin, et al. (2015), KL (2019) (reviews)



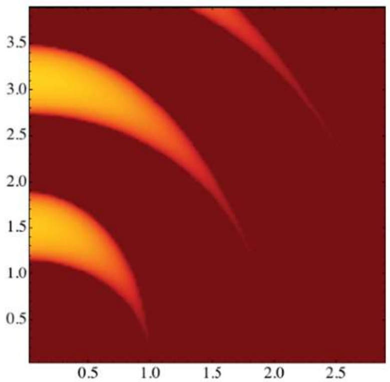
time

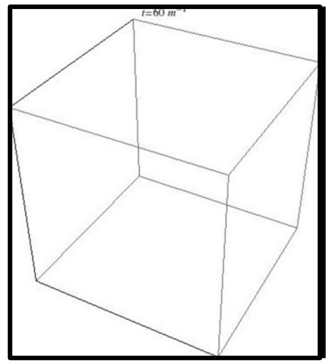
linear regime
(preheating)

inflation

Time-line of Reheating

Allahverdi, Brandenberger, et al. (2010), Amin, et al. (2015), KL (2019) (reviews)





time

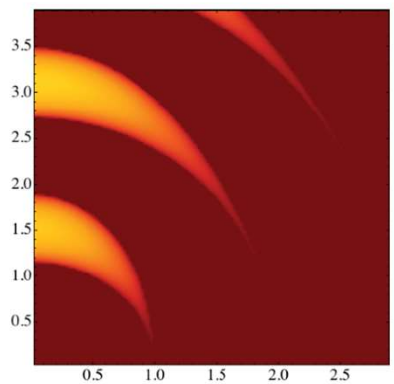
inflation

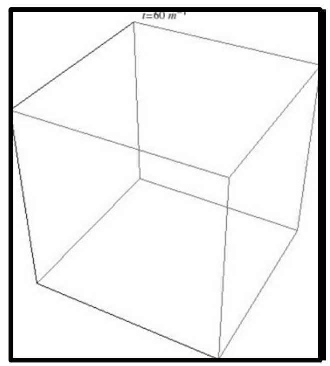
linear regime
(preheating)

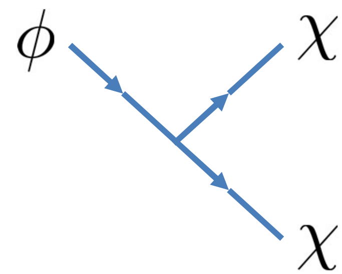
non-linear
regime

Time-line of Reheating

Allahverdi, Brandenberger, et al. (2010), Amin, et al. (2015), KL (2019) (reviews)







time

inflation

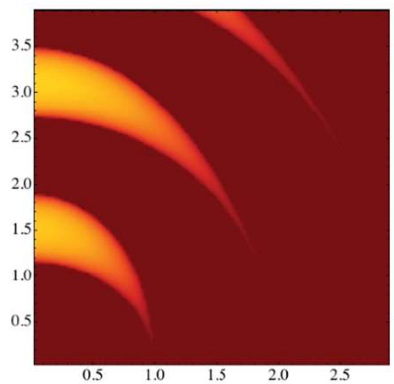
linear regime
(preheating)

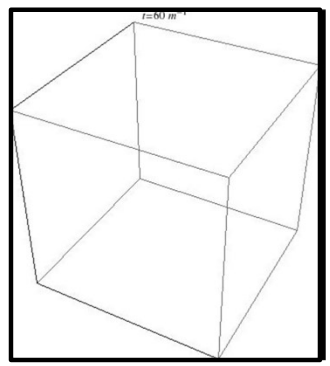
non-linear
regime

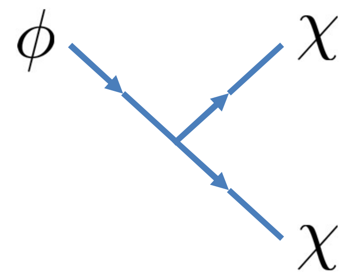
perturbative
regime

Time-line of Reheating

Allahverdi, Brandenberger, et al. (2010), Amin, et al. (2015), KL (2019) (reviews)







time

inflation

linear regime (preheating)

non-linear regime

perturbative regime

thermalization

nucleosynthesis

Observational signatures

- Difficult

  - very small length scales

  - thermalization

Observational consequences

Allahverdi, Brandenberger, et al. (2010), Amin, et al. (2015), KL (2019) (reviews)

- Expansion history effects

- Potentially observable remnants:
  - Stochastic GWs
  - Non-gaussianities
  - Solitons

- Constraints on high energy physics models:
  - Overproduction of defects
  - Complete decay of $\bar{\phi}(t)$
  - Primordial BHs

The way forward

- Theory
  - more realistic models (couplings)

- Phenomenology
  - detailed (numerical) analysis
  - observational signatures

- Observations
  - non-Gaussianities, high-freq GWs
  - expansion history effects

Our works

The equation of state after inflation

KL and M. Amin, PRD 99 123504 (2019)
KL and M. Amin, PRD 97 023533 (2018)
KL and M. Amin, PRL 119 061301 (2017)

What is w after inflation?

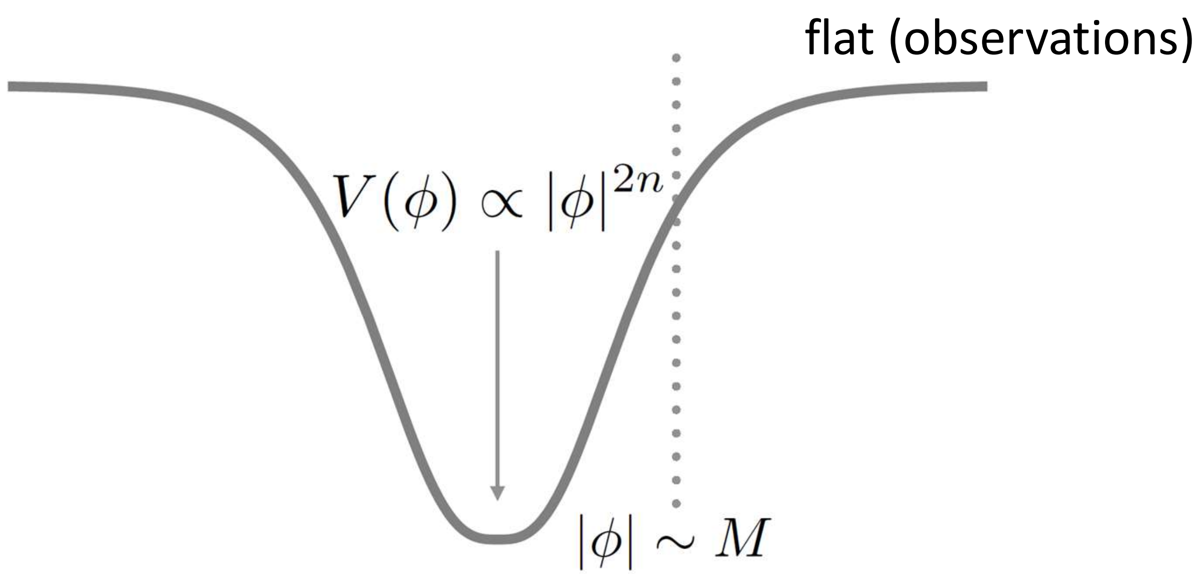
$$w = \frac{\text{pressure}}{\text{energy density}}$$

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\Big]$$

What is w after inflation?

$$w = \frac{\text{pressure}}{\text{energy density}}$$

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\Big]$$

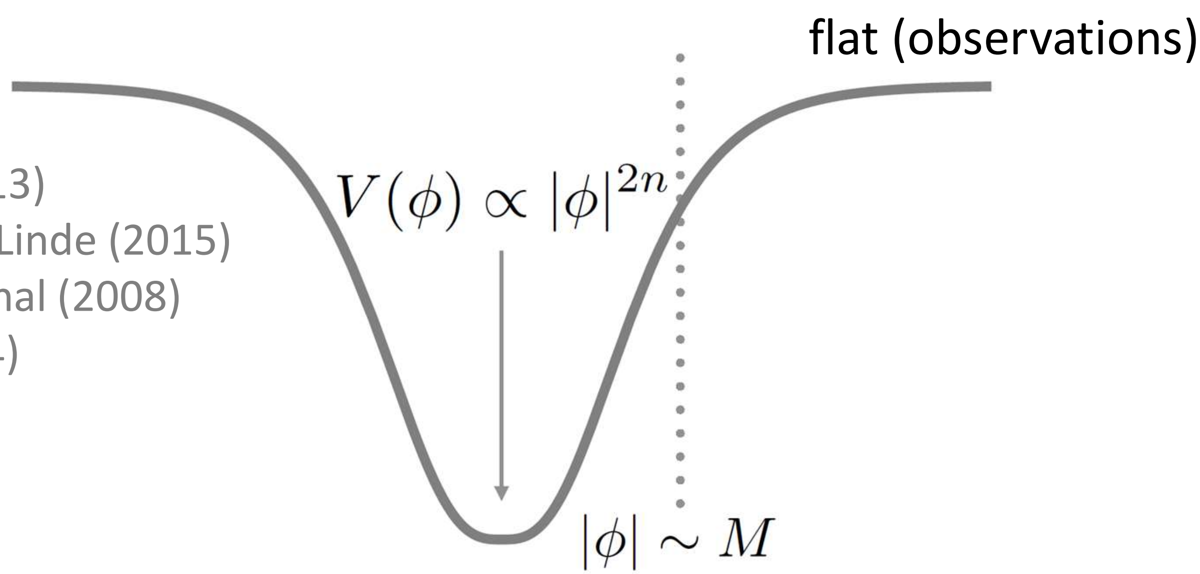


What is w after inflation?

$$w = \frac{\text{pressure}}{\text{energy density}}$$

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\Big]$$

Kallosh and Linde (2013)
Carrasco, Kallosh and Linde (2015)
Silverstein and Westphal (2008)
McAllister, et al. (2014)



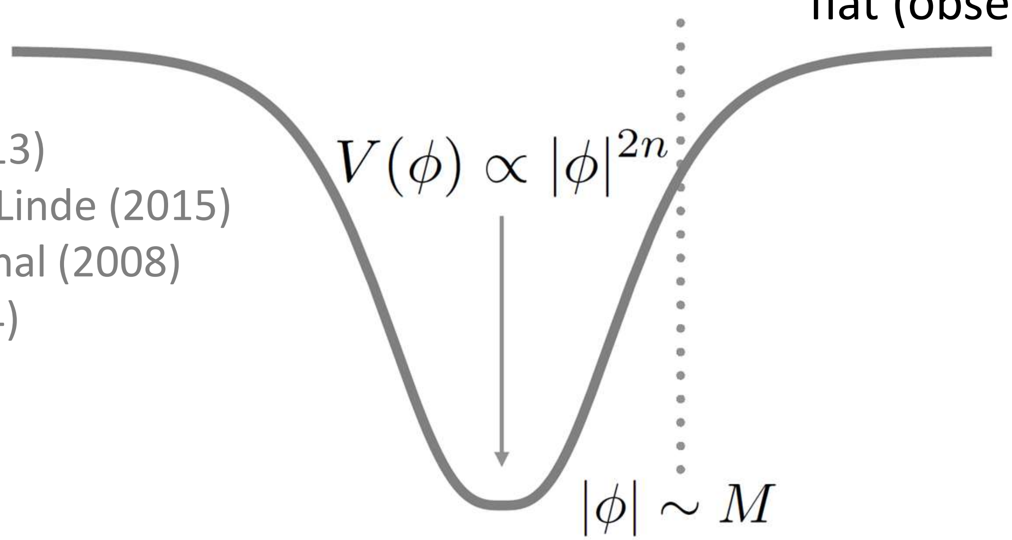
What is w after inflation?

$$w = \frac{\text{pressure}}{\text{energy density}}$$

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\Big]$$

flat (observations)

Kallosh and Linde (2013)
Carrasco, Kallosh and Linde (2015)
Silverstein and Westphal (2008)
McAllister, et al. (2014)



assumption: self-couplings dominate over others

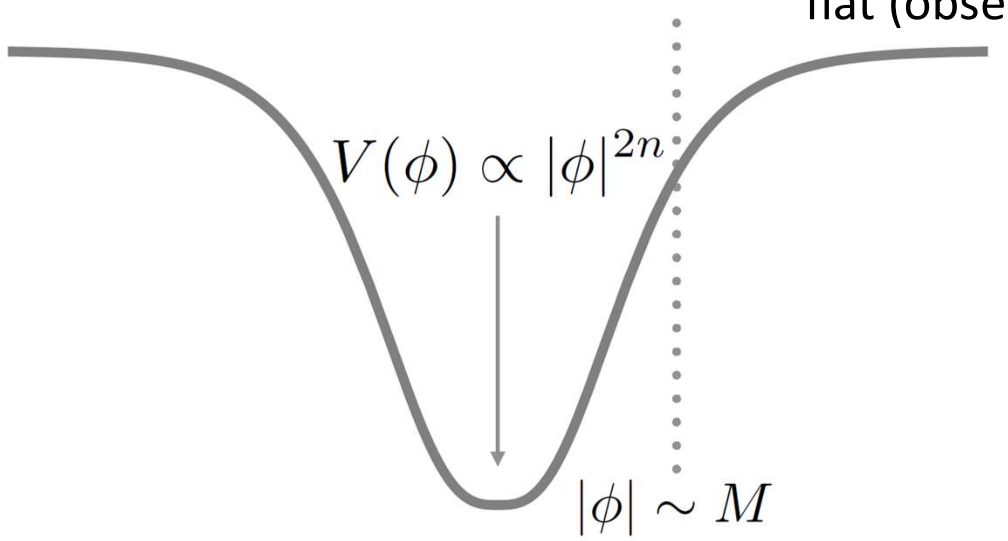
What is w after inflation?

at sufficiently late times:

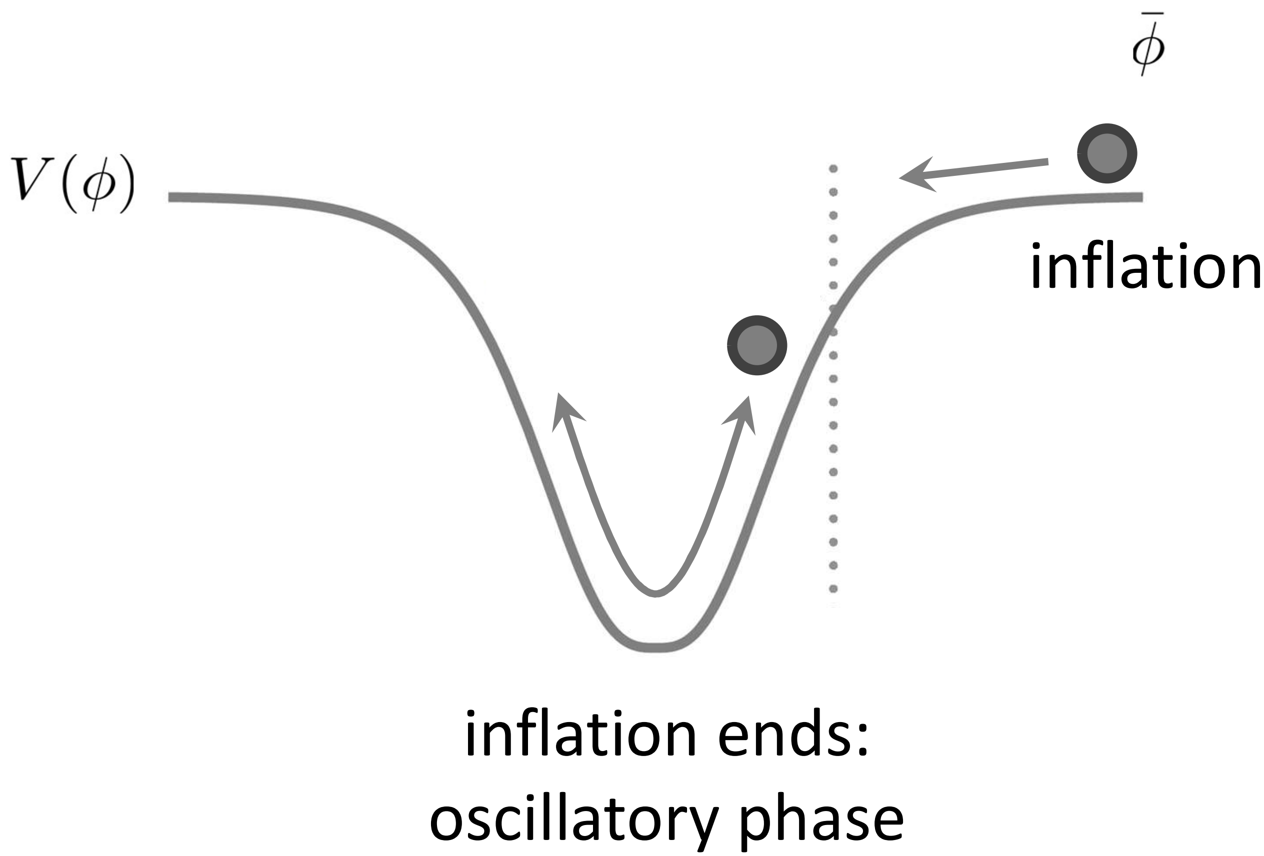
$$w = \begin{cases} 0 & \text{if } n = 1 \\ 1/3 & \text{if } n > 1 \end{cases}$$

(even without couplings to other fields!)

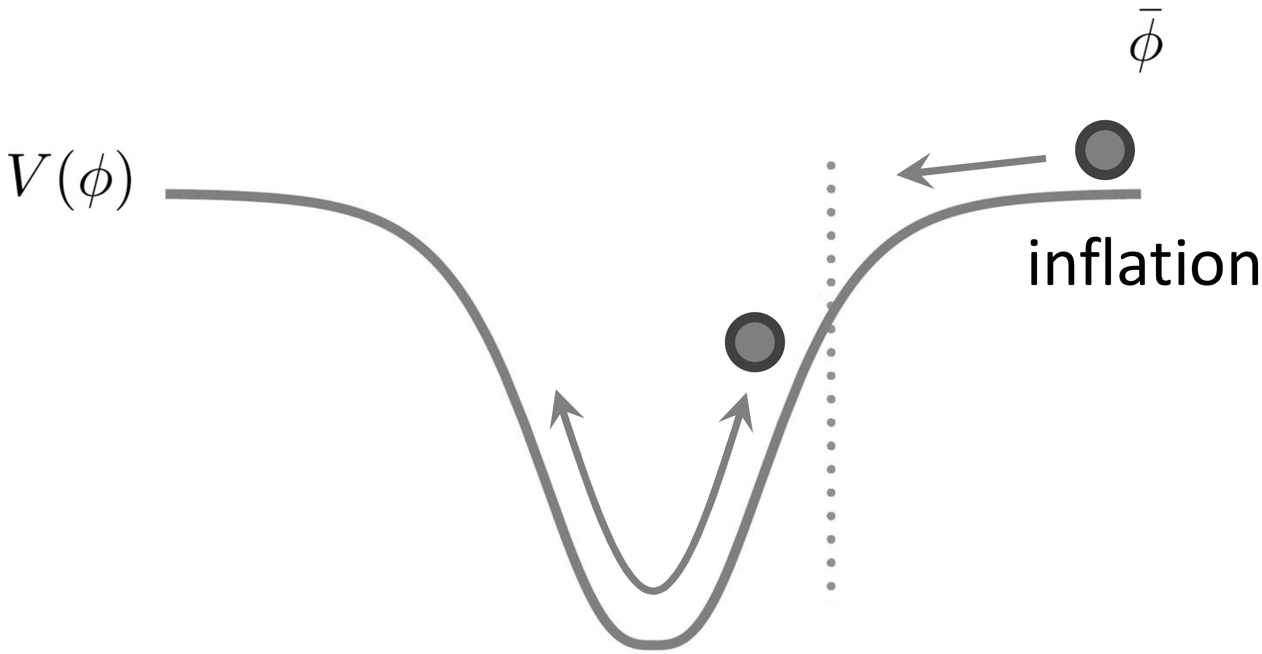
flat (observations)



Inflaton dynamics



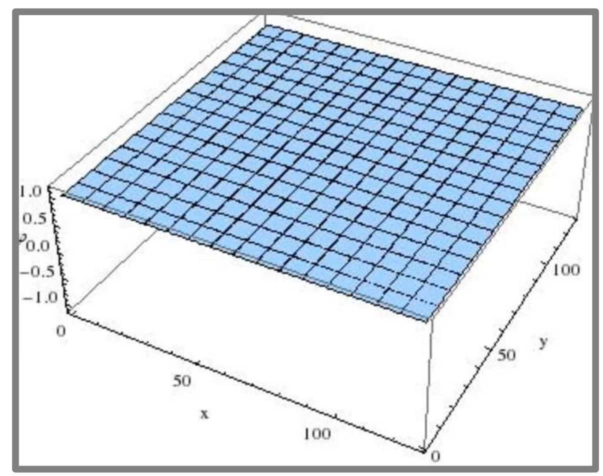
Inflaton (homogeneous) dynamics



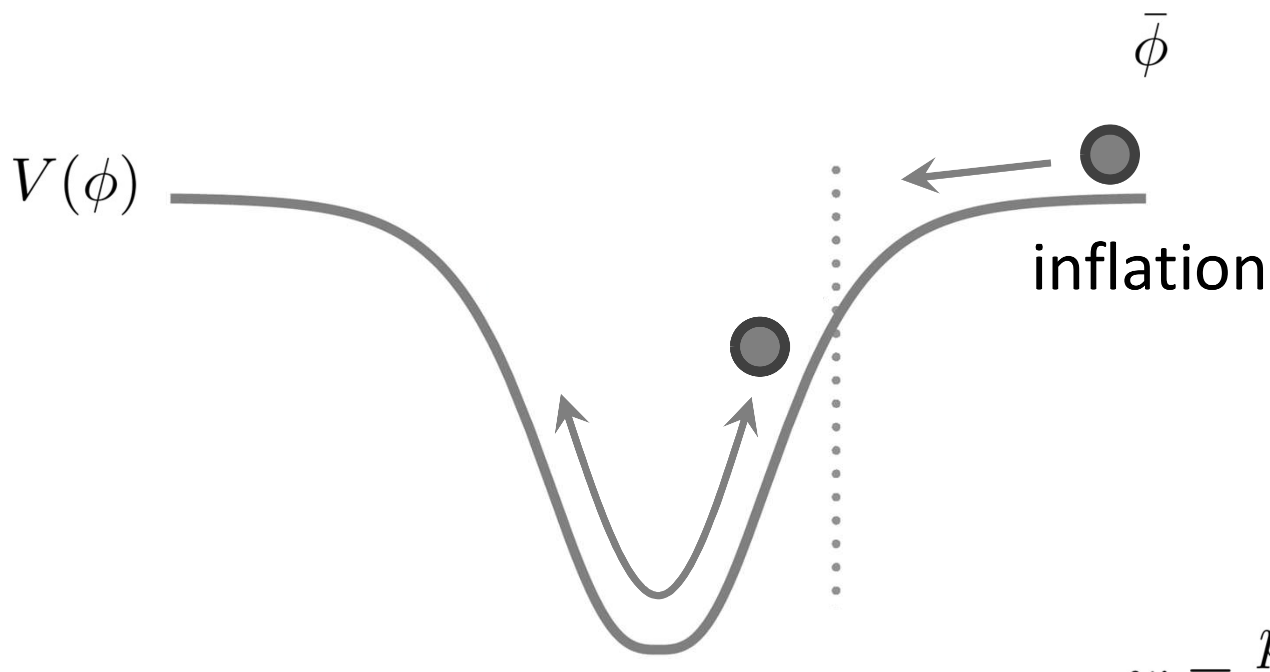
inflation ends:
oscillatory phase

$$\bar{\phi}(t) \propto a(t)^{-3/(n+1)}$$

$$\phi \times a^{3/(n+1)}$$



Inflaton (homogeneous) dynamics

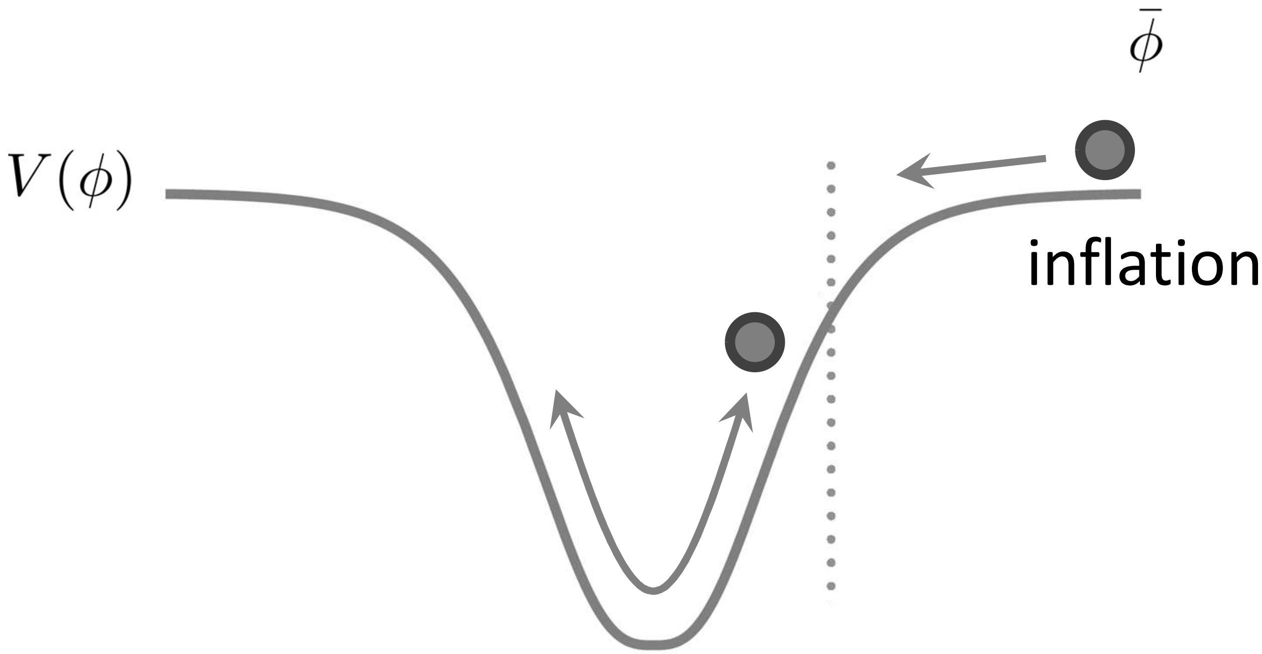


inflation ends:
oscillatory phase

$$\bar{\phi}(t) \propto a(t)^{-3/(n+1)}$$

$$w \equiv \frac{p}{\rho} = \frac{\dot{\phi}^2/2 - (\nabla\phi)^2/6 - V(\phi)}{\dot{\phi}^2/2 + (\nabla\phi)^2/2 + V(\phi)}$$

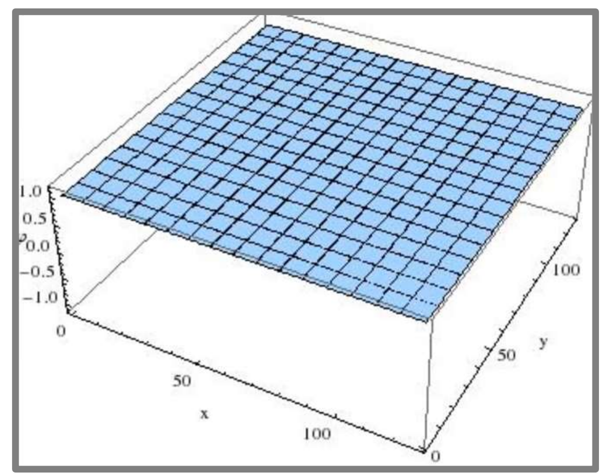
Inflaton (homogeneous) dynamics



inflation ends:
oscillatory phase

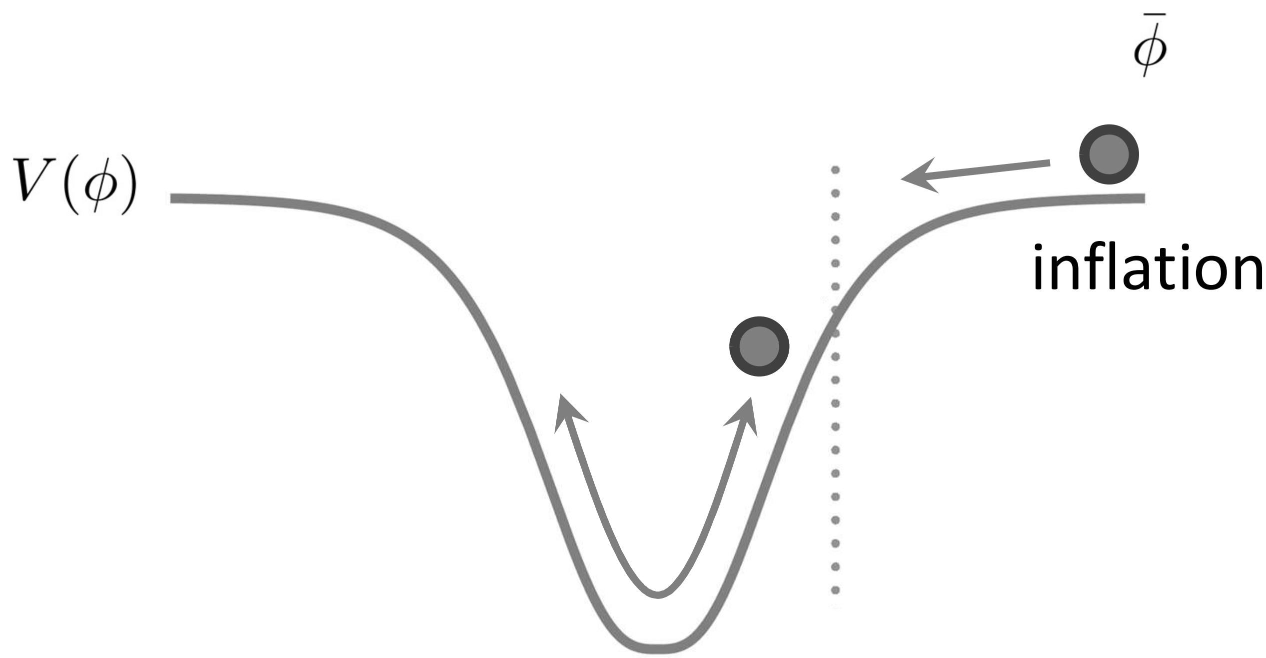
$$w_{\rm hom} = \frac{n-1}{n+1}$$

$\phi \times a^{3/(n+1)}$



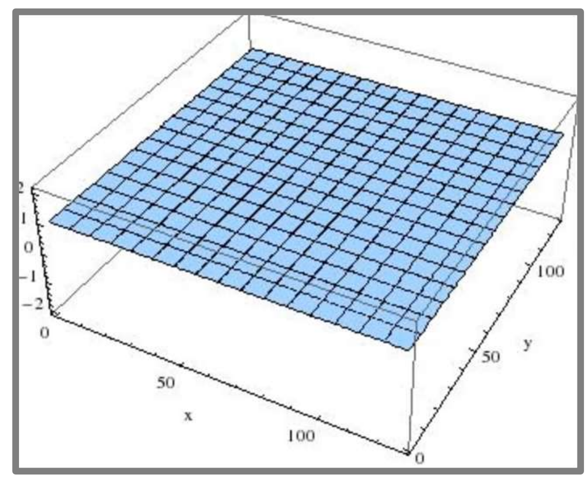
Turner (1983)

Inflaton (actual) dynamics



inflation ends:
oscillatory phase

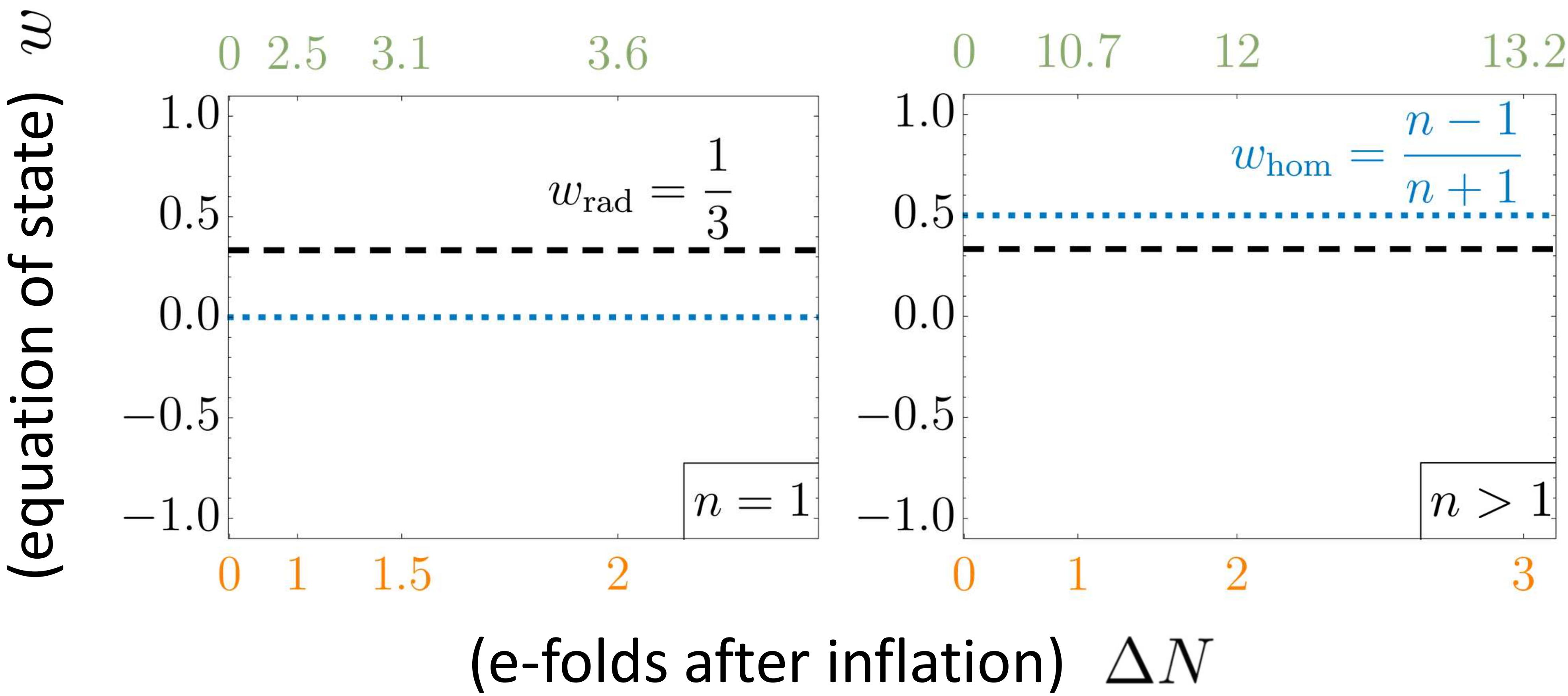
$\phi \times a^{3/(n+1)}$



- parametric resonance of $\delta\phi(t, \mathbf{x})$
- $\bar{\phi}$ fragments

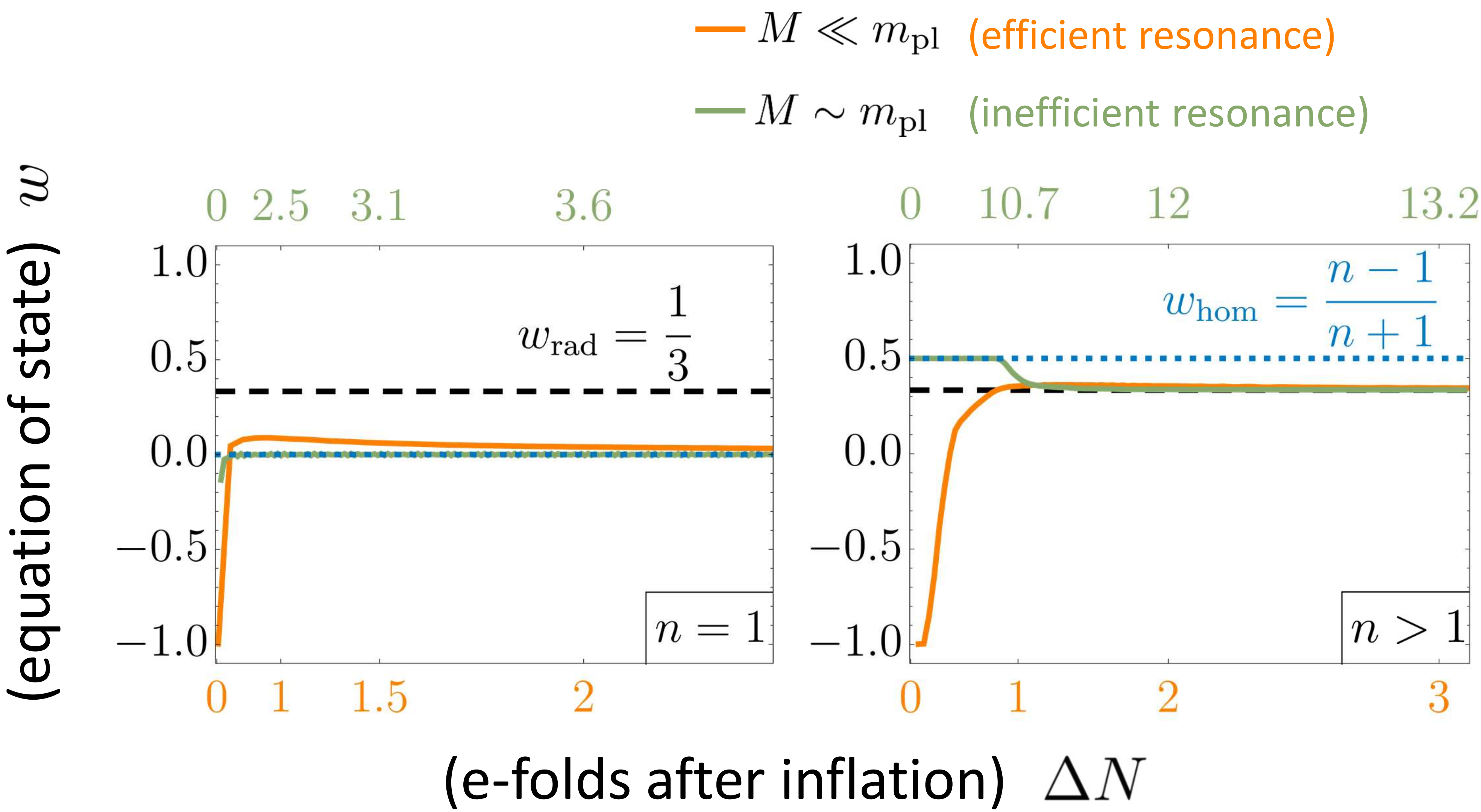
KL and M. Amin (2017, 18, 19)

Equation of state



$\Delta N \equiv \int_{a_{\rm end}}^{a} d\ln a$

Equation of state



$\Delta N \equiv \int_{a_{\rm end}}^{a} d\ln a$

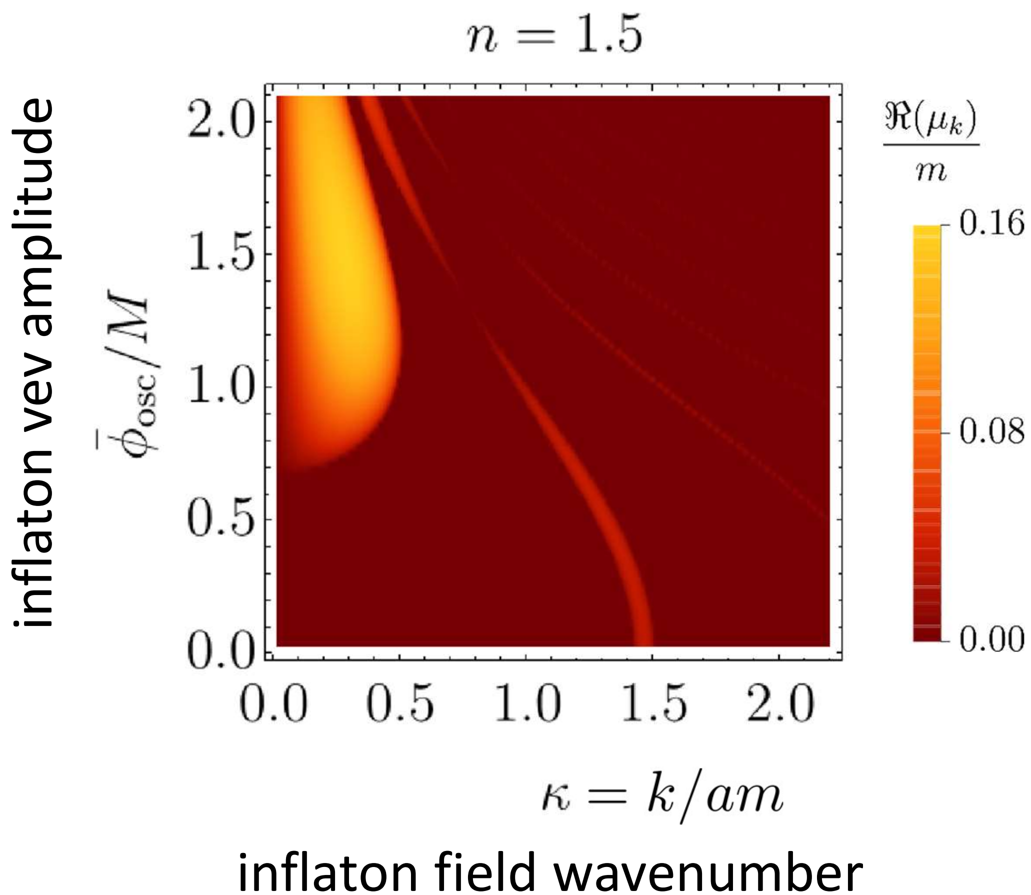
Towards radiation domination

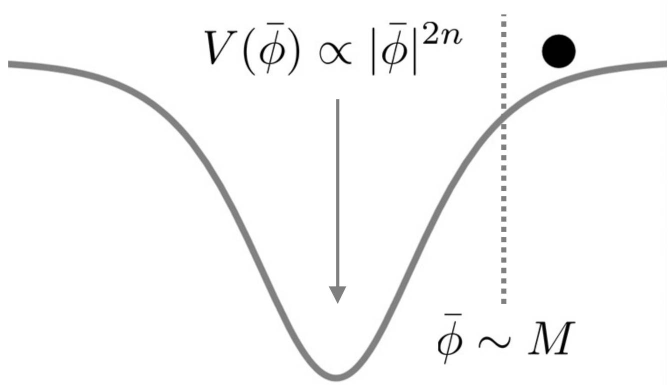
$$n > 1$$

Towards radiation domination

$$n > 1$$

Non-perturbative decay (parametric self-resonance)





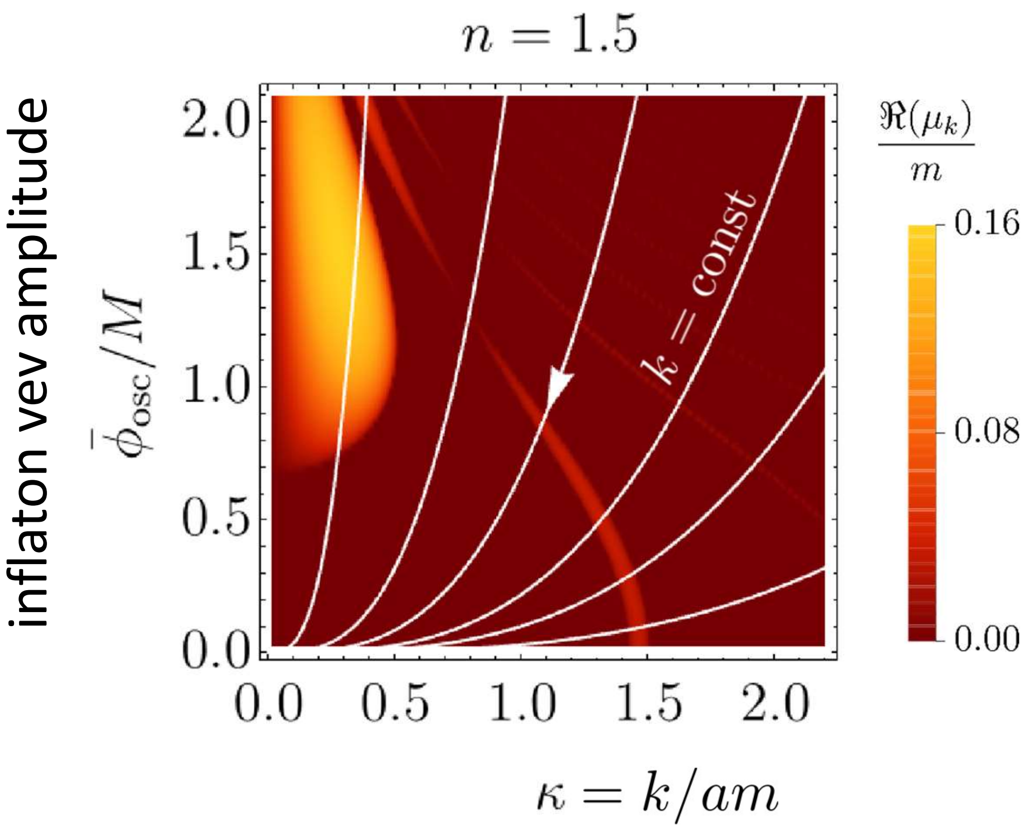
$$\delta\phi_k \propto \exp(\pm\mu_k t)$$

$m^2 \equiv V'(\bar{\phi}_{\rm osc})/\bar{\phi}_{\rm osc}$

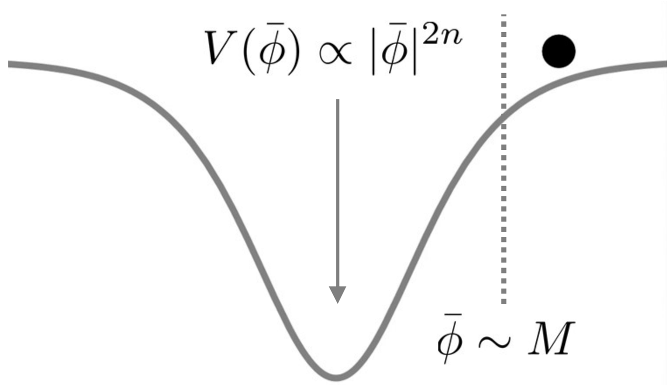
Towards radiation domination

$$n > 1$$

Non-perturbative decay (parametric self-resonance)



inflaton field wavenumber



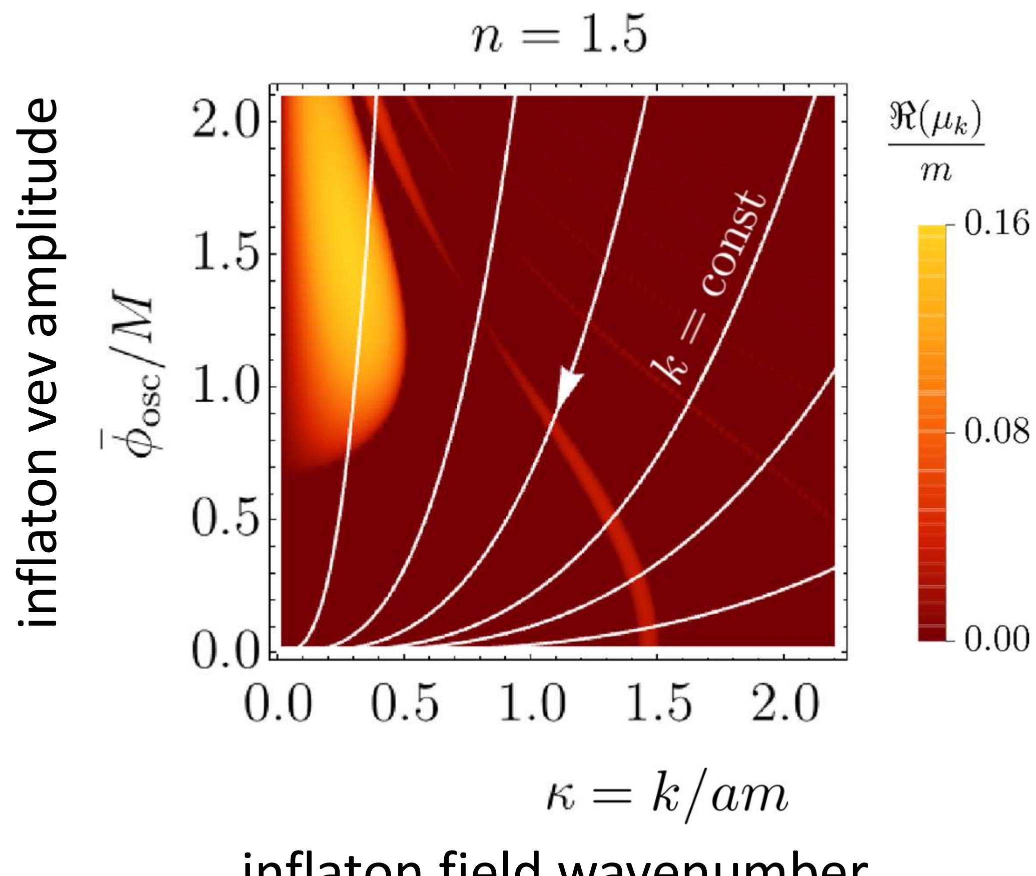
$$\delta\phi_k \propto \exp(\pm\mu_k t)$$

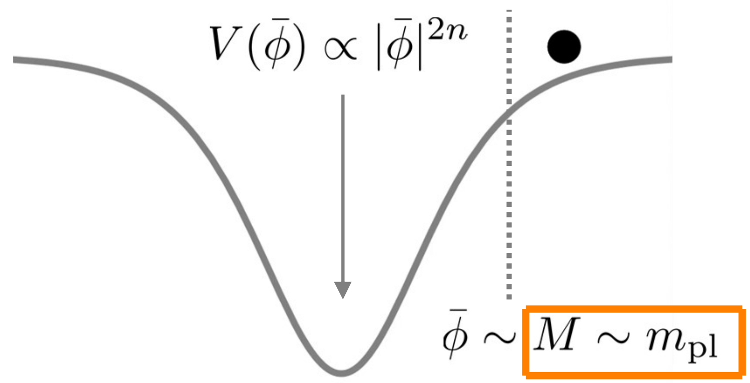
$$m^2 \equiv V'(\bar{\phi}_{\rm osc})/\bar{\phi}_{\rm osc}$$

Towards radiation domination

$$n > 1$$

Non-perturbative decay (parametric self-resonance)





$$\delta\phi_k \propto \exp(\pm \mu_k t)$$

$m^2 \equiv V'(\bar{\phi}_{\rm osc})/\bar{\phi}_{\rm osc}$

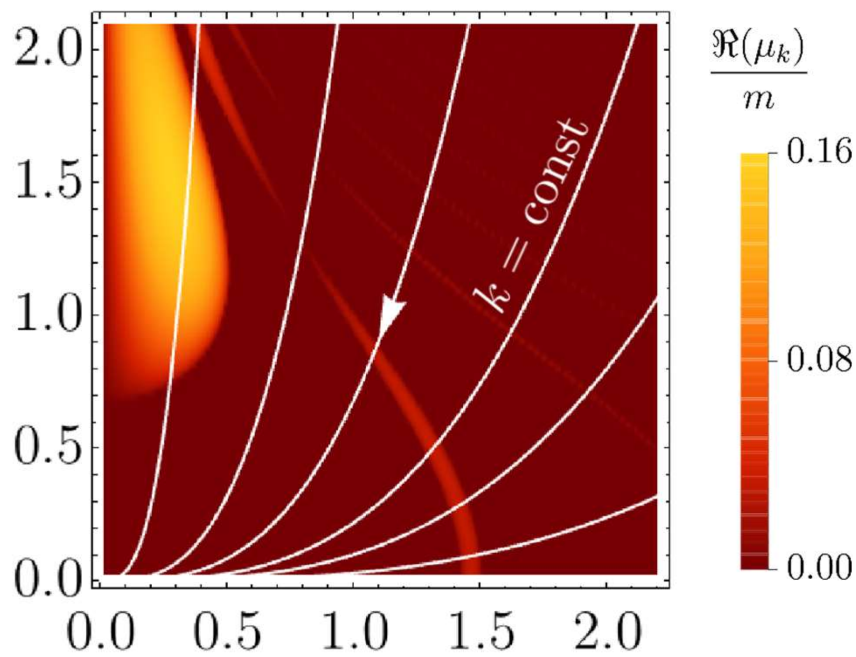
Towards radiation domination

$$n > 1$$

Non-perturbative decay (parametric self-resonance)

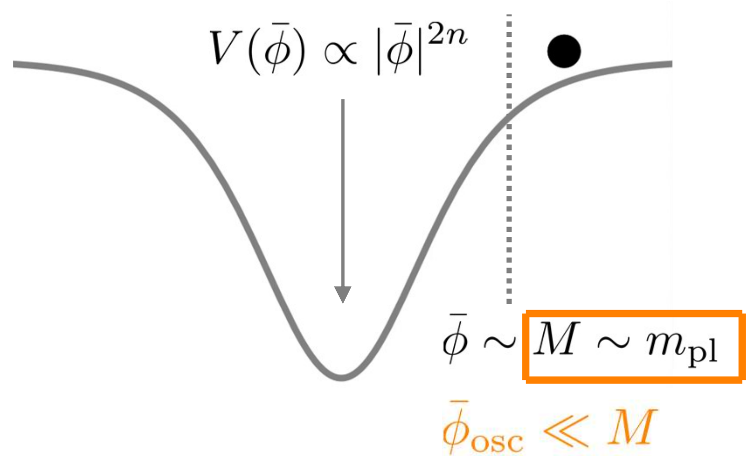
$n = 1.5$

inflaton vev amplitude



$\kappa = k/am$

inflaton field wavenumber



$$\delta\phi_k \propto \exp(\pm\mu_k t)$$

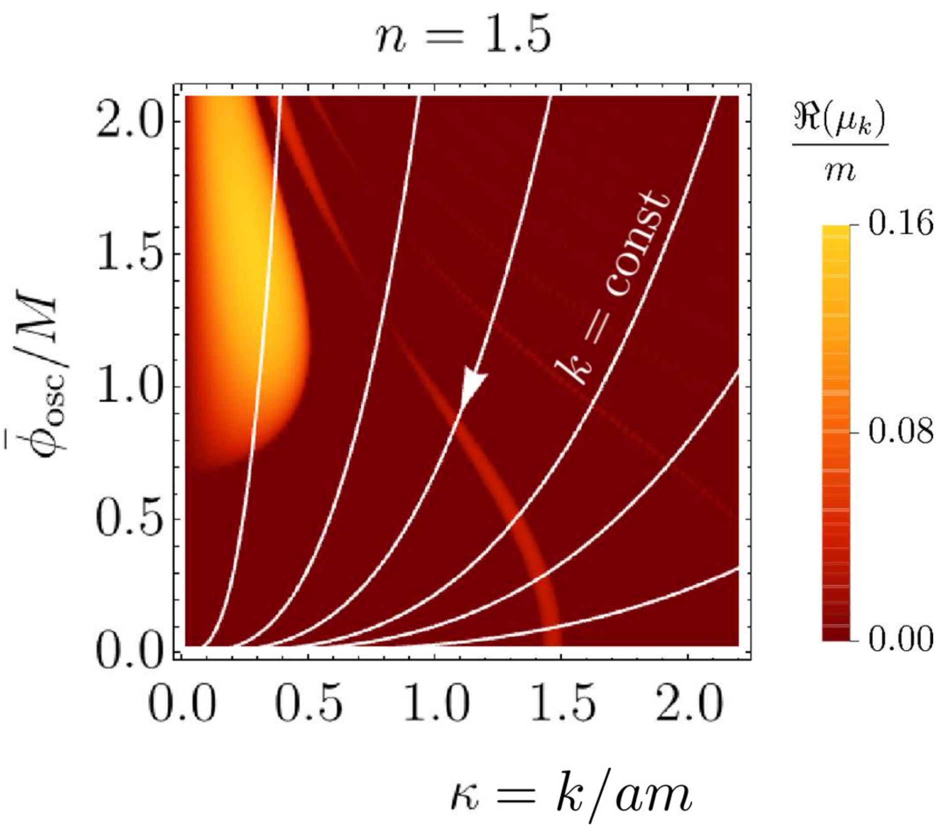
$m^2 \equiv V'(\bar{\phi}_{\rm osc})/\bar{\phi}_{\rm osc}$

Towards radiation domination

$$n > 1$$

Non-perturbative decay (parametric self-resonance)

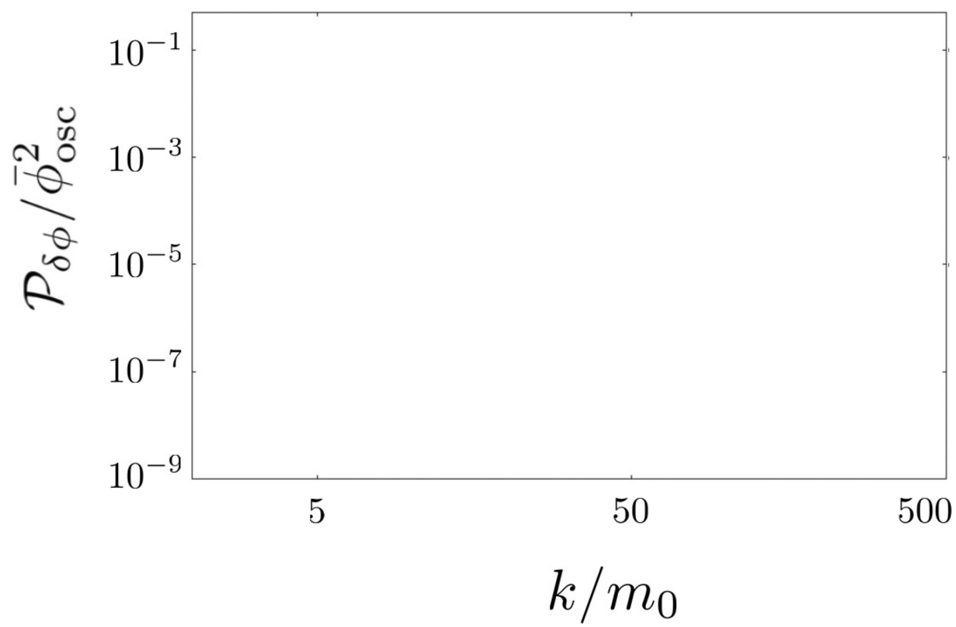
inflaton vev amplitude



inflaton field wavenumber

Power spectrum:

$$\langle \delta\phi(x)^2 \rangle \equiv \int \mathcal{P}_{\delta\phi}\, d\ln k$$



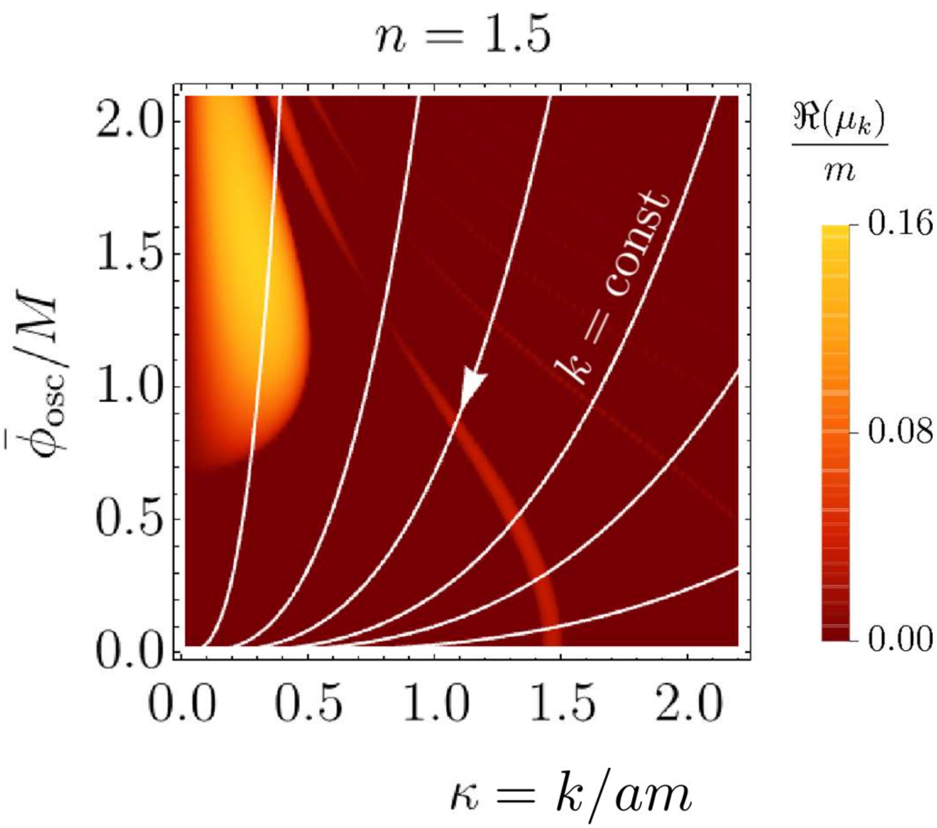
comoving wavenumber

Towards radiation domination

$$n > 1$$

Non-perturbative decay (parametric self-resonance)

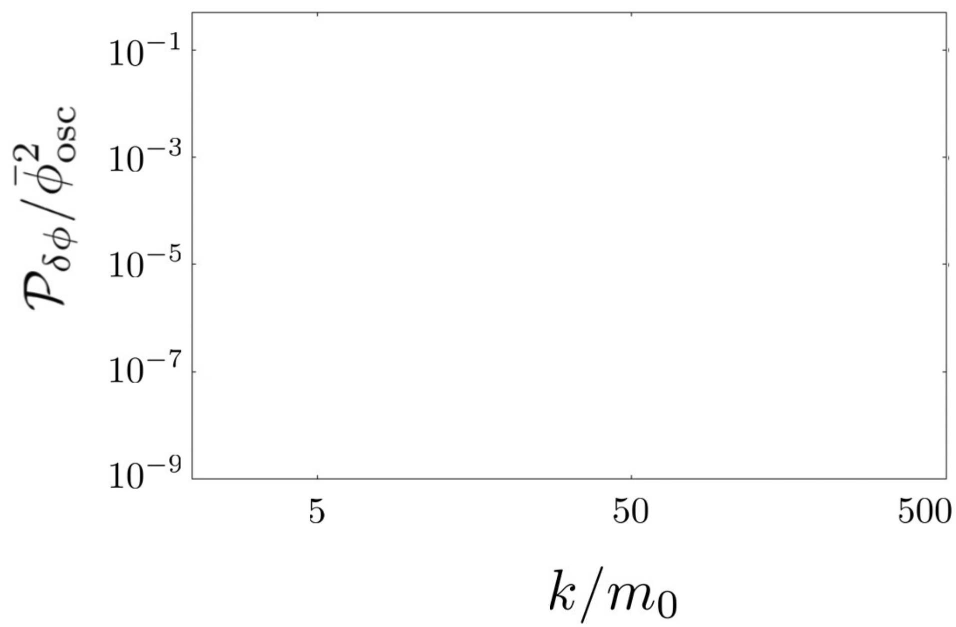
inflaton vev amplitude



inflaton field wavenumber

Power spectrum:

$$\langle \delta\phi(x)^2 \rangle \equiv \int \mathcal{P}_{\delta\phi}\, d\ln k$$



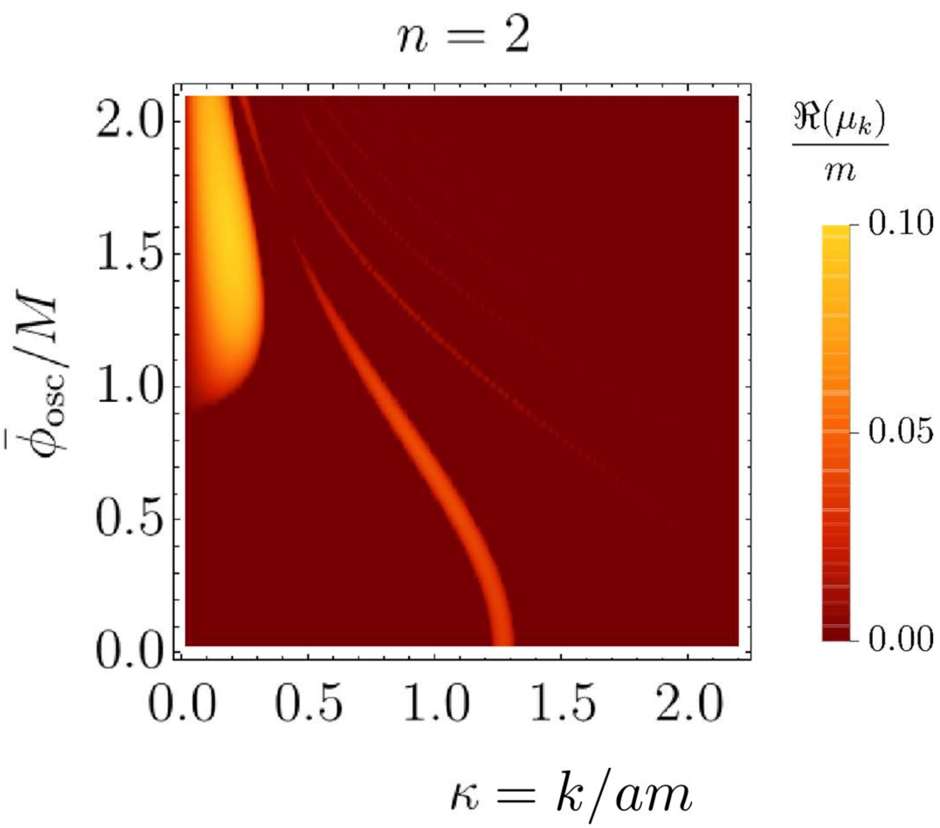
comoving wavenumber

Towards radiation domination

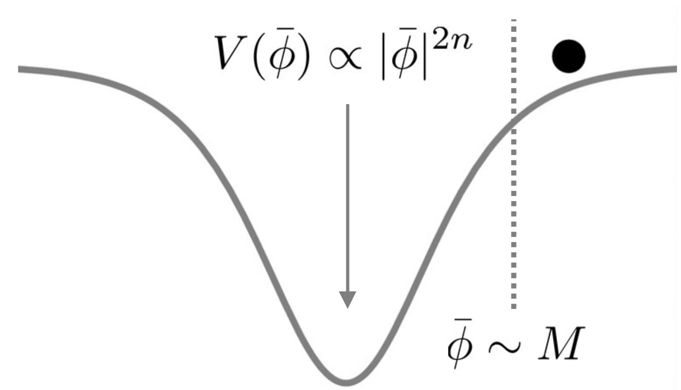
$$n > 1$$

Non-perturbative decay (parametric self-resonance)

inflaton vev amplitude



inflaton field wavenumber



$$\delta\phi_k \propto \exp(\pm\mu_k t)$$

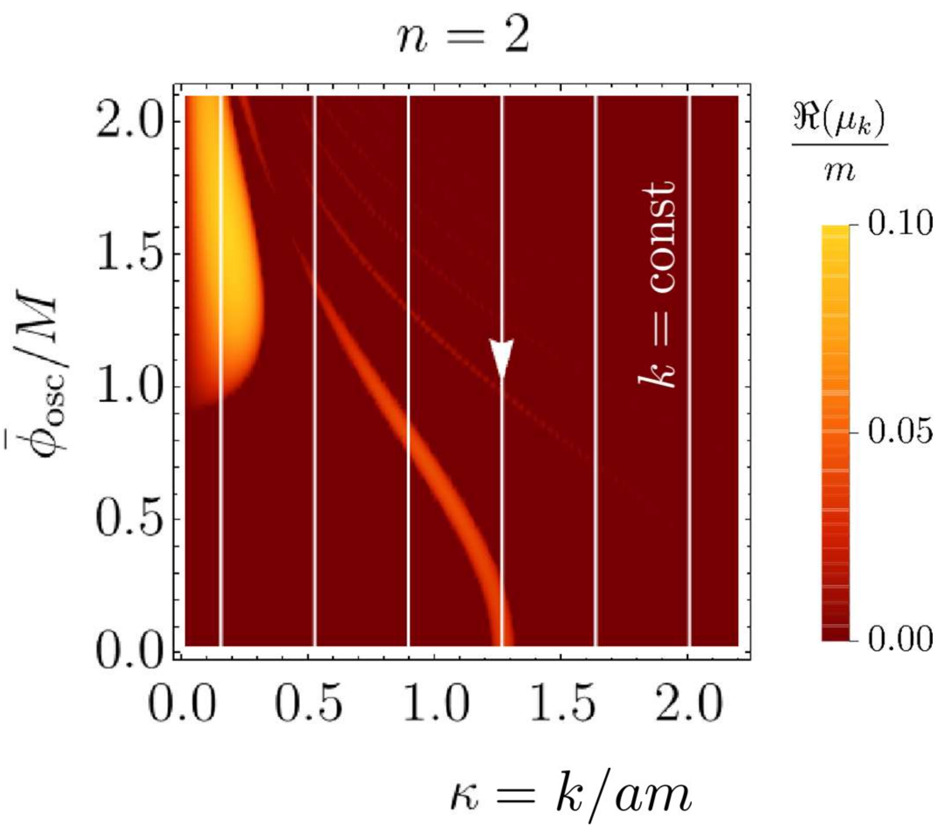
$m^2 \equiv V'(\bar{\phi}_{\rm osc})/\bar{\phi}_{\rm osc}$

Towards radiation domination

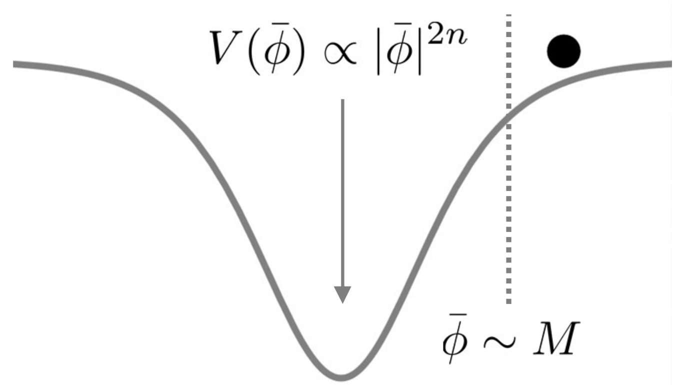
$$n > 1$$

Non-perturbative decay (parametric <u>self</u>-resonance)

inflaton vev amplitude



inflaton field wavenumber



$$\delta\phi_k \propto \exp(\pm\mu_k t)$$

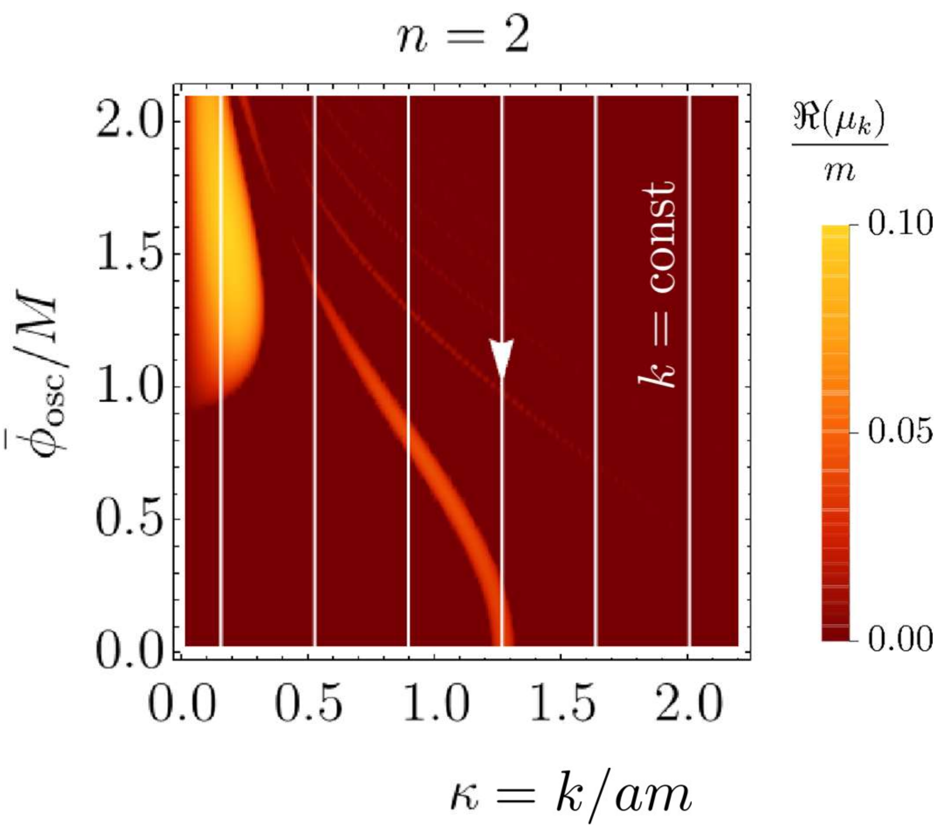
$m^2 \equiv V'(\bar{\phi}_{\rm osc})/\bar{\phi}_{\rm osc}$

Towards radiation domination

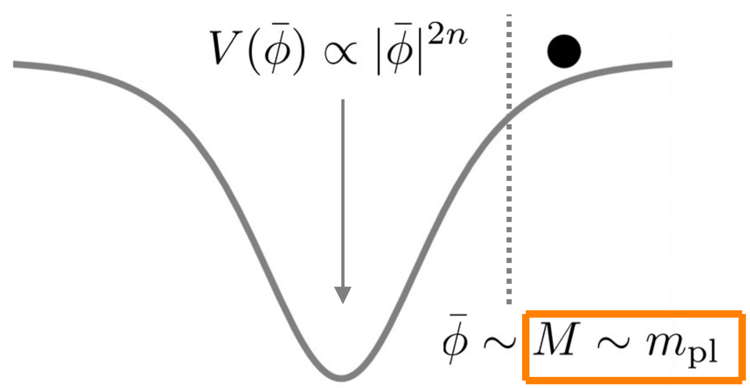
$$n > 1$$

Non-perturbative decay (parametric self-resonance)

inflaton vev amplitude



inflaton field wavenumber



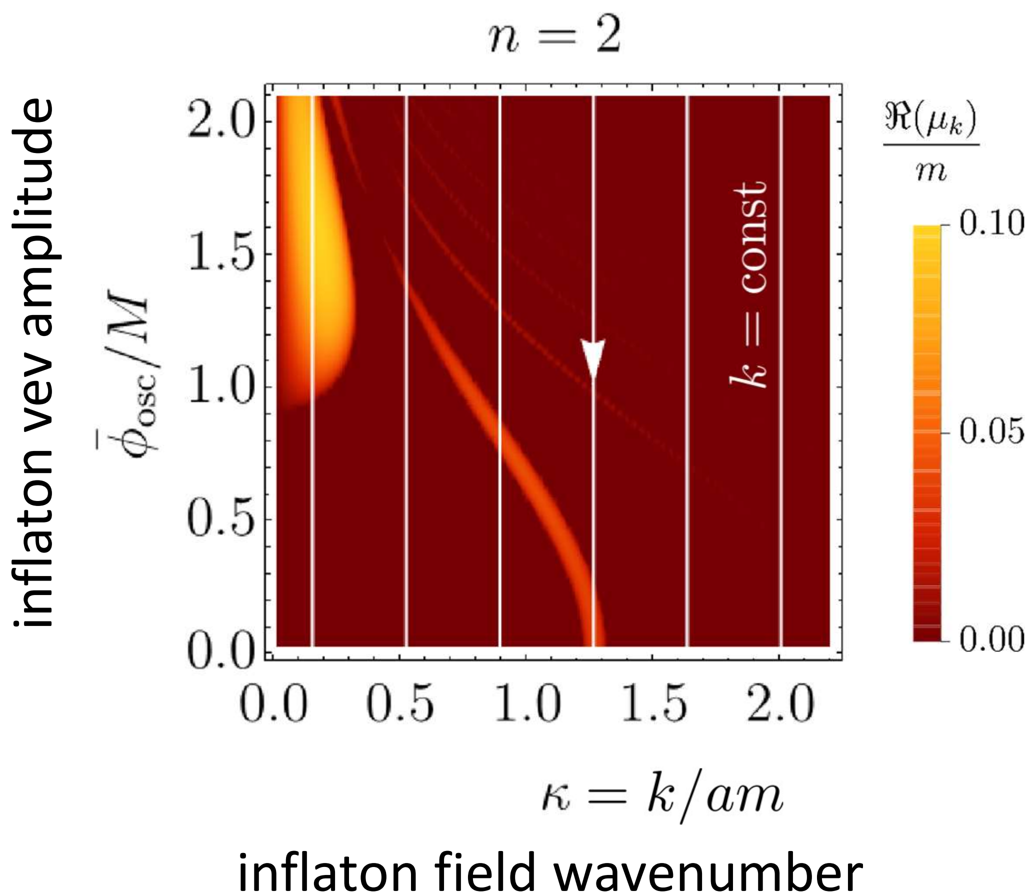
$$\delta\phi_k \propto \exp(\pm\mu_k t)$$

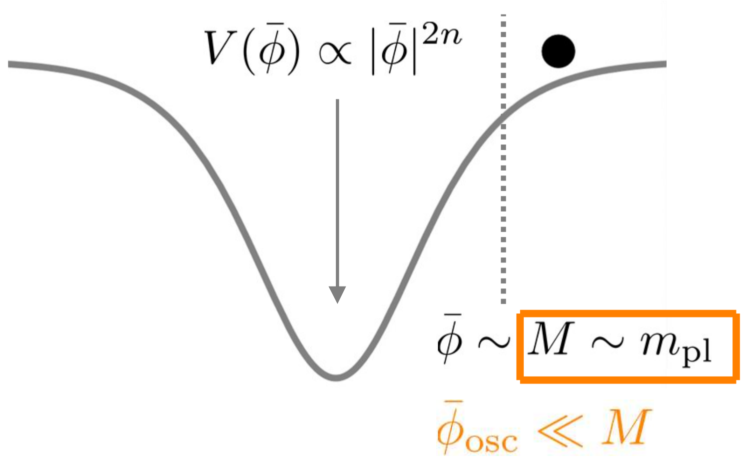
$m^2 \equiv V'(\bar{\phi}_{\rm osc})/\bar{\phi}_{\rm osc}$

Towards radiation domination

$$n > 1$$

Non-perturbative decay (parametric self-resonance)





$$\delta\phi_k \propto \exp(\pm\mu_k t)$$

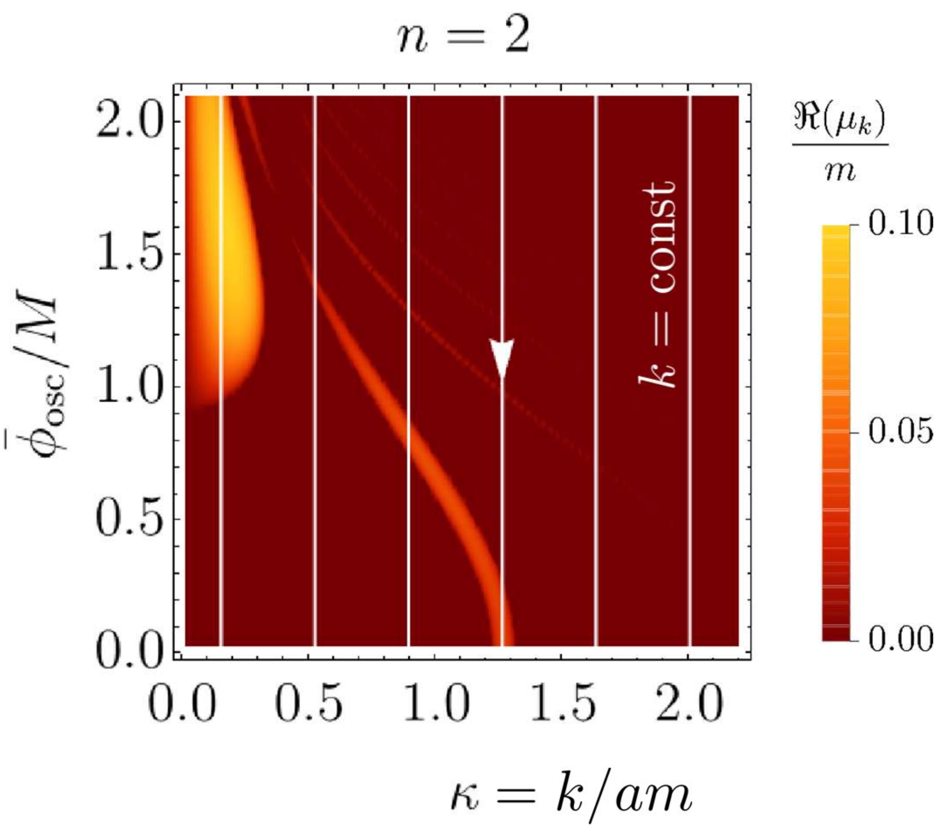
$m^2 \equiv V'(\bar{\phi}_{\rm osc})/\bar{\phi}_{\rm osc}$

Towards radiation domination

$$n > 1$$

Non-perturbative decay (parametric self-resonance)

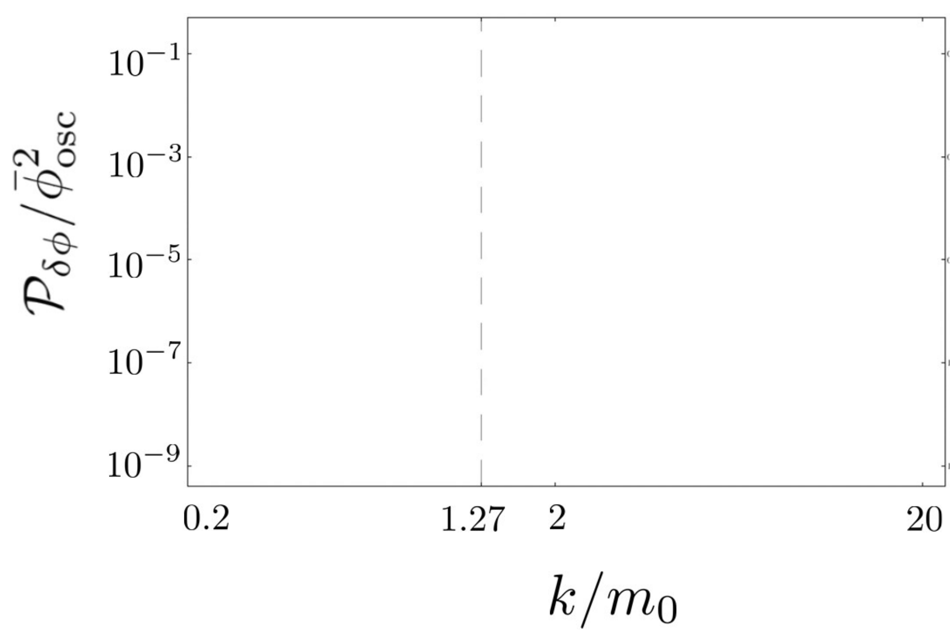
inflaton vev amplitude



inflaton field wavenumber

Power spectrum:

$$\langle \delta\phi(x)^2 \rangle \equiv \int \mathcal{P}_{\delta\phi}\, d\ln k$$



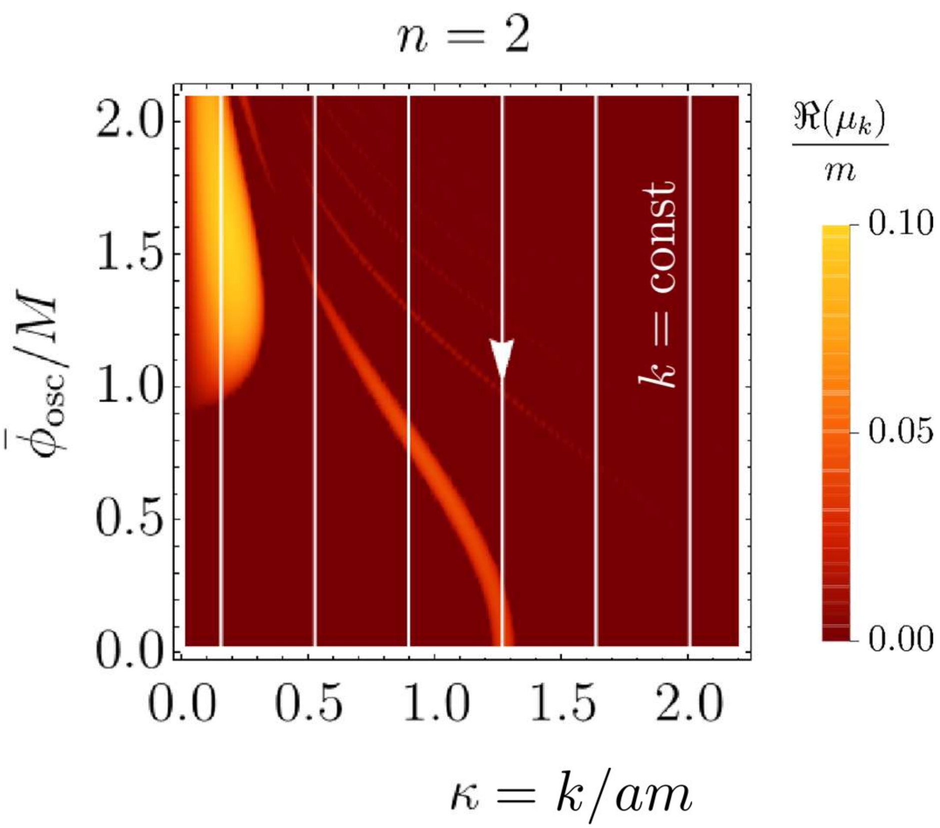
comoving wavenumber

Towards radiation domination

$$n > 1$$

Non-perturbative decay (parametric self-resonance)

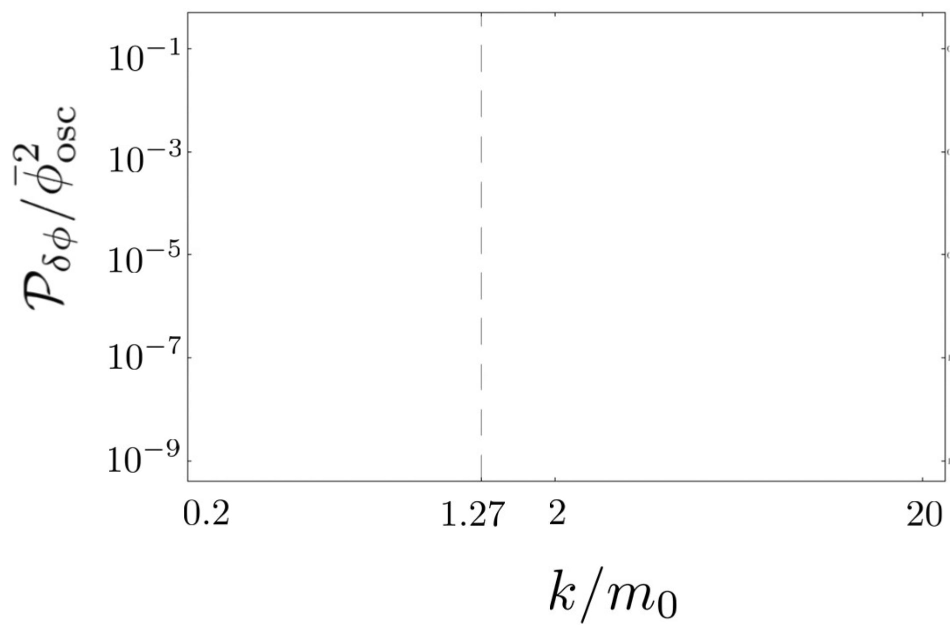
inflaton vev amplitude



inflaton field wavenumber

Power spectrum:

$$\langle \delta\phi(x)^2 \rangle \equiv \int \mathcal{P}_{\delta\phi} \, d\ln k$$



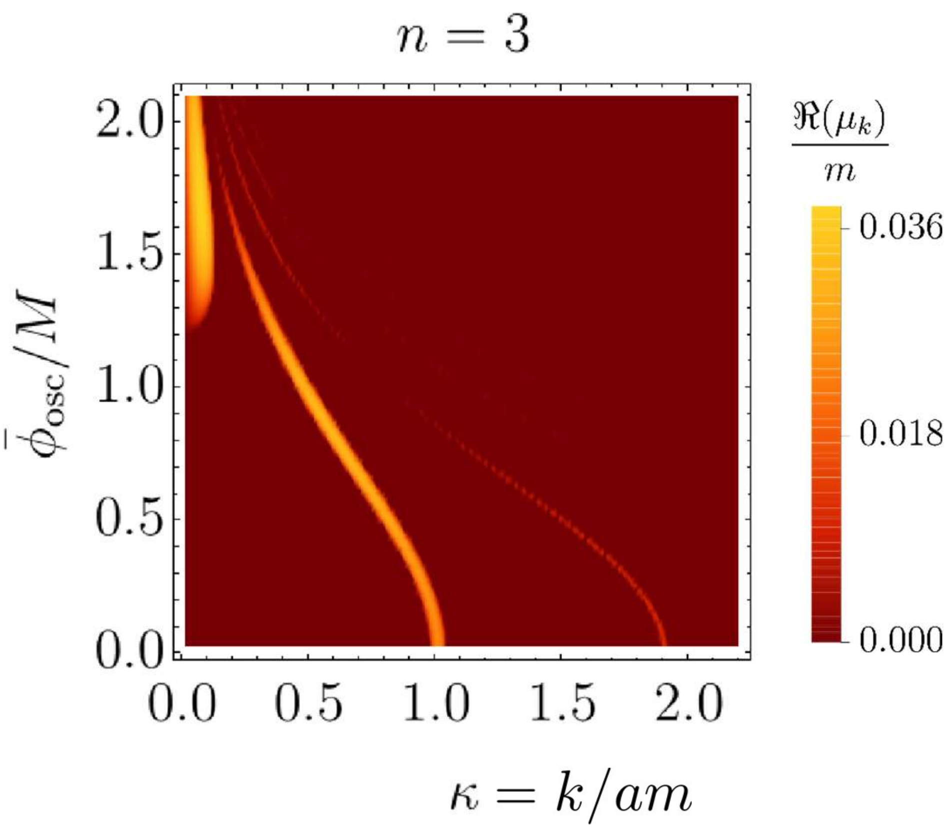
comoving wavenumber

Towards radiation domination

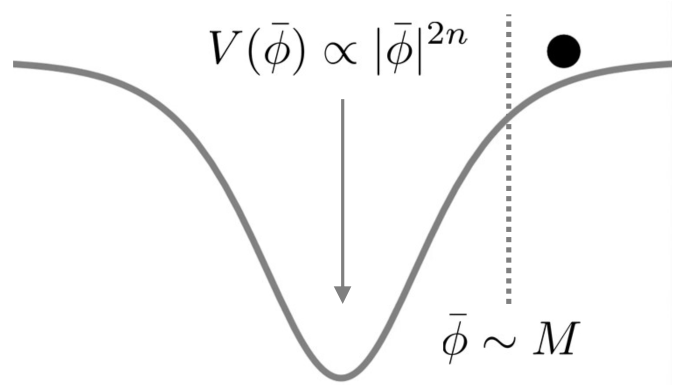
$$n > 1$$

Non-perturbative decay (parametric self-resonance)

inflaton vev amplitude



inflaton field wavenumber



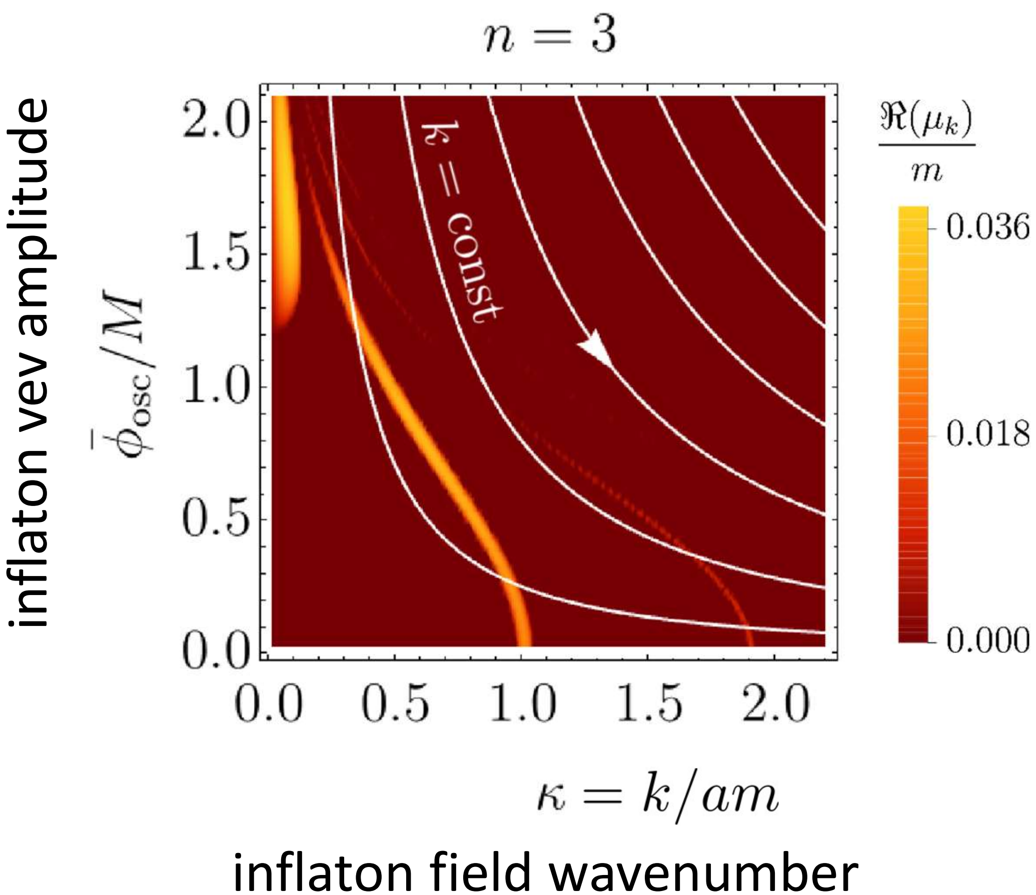
$$\delta\phi_k \propto \exp(\pm\mu_k t)$$

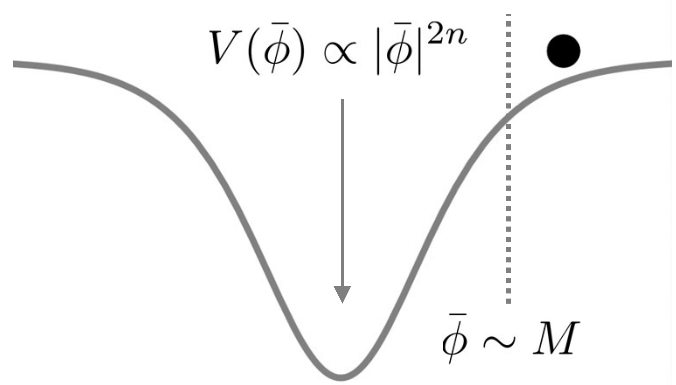
$m^2 \equiv V'(\bar{\phi}_{\rm osc})/\bar{\phi}_{\rm osc}$

Towards radiation domination

$$n > 1$$

Non-perturbative decay (parametric self-resonance)





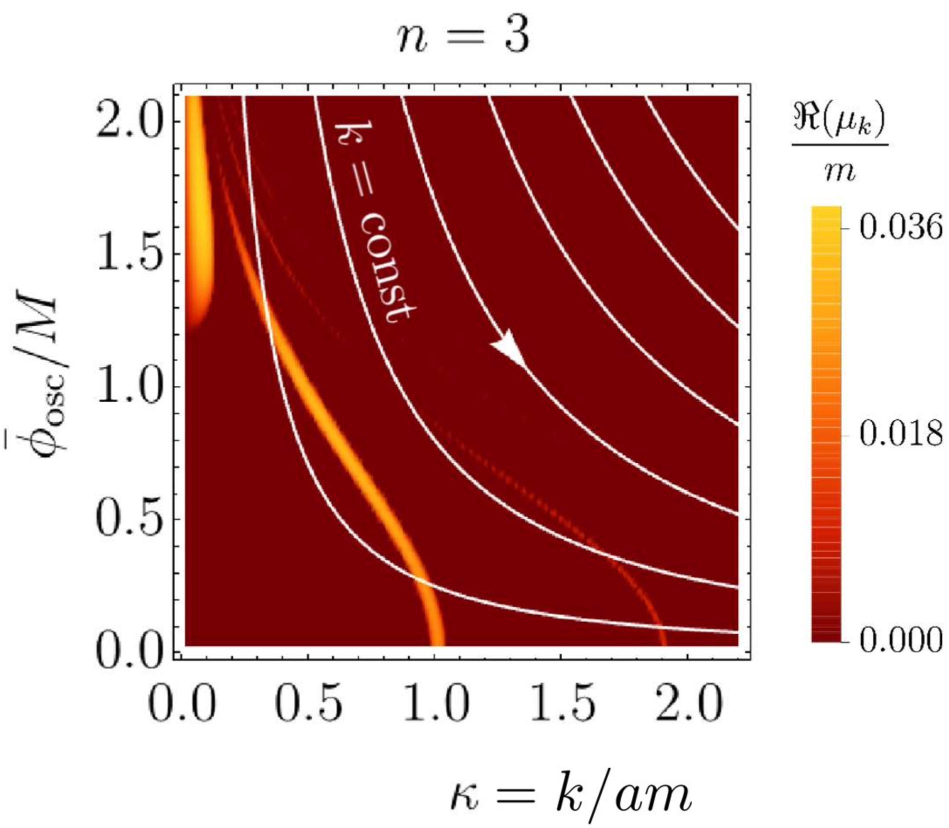
$$\delta\phi_k \propto \exp(\pm\mu_k t)$$

$m^2 \equiv V'(\bar{\phi}_{\rm osc})/\bar{\phi}_{\rm osc}$

Towards radiation domination

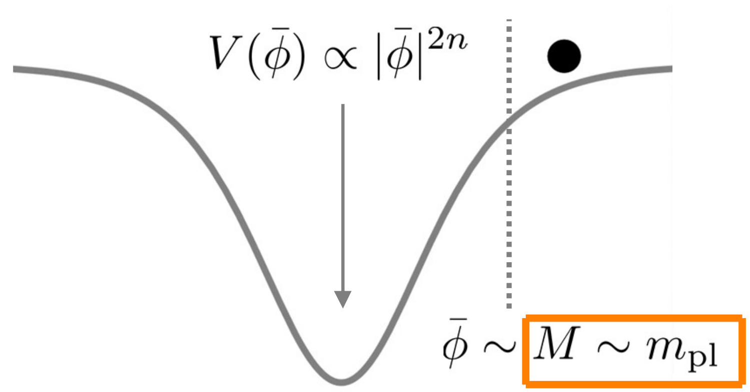
$$n > 1$$

Non-perturbative decay (parametric self-resonance)



inflaton vev amplitude

inflaton field wavenumber



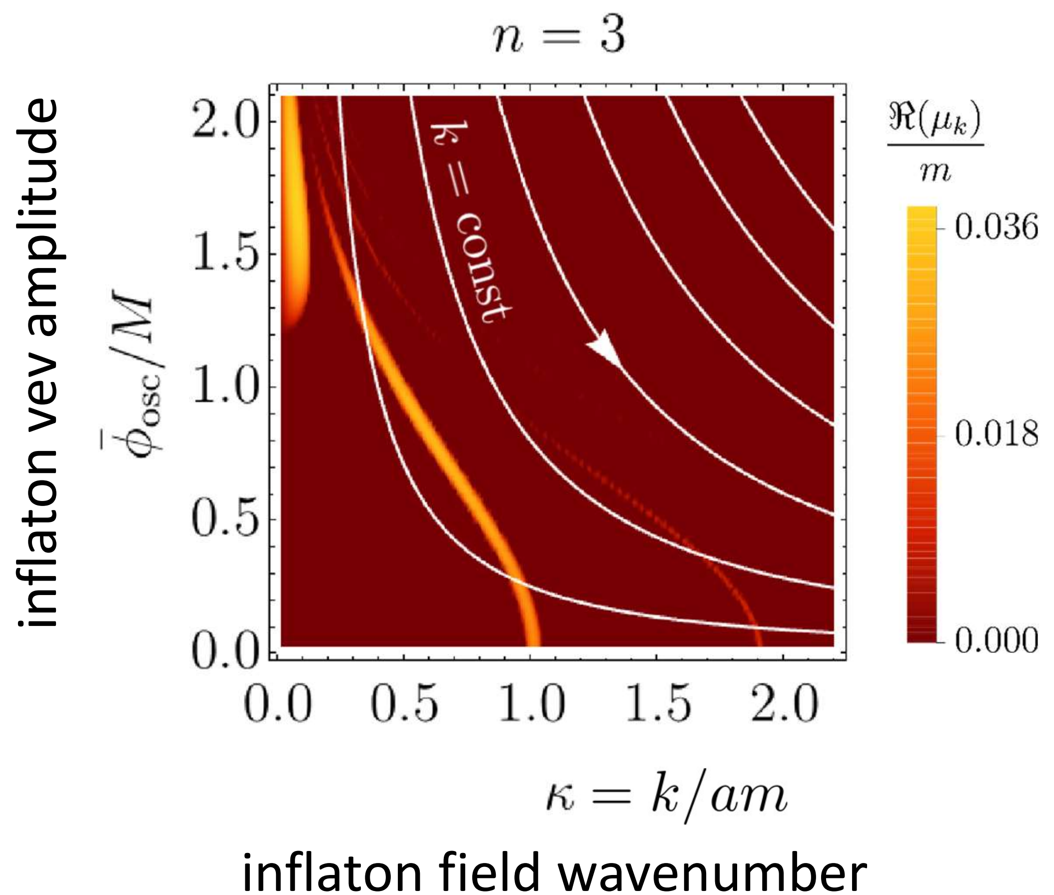
$$\delta\phi_k \propto \exp(\pm\mu_k t)$$

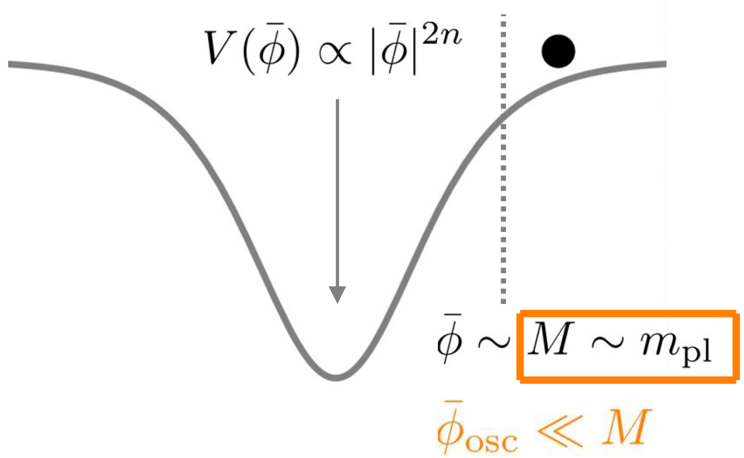
$m^2 \equiv V'(\bar{\phi}_{\rm osc})/\bar{\phi}_{\rm osc}$

Towards radiation domination

$$n > 1$$

Non-perturbative decay (parametric self-resonance)





$$\delta\phi_k \propto \exp(\pm\mu_k t)$$

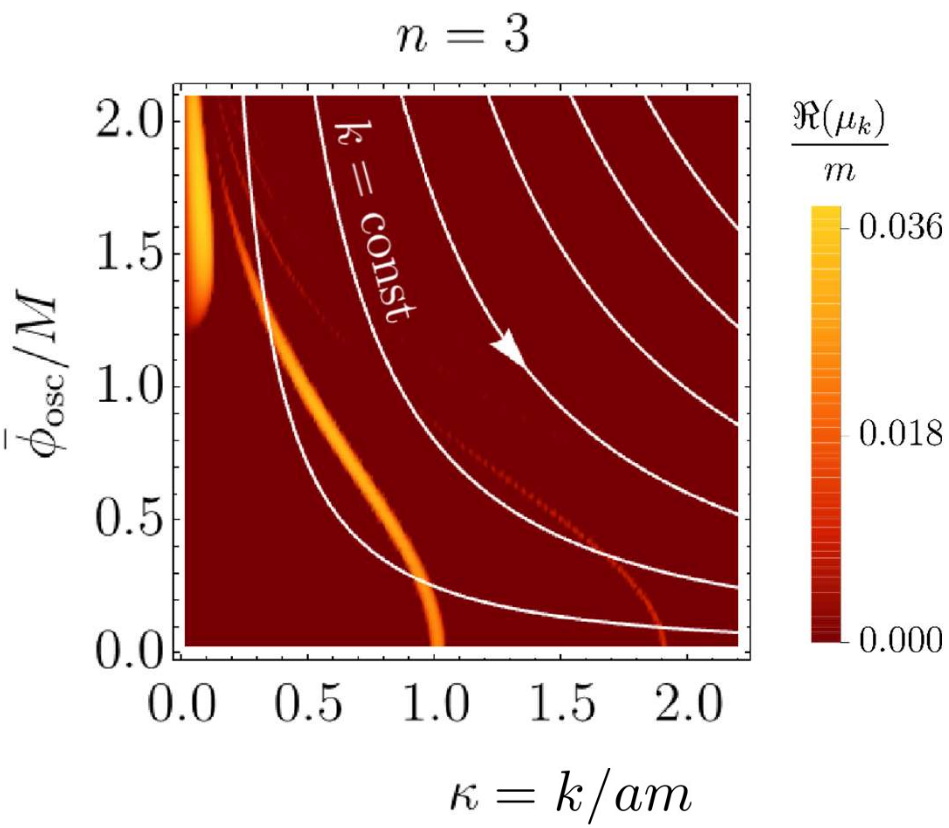
$m^2 \equiv V'(\bar{\phi}_{\rm osc})/\bar{\phi}_{\rm osc}$

Towards radiation domination

$$n > 1$$

Non-perturbative decay (parametric self-resonance)

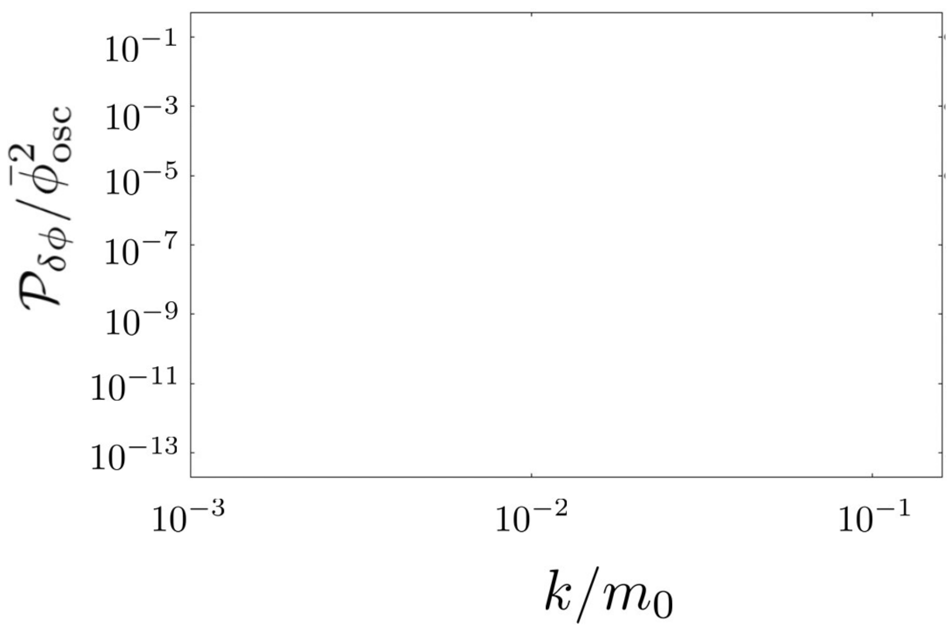
inflaton vev amplitude



inflaton field wavenumber

Power spectrum:

$$\langle \delta\phi(x)^2 \rangle \equiv \int \mathcal{P}_{\delta\phi}\, d\ln k$$

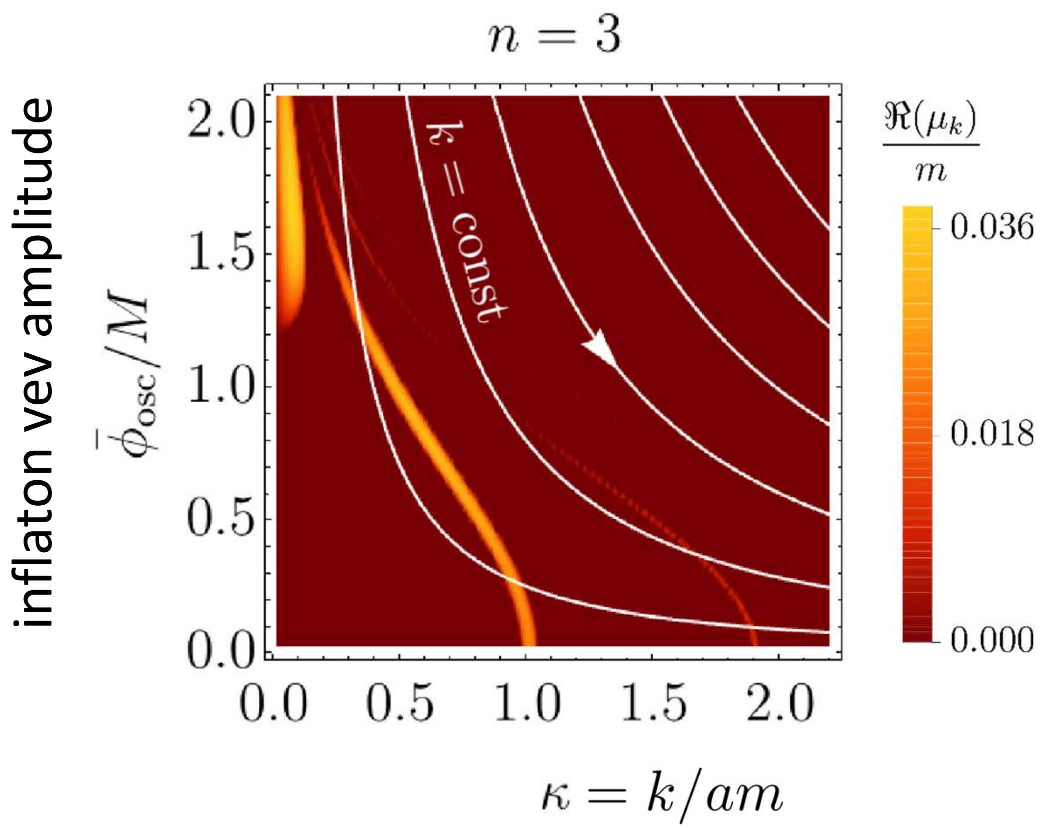


comoving wavenumber

Towards radiation domination

$$n > 1$$

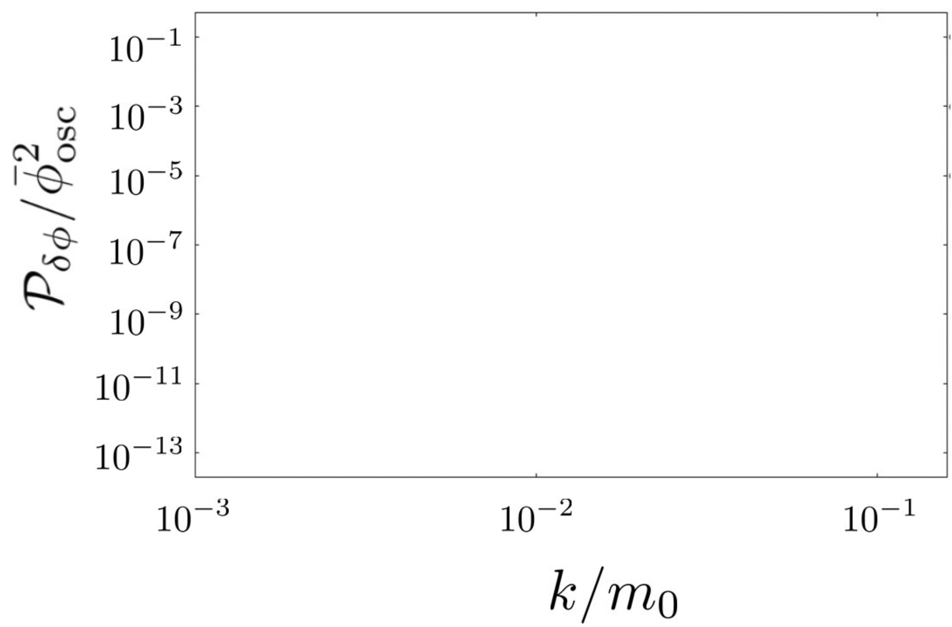
Non-perturbative decay (parametric self-resonance)



inflaton field wavenumber

Power spectrum:

$$\langle \delta\phi(x)^2 \rangle \equiv \int \mathcal{P}_{\delta\phi}\, d\ln k$$

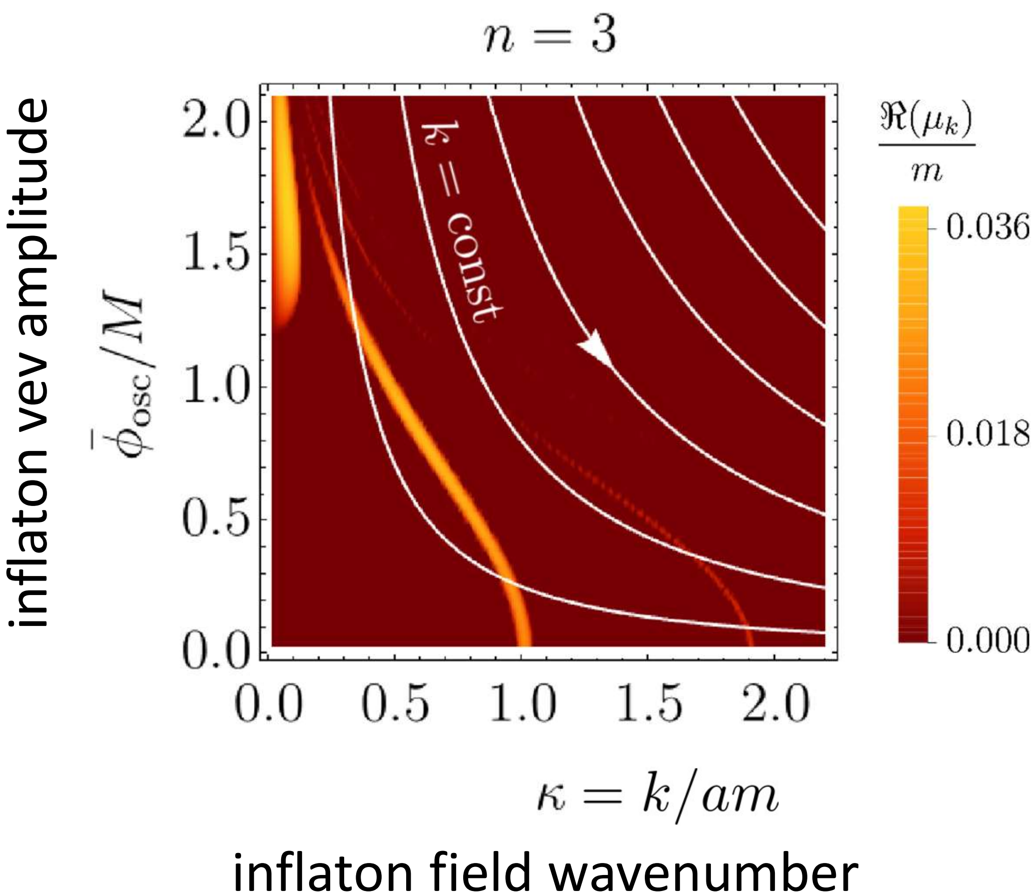


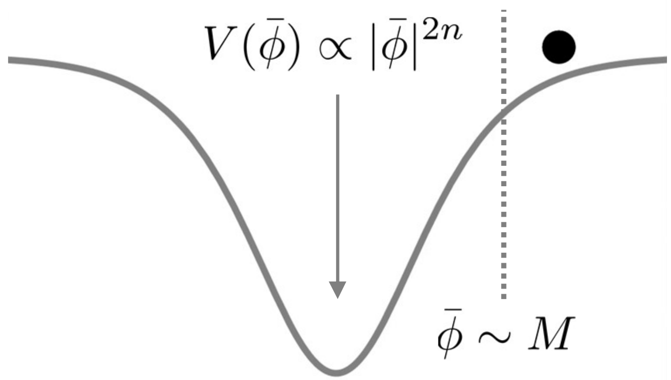
comoving wavenumber

Towards radiation domination

$$n > 1$$

Non-perturbative decay (parametric self-resonance)





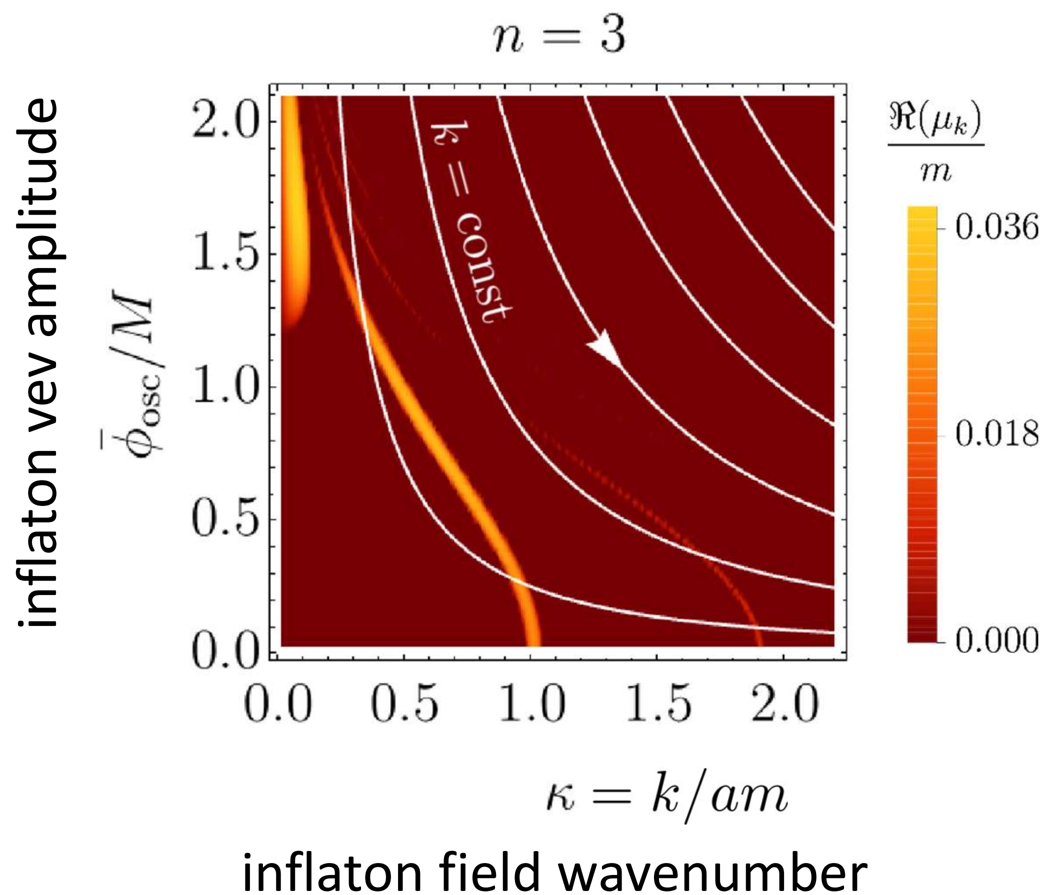
$$\delta\phi_k \propto \exp(\pm\mu_k t)$$

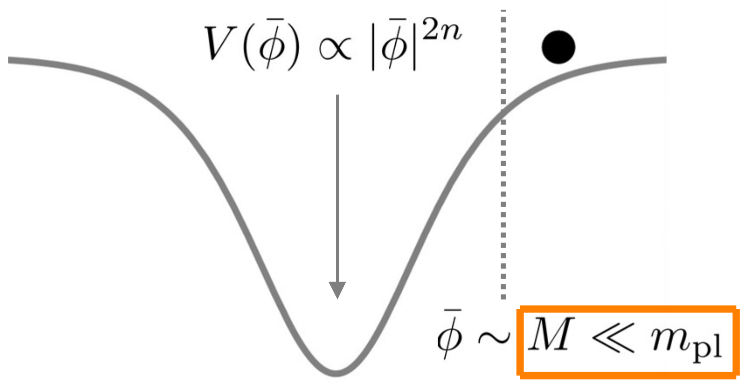
$m^2 \equiv V'(\bar{\phi}_{\rm osc})/\bar{\phi}_{\rm osc}$

Towards radiation domination

$$n > 1$$

Non-perturbative decay (parametric self-resonance)





$$\delta\phi_k \propto \exp(\pm\mu_k t)$$

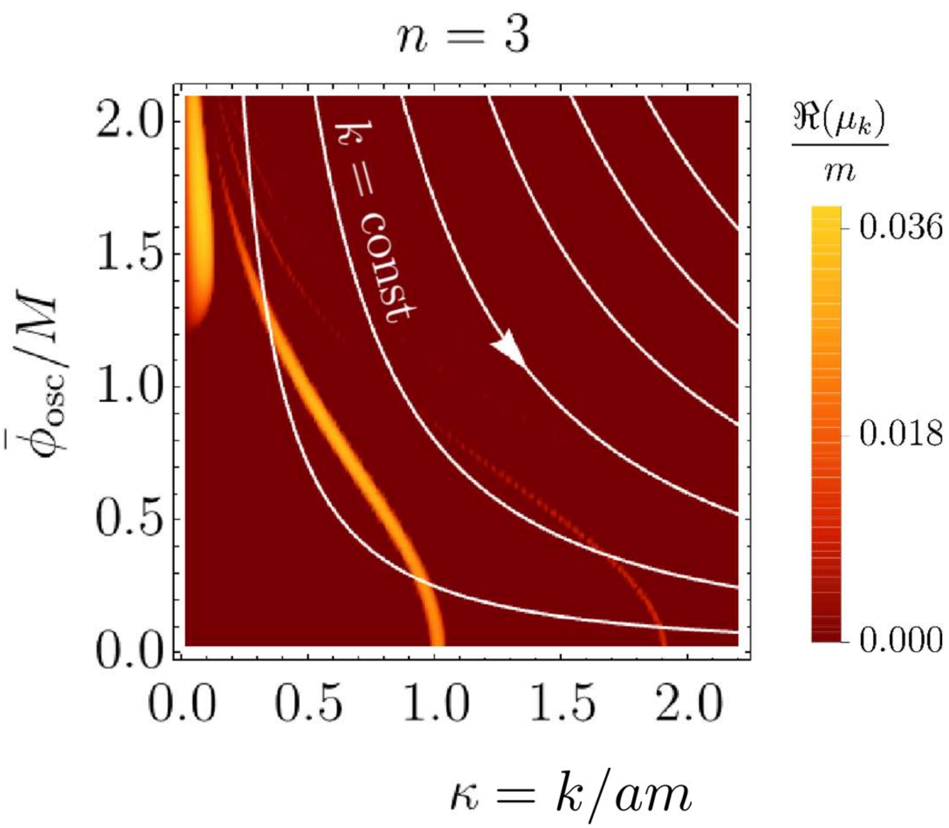
$m^2 \equiv V'(\bar{\phi}_{\text{osc}})/\bar{\phi}_{\text{osc}}$

Towards radiation domination

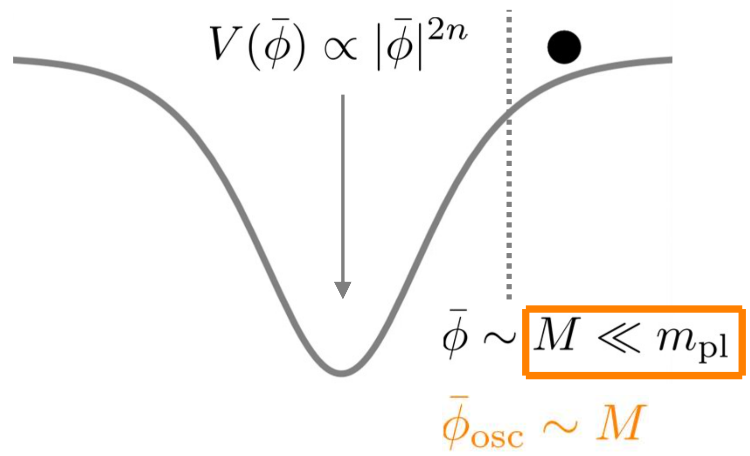
$$n > 1$$

Non-perturbative decay (parametric self-resonance)

inflaton vev amplitude



inflaton field wavenumber



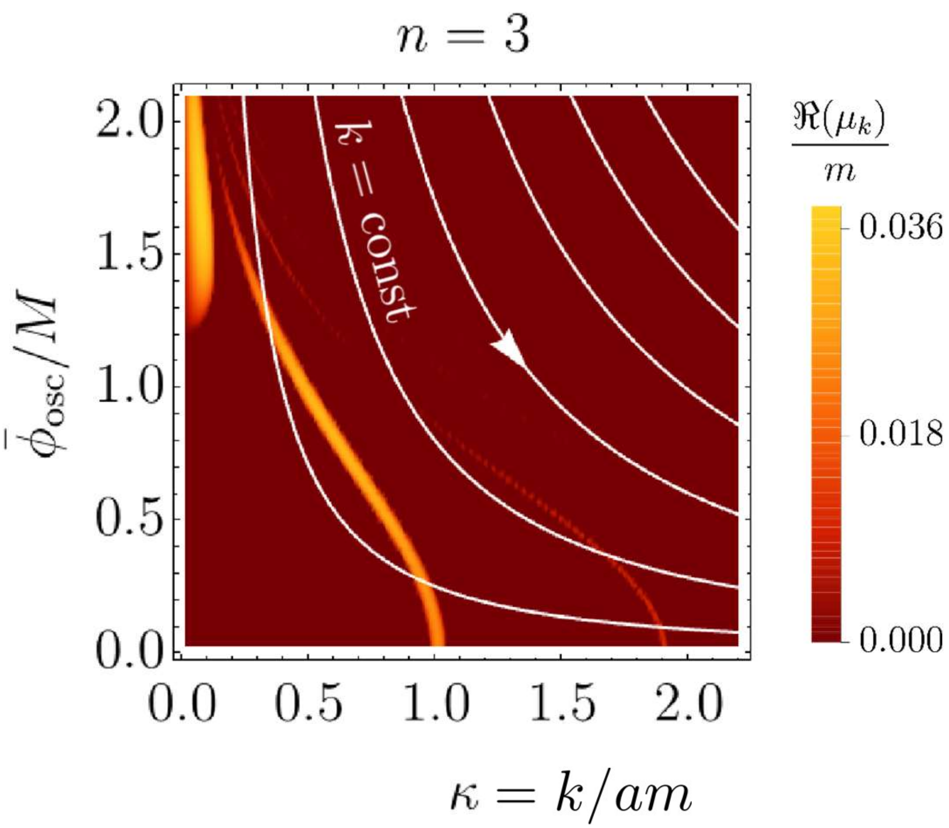
$$\delta\phi_k \propto \exp(\pm\mu_k t)$$

$m^2 \equiv V'(\bar{\phi}_{\rm osc})/\bar{\phi}_{\rm osc}$

Towards radiation domination

$$n > 1$$

Non-perturbative decay (parametric self-resonance)

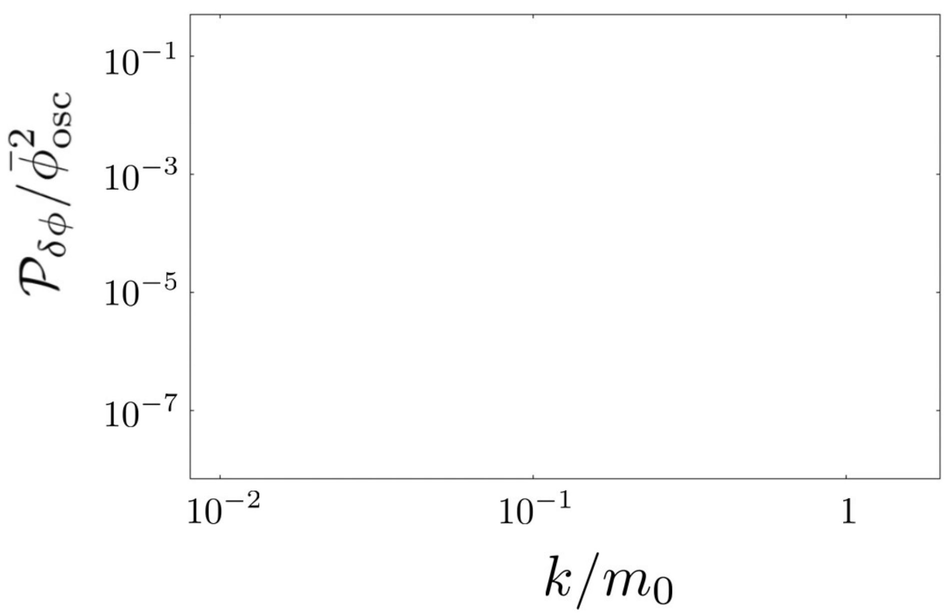


inflaton vev amplitude

inflaton field wavenumber

Power spectrum:

$$\langle \delta\phi(x)^2 \rangle \equiv \int \mathcal{P}_{\delta\phi}\, d\ln k$$



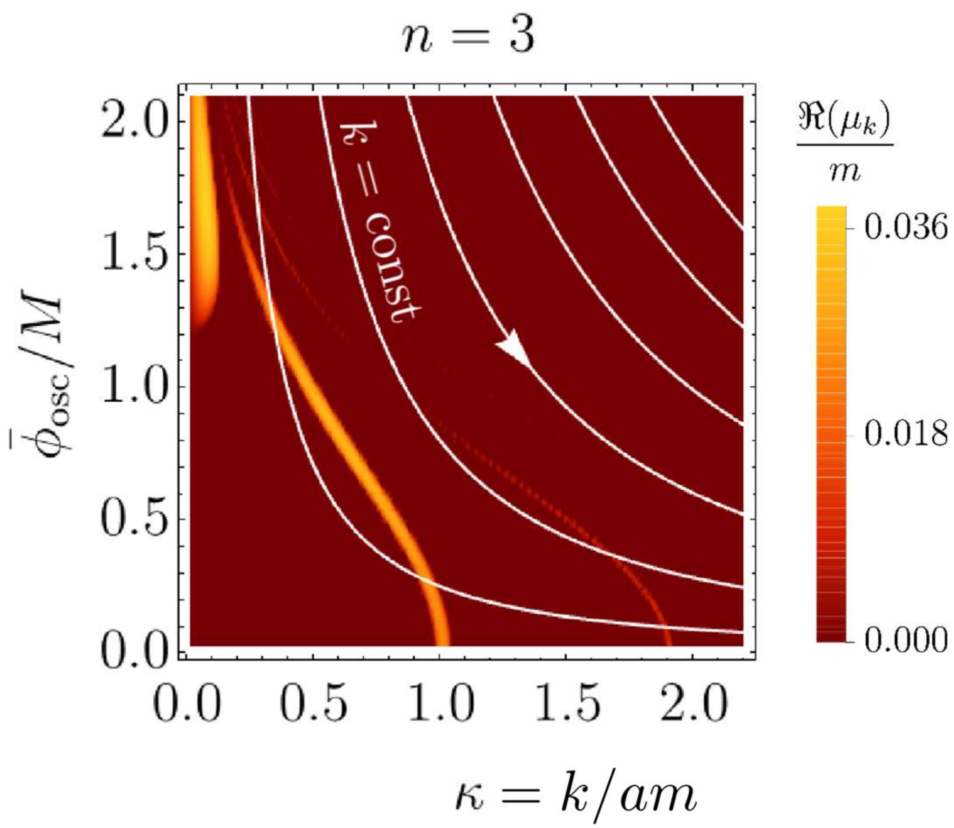
comoving wavenumber

Towards radiation domination

$$n > 1$$

Non-perturbative decay (parametric self-resonance)

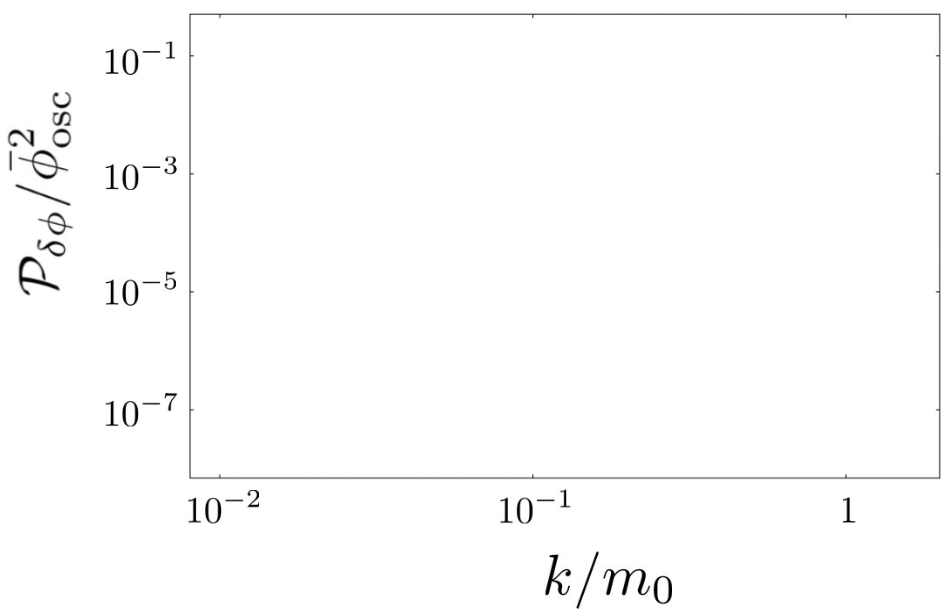
inflaton vev amplitude



inflaton field wavenumber

Power spectrum:

$$\langle \delta\phi(x)^2 \rangle \equiv \int \mathcal{P}_{\delta\phi}\, d\ln k$$

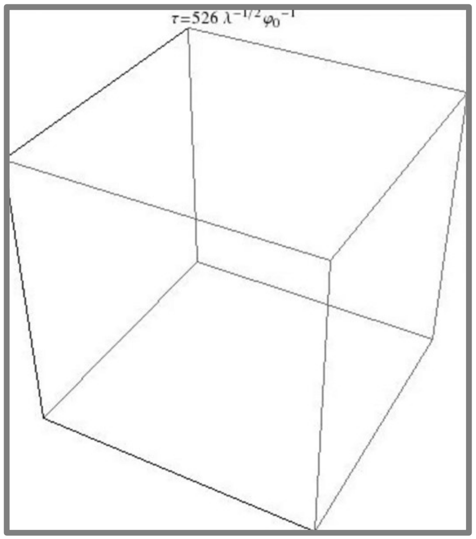


comoving wavenumber

Towards radiation domination

$$n > 1$$

$M \ll m_{\rm pl}$



(transients)

- $\bar{\phi}$ fragments quickly

$M \sim m_{\rm pl}$

- slow production of $\delta\phi(t, \mathbf{x})$

- $\bar{\phi}$ fragments gradually

$$\Delta N_{\rm fr} \approx \frac{n+1}{3} \ln\left(10^3 \frac{M}{m_{\rm pl}}\right)$$

at sufficiently late times:

ϕ virialized + turbulent $\rightarrow w = \frac{1}{3}$

Expansion history effects

Expansion history effects

Spectral index: $n_{\rm s}$

Tensor-to-scalar ratio: r

Expansion history effects

Spectral index: $n_{\rm s} = n_{\rm s}(M, n, N_*)$

Tensor-to-scalar ratio: $r = r(M, n, N_*)$

Expansion history effects

Spectral index: $$n_{\mathrm{s}} = n_{\mathrm{s}}(M, n, N_*)$$

Tensor-to-scalar ratio: $$r = r(M, n, N_*)$$??

Expansion history effects

Spectral index: $n_{\rm s} = n_{\rm s}(M, n, N_*)$

Tensor-to-scalar ratio: $r = r(M, n, N_*)$??

$$50 \leq N_* \leq 60$$

Planck Collaboration (2015)

Expansion history effects

Spectral index: $n_{\mathrm{s}} = n_{\mathrm{s}}(M, n, N_*)$

Tensor-to-scalar ratio: $r = r(M, n, N_*)$ ✓

$$50 \leq N_* \leq 60$$

KL and M. Amin (2017, 18, 19)

Expansion history effects

Spectral index: $n_{\rm s} = n_{\rm s}(M, n, N_*)$

Tensor-to-scalar ratio: $r = r(M, n, N_*)$ ✓

$$N_* = 66.89 - \frac{1}{12}\ln g_{\rm th} + \frac{1}{4}\ln\frac{V_*^4}{m_{\rm pl}^4\rho_{\rm end}} - \ln\frac{k_*}{a_0 H_0} + \frac{3\bar{w}_{\rm int} - 1}{4}\Delta N_{\rm rad}$$

$$\Delta N_{\rm rad} \equiv \int_{a_{\rm end}}^{a_{\rm rad}} d\ln a$$

KL and M. Amin (2017, 18, 19)

Expansion history effects

Spectral index: $n_\mathrm{s} = n_\mathrm{s}(M, n, N_*)$

Tensor-to-scalar ratio: $r = r(M, n, N_*)$ ✓

$$N_* = 66.89 - \frac{1}{12}\ln g_\mathrm{th} + \frac{1}{4}\ln\frac{V_*^4}{m_\mathrm{pl}^4 \rho_\mathrm{end}} - \ln\frac{k_*}{a_0 H_0} + \frac{3\bar{w}_\mathrm{int} - 1}{4}\Delta N_\mathrm{rad}$$

reheating

$$\Delta N_\mathrm{rad} \equiv \int_{a_\mathrm{end}}^{a_\mathrm{rad}} d\ln a$$

KL and M. Amin (2017, 18, 19)

Expansion history effects

Spectral index: $n_{\rm s} = n_{\rm s}(M, n, N_*)$

Tensor-to-scalar ratio: $r = r(M, n, N_*)$ ✓

$$N_* = 66.89 - \frac{1}{12}\ln g_{\rm th} + \frac{1}{4}\ln\frac{V_*^4}{m_{\rm pl}^4\rho_{\rm end}} - \ln\frac{k_*}{a_0 H_0} + \frac{3\bar{w}_{\rm int} - 1}{4}\Delta N_{\rm rad}$$

$$\Delta N_{\rm rad} \equiv \int_{a_{\rm end}}^{a_{\rm rad}} d\ln a$$

KL and M. Amin (2017, 18, 19)

Expansion history effects

Spectral index: $n_\mathrm{s} = n_\mathrm{s}(M, n, N_*)$

Tensor-to-scalar ratio: $r = r(M, n, N_*)$ ✓

$$N_* = 66.89 - \frac{1}{12}\ln g_\mathrm{th} + \frac{1}{4}\ln\frac{V_*^4}{m_\mathrm{pl}^4\rho_\mathrm{end}} - \ln\frac{k_*}{a_0 H_0} + \frac{3\bar{w}_\mathrm{int}-1}{4}\Delta N_\mathrm{rad}$$

$$\Delta N_\mathrm{fr} = \begin{cases} 1 & \text{if} \quad M \ll m_\mathrm{pl} \\ \frac{n+1}{3}\ln\left(10\frac{\kappa}{\Delta\kappa}\frac{M}{m_\mathrm{pl}}\right) & \text{if} \quad M \sim m_\mathrm{pl} \end{cases}$$

$$\Delta N_\mathrm{rad} \equiv \int_{a_\mathrm{end}}^{a_\mathrm{rad}} d\ln a$$

KL and M. Amin (2017, 18, 19)

Expansion history effects

Spectral index: $n_s = n_s(M, n, N_*)$

Tensor-to-scalar ratio: $r = r(M, n, N_*)$

$$N_* = 66.89 - \frac{1}{12}\ln g_{\rm th} + \frac{1}{4}\ln\frac{V_*^4}{m_{\rm pl}^4\rho_{\rm end}} - \ln\frac{k_*}{a_0 H_0} + \frac{3\bar{w}_{\rm int}-1}{4}\Delta N_{\rm rad}$$

$$\Delta N_{\rm rad} \leq \Delta N_{\rm fr} = \begin{cases} 1 & \text{if} \quad M \ll m_{\rm pl} \\ \frac{n+1}{3}\ln\left(10\frac{\kappa}{\Delta\kappa}\frac{M}{m_{\rm pl}}\right) & \text{if} \quad M \sim m_{\rm pl} \end{cases}$$

$$\Delta N_{\rm rad} \equiv \int_{a_{\rm end}}^{a_{\rm rad}} d\ln a$$

KL and M. Amin (2017, 18, 19)

Expansion history effects

Spectral index: $n_s = n_s(M, n, N_*)$

Tensor-to-scalar ratio: $r = r(M, n, N_*)$ ✓

$$N_* = 66.89 - \frac{1}{12}\ln g_{\rm th} + \frac{1}{4}\ln\frac{V_*^4}{m_{\rm pl}^4\rho_{\rm end}} - \ln\frac{k_*}{a_0H_0} + \frac{3\bar{w}_{\rm int}-1}{4}\Delta N_{\rm rad}$$

$$\Delta N_{\rm rad} \leq \Delta N_{\rm fr} = \begin{cases} 1 & \text{if} \quad M \ll m_{\rm pl} \\ \frac{n+1}{3}\ln\left(10\frac{\kappa}{\Delta\kappa}\frac{M}{m_{\rm pl}}\right) & \text{if} \quad M \sim m_{\rm pl} \end{cases}$$

$$\Delta N_{\rm rad} \equiv \int_{a_{\rm end}}^{a_{\rm rad}} d\ln a$$

KL and M. Amin (2017, 18, 19)

Expansion history effects

Spectral index: $n_s = n_s(M, n, N_*)$

Tensor-to-scalar ratio: $r = r(M, n, N_*)$ ✓

$$N_* = 66.89 - \frac{1}{12}\ln g_{\rm th} + \frac{1}{4}\ln\frac{V_*^4}{m_{\rm pl}^4\rho_{\rm end}} - \ln\frac{k_*}{a_0 H_0} + \frac{3\bar{w}_{\rm int} - 1}{4}\Delta N_{\rm rad}$$

$$\Delta N_{\rm rad} \leq \Delta N_{\rm fr} = \begin{cases} 1 & \text{if} \quad M \ll m_{\rm pl} \\ \frac{n+1}{3}\ln\left(10\frac{\kappa}{\Delta\kappa}\frac{M}{m_{\rm pl}}\right) & \text{if} \quad M \sim m_{\rm pl} \end{cases}$$

$$w_{\rm int}(\Delta N) = \begin{cases} \frac{n-1}{n+1} & \text{if} \quad 0 < \Delta N < \Delta N_{\rm rad} \\ \frac{1}{3} & \text{if} \quad \Delta N > \Delta N_{\rm rad} \end{cases}$$

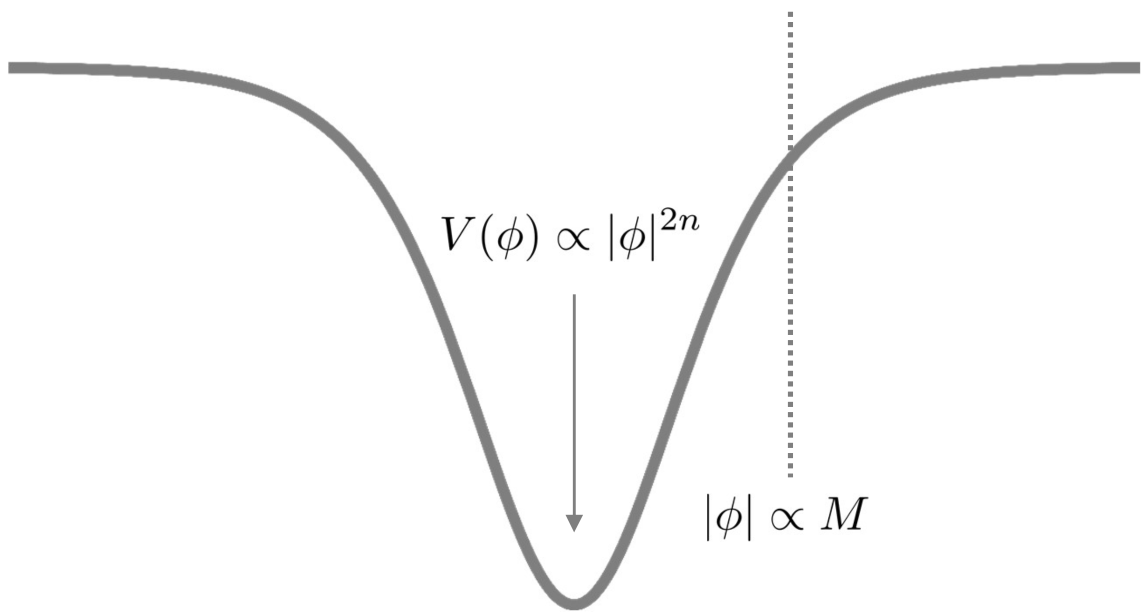
$$\Delta N_{\rm rad} \equiv \int_{a_{\rm end}}^{a_{\rm rad}} d\ln a$$

KL and M. Amin (2017, 18, 19)

Expansion history effects

$$V(\phi) \propto \tanh^{2n}\left(\frac{|\phi|}{M}\right)$$

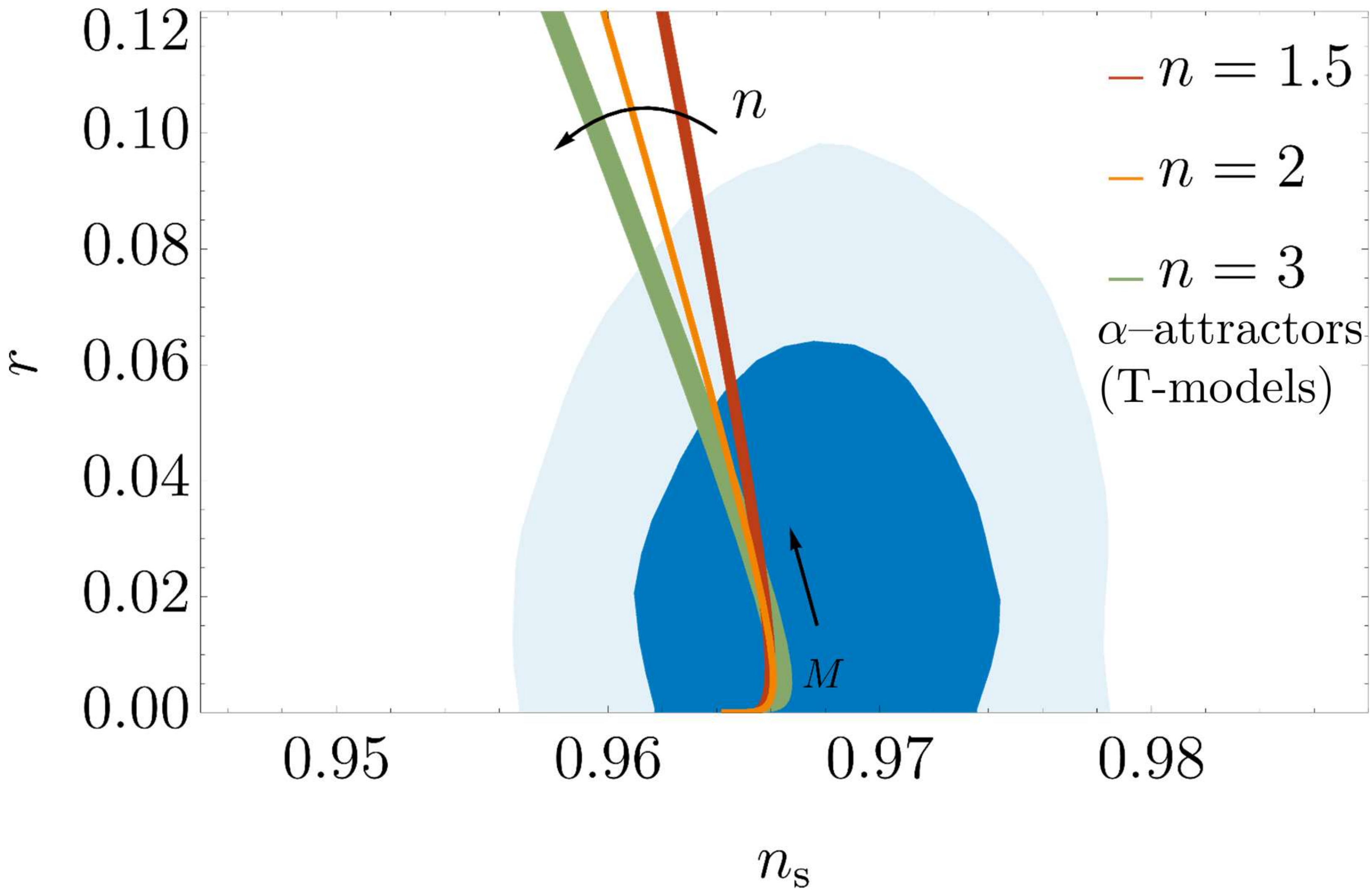
α–attractors
(T-models)



Kallosh and Linde (2013)
Carrasco, Kallosh and Linde (2015)

Expansion history effects

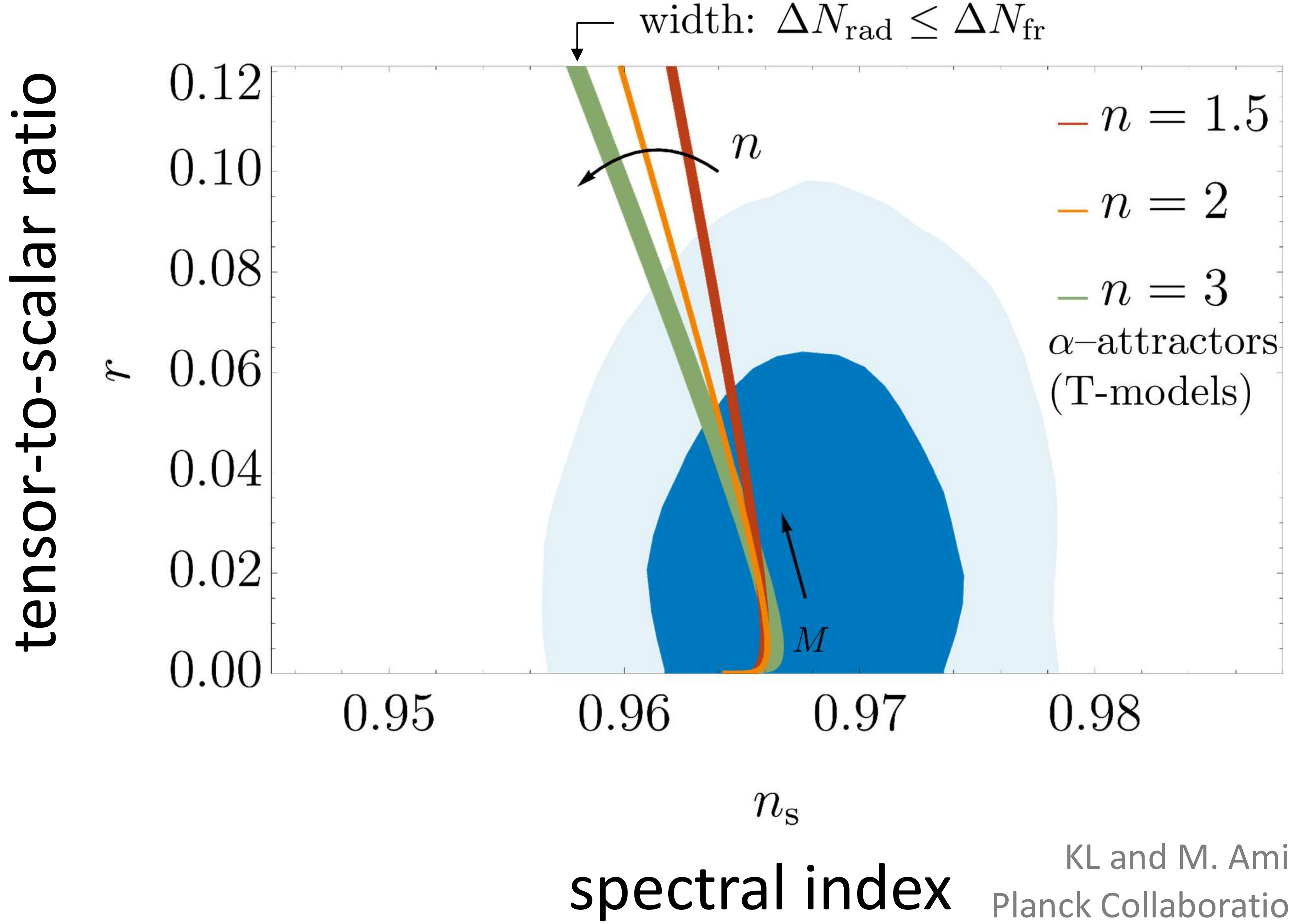
tensor-to-scalar ratio



spectral index

KL and M. Amin (2017)
Planck Collaboration (2015)

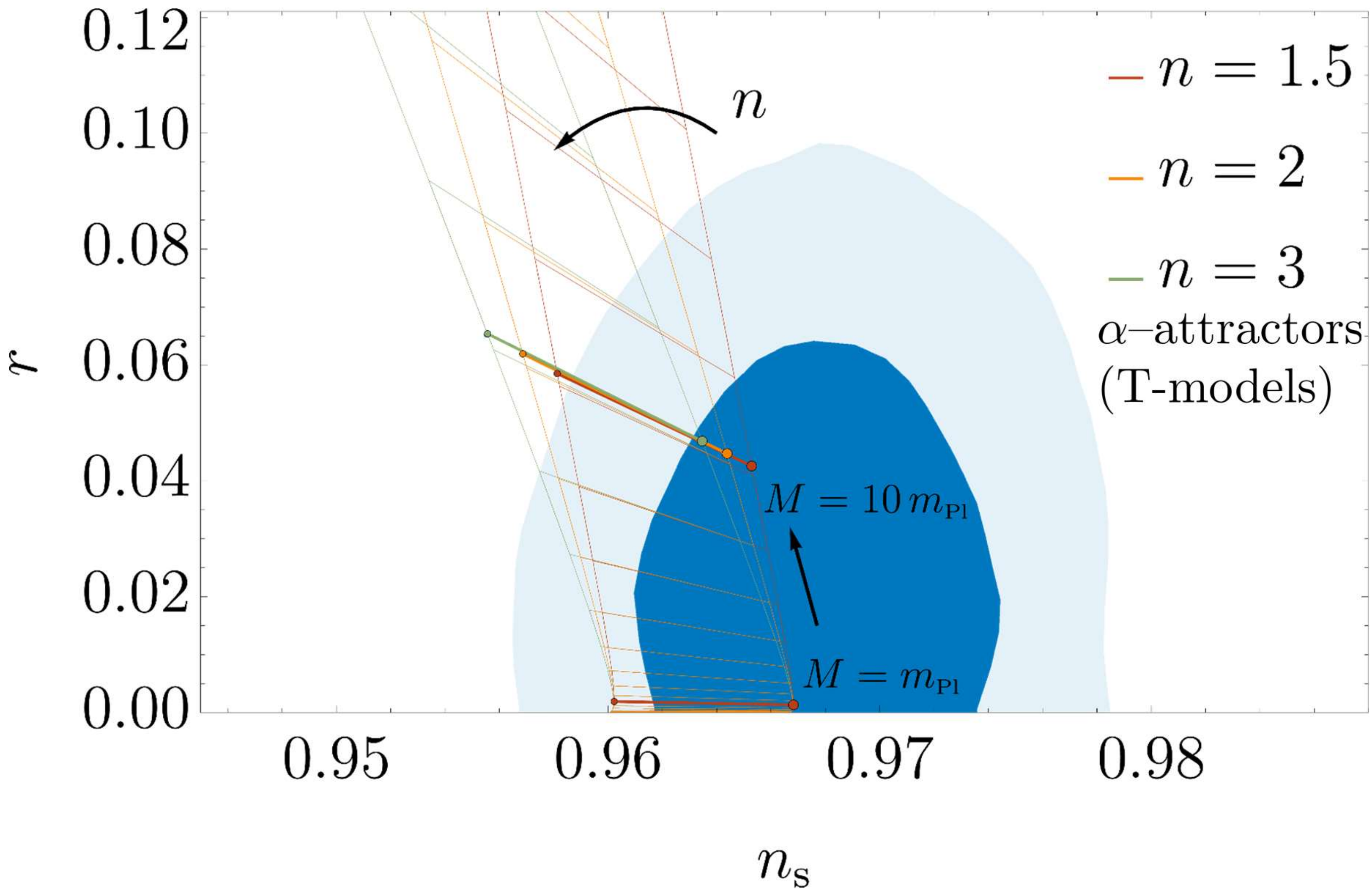
Expansion history effects



KL and M. Amin (2017)
Planck Collaboration (2015)

Expansion history effects

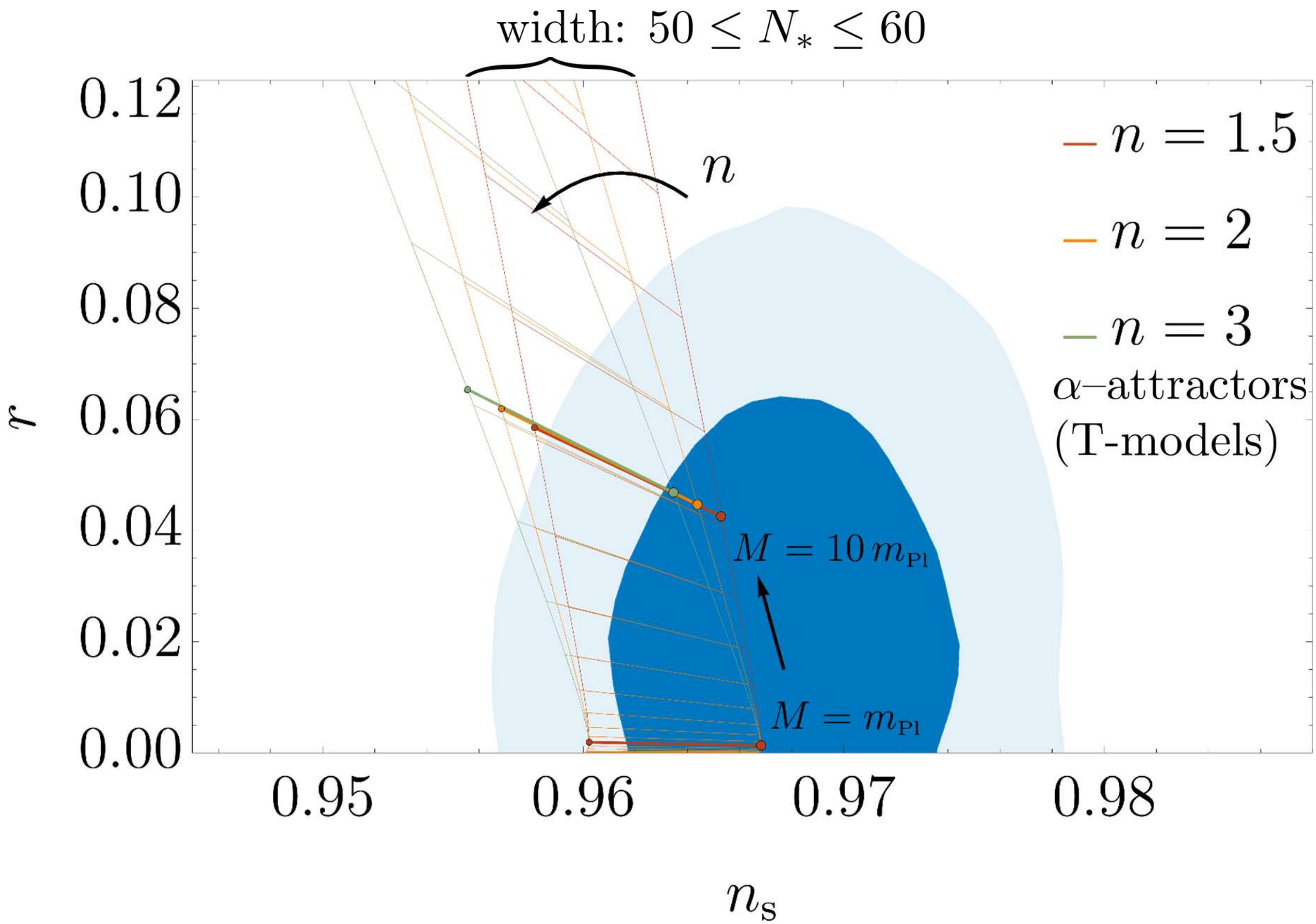
tensor-to-scalar ratio



spectral index

KL and M. Amin (2017)
Planck Collaboration (2015)

Expansion history effects



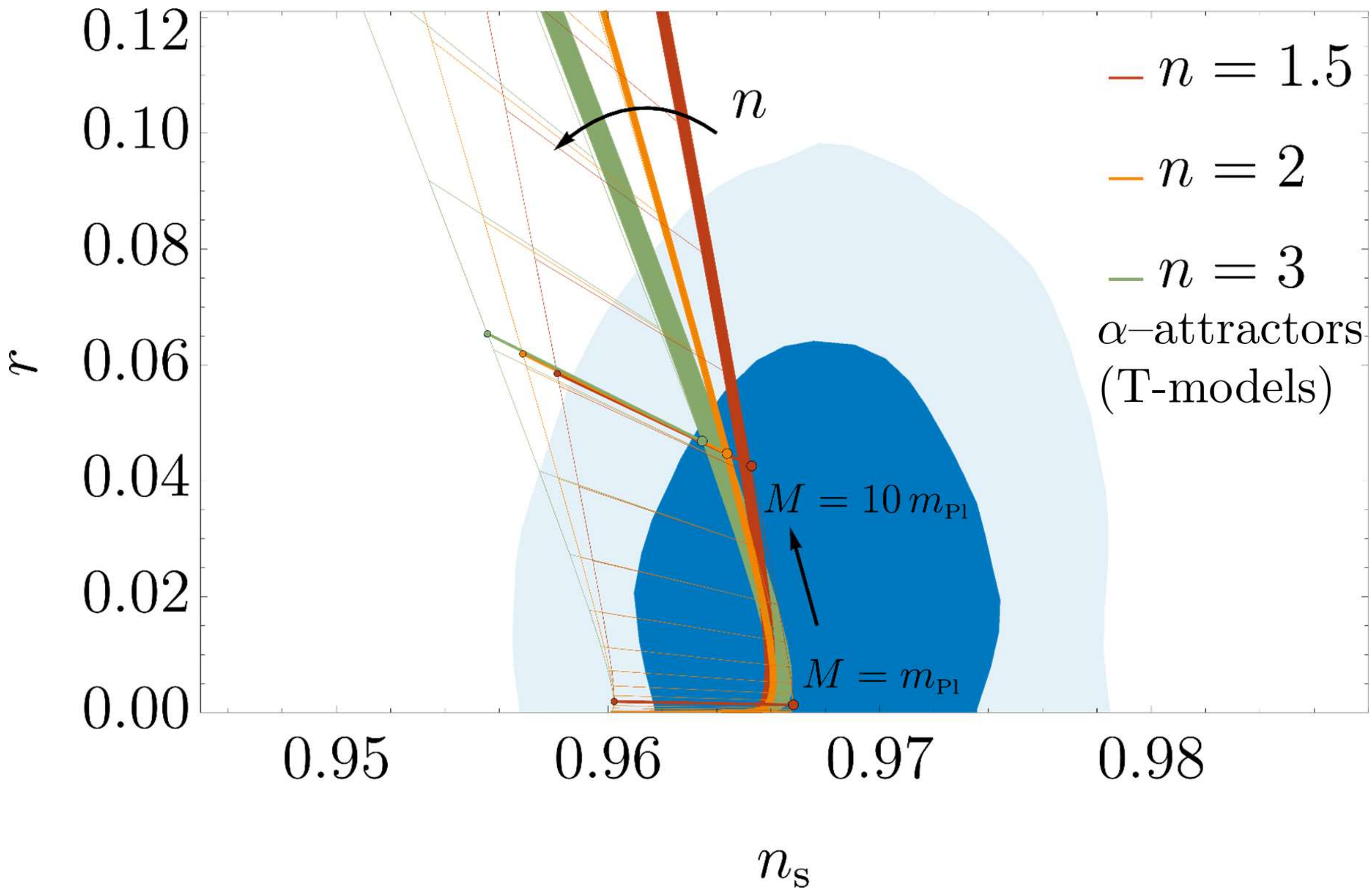
tensor-to-scalar ratio

spectral index

KL and M. Amin (2017)
Planck Collaboration (2015)

Expansion history effects

tensor-to-scalar ratio



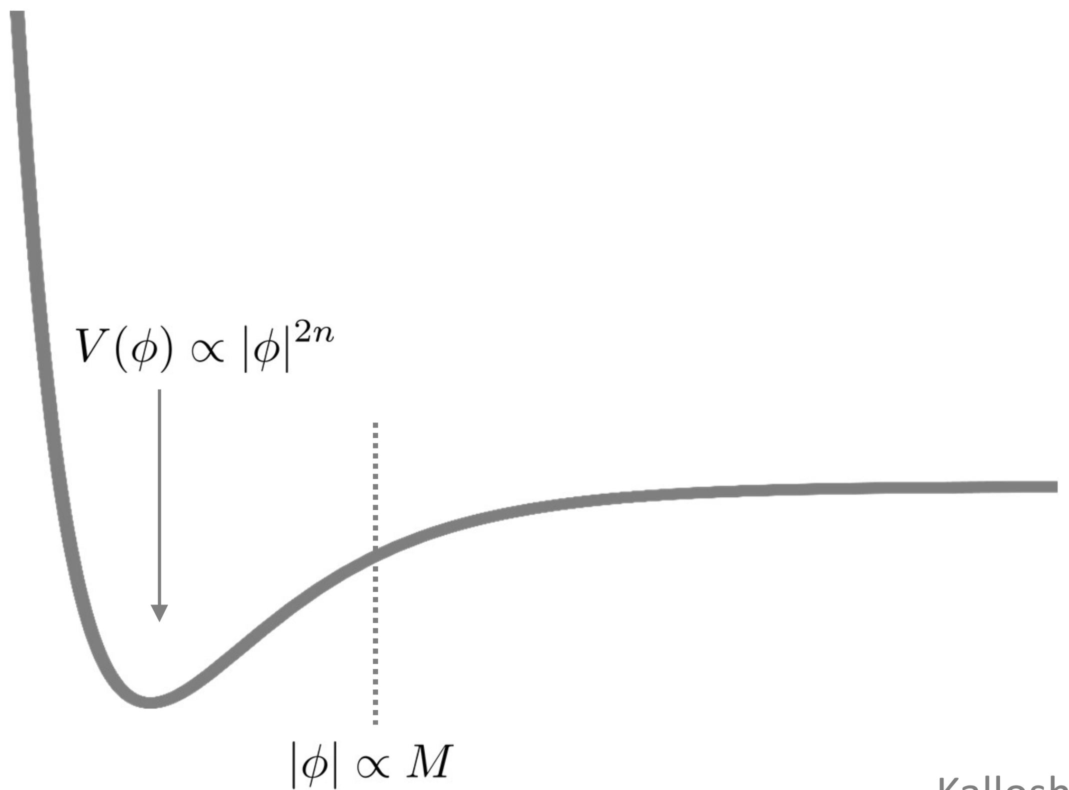
spectral index

KL and M. Amin (2017)
Planck Collaboration (2015)

Expansion history effects

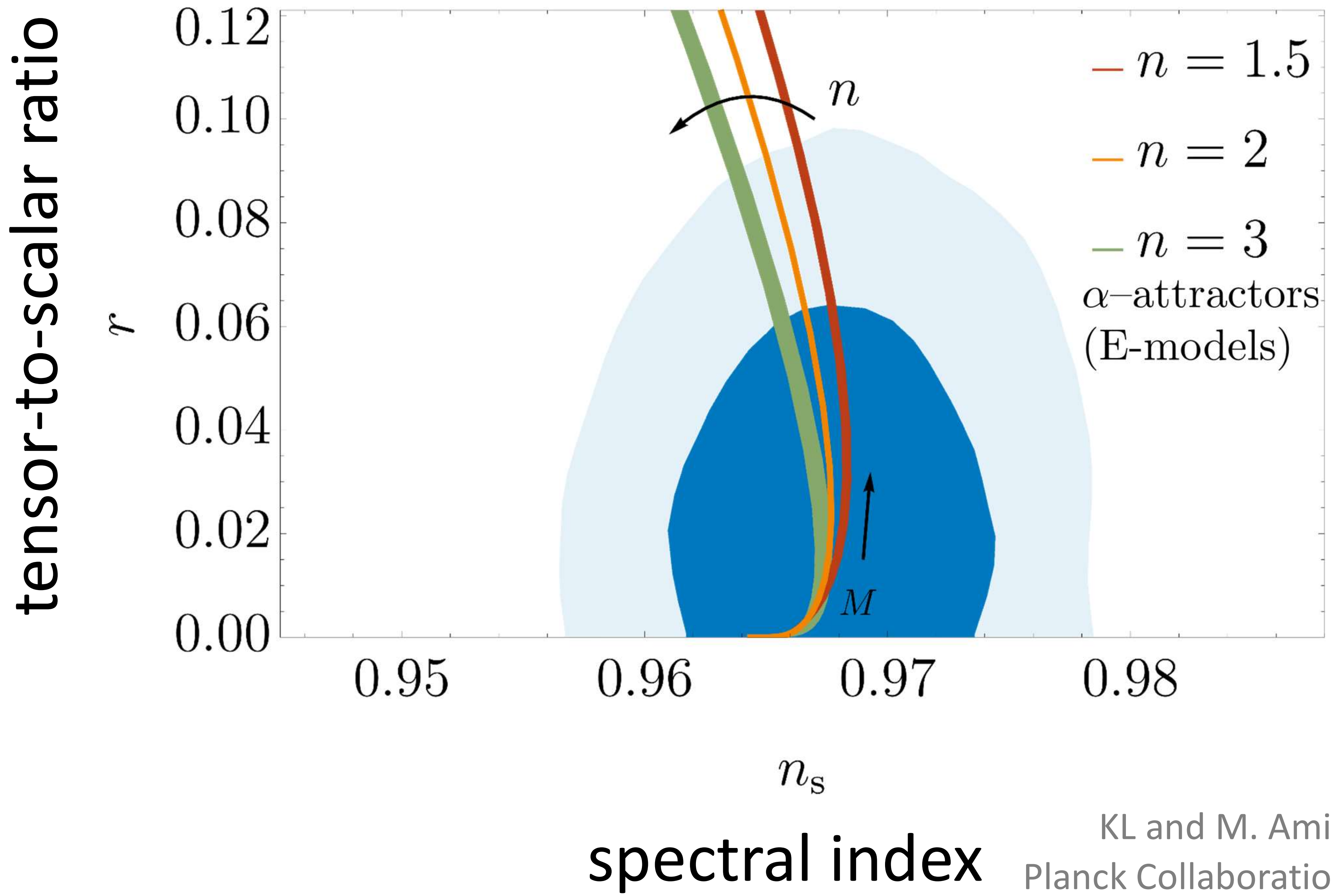
$$V(\phi) \propto \left|1 - e^{-\phi/M}\right|^{2n}$$

α–attractors
(E-models)



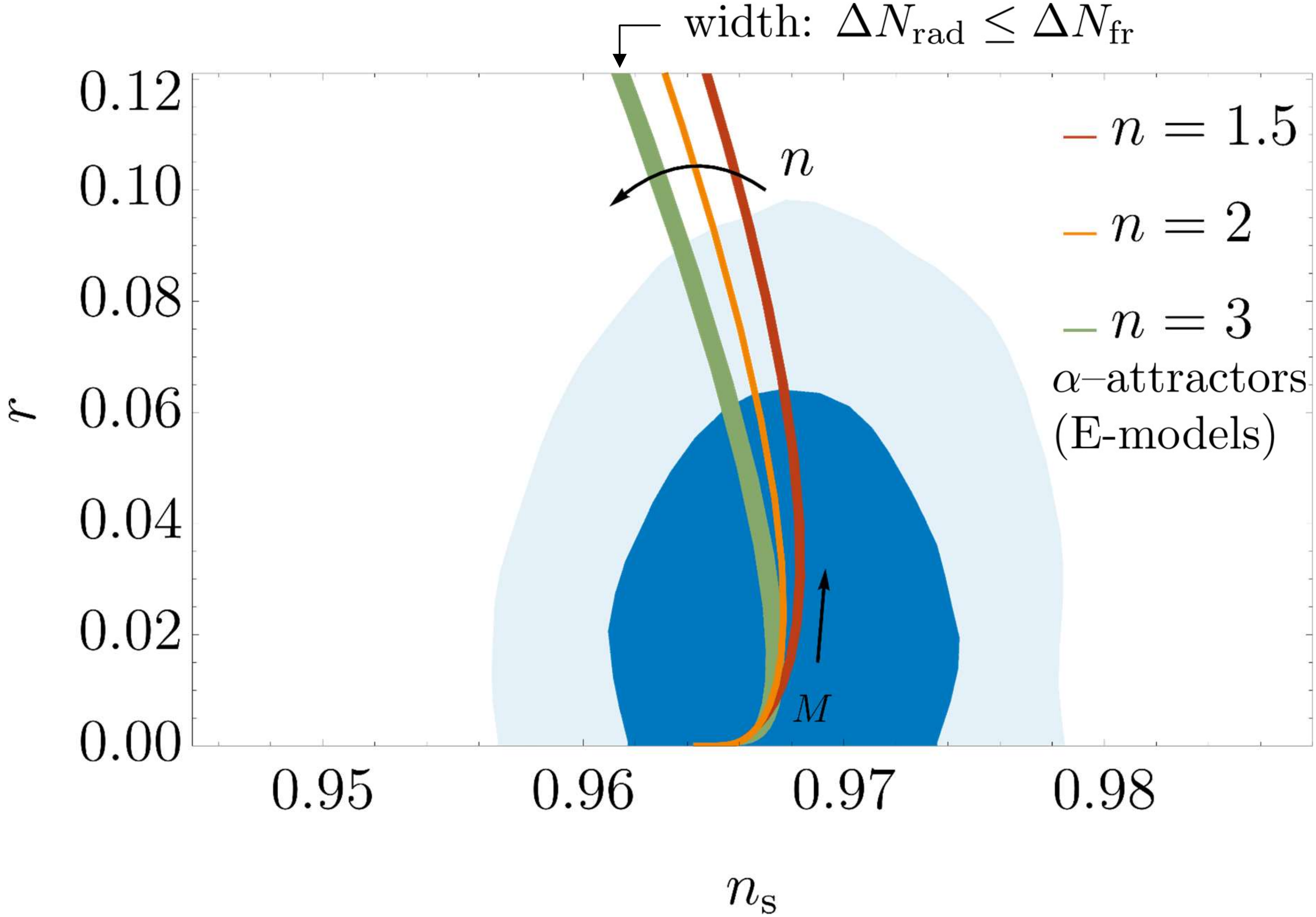
Kallosh and Linde (2013)
Carrasco, Kallosh and Linde (2015)

Expansion history effects



KL and M. Amin (2018)
Planck Collaboration (2015)

Expansion history effects

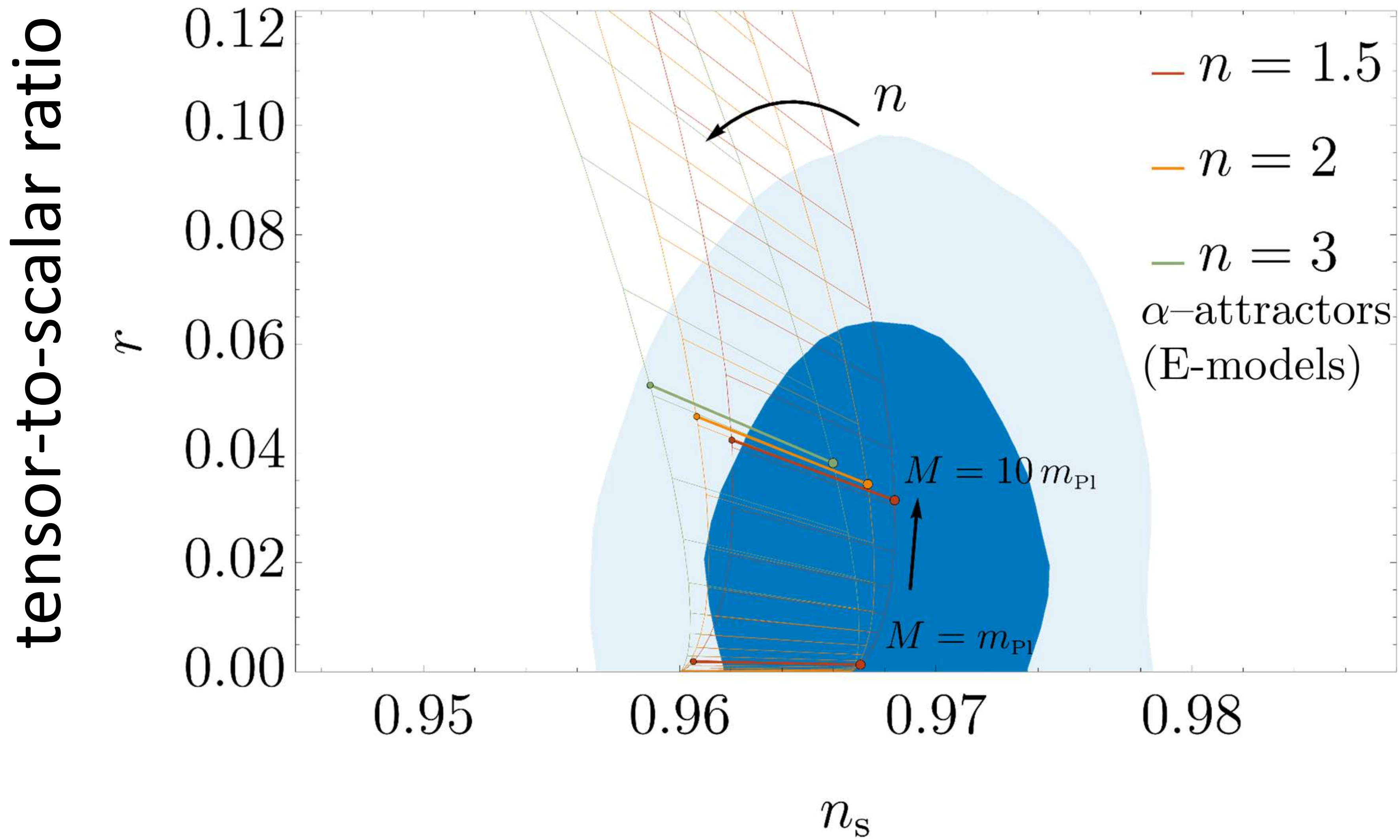


tensor-to-scalar ratio

spectral index

KL and M. Amin (2018)
Planck Collaboration (2015)

Expansion history effects

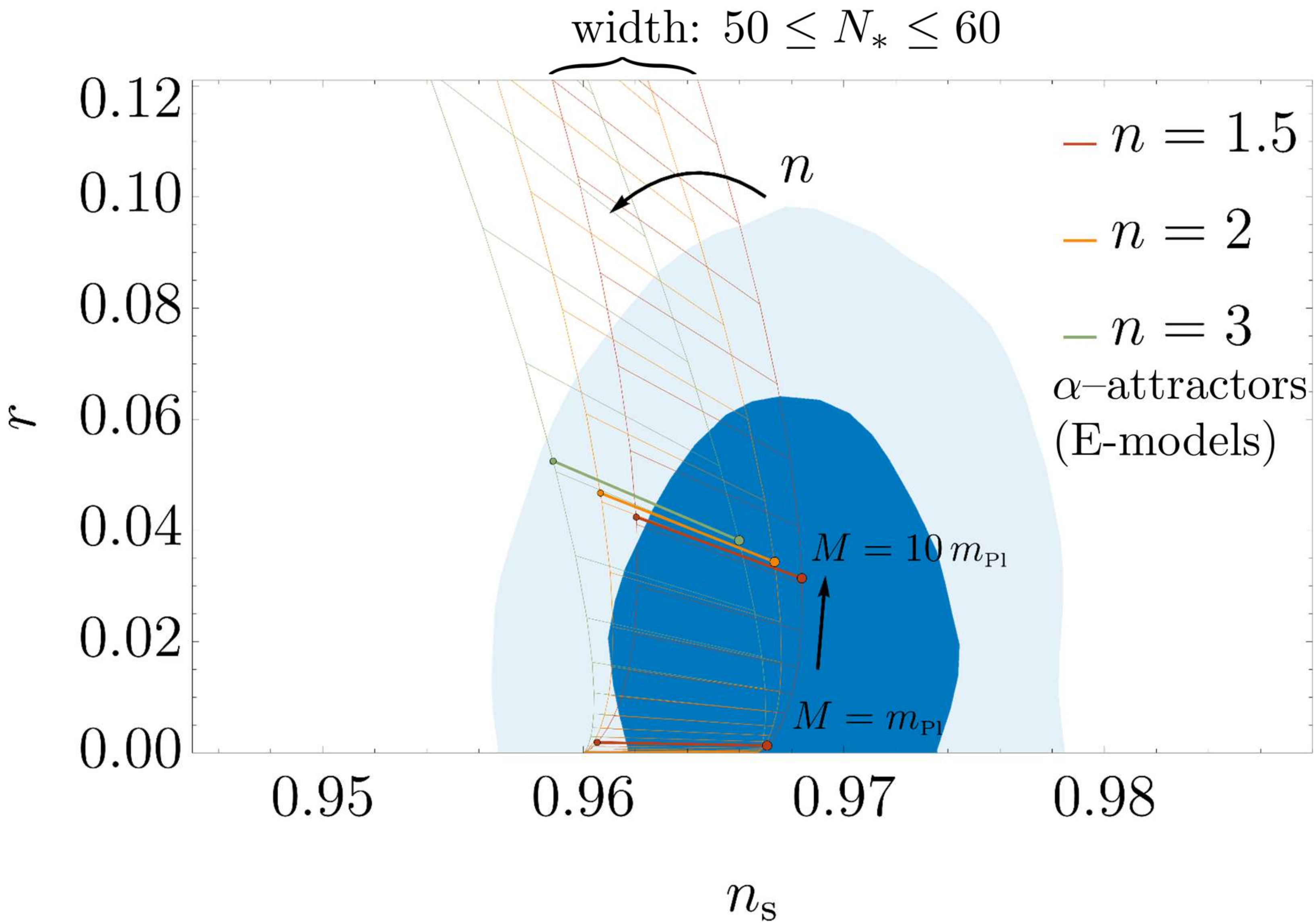


spectral index

KL and M. Amin (2018)
Planck Collaboration (2015)

Expansion history effects

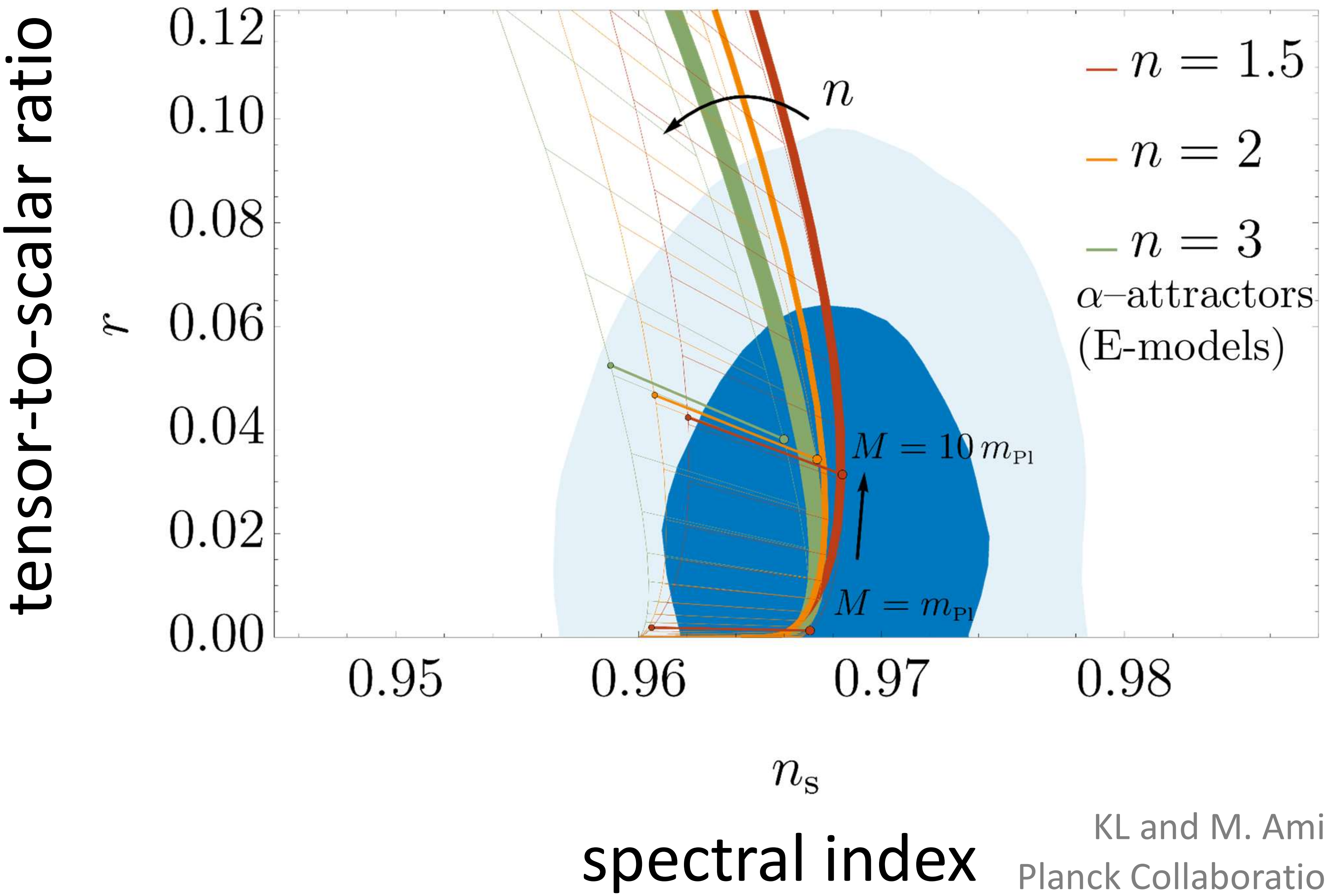
tensor-to-scalar ratio



spectral index

KL and M. Amin (2018)
Planck Collaboration (2015)

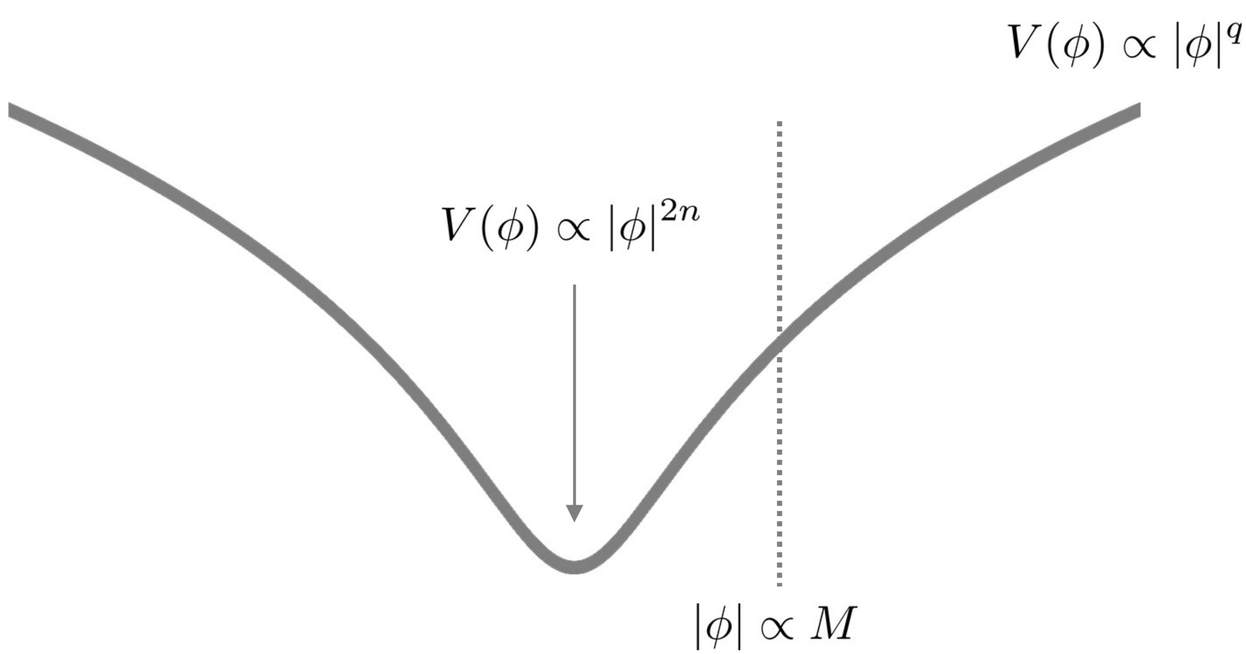
Expansion history effects



KL and M. Amin (2018)
Planck Collaboration (2015)

Expansion history effects

$$V(\phi) \propto \left[1 + \left|\frac{\phi}{M}\right|^{2n}\right]^{\frac{q}{2n}} - 1 \qquad \text{Monodromy}$$

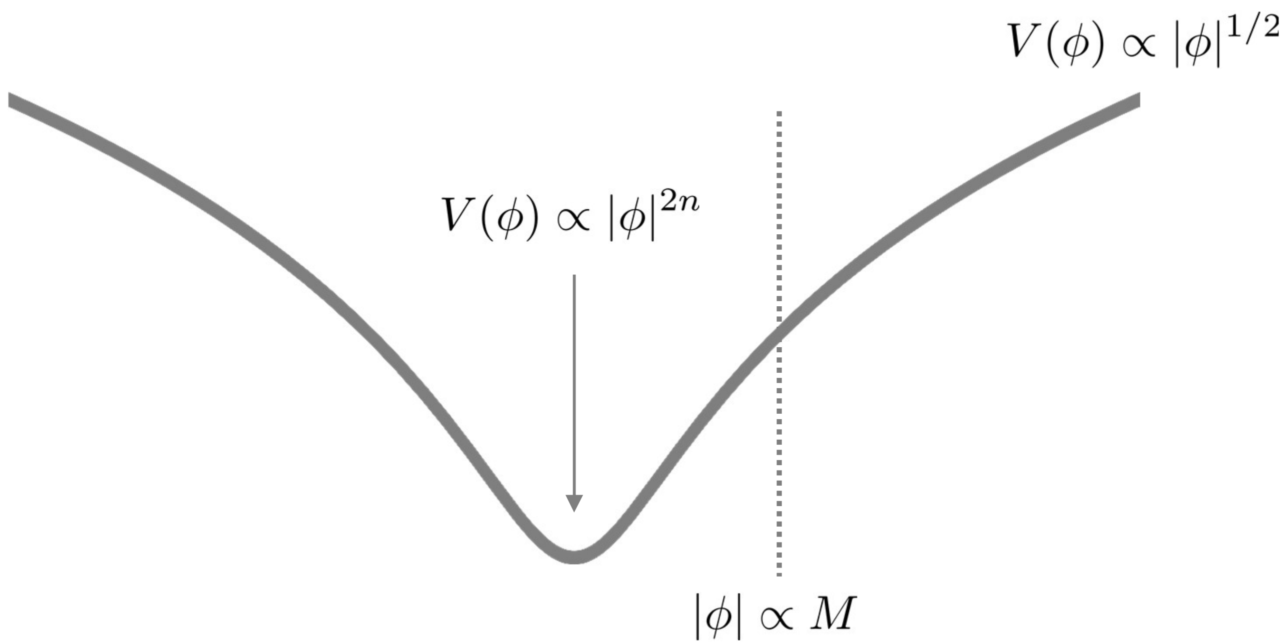


Silverstein and Westphal (2008)
McAllister, et al. (2014)

Expansion history effects

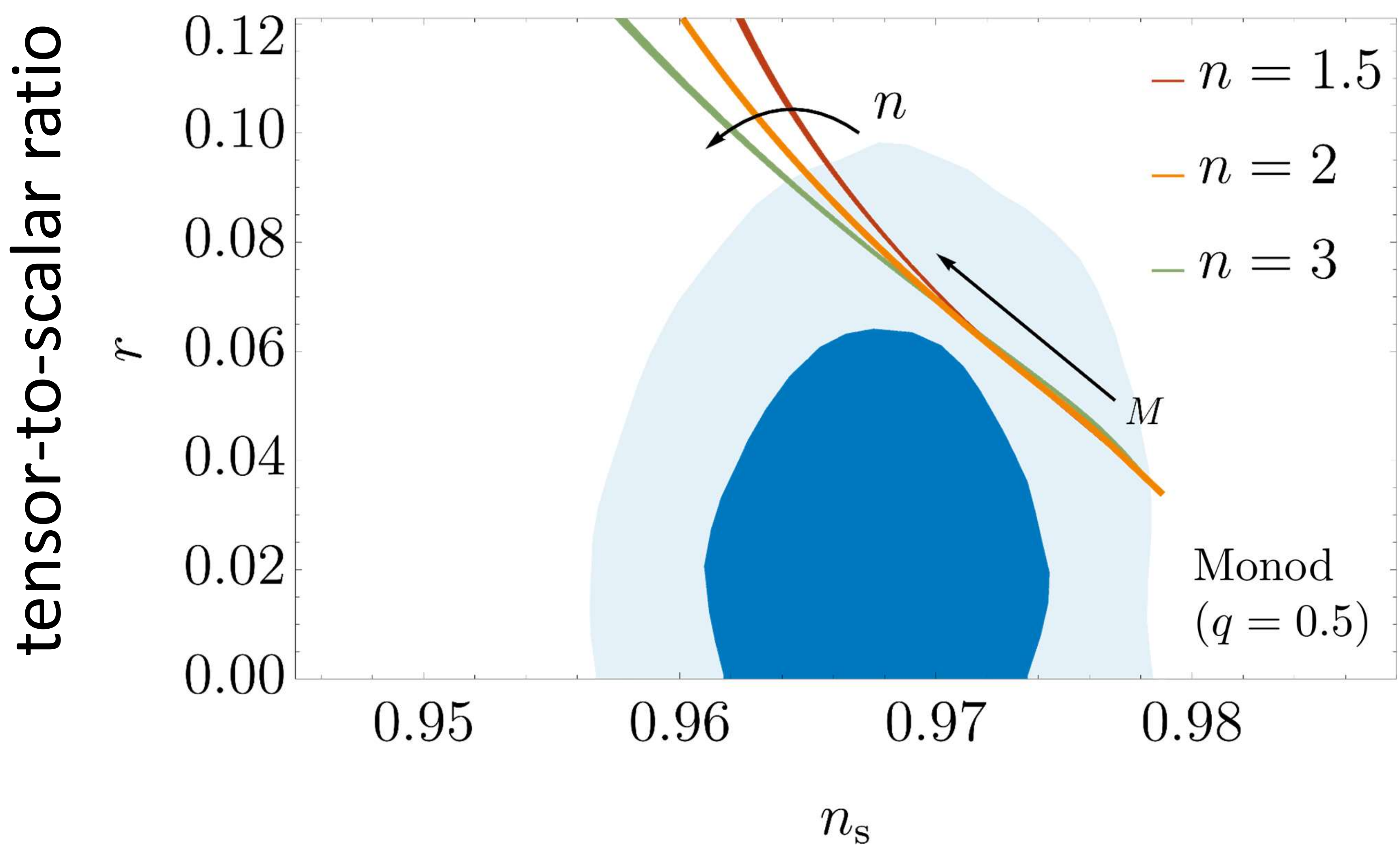
$$V(\phi) \propto \left[1 + \left|\frac{\phi}{M}\right|^{2n}\right]^{\frac{1}{4n}} - 1$$

Monodromy
$q = 1/2$



Silverstein and Westphal (2008)
McAllister, et al. (2014)

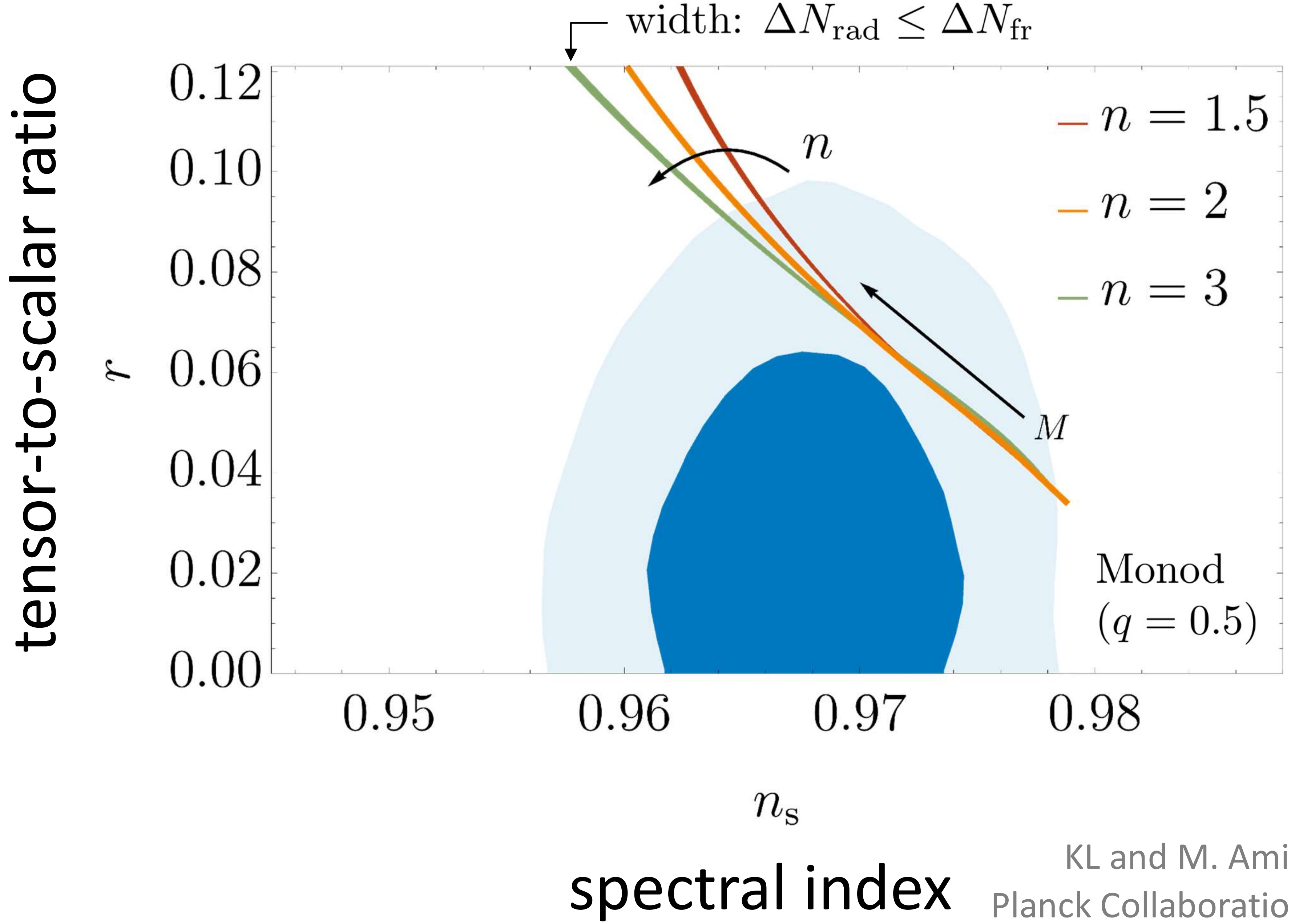
Expansion history effects



spectral index

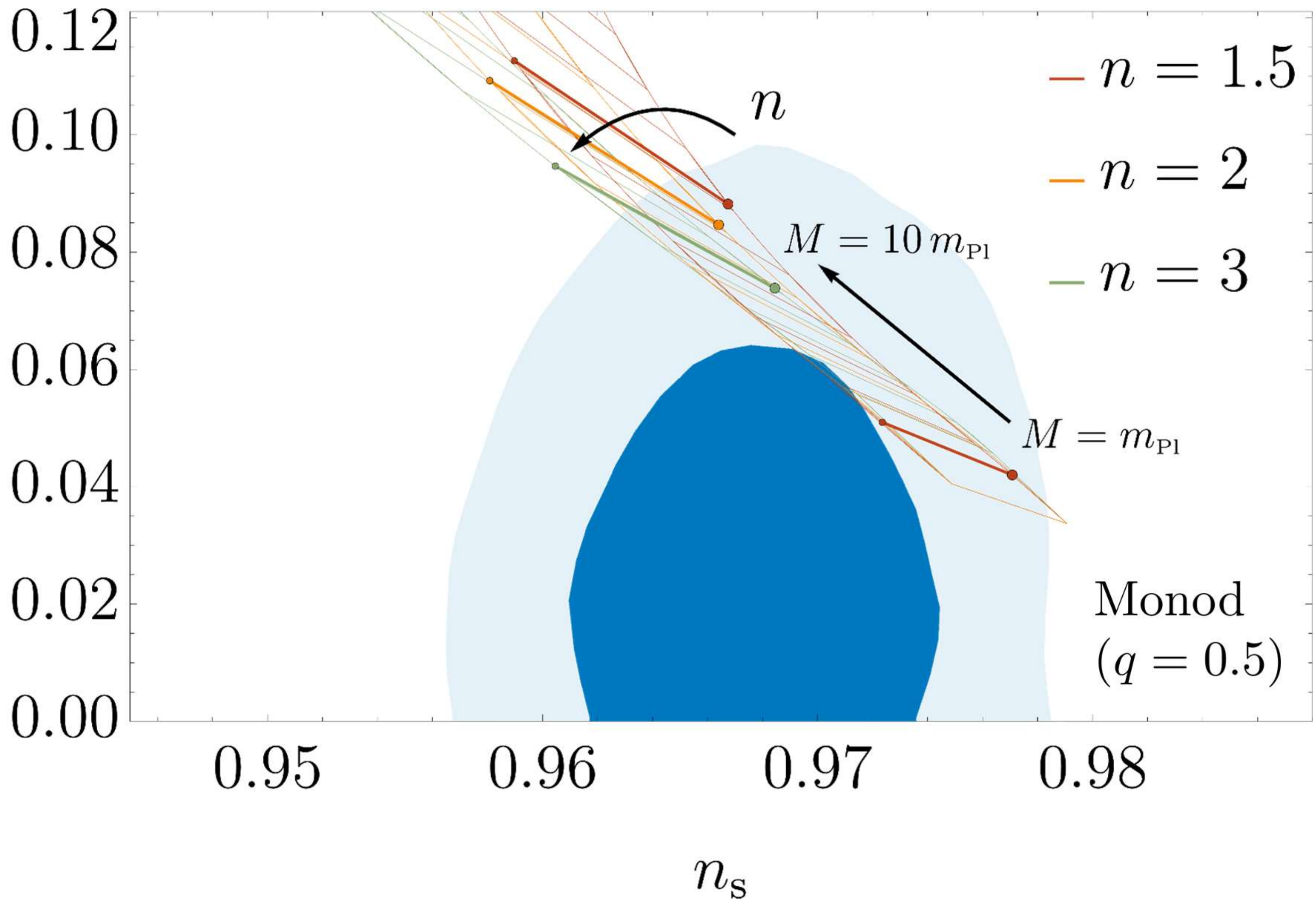
KL and M. Amin (2018)
Planck Collaboration (2015)

Expansion history effects



KL and M. Amin (2018)
Planck Collaboration (2015)

Expansion history effects

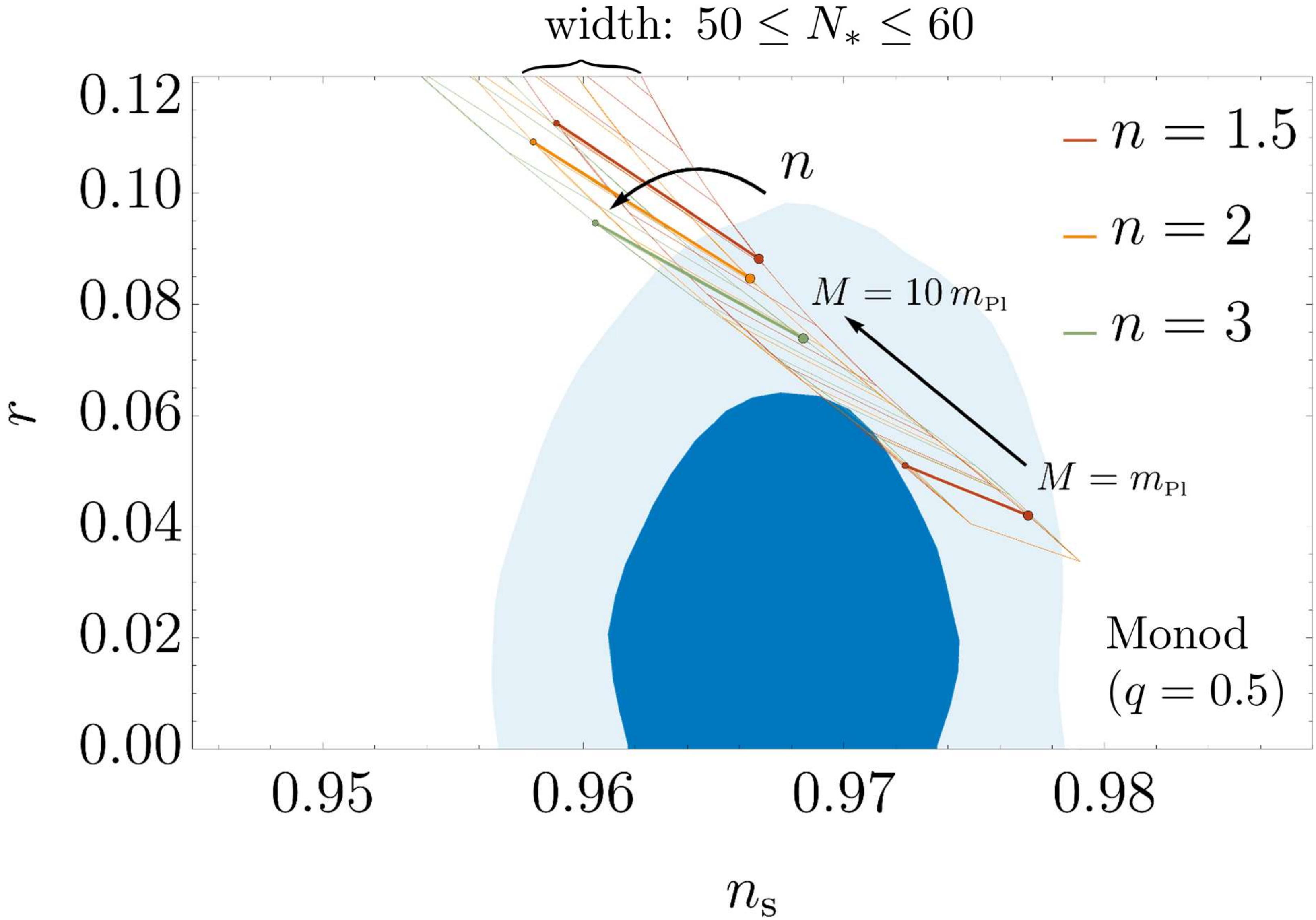


tensor-to-scalar ratio

spectral index

KL and M. Amin (2018)
Planck Collaboration (2015)

Expansion history effects

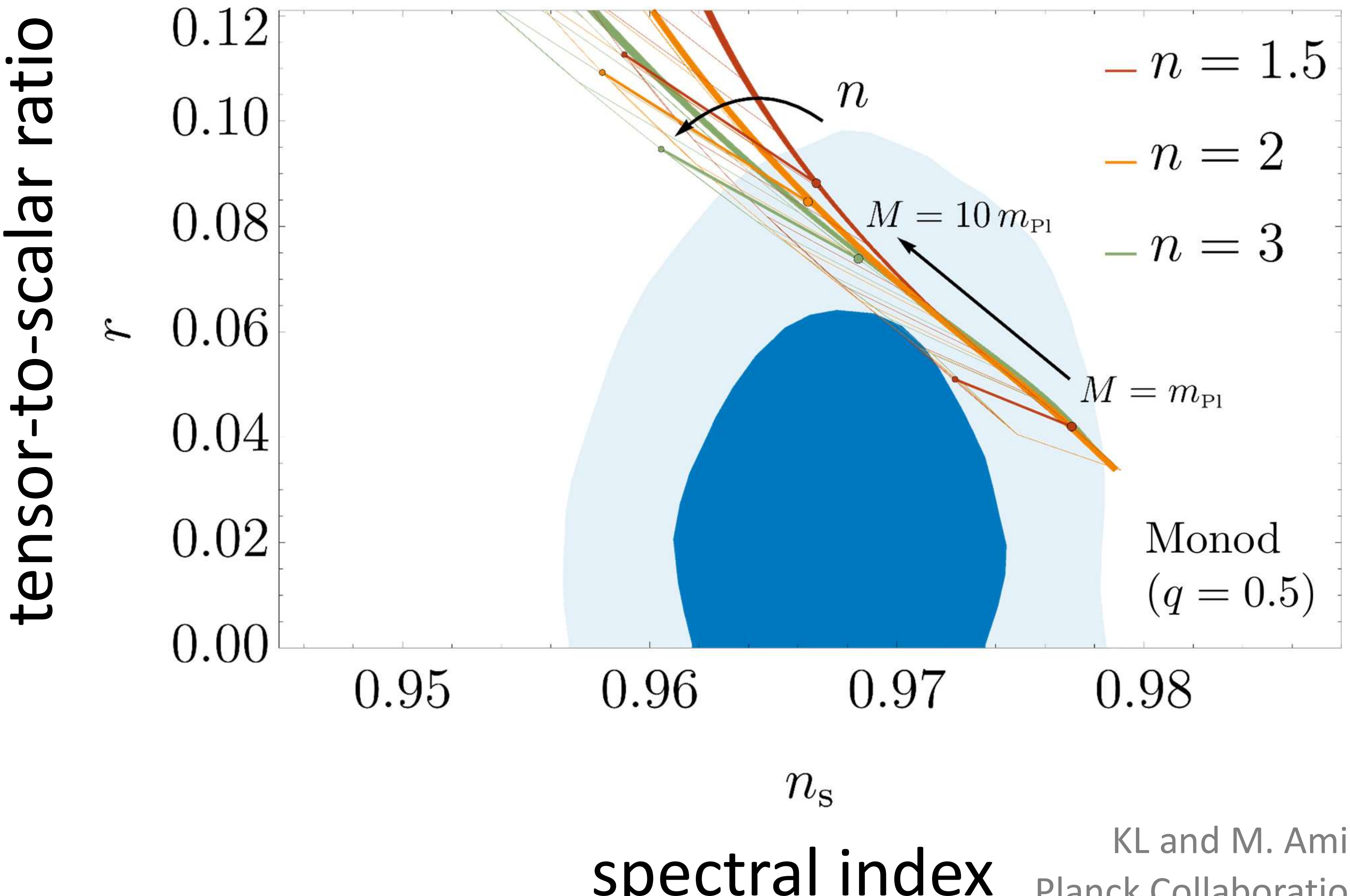


tensor-to-scalar ratio

spectral index

KL and M. Amin (2018)
Planck Collaboration (2015)

Expansion history effects

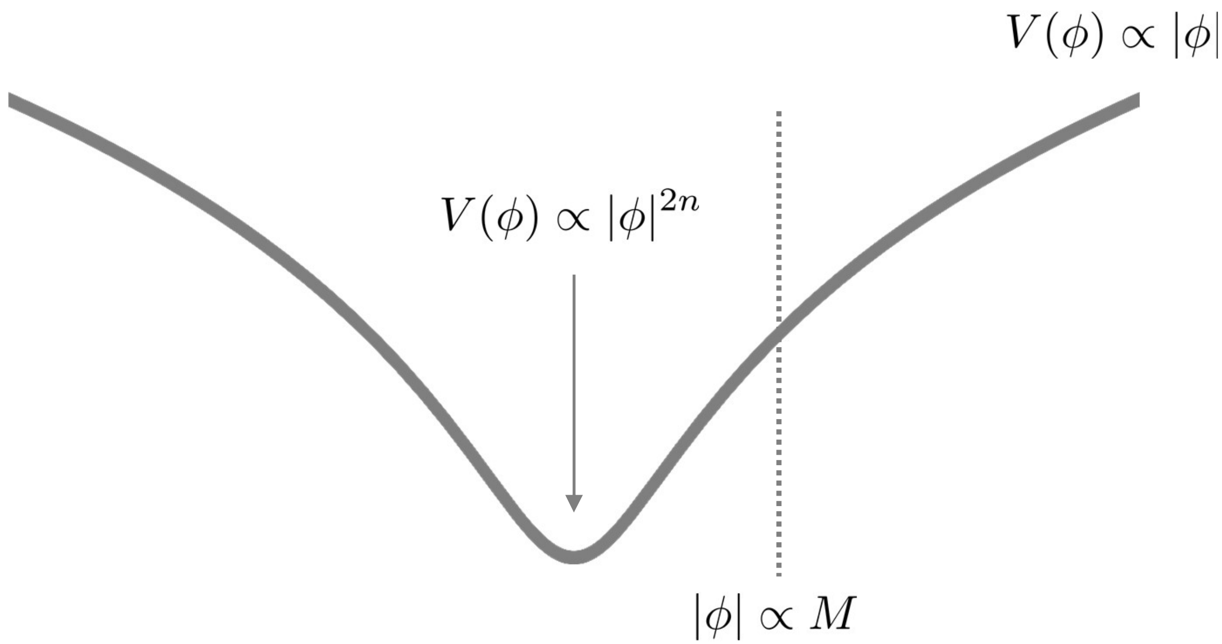


KL and M. Amin (2018)
Planck Collaboration (2015)

Expansion history effects

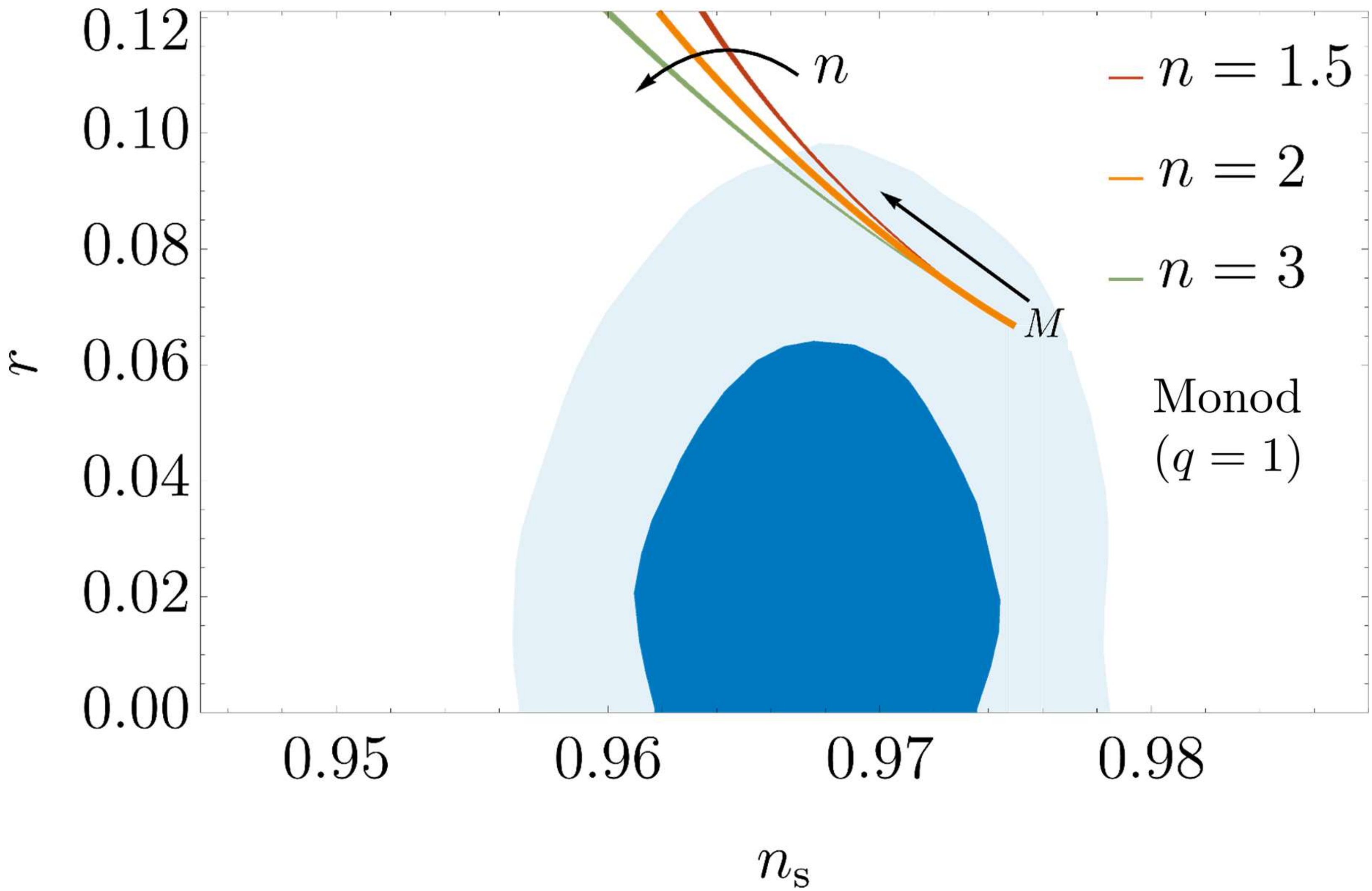
$$V(\phi) \propto \left[1 + \left|\frac{\phi}{M}\right|^{2n}\right]^{\frac{1}{2n}} - 1$$

Monodromy
$q = 1$



Silverstein and Westphal (2008)
McAllister, et al. (2014)

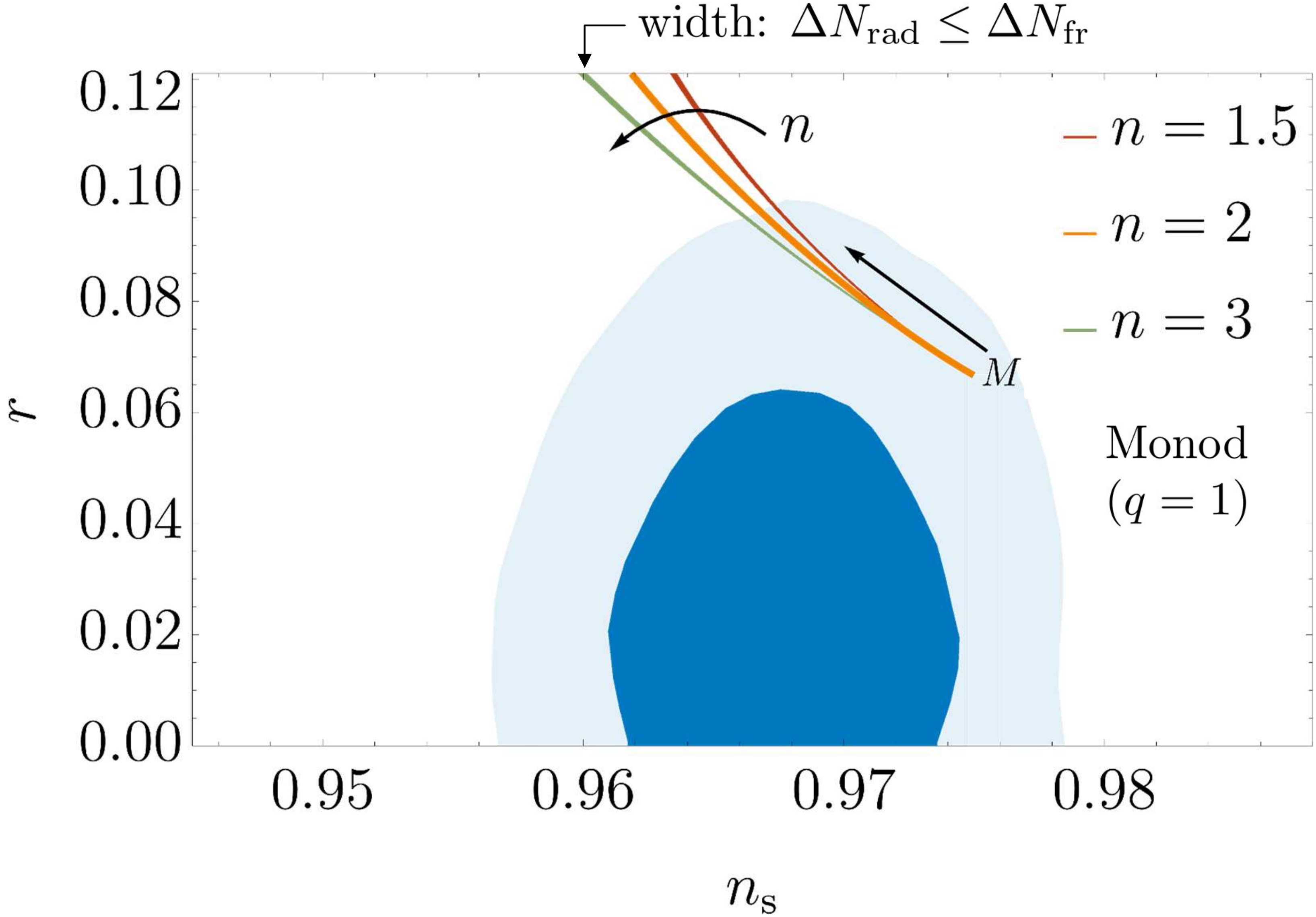
Expansion history effects



spectral index

KL and M. Amin (2018)
Planck Collaboration (2015)

Expansion history effects

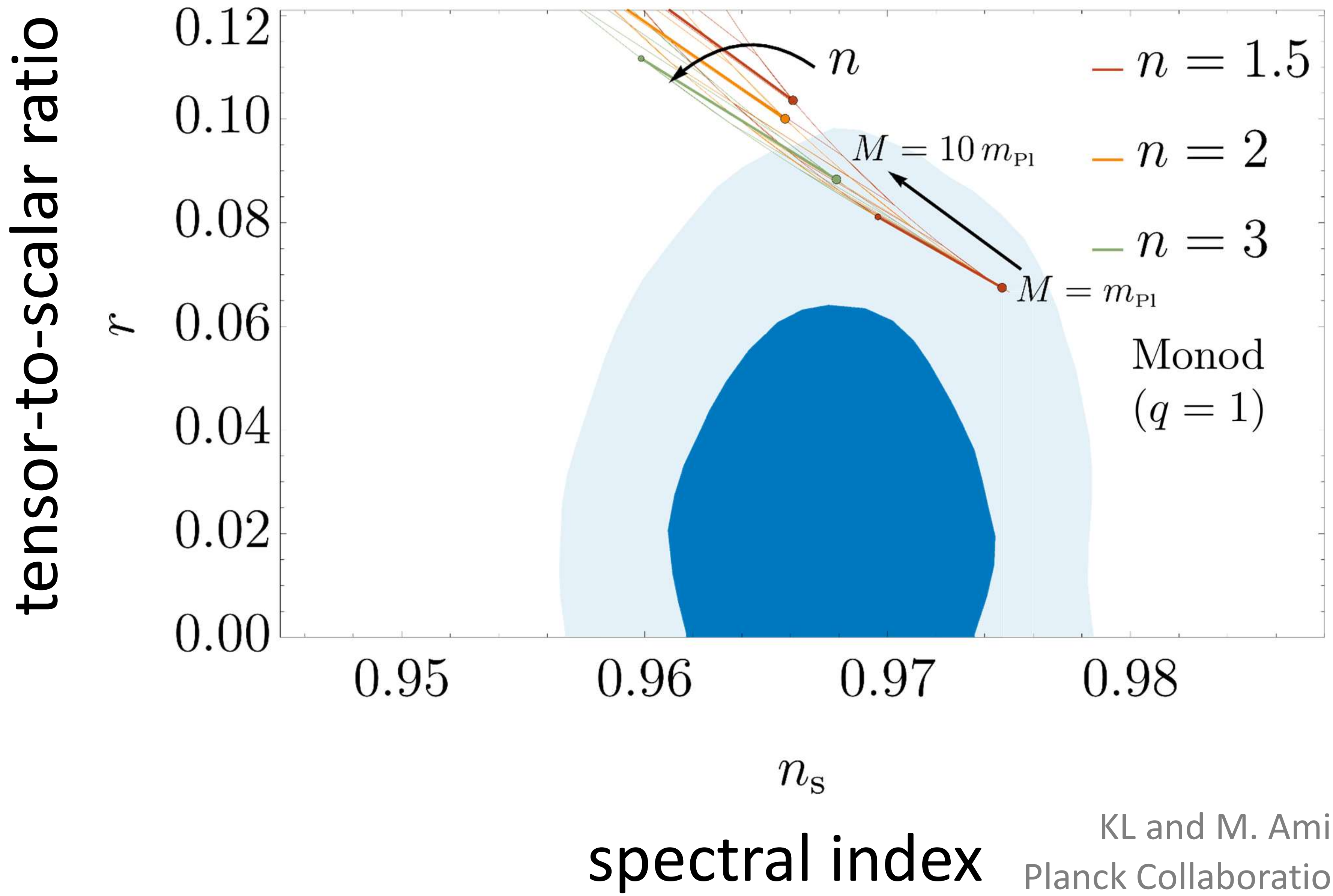


tensor-to-scalar ratio

spectral index

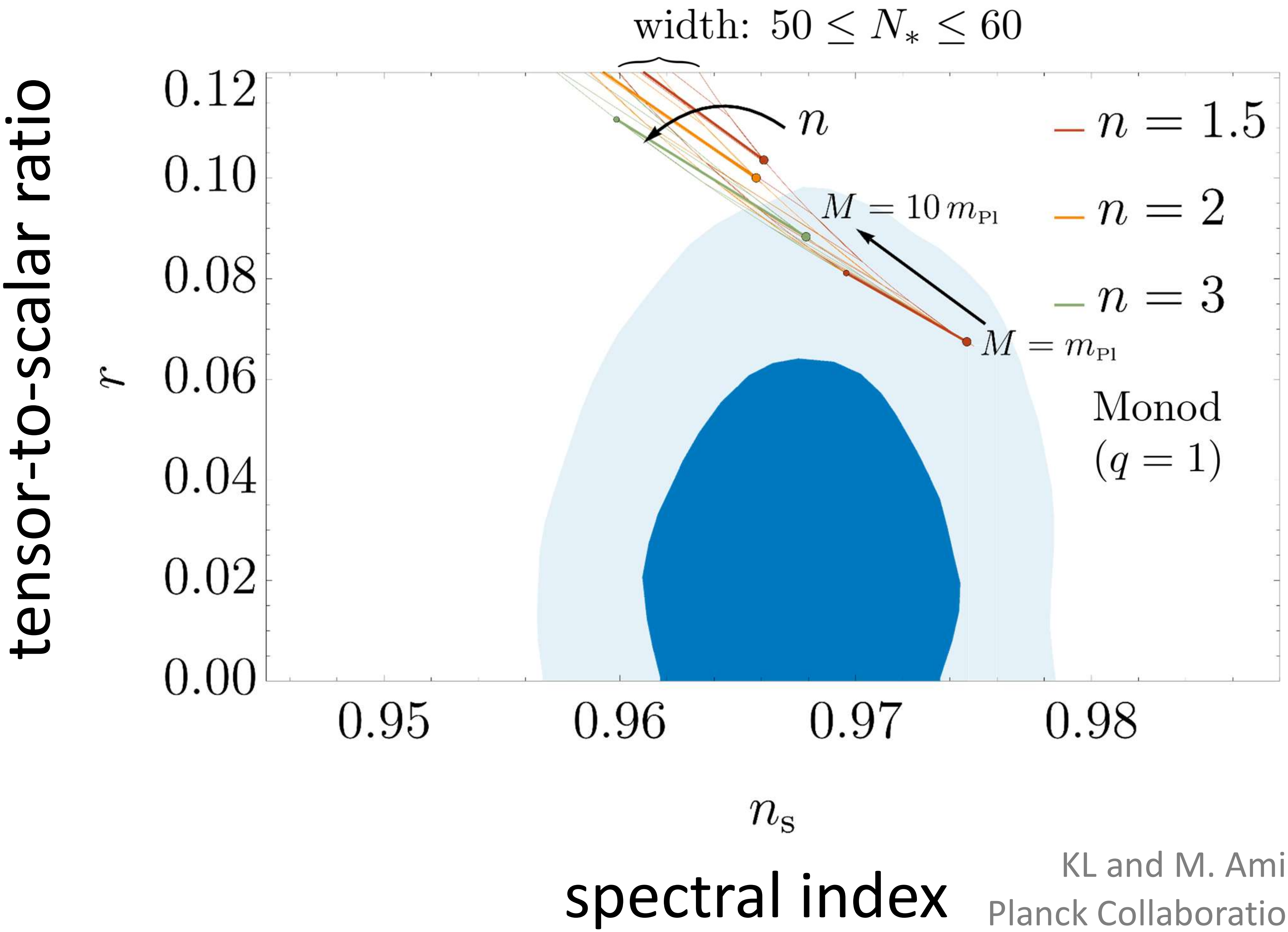
KL and M. Amin (2018)
Planck Collaboration (2015)

Expansion history effects



KL and M. Amin (2018)
Planck Collaboration (2015)

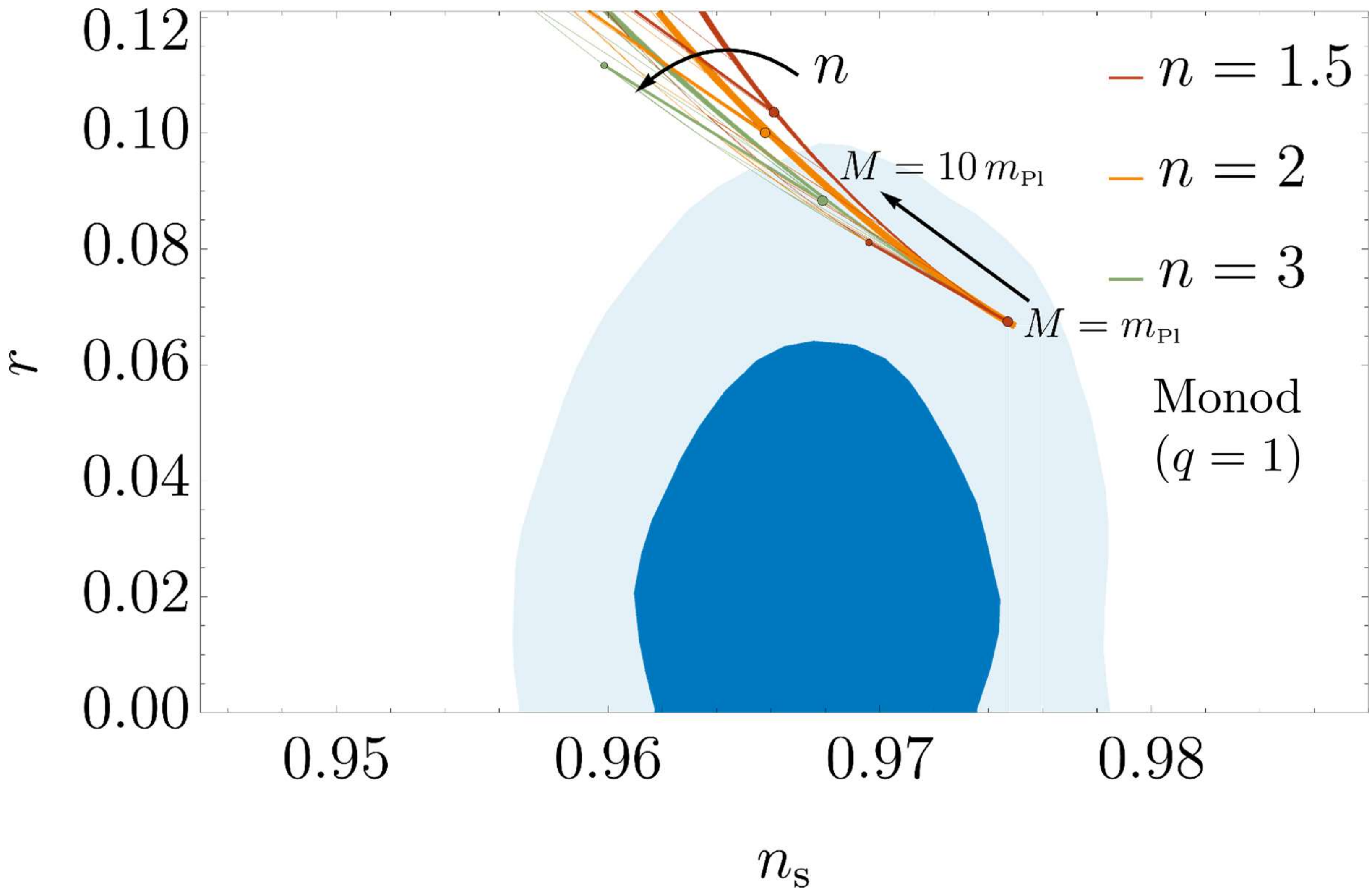
Expansion history effects



KL and M. Amin (2018)
Planck Collaboration (2015)

Expansion history effects

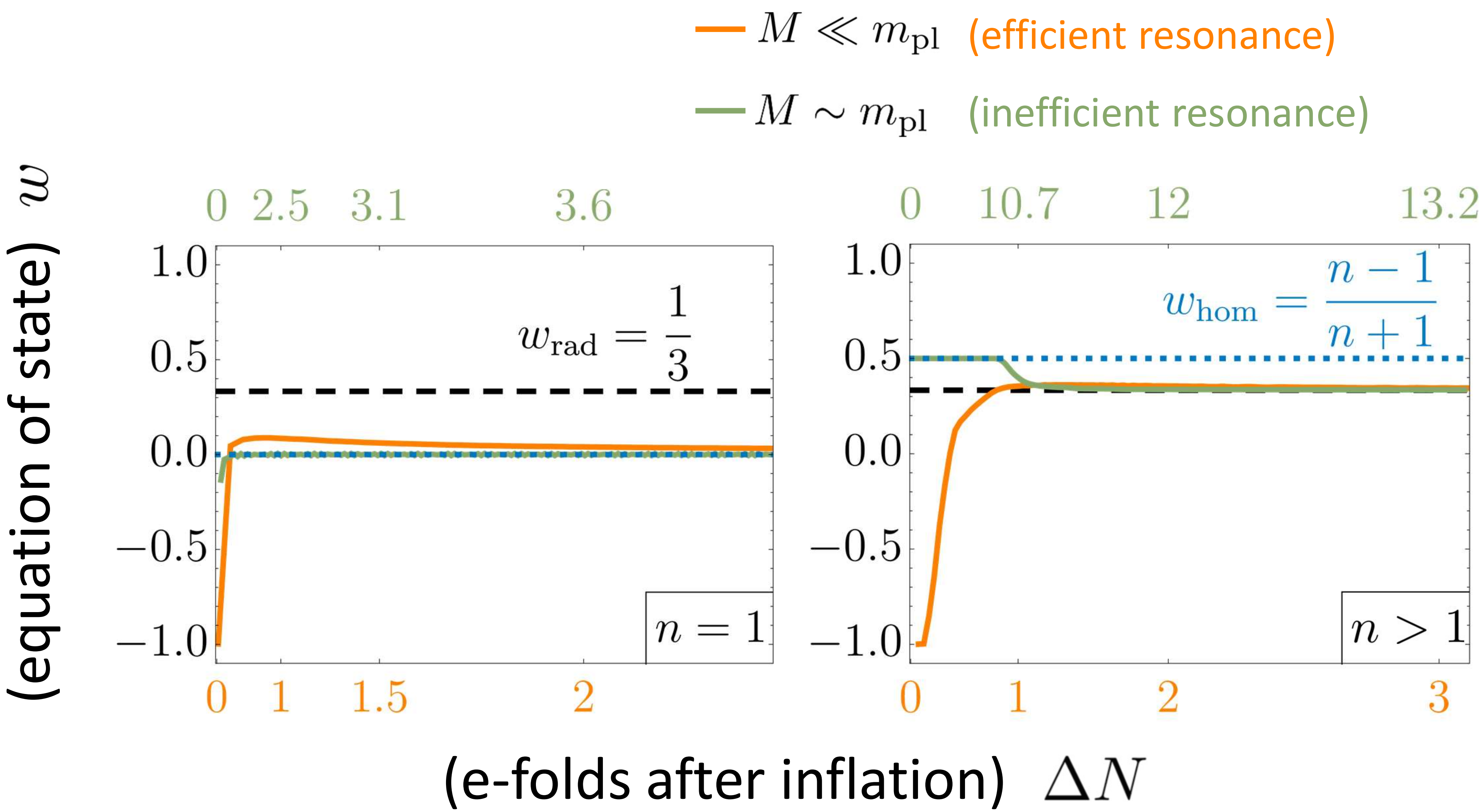
tensor-to-scalar ratio



spectral index

KL and M. Amin (2018)
Planck Collaboration (2015)

Equation of state



$\Delta N \equiv \int_{a_{\rm end}}^{a} d \ln a$

Matter domination?

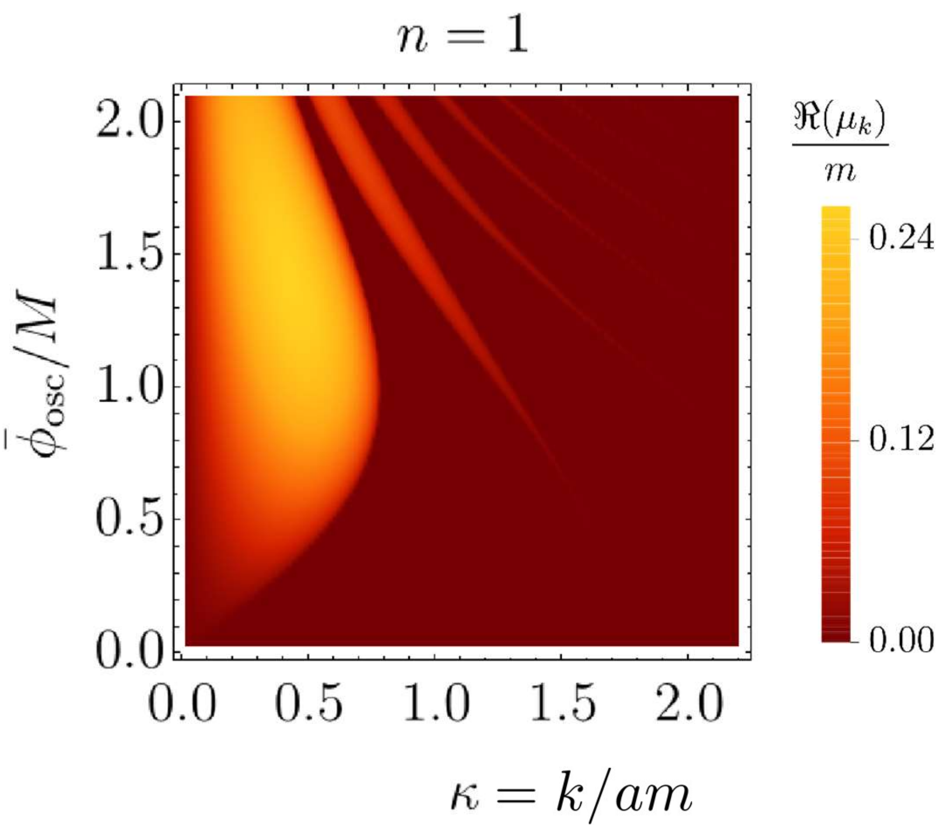
$$n = 1$$

Matter domination?

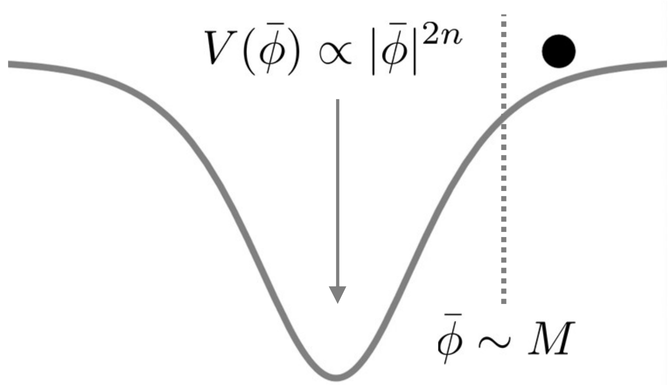
$$n = 1$$

Non-perturbative decay (parametric self-resonance)

inflaton vev amplitude



inflaton field wavenumber



$$\delta\phi_k \propto \exp(\pm\mu_k t)$$

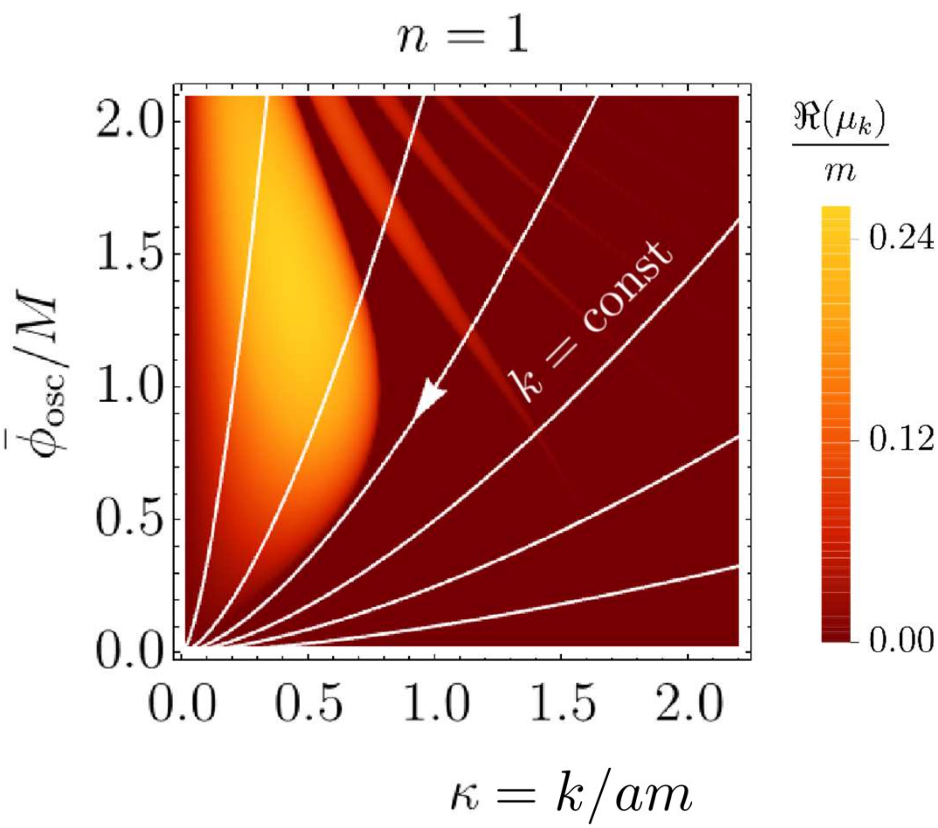
$m^2 \equiv V'(\bar{\phi}_{\rm osc})/\bar{\phi}_{\rm osc}$

Matter domination?

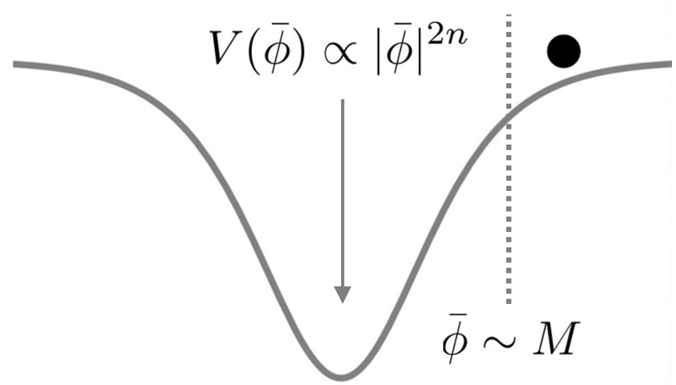
$$n = 1$$

Non-perturbative decay (parametric self-resonance)

inflaton vev amplitude



inflaton field wavenumber



$$\delta\phi_k \propto \exp(\pm\mu_k t)$$

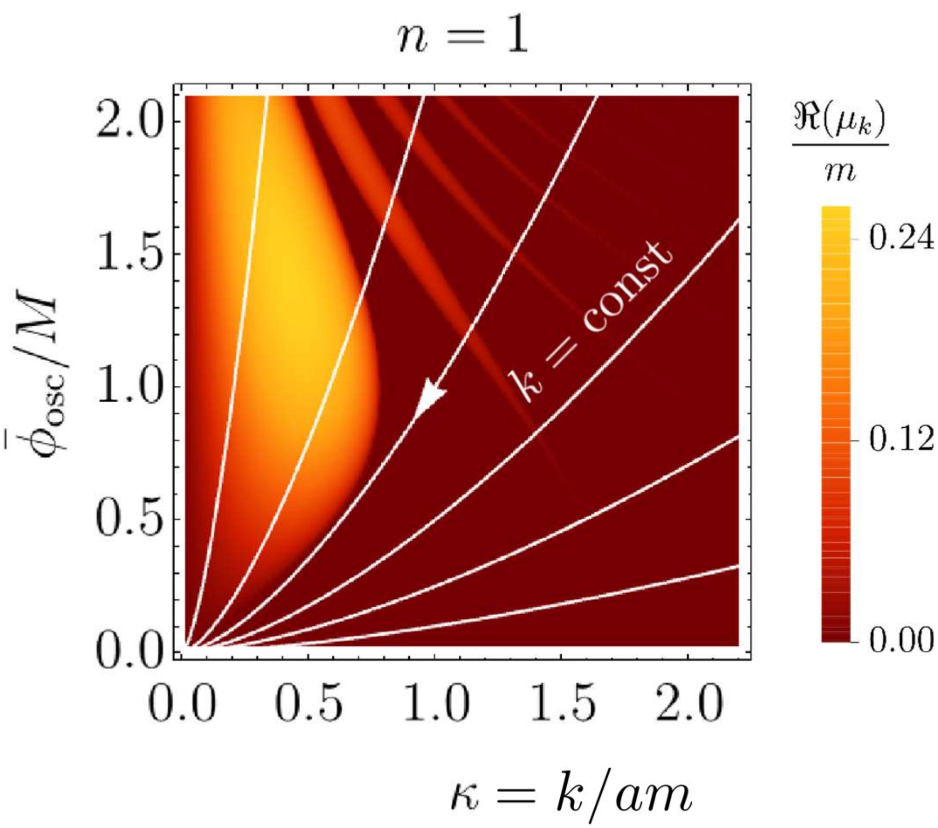
$m^2 \equiv V'(\bar{\phi}_{\rm osc})/\bar{\phi}_{\rm osc}$

Matter domination?

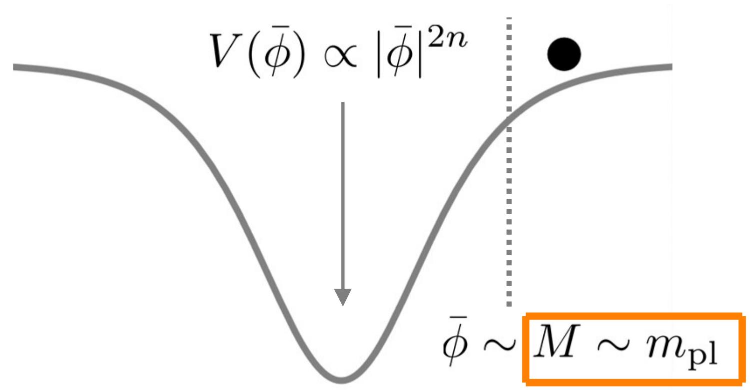
$$n = 1$$

Non-perturbative decay (parametric self-resonance)

inflaton vev amplitude



inflaton field wavenumber



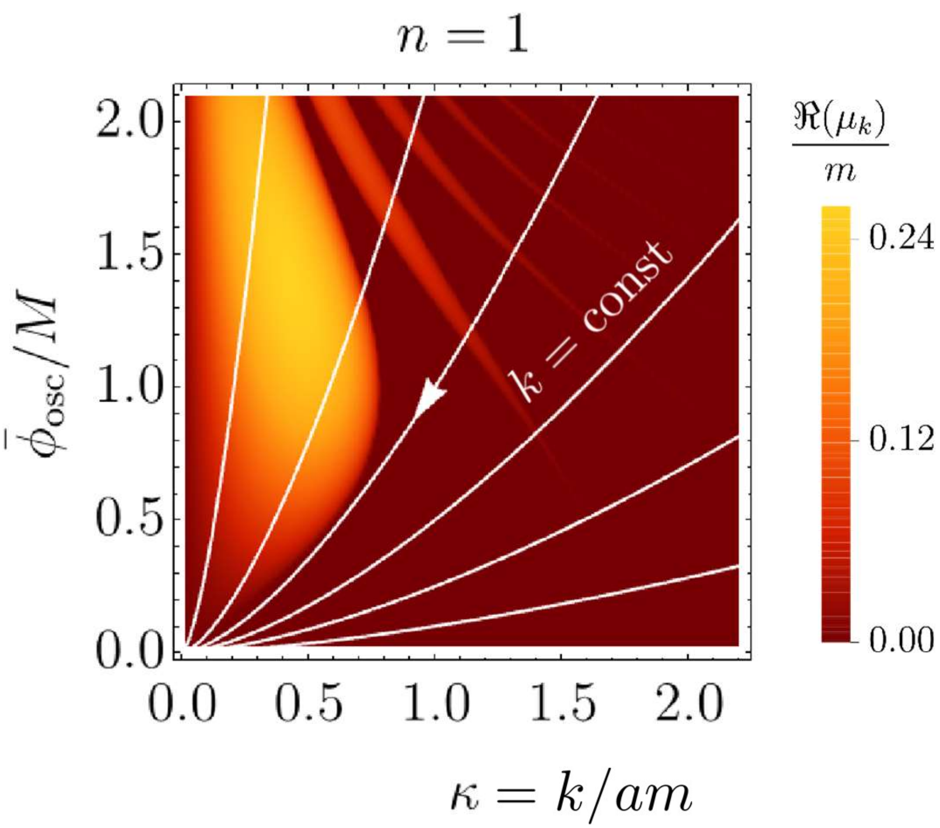
$$\delta\phi_k \propto \exp(\pm\mu_k t)$$

$m^2 \equiv V'(\bar{\phi}_{\rm osc})/\bar{\phi}_{\rm osc}$

Matter domination?

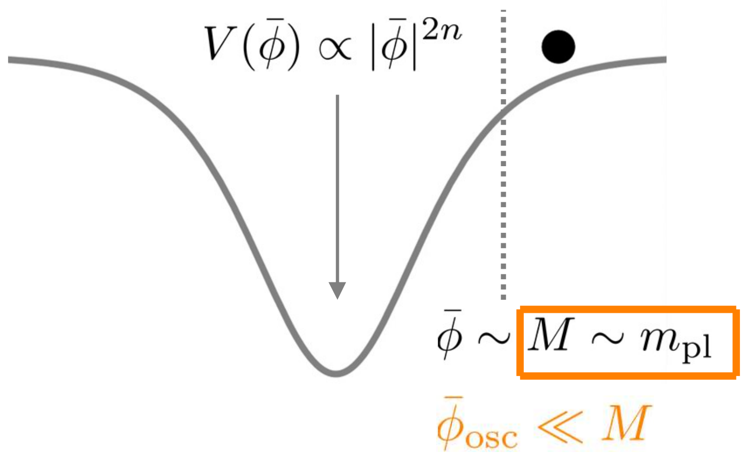
$$n = 1$$

Non-perturbative decay (parametric self-resonance)



inflaton vev amplitude

inflaton field wavenumber



$$\delta\phi_k \propto \exp(\pm\mu_k t)$$

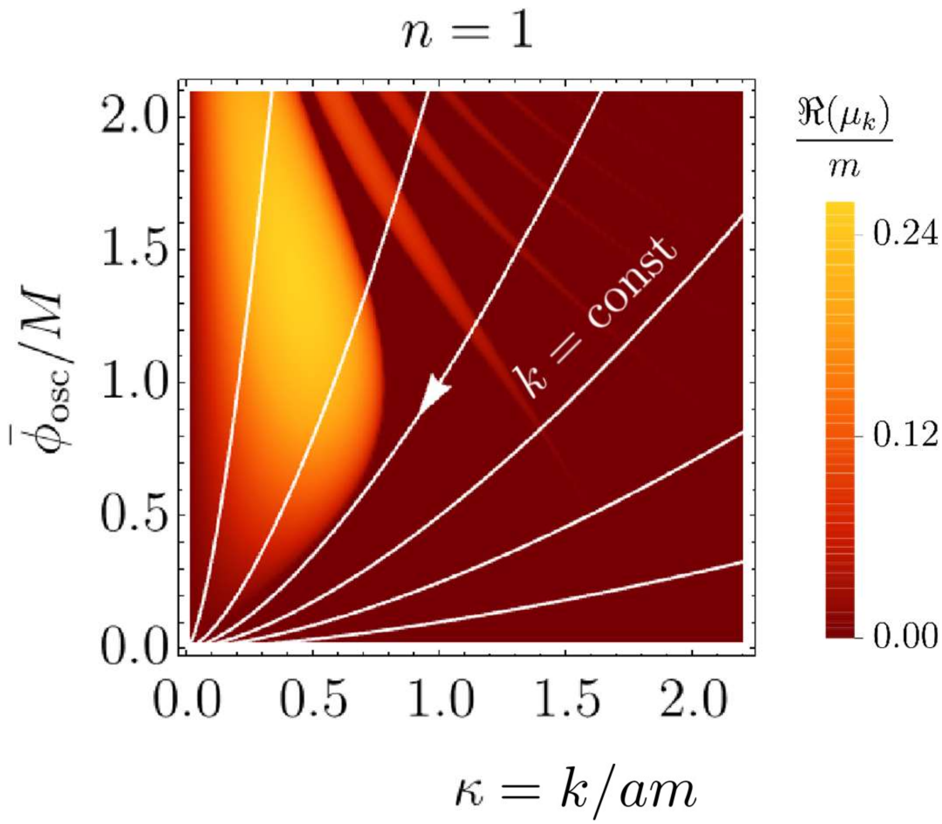
$m^2 \equiv V'(\bar{\phi}_{\rm osc})/\bar{\phi}_{\rm osc}$

Matter domination?

$$n = 1$$

Non-perturbative decay (parametric self-resonance)

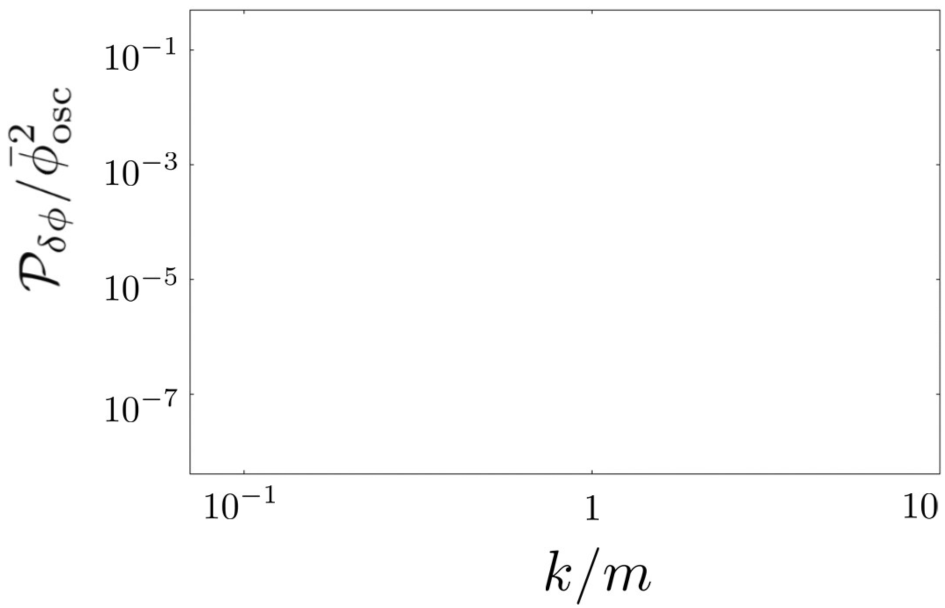
inflaton vev amplitude



inflaton field wavenumber

Power spectrum:

$$\langle \delta\phi(x)^2 \rangle \equiv \int \mathcal{P}_{\delta\phi}\, d\ln k$$



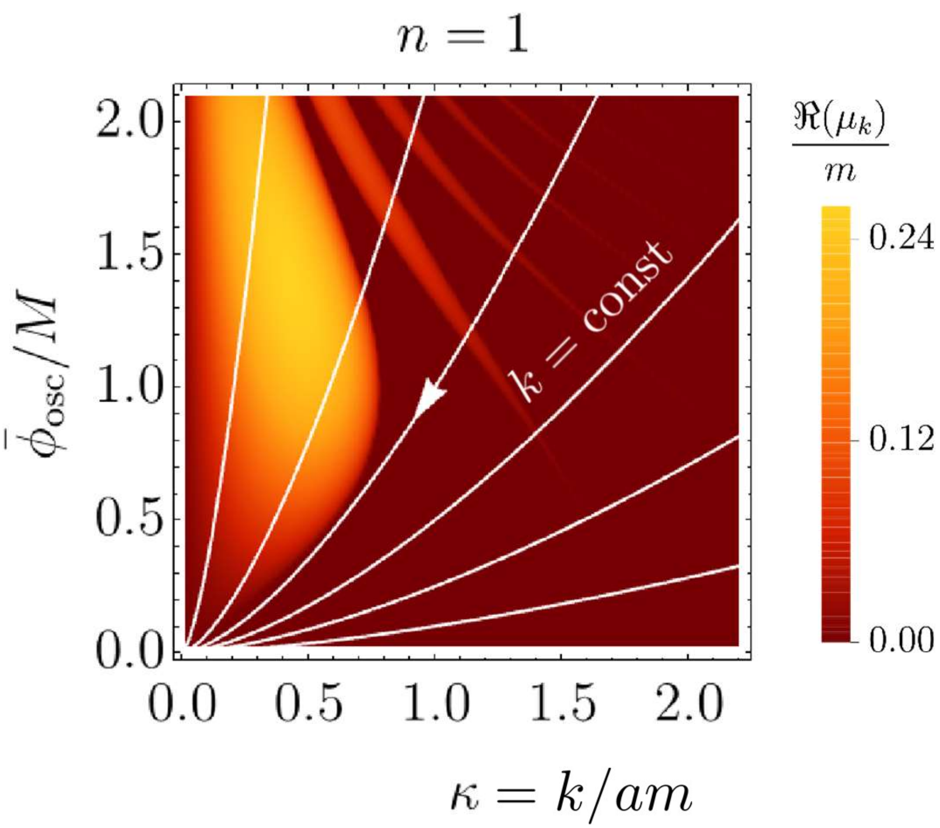
comoving wavenumber

Matter domination?

$$n = 1$$

Non-perturbative decay (parametric self-resonance)

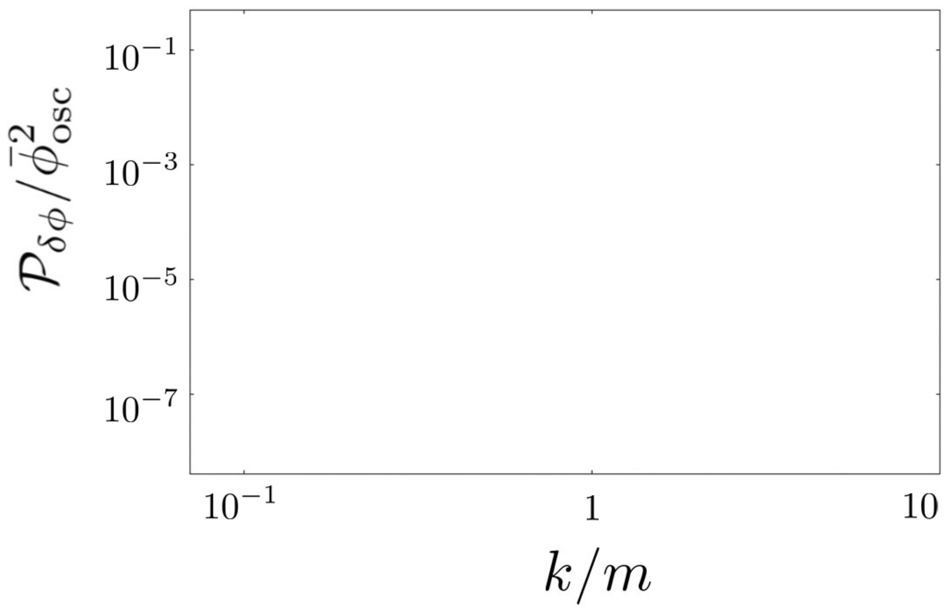
inflaton vev amplitude



inflaton field wavenumber

Power spectrum:

$$\langle \delta\phi(x)^2 \rangle \equiv \int \mathcal{P}_{\delta\phi} \, d\ln k$$



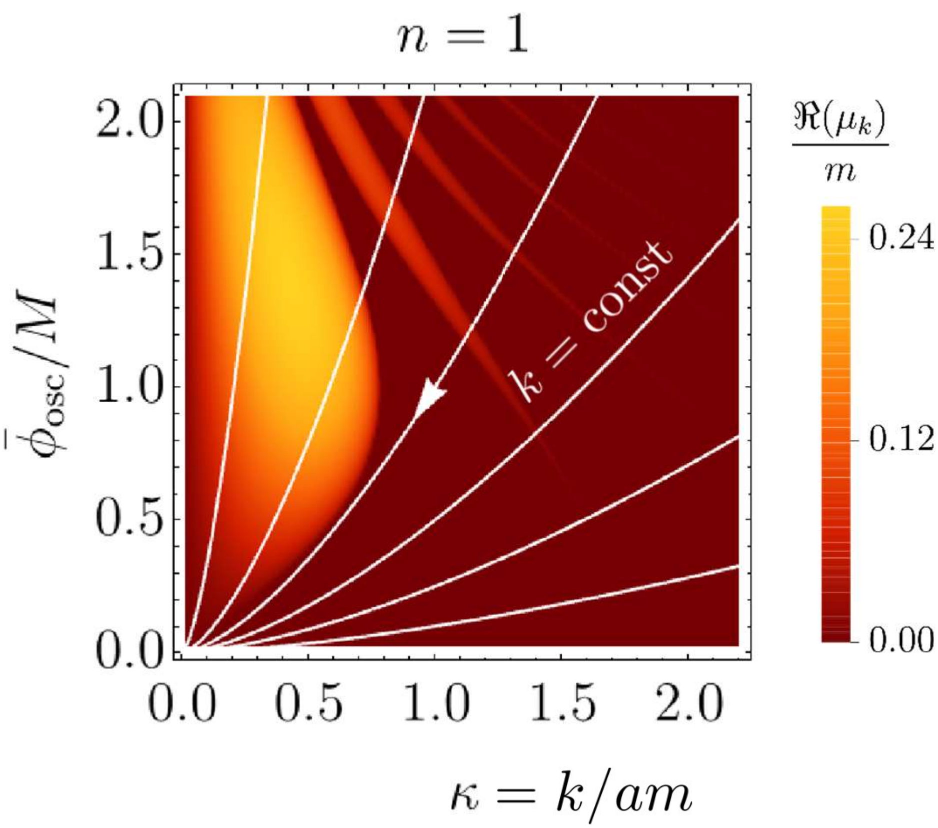
comoving wavenumber

Matter domination?

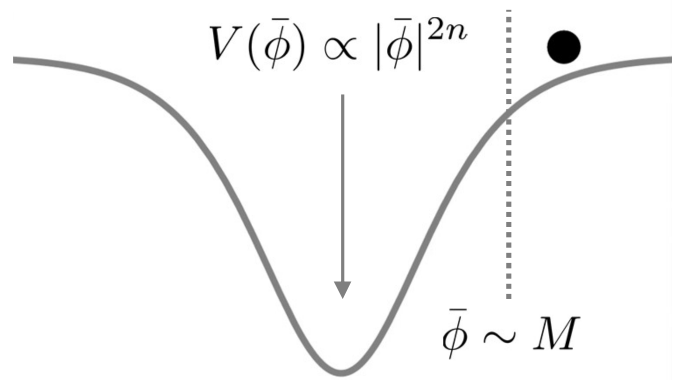
$$n = 1$$

Non-perturbative decay (parametric self-resonance)

inflaton vev amplitude



inflaton field wavenumber



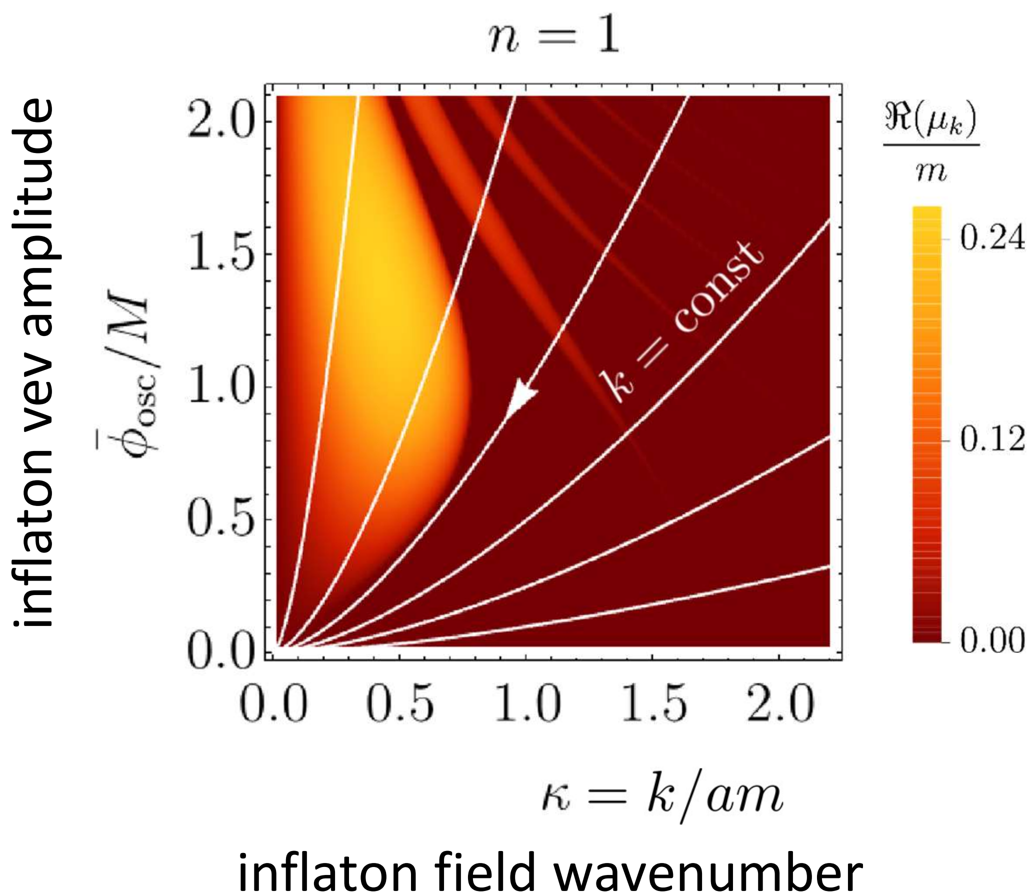
$$\delta\phi_k \propto \exp(\pm\mu_k t)$$

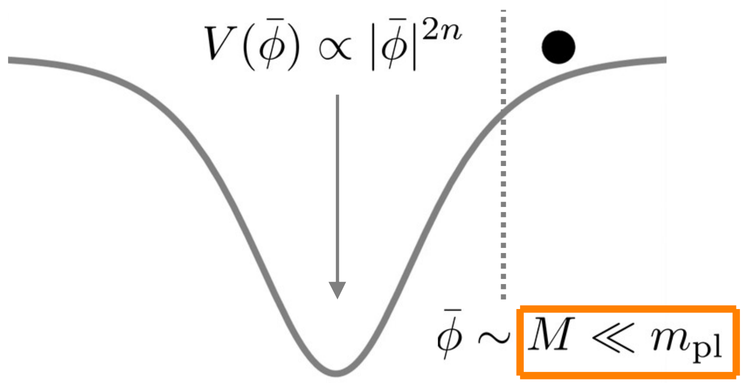
$m^2 \equiv V'(\bar{\phi}_{\rm osc})/\bar{\phi}_{\rm osc}$

Matter domination?

$$n = 1$$

Non-perturbative decay (parametric self-resonance)





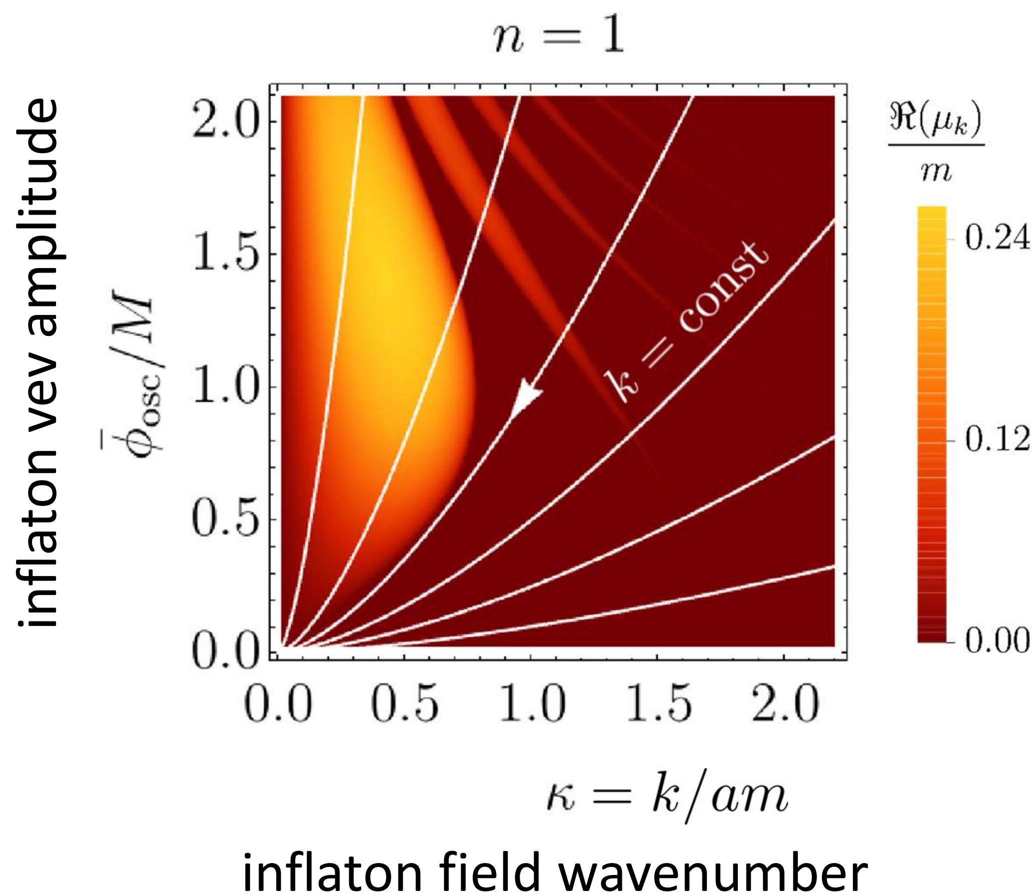
$$\delta\phi_k \propto \exp(\pm\mu_k t)$$

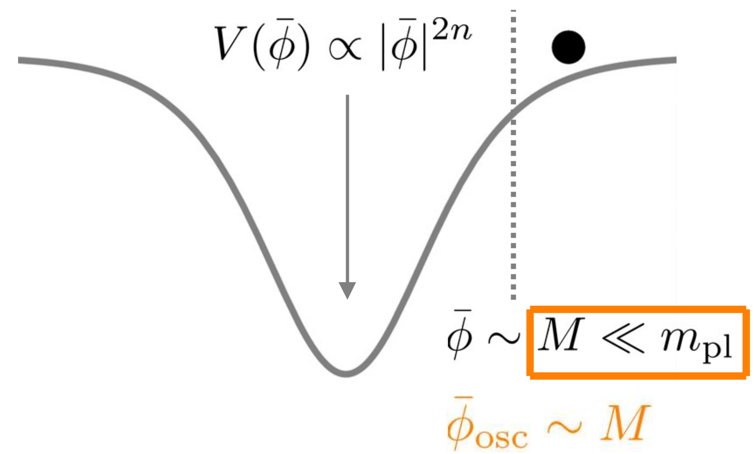
$m^2 \equiv V'(\bar{\phi}_{\rm osc})/\bar{\phi}_{\rm osc}$

Matter domination?

$$n = 1$$

Non-perturbative decay (parametric self-resonance)





$$\delta\phi_k \propto \exp(\pm \mu_k t)$$

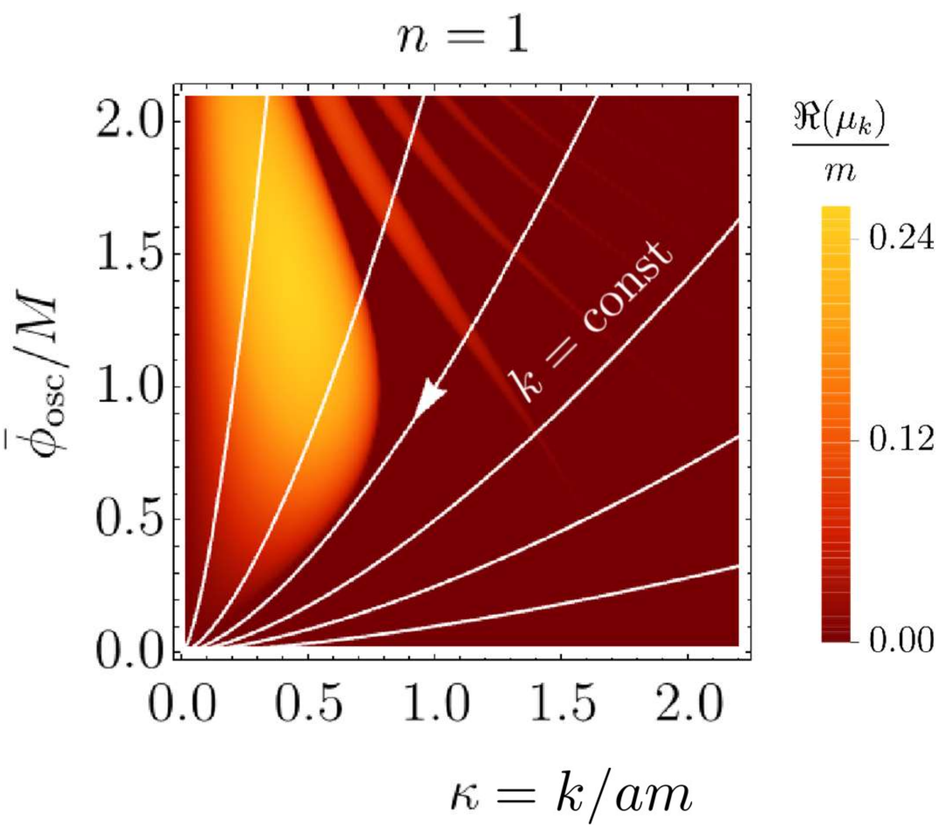
$m^2 \equiv V'(\bar{\phi}_{\rm osc})/\bar{\phi}_{\rm osc}$

Matter domination?

$$n = 1$$

Non-perturbative decay (parametric self-resonance)

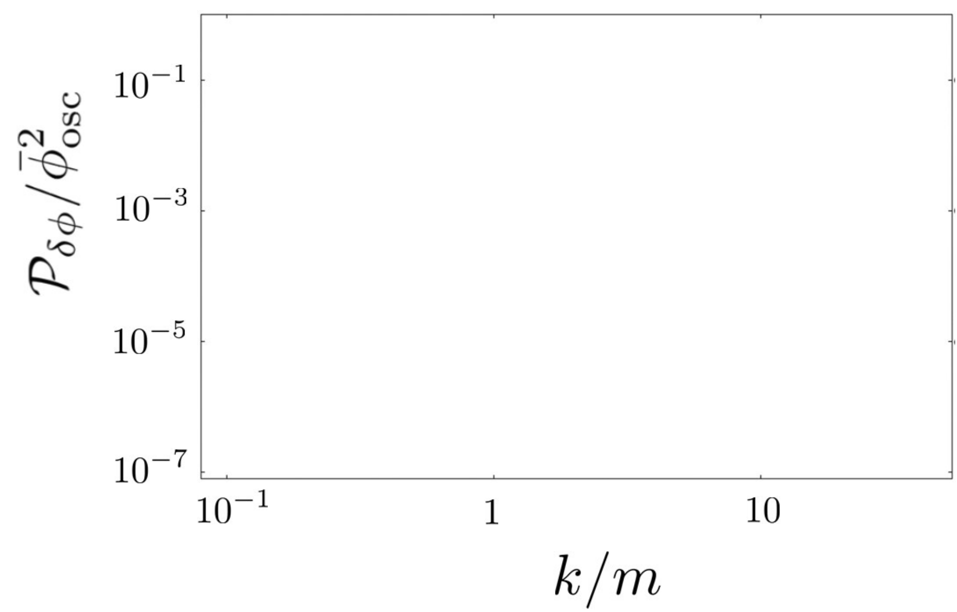
inflaton vev amplitude



inflaton field wavenumber

Power spectrum:

$$\langle \delta\phi(x)^2 \rangle \equiv \int \mathcal{P}_{\delta\phi}\, d\ln k$$



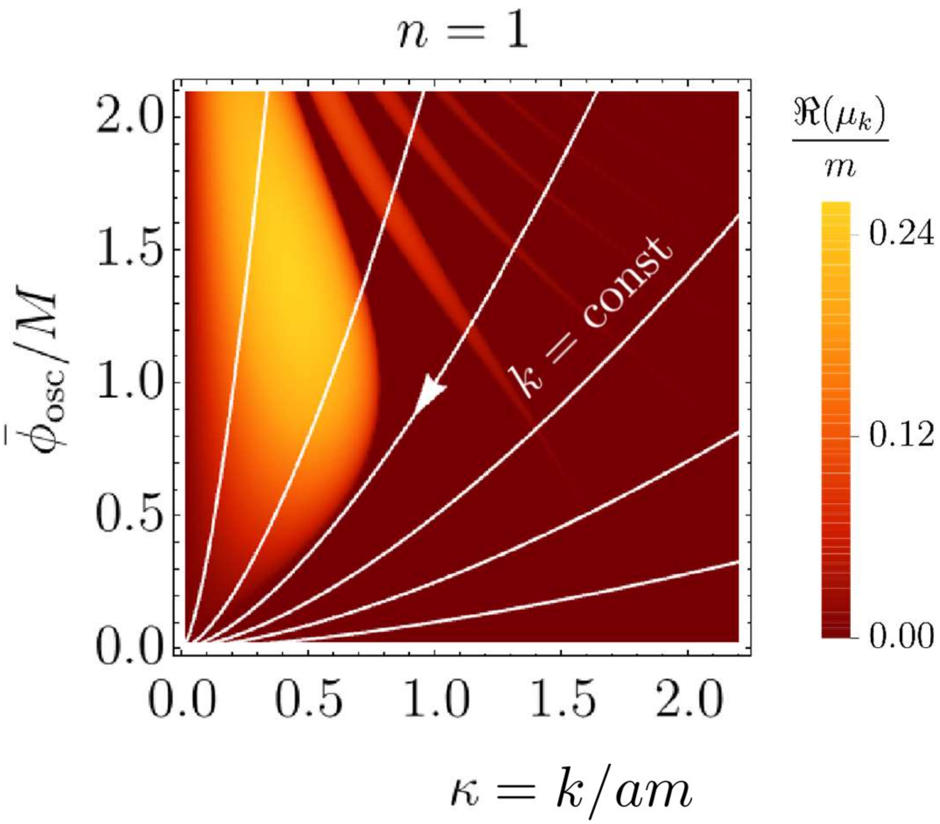
comoving wavenumber

Matter domination?

$$n = 1$$

Non-perturbative decay (parametric self-resonance)

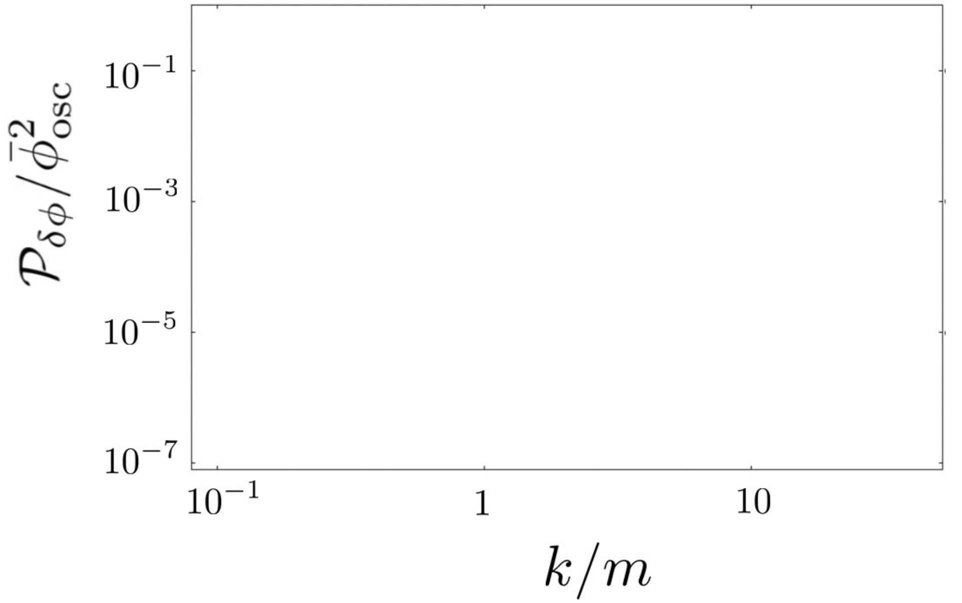
inflaton vev amplitude



inflaton field wavenumber

Power spectrum:

$$\langle \delta\phi(x)^2 \rangle \equiv \int \mathcal{P}_{\delta\phi}\, d\ln k$$

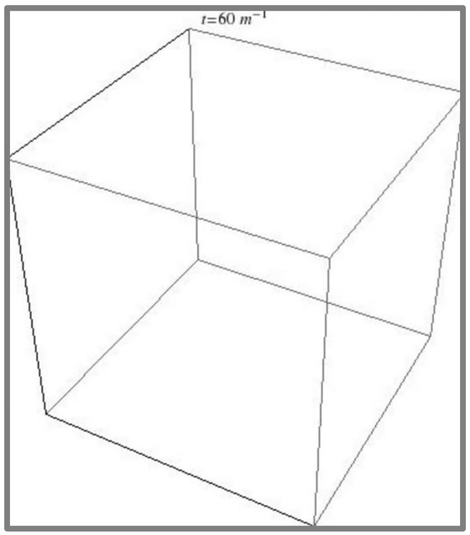


comoving wavenumber

Matter domination?

$$n = 1$$

$M \ll m_{\mathrm{pl}}$



- $\bar{\phi}$ forms oscillons (stable)

KL and Amin (2017, 18, 19)

See also Amin, Easther, Finkel, Flauger, Hertzberg (2011)

$M \sim m_{\mathrm{pl}}$

- $\delta\phi(t, \mathbf{x})$ production shut off
- $\bar{\phi}_{\mathrm{osc}}(t) =$ pressureless dust

matter-like eos: $w = 0$

couplings to other fields?

Other connections …

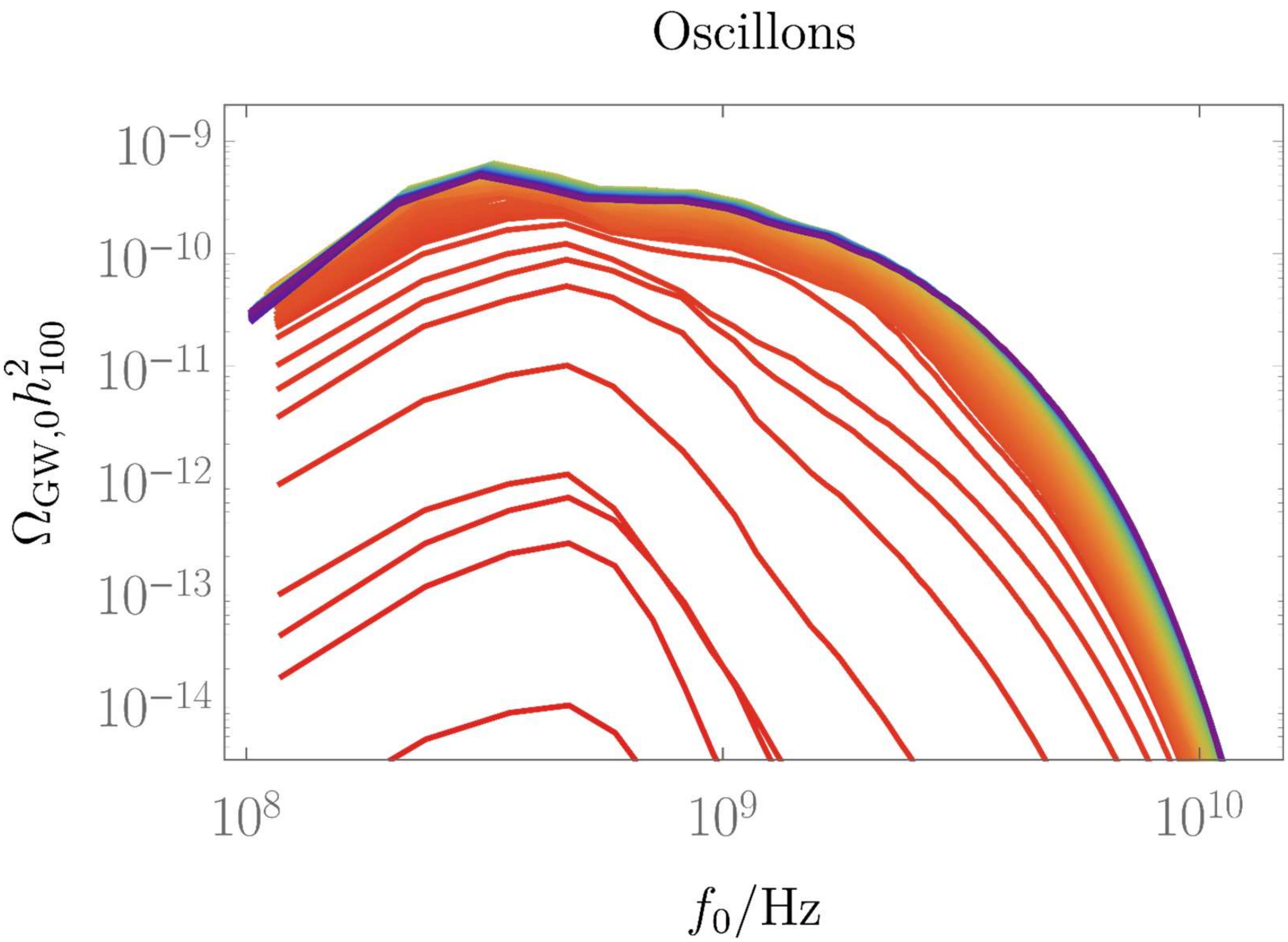
Other connections …

- stochastic GWs from fragmentation KL and M. Amin (2019)

Other connections …

- stochastic GWs from fragmentation

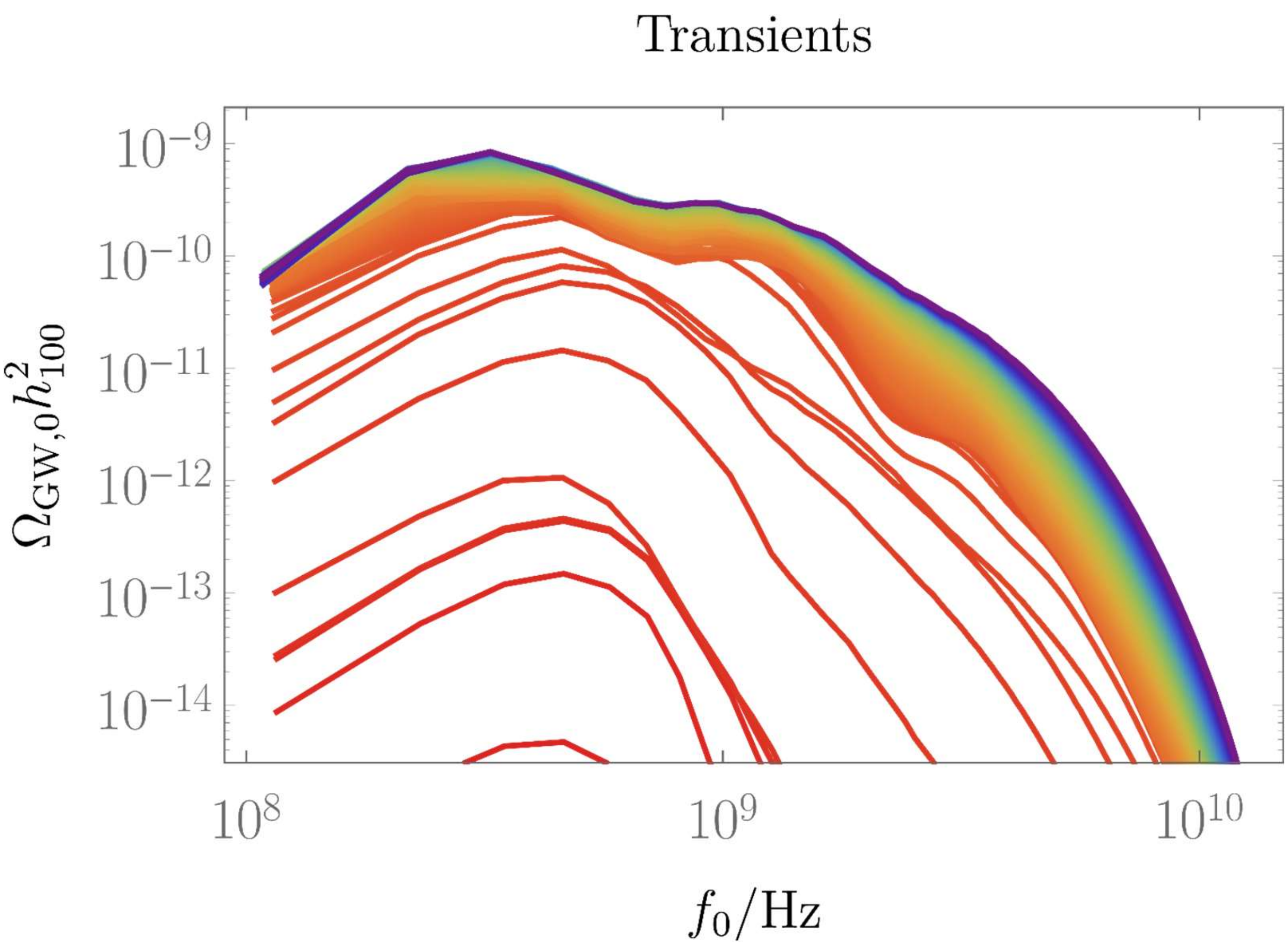
KL and M. Amin (2019)



Other connections …

- stochastic GWs from fragmentation

KL and M. Amin (2019)

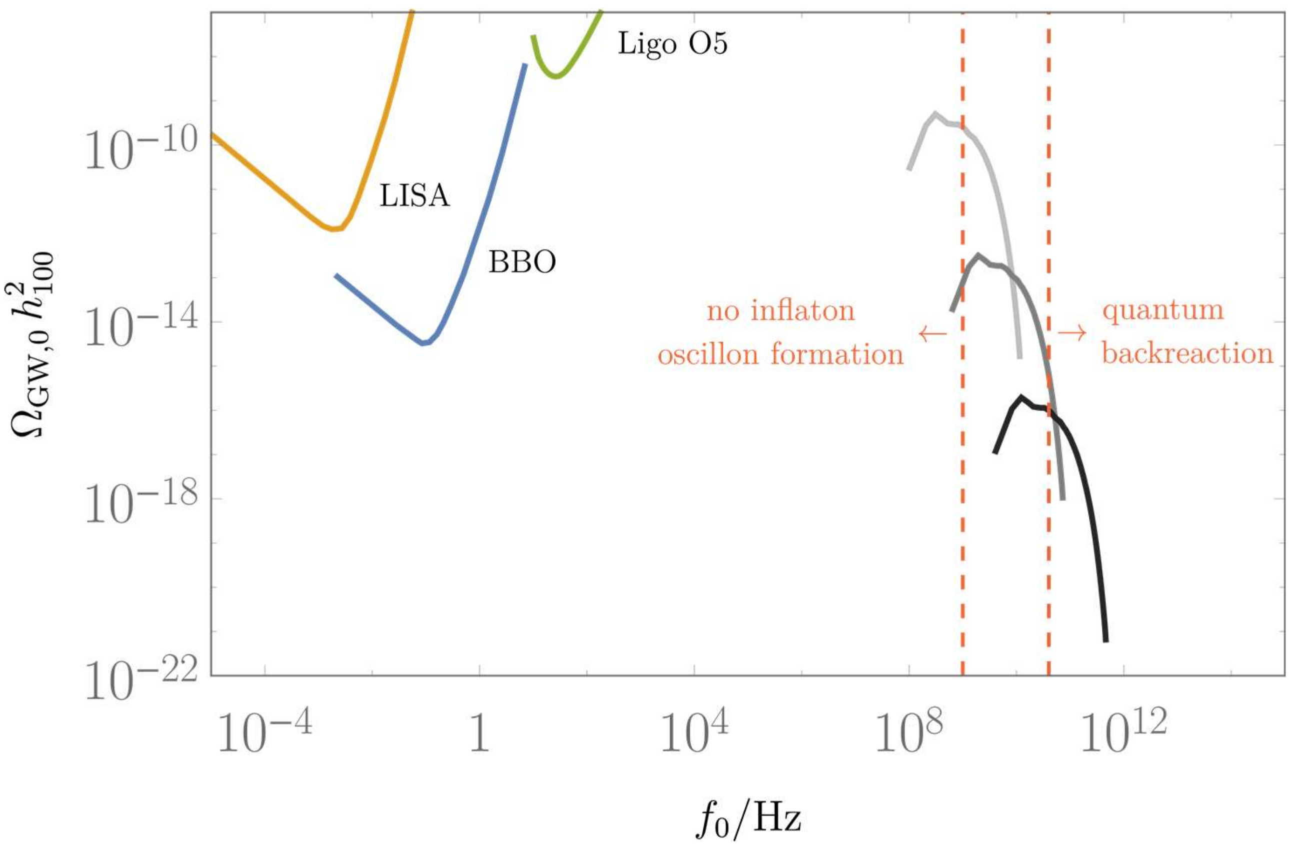


Other connections …

- stochastic GWs from fragmentation

KL and M. Amin (2019)

INFLATON oscillons

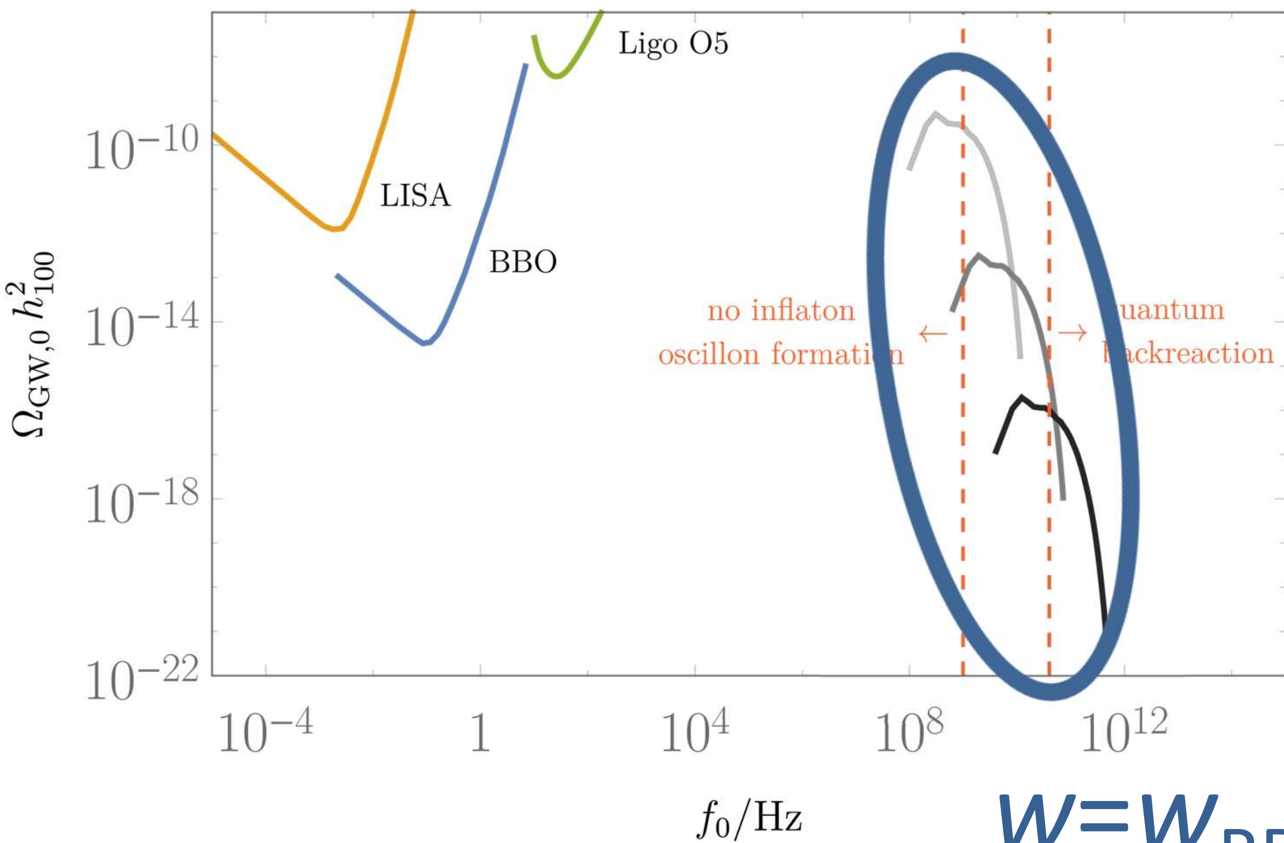


Other connections …

- stochastic GWs from fragmentation

KL and M. Amin (2019)

INFLATON oscillons



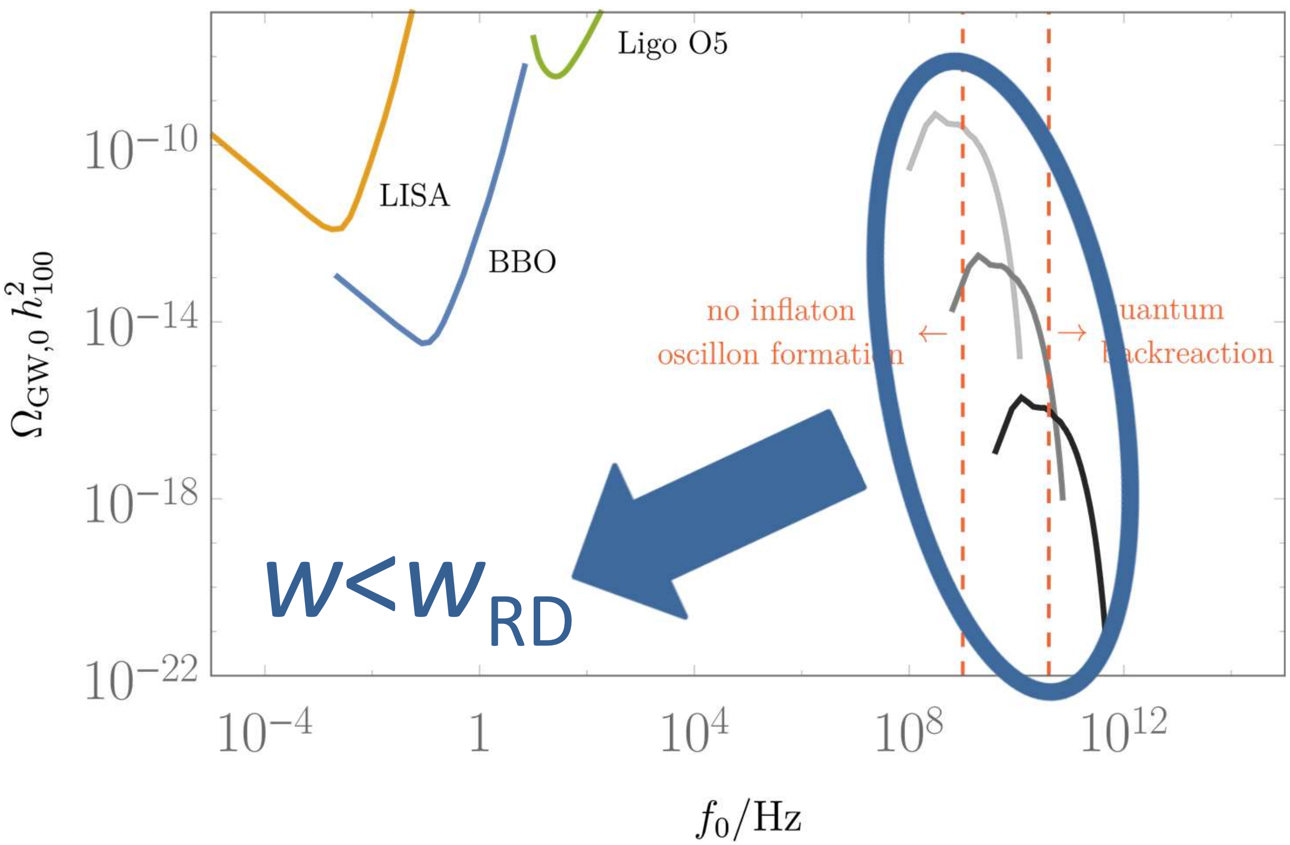
$w = w_{\mathrm{RD}}$

Other connections …

- stochastic GWs from fragmentation

KL and M. Amin (2019)

INFLATON oscillons

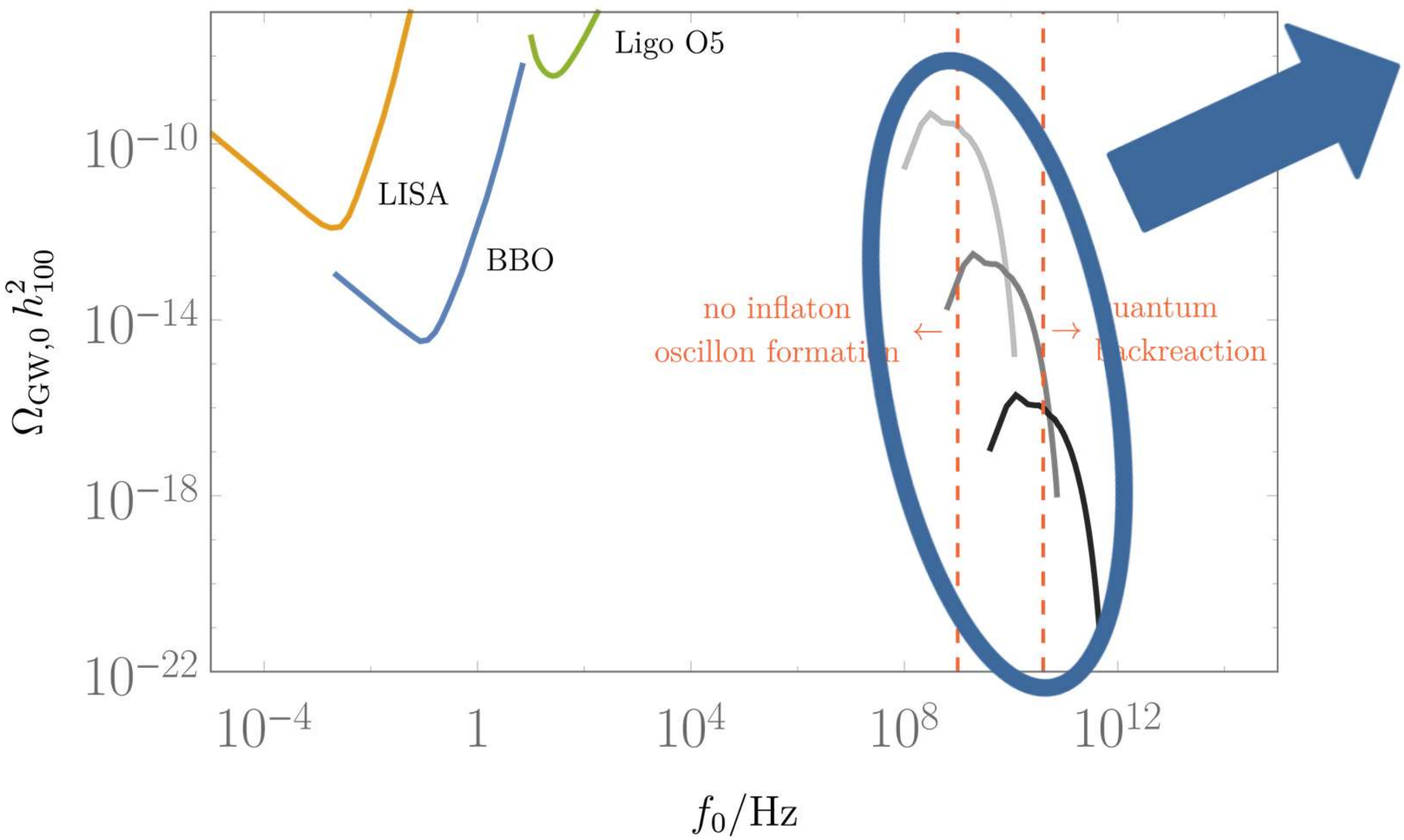


Other connections …

- stochastic GWs from fragmentation

KL and M. Amin (2019)

INFLATON oscillons



$w > w_{\rm RD}$

Other connections …

- stochastic GWs from fragmentation — KL and M. Amin (2019)
- (early) dark energy (n=2) — Agrawal, Cyr-Racine, Pinner, Randall (2019); M. Amin, KL and T. Smith (in progress)

Other connections …

- stochastic GWs from fragmentation — KL and M. Amin (2019)
- (early) dark energy (n=2) — Agrawal, Cyr-Racine, Pinner, Randall (2019); M. Amin, KL and T. Smith (in progress)
- dark matter (n=1 oscillons → Q-balls)

Other connections …

- stochastic GWs from fragmentation — KL and M. Amin (2019)
- (early) dark energy (n=2) — Agrawal, Cyr-Racine, Pinner, Randall (2019); M. Amin, KL and T. Smith (in progress)
- dark matter (n=1 oscillons → Q-balls)
- matter-antimatter asymmetry… — KL and M. Amin (2014)

Oscillons & baryogenesis

KL and M. Amin, PRD 90, 083528 (2014)

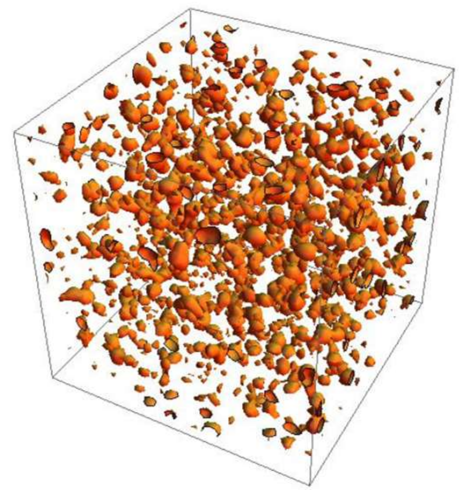
complex
inflaton ϕ
+ ~~U(1)~~

↓ dynamics at the end of inflation

inflaton/anti-inflaton
asymmetry

↓ decay

matter-antimatter
asymmetry
$\eta \approx 6 \times 10^{-10}$



very different dynamics from homogeneous case!

Gauge fields, inflation & reheating

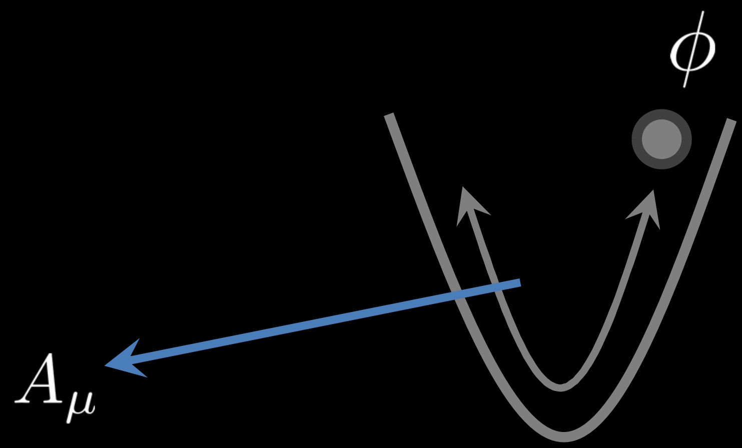
KL and M. Amin, JCAP 1606 032 (2016)

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - (D_\mu\phi)^\dagger D^\mu\phi - V(|\phi|) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\Big]$$

Gauge fields, inflation & reheating

KL and M. Amin, JCAP 1606 032 (2016)

$$S = \int d^4x \sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - (D_\mu \phi)^\dagger D^\mu \phi - V(|\phi|) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\Big]$$

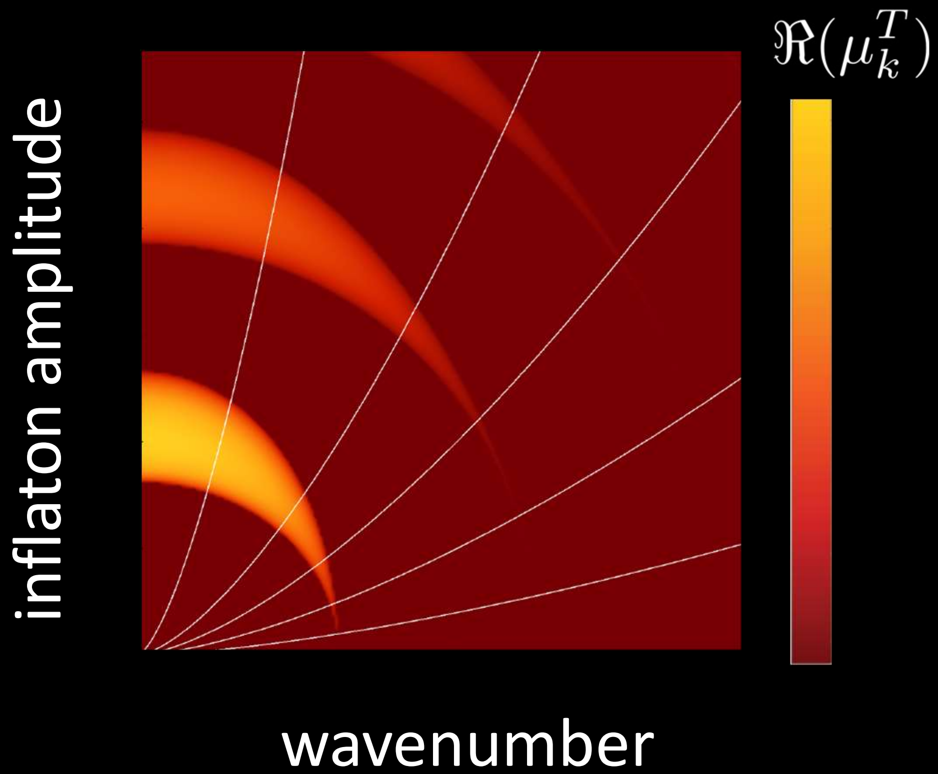


Gauge fields, inflation & reheating

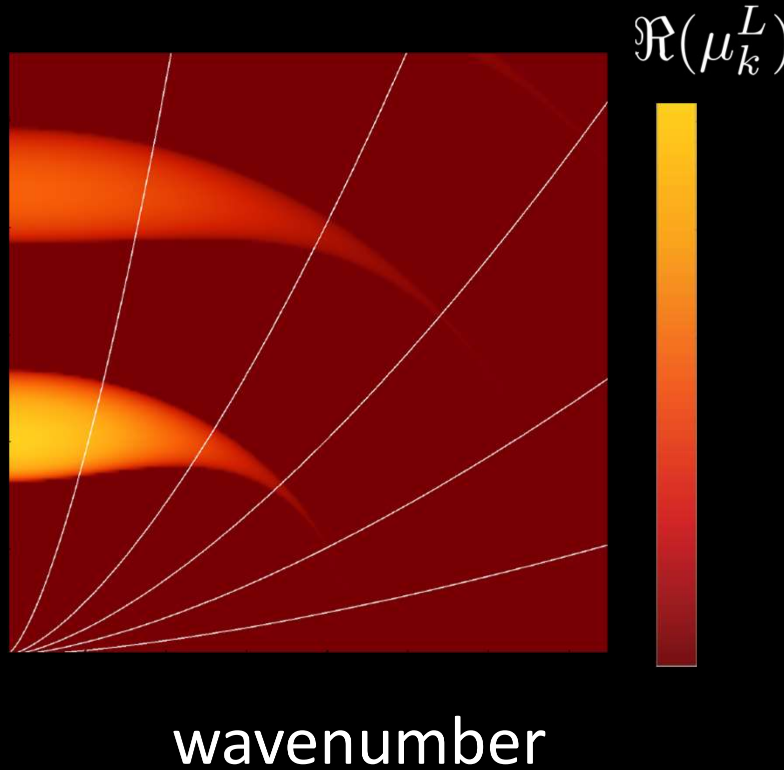
KL and M. Amin, JCAP 1606 032 (2016)

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - (D_\mu\phi)^\dagger D^\mu\phi - V(|\phi|) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\Big]$$

$A_k^T \propto \exp(\pm\mu_k^T t)$



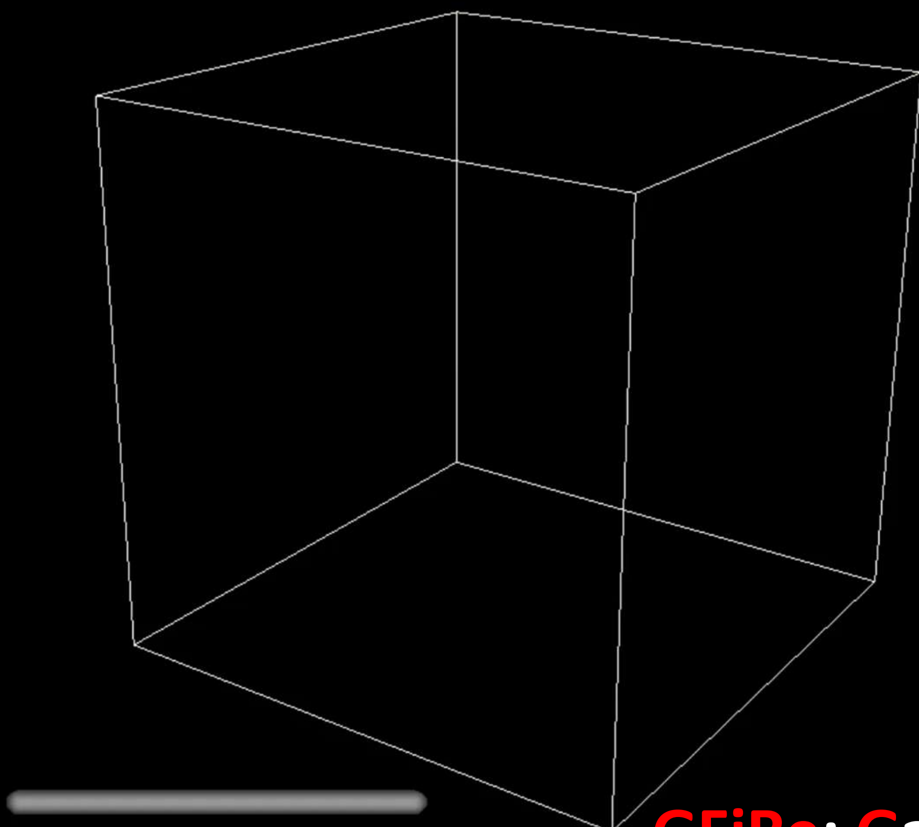
$A_k^L \propto \exp(\pm\mu_k^L t)$

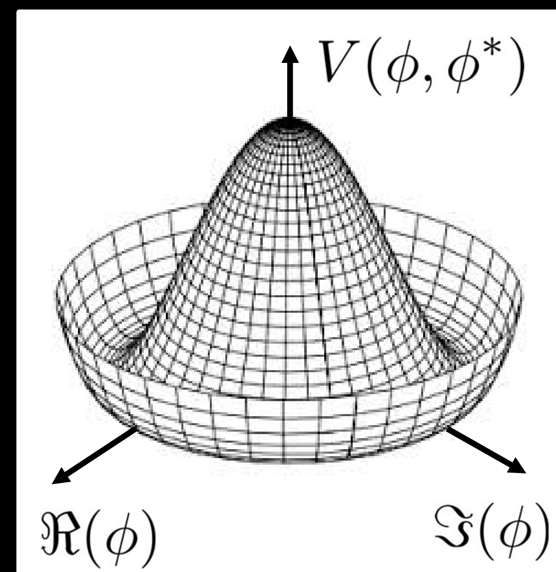


Gauge fields, inflation & reheating

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - (D_\mu\phi)^\dagger D^\mu\phi - V(|\phi|) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\Big]$$

Non-Abelian code in collaboration with A. Caravano, KL, Eiichiro Komatsu, J. Weller (2021a, 2021b, in progress)





Abelian code: KL, M. Amin (2020)

GFiRe: Gauge Field integrator for Reheating

NEW LATTICE CODE!!! JCAP 058 04 (2020)

Gauge fields, inflation & reheating

Adshead and Wyman (2012)

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\Big]$$

Gauge fields, inflation & reheating

Adshead and Wyman (2012)

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\Big]$$

Non-trivial vevs:

Gauge fields, inflation & reheating

Adshead and Wyman (2012)

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\Big]$$

Non-trivial vevs:

$$\overline{\phi}(t)$$

Gauge fields, inflation & reheating

Adshead and Wyman (2012)

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\Big]$$

Non-trivial vevs:

$$\overline{\phi}(t)$$
$$\overline{A^b_i}(t) = a(t)Q(t)\delta^b_i$$

Gauge fields, inflation & reheating

Adshead and Wyman (2012)

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\Big]$$

Non-trivial vevs:

$$\overline{\phi}(t)$$

$$\overline{A^b_i}(t) = a(t)Q(t)\delta^b_i$$

Drive inflation

Gauge fields, inflation & reheating

Adshead and Wyman (2012)

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\Big]$$

Non-trivial vevs:

$$\overline{\phi}(t)$$

$$\overline{A^b_i}(t) = a(t)Q(t)\delta^b_i$$

Drive inflation
(+extensions)

Adshead, Sfakianakis (2017)
Adshead, Martinec, Sfakianakis, Wyman (2016)

Gauge fields, inflation & reheating

Adshead and Wyman (2012)

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\Big]$$

Non-trivial vevs:

$$\overline{\phi}(t)$$
$$\overline{A^b_i}(t) = a(t)Q(t)\delta^b_i$$

Drive inflation (+extensions) — Spectator sector

Adshead, Sfakianakis (2017)
Adshead, Martinec, Sfakianakis, Wyman (2016)

Gauge fields, inflation & reheating

Adshead and Wyman (2012)

$$S = \int d^4x \sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\Big]$$

Non-trivial vevs:

$$\overline{\phi}(t)$$
$$\overline{A^b_i}(t) = a(t)Q(t)\delta^b_i$$

Drive inflation
(+extensions)

Adshead, Sfakianakis (2017)
Adshead, Martinec, Sfakianakis, Wyman (2016)

Spectator sector

Dimastrogiovanni et al (2016)
Maleknejad (2016, 2018)
Fujita, Sfakianakis, Shiraishi (2018)
A. Agrawal, Komatsu et al (2017, 2018)

Gauge fields, inflation & reheating

Adshead and Wyman (2012)

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\right]$$

Non-trivial vevs:

$$\overline{\phi}(t)$$

$$\overline{A^b_i}(t) = a(t)Q(t)\delta^b_i$$

Linear coupling between Gauge Fields and GWs!

Gauge fields, inflation & reheating

Adshead and Wyman (2012)

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\Big]$$

Non-trivial vevs:

$$\overline{\phi}(t)$$

$$\overline{A^b_i}(t) = a(t)Q(t)\delta^b_i$$

Linear coupling between Gauge Fields and GWs!

Gauge fields, inflation & reheating

Adshead and Wyman (2012)

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\Big]$$

Non-trivial vevs:

$$\overline{\phi}(t)$$

$$\overline{A^b_i}(t) = a(t)Q(t)\delta^b_i$$

Linear coupling between Gauge Fields and GWs!

$$\delta A^b_i$$

Gauge fields, inflation & reheating

Adshead and Wyman (2012)

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\Big]$$

Non-trivial vevs:

$$\overline{\phi}(t)$$

$$\overline{A^b_i}(t) = a(t)Q(t)\delta^b_i$$

Linear coupling between Gauge Fields and GWs!

δA^b_i and

Gauge fields, inflation & reheating

Adshead and Wyman (2012)

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\Big]$$

Non-trivial vevs:

$$\overline{\phi}(t)$$

$$\overline{A^b_i}(t) = a(t)Q(t)\delta^b_i$$

Linear coupling between Gauge Fields and GWs!

δA^b_i and h_{ij}

Gauge fields, inflation & reheating

Adshead and Wyman (2012)

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\Big]$$

Non-trivial vevs:

$$\overline{\phi}(t)$$

$$\overline{A^b_i}(t) = a(t)Q(t)\delta^b_i$$

Linear coupling between Gauge Fields and GWs!

δA^b_i and h_{ij}

Enhanced and chiral GWs!

Gauge fields, inflation & reheating

Adshead and Wyman (2012)

$$S = \int d^4x \sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\Big]$$

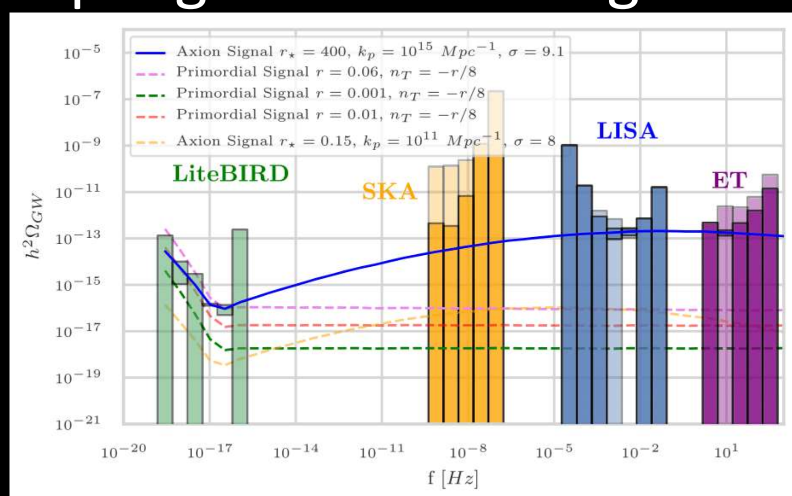
Non-trivial vevs:

$$\overline{\phi}(t)$$

$$\overline{A^b_i}(t) = a(t)Q(t)\delta^b_i$$

Linear coupling between Gauge Fields and GWs!

GW power sp.



log(frequency)

Figure from
Campeti, Komatsu et al.
arXiv:2007.04241

Gauge fields, inflation & reheating

Adshead and Wyman (2012)

$$S = \int d^4x \sqrt{-g} \Big[\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu} \Big]$$

Non-trivial vevs:

$$\overline{\phi}(t)$$

$$\overline{A^b_i}(t) = a(t)Q(t)\delta^b_i$$

Gauge fields, inflation & reheating

Adshead and Wyman (2012)

$$S = \int d^4x \sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\Big]$$

Non-trivial vevs:

$$\overline{\phi}(t)$$

$$\overline{A^b_i}(t) = a(t)Q(t)\delta^b_i$$

Schwinger effect?

Gauge fields, inflation & reheating

Adshead and Wyman (2012)

$$S=\int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R}-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-V(\phi)-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}-\frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\Big]$$

Non-trivial vevs:

$$\overline{\phi}(t)$$

$$\overline{A^b_i}(t)=a(t)Q(t)\delta^b_i$$

Schwinger effect?
Backreaction?

Gauge fields, inflation & reheating

Adshead and Wyman (2012)

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\Big]$$

Non-trivial vevs:

$$\overline{\phi}(t)$$

$$\overline{A^b_i}(t) = a(t)Q(t)\delta^b_i$$

→ Schwinger effect? Backreaction?

KL, Maleknejad and Komatsu (2018)

$$S_1 = \int d^4x\sqrt{-g}\Big[-(D_\mu\varphi)^\dagger D^\mu\varphi - V(|\varphi|)\Big]$$

Gauge fields, inflation & reheating

Adshead and Wyman (2012)

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\Big]$$

Non-trivial vevs:

$$\overline{\phi}(t)$$

$$\overline{A^b_i}(t) = a(t)Q(t)\delta^b_i$$

Schwinger effect?
Backreaction?

KL, Maleknejad and Komatsu (2018)

$$S_1 = \int d^4x\sqrt{-g}\Big[-(D_\mu\varphi)^\dagger D^\mu\varphi - V(|\varphi|)\Big] \qquad \varphi = \begin{pmatrix}\varphi^1\\ \varphi^2\end{pmatrix}$$

Gauge fields, inflation & reheating

Adshead and Wyman (2012)

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\Big]$$

Non-trivial vevs:

$$\overline{\phi}(t)$$

$$\overline{A^b_i}(t) = a(t)Q(t)\delta^b_i$$

Schwinger effect?
Backreaction?

KL, Maleknejad and Komatsu (2018)

$$S_1 = \int d^4x\sqrt{-g}\Big[-(D_\mu\varphi)^\dagger D^\mu\varphi - V(|\varphi|)\Big] \qquad \varphi = \begin{pmatrix}\varphi^1\\ \varphi^2\end{pmatrix}$$

Mirzagoli, Maleknejad and KL (2019)

$$S_2 = \int d^4x\sqrt{-g}\Big[i\overline{\psi}\gamma^\mu D_\mu\psi - m\overline{\psi}\psi - \lambda\frac{\partial_\mu\phi}{f}J^{5\mu}\Big]$$

Gauge fields, inflation & reheating

Adshead and Wyman (2012)

$$S = \int d^4x \sqrt{-g} \Big[\frac{1}{16\pi G} \mathcal{R} - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - \frac{\phi}{4f} F^a_{\mu\nu} \tilde{F}^{a\mu\nu} \Big]$$

Non-trivial vevs:

$$\overline{\phi}(t)$$

$$\overline{A^b_i}(t) = a(t) Q(t) \delta^b_i$$

Schwinger effect?
Backreaction?

KL, Maleknejad and Komatsu (2018)

$$S_1 = \int d^4x \sqrt{-g} \Big[- (D_\mu \varphi)^\dagger D^\mu \varphi - V(|\varphi|) \Big] \qquad \varphi = \begin{pmatrix} \varphi^1 \\ \varphi^2 \end{pmatrix}$$

Mirzagoli, Maleknejad and KL (2019)

$$S_2 = \int d^4x \sqrt{-g} \Big[i\overline{\psi} \gamma^\mu D_\mu \psi - m\overline{\psi}\psi - \lambda \frac{\partial_\mu \phi}{f} J^{5\mu} \Big] \qquad \psi = \begin{pmatrix} \psi^1 \\ \psi^2 \end{pmatrix}$$

Gauge fields, inflation & reheating

Adshead and Wyman (2012)

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\Big]$$

Non-trivial vevs:

$$\overline{\phi}(t)$$

$$\overline{A^b_i}(t) = a(t)Q(t)\delta^b_i$$

Schwinger effect?

Backreaction?

KL, Maleknejad and Komatsu (2018)

$$S_1 = \int d^4x\sqrt{-g}\Big[-(D_\mu\varphi)^\dagger D^\mu\varphi - V(|\varphi|)\Big] \qquad \varphi = \begin{pmatrix}\varphi^1\\ \varphi^2\end{pmatrix}$$

Mirzagoli, Maleknejad and KL (2019)

$$S_2 = \int d^4x\sqrt{-g}\Big[i\overline{\psi}\gamma^\mu D_\mu\psi - m\overline{\psi}\psi - \lambda\frac{\partial_\mu\phi}{f}J^{5\mu}\Big] \qquad \psi = \begin{pmatrix}\psi^1\\ \psi^2\end{pmatrix}$$

See also Domcke et al (2018), Adshead, Sfakianakis (2015)

Gauge fields, inflation & reheating

Adshead and Wyman (2012)

$$S = \int d^4x \sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\Big]$$

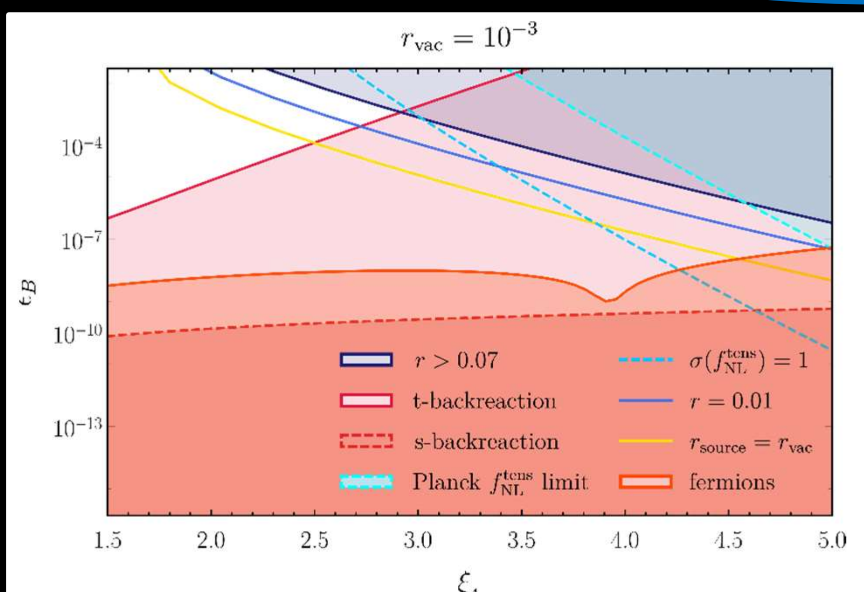
Non-trivial vevs:

$$\overline{\phi}(t)$$

$$\overline{A^b_i}(t) = a(t)Q(t)\delta^b_i$$

Schwinger effect?
Backreaction?

Fraction of energy in Gauge Fields



Gauge field coupling strength

KL, Maleknejad and Komatsu (2018)

$$\varphi = \begin{pmatrix} \varphi^1 \\ \varphi^2 \end{pmatrix}$$

Mirzagoli, Maleknejad and KL (2019)

$$\psi = \begin{pmatrix} \psi^1 \\ \psi^2 \end{pmatrix}$$

Gauge fields, inflation & reheating

Adshead and Wyman (2012)

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\Big]$$

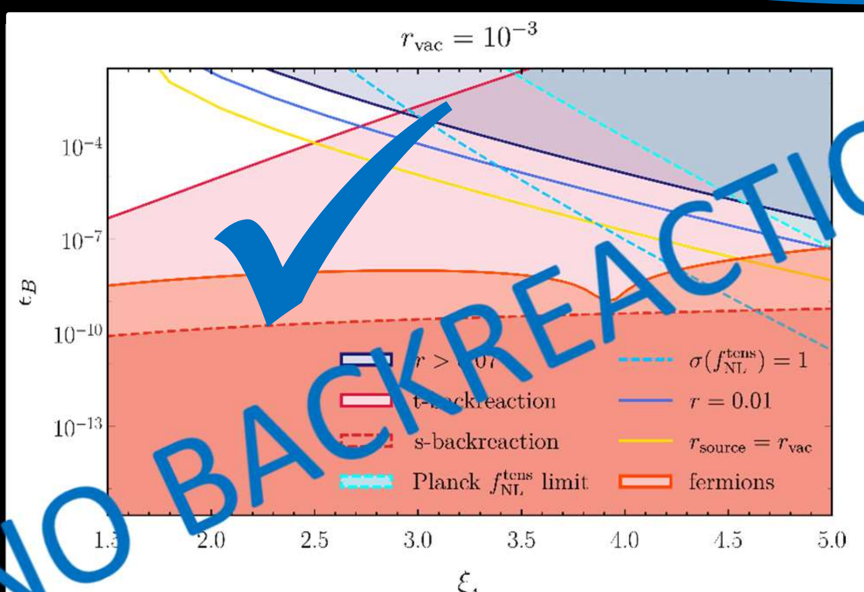
Non-trivial vevs:

$$\overline{\phi}(t)$$

$$\overline{A^b_i}(t) = a(t)Q(t)\delta^b_i$$

Schwinger effect?
Backreaction?

Fraction of energy in Gauge Fields



Gauge field coupling strength

KL, Maleknejad and Komatsu (2018)

$$\varphi = \begin{pmatrix} \varphi^1 \\ \varphi^2 \end{pmatrix}$$

Mirzagoli, Maleknejad and KL (2019)

$$\psi = \begin{pmatrix} \psi^1 \\ \psi^2 \end{pmatrix}$$

See also
Mirzagoli, Komatsu, KL, Watanabe (2020)
(on gravitational CS)

Fuzzy
Vector Dark Matter

UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN

Kaloian Lozanov

UIUC, Cosmology and HEPheno

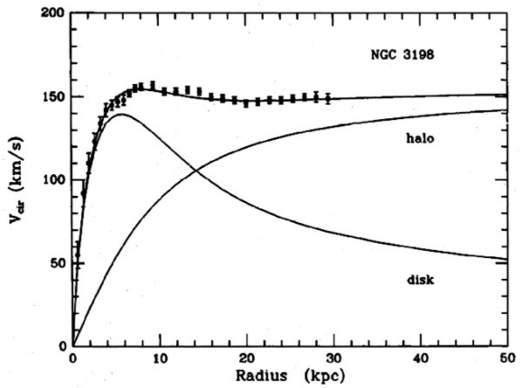
ILLINOIS Physics
GRAINGER COLLEGE OF ENGINEERING

Based on: PRD 103, 103501 (2021) (Adshead and KL)

Dark Matter

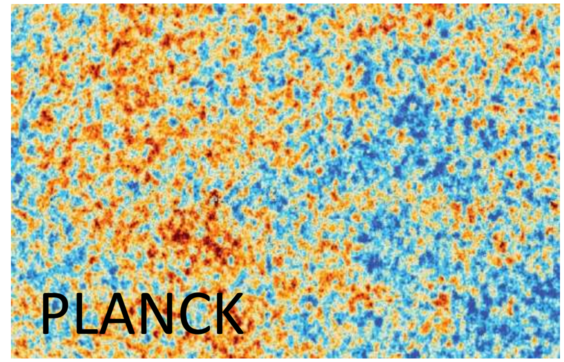
Dark Matter

Rotation curves



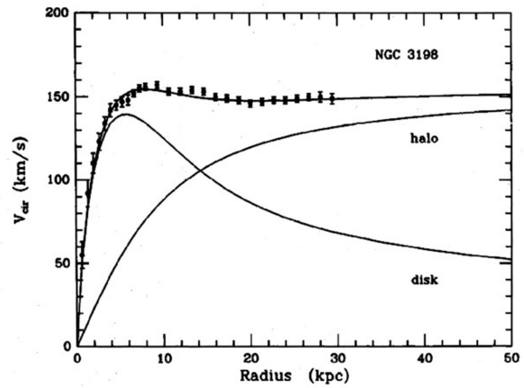
LSS

CMB



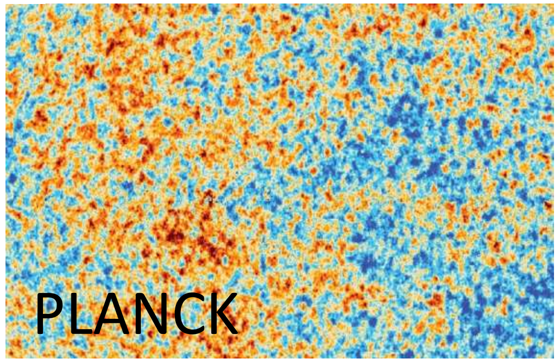
Dark Matter

Rotation curves



LSS

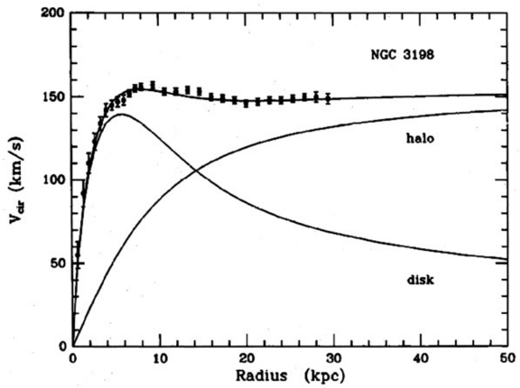
CMB



DM mass ??

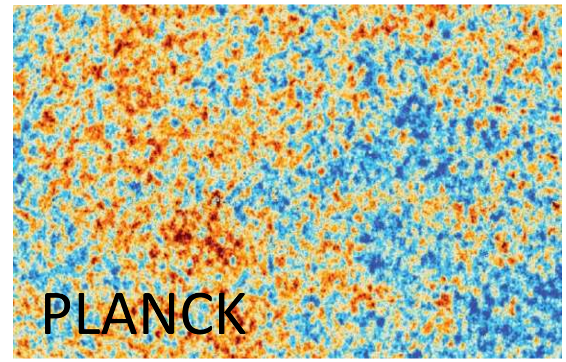
Dark Matter

Rotation curves

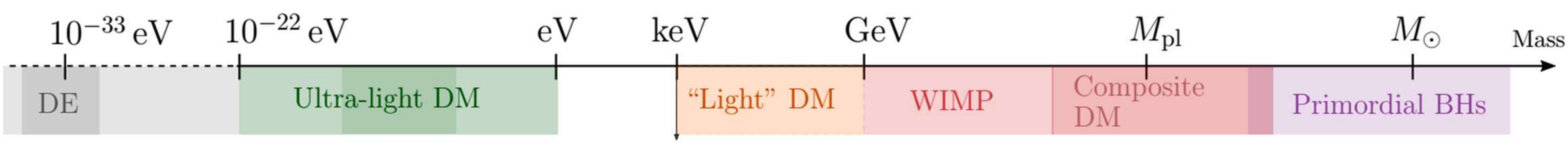


LSS

CMB



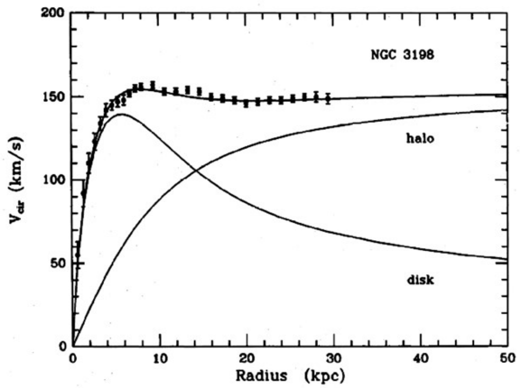
DM mass ??



Ferreira (review) (2020)

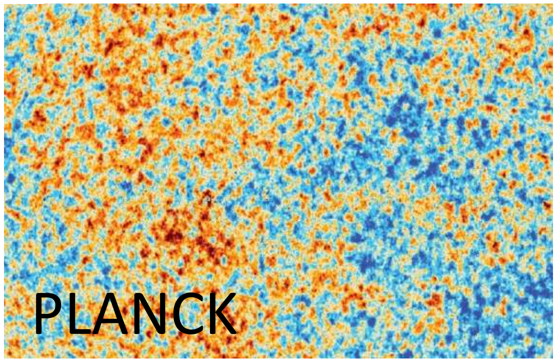
Dark Matter

Rotation curves



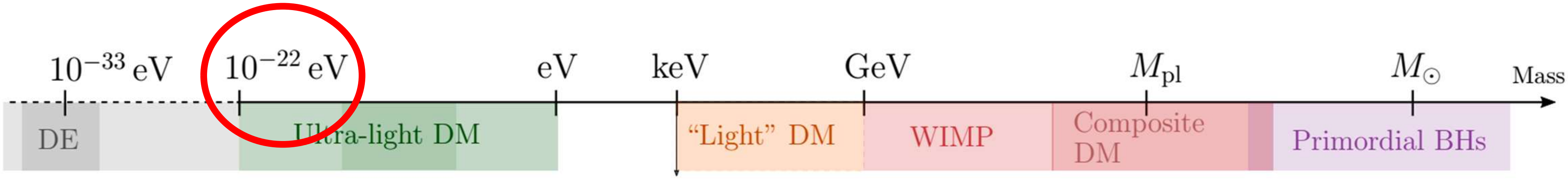
LSS

CMB



DM mass ??

Fuzzy DM



Ferreira (review) (2020)

Fuzzy Dark Matter (FDM)

Fuzzy Dark Matter (FDM)

FDM is

Fuzzy Dark Matter (FDM)

FDM is

- Cold and Bosonic: forms Bose-Einstein Condensate

Fuzzy Dark Matter (FDM)

FDM is

- Cold and Bosonic: forms Bose-Einstein Condensate

- Ultralight: macroscopic wave properties

Fuzzy Dark Matter (FDM)

FDM is

- Cold and Bosonic: forms Bose-Einstein Condensate
- Ultralight: macroscopic wave properties

$$\lambda_{\mathrm{dB}} \sim \left(\frac{M_{\mathrm{halo}}}{10^8 M_\odot}\right)^{-1} \left(\frac{m}{10^{-22}\mathrm{eV}}\right)^{-2} \mathrm{kpc}$$

Motivation for FDM

Hu, Barkana, Gruzinov (2000)
Hui, Ostriker, Tremaine, Witten (review) (2016)

Motivation for FDM

ΛCDM small-scale challenges

Hu, Barkana, Gruzinov (2000)
Hui, Ostriker, Tremaine, Witten (review) (2016)

Motivation for FDM

ΛCDM small-scale challenges

- Missing-satellite/too-big-to-fail problems

Hu, Barkana, Gruzinov (2000)
Hui, Ostriker, Tremaine, Witten (review) (2016)

Motivation for FDM

ΛCDM small-scale challenges

- Missing-satellite/too-big-to-fail problems

- Cusp-core problems

Hu, Barkana, Gruzinov (2000)
Hui, Ostriker, Tremaine, Witten (review) (2016)

Motivation for FDM

ΛCDM small-scale challenges

- Missing-satellite/too-big-to-fail problems

- Cusp-core problems

FDM solution

Hu, Barkana, Gruzinov (2000)
Hui, Ostriker, Tremaine, Witten (review) (2016)

Motivation for FDM

ΛCDM small-scale challenges

- Missing-satellite/too-big-to-fail problems

- Cusp-core problems

FDM solution

- No grav. collapse below λ_{dB}

Hu, Barkana, Gruzinov (2000)
Hui, Ostriker, Tremaine, Witten (review) (2016)

Scalar FDM

Scalar FDM

$$S = \int d^4x \sqrt{-g} \left[-\frac{m_{\rm pl}^2 R}{2} + \frac{\partial_\mu \phi \partial^\mu \phi}{2} - \frac{m^2 \phi^2}{2} \right]$$

Scalar FDM

$$S = \int d^4x \sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} + \frac{\partial_\mu \phi \partial^\mu \phi}{2} - \frac{m^2\phi^2}{2}\right]$$

↓

Scalar FDM

$$S = \int d^4x \sqrt{-g} \left[-\frac{m_{\rm pl}^2 R}{2} + \frac{\partial_\mu \phi \partial^\mu \phi}{2} - \frac{m^2 \phi^2}{2} \right]$$

↓

$$\Box \phi + m^2 \phi = 0$$

Scalar FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} + \frac{\partial_\mu\phi\partial^\mu\phi}{2} - \frac{m^2\phi^2}{2}\right]$$

↓

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) + m^2\phi = 0$$

Scalar FDM

$$S = \int d^4x \sqrt{-g} \left[-\frac{m_{\rm pl}^2 R}{2} + \frac{\partial_\mu \phi \partial^\mu \phi}{2} - \frac{m^2 \phi^2}{2} \right]$$

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \phi) + m^2 \phi = 0$$

Scalar FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} + \frac{\partial_\mu\phi\partial^\mu\phi}{2} - \frac{m^2\phi^2}{2}\right]$$

↓

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) + m^2\phi = 0$$

↓ $g_{\mu\nu} = ?$

Scalar FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} + \frac{\partial_\mu\phi\partial^\mu\phi}{2} - \frac{m^2\phi^2}{2}\right]$$

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) + m^2\phi = 0$$

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Scalar FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} + \frac{\partial_\mu\phi\partial^\mu\phi}{2} - \frac{m^2\phi^2}{2}\right]$$

↓

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) + m^2\phi = 0$$

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\phi = \sqrt{\frac{2}{m}}\Re[\psi e^{-imt}]$$

↓

Scalar FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} + \frac{\partial_\mu\phi\partial^\mu\phi}{2} - \frac{m^2\phi^2}{2}\right]$$

$$\downarrow$$

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) + m^2\phi = 0$$

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\phi = \sqrt{\frac{2}{m}}\Re[\psi e^{-imt}]$$

$$|\partial_t\psi| \ll m|\psi|$$

$$\downarrow$$

Scalar FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} + \frac{\partial_\mu\phi\partial^\mu\phi}{2} - \frac{m^2\phi^2}{2}\right]$$

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) + m^2\phi = 0$$

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\phi = \sqrt{\frac{2}{m}}\Re[\psi e^{-imt}]$$

$$|\partial_t\psi| \ll m|\psi| \iff \frac{\Delta}{m^2} \ll 1$$

Scalar FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} + \frac{\partial_\mu\phi\partial^\mu\phi}{2} - \frac{m^2\phi^2}{2}\right]$$

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) + m^2\phi = 0$$

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\phi = \sqrt{\frac{2}{m}}\Re[\psi e^{-imt}]$$

$$|\partial_t\psi| \ll m|\psi| \iff \frac{\Delta}{m^2} \ll 1$$

$$\boxed{\left[i\partial_t + \frac{\Delta}{2m}\right]\psi = 0}$$

Scalar FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} + \frac{\partial_\mu\phi\partial^\mu\phi}{2} - \frac{m^2\phi^2}{2}\right]$$

$$\downarrow$$

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) + m^2\phi = 0$$

$$\downarrow \quad g_{\mu\nu} = ?$$

Scalar FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} + \frac{\partial_\mu\phi\partial^\mu\phi}{2} - \frac{m^2\phi^2}{2}\right]$$

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) + m^2\phi = 0$$

$$g_{\mu\nu} = \begin{pmatrix} 1+2\Phi & 0 & 0 & 0 \\ 0 & 2\Psi-1 & 0 & 0 \\ 0 & 0 & 2\Psi-1 & 0 \\ 0 & 0 & 0 & 2\Psi-1 \end{pmatrix}$$

Scalar FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} + \frac{\partial_\mu\phi\partial^\mu\phi}{2} - \frac{m^2\phi^2}{2}\right]$$

$$\downarrow$$

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) + m^2\phi = 0$$

$$g_{\mu\nu} = \begin{pmatrix} 1+2\Phi & 0 & 0 & 0 \\ 0 & 2\Psi-1 & 0 & 0 \\ 0 & 0 & 2\Psi-1 & 0 \\ 0 & 0 & 0 & 2\Psi-1 \end{pmatrix}$$

$$\phi = \sqrt{\frac{2}{m}}\Re[\psi e^{-imt}]$$

$$\downarrow$$

Scalar FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} + \frac{\partial_\mu\phi\partial^\mu\phi}{2} - \frac{m^2\phi^2}{2}\right]$$

↓

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) + m^2\phi = 0$$

$$g_{\mu\nu} = \begin{pmatrix} 1+2\Phi & 0 & 0 & 0 \\ 0 & 2\Psi-1 & 0 & 0 \\ 0 & 0 & 2\Psi-1 & 0 \\ 0 & 0 & 0 & 2\Psi-1 \end{pmatrix}$$

$$\phi = \sqrt{\frac{2}{m}}\Re[\psi e^{-imt}]$$

$$|\partial_t\psi| \ll m|\psi|$$

↓

Scalar FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} + \frac{\partial_\mu\phi\partial^\mu\phi}{2} - \frac{m^2\phi^2}{2}\right]$$

↓

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) + m^2\phi = 0$$

$$g_{\mu\nu} = \begin{pmatrix} 1+2\Phi & 0 & 0 & 0 \\ 0 & 2\Psi-1 & 0 & 0 \\ 0 & 0 & 2\Psi-1 & 0 \\ 0 & 0 & 0 & 2\Psi-1 \end{pmatrix}$$

$$\phi = \sqrt{\frac{2}{m}}\Re[\psi e^{-imt}]$$

$$|\partial_t\psi| \ll m|\psi| \qquad |\Phi|, |\Psi| \ll 1$$

↓

Scalar FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} + \frac{\partial_\mu\phi\partial^\mu\phi}{2} - \frac{m^2\phi^2}{2}\right]$$

↓

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) + m^2\phi = 0$$

$$g_{\mu\nu} = \begin{pmatrix} 1+2\Phi & 0 & 0 & 0 \\ 0 & 2\Psi-1 & 0 & 0 \\ 0 & 0 & 2\Psi-1 & 0 \\ 0 & 0 & 0 & 2\Psi-1 \end{pmatrix}$$

$$\phi = \sqrt{\frac{2}{m}}\Re[\psi e^{-imt}]$$

$$|\partial_t\psi| \ll m|\psi| \qquad |\Phi|, |\Psi| \ll 1$$

↓

$$\boxed{\left[i\partial_t + \frac{\Delta}{2m} - \Phi m\right]\psi = 0}$$

Scalar FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} + \frac{\partial_\mu\phi\partial^\mu\phi}{2} - \frac{m^2\phi^2}{2}\right]$$

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) + m^2\phi = 0$$

$$g_{\mu\nu} = \begin{pmatrix} 1+2\Phi & 0 & 0 & 0 \\ 0 & 2\Psi - 1 & 0 & 0 \\ 0 & 0 & 2\Psi - 1 & 0 \\ 0 & 0 & 0 & 2\Psi - 1 \end{pmatrix}$$

$$\phi = \sqrt{\frac{2}{m}}\Re[\psi e^{-imt}]$$

$$|\partial_t\psi| \ll m|\psi| \qquad |\Phi|, |\Psi| \ll 1$$

$$\left[i\partial_t + \frac{\Delta}{2m} - \Phi m\right]\psi = 0 \qquad \Delta\Phi = \frac{m|\psi|^2}{2m_{\rm pl}^2} \qquad \Phi = \Psi$$

Scalar FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} + \frac{\partial_\mu\phi\partial^\mu\phi}{2} - \frac{m^2\phi^2}{2}\right]$$

$$\downarrow$$

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) + m^2\phi = 0$$

$$\downarrow \quad g_{\mu\nu} = ?$$

Scalar FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} + \frac{\partial_\mu\phi\partial^\mu\phi}{2} - \frac{m^2\phi^2}{2}\right]$$

↓

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) + m^2\phi = 0$$

$$g_{\mu\nu} = a^2(\tau)\begin{pmatrix} 1+2\Phi & 0 & 0 & 0 \\ 0 & 2\Psi-1 & 0 & 0 \\ 0 & 0 & 2\Psi-1 & 0 \\ 0 & 0 & 0 & 2\Psi-1 \end{pmatrix}$$

↓

Scalar FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} + \frac{\partial_\mu\phi\partial^\mu\phi}{2} - \frac{m^2\phi^2}{2}\right]$$

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) + m^2\phi = 0$$

$$g_{\mu\nu} = a^2(\tau)\begin{pmatrix} 1+2\Phi & 0 & 0 & 0 \\ 0 & 2\Psi-1 & 0 & 0 \\ 0 & 0 & 2\Psi-1 & 0 \\ 0 & 0 & 0 & 2\Psi-1 \end{pmatrix}$$

$$\phi = \sqrt{\frac{2}{m}}\Re[\psi e^{-imt}] \qquad t = \int_\tau a(\tau')d\tau'$$

Scalar FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} + \frac{\partial_\mu\phi\partial^\mu\phi}{2} - \frac{m^2\phi^2}{2}\right]$$

↓

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) + m^2\phi = 0$$

$$g_{\mu\nu} = a^2(\tau)\begin{pmatrix} 1+2\Phi & 0 & 0 & 0 \\ 0 & 2\Psi-1 & 0 & 0 \\ 0 & 0 & 2\Psi-1 & 0 \\ 0 & 0 & 0 & 2\Psi-1 \end{pmatrix}$$

$$\phi = \sqrt{\frac{2}{m}}\Re[\psi e^{-imt}] \qquad t = \int_\tau a(\tau')d\tau'$$

$$|\partial_t\psi| \ll m|\psi|$$

↓

Scalar FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} + \frac{\partial_\mu\phi\partial^\mu\phi}{2} - \frac{m^2\phi^2}{2}\right]$$

↓

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) + m^2\phi = 0$$

$$g_{\mu\nu} = a^2(\tau)\begin{pmatrix}1+2\Phi & 0 & 0 & 0\\ 0 & 2\Psi-1 & 0 & 0\\ 0 & 0 & 2\Psi-1 & 0\\ 0 & 0 & 0 & 2\Psi-1\end{pmatrix}$$

$$\phi = \sqrt{\frac{2}{m}}\Re[\psi e^{-imt}] \qquad t = \int_\tau a(\tau')d\tau'$$

$$|\partial_t\psi| \ll m|\psi| \qquad |\Phi|, |\Psi| \ll 1$$

↓

Scalar FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} + \frac{\partial_\mu\phi\partial^\mu\phi}{2} - \frac{m^2\phi^2}{2}\right]$$

↓

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) + m^2\phi = 0$$

↓

$$g_{\mu\nu} = a^2(\tau)\begin{pmatrix} 1+2\Phi & 0 & 0 & 0 \\ 0 & 2\Psi-1 & 0 & 0 \\ 0 & 0 & 2\Psi-1 & 0 \\ 0 & 0 & 0 & 2\Psi-1 \end{pmatrix}$$

$$\phi = \sqrt{\frac{2}{m}}\Re[\psi e^{-imt}] \qquad t = \int_\tau a(\tau')d\tau'$$

$$|\partial_t\psi| \ll m|\psi| \qquad |\Phi|, |\Psi| \ll 1 \qquad \mathcal{H} \ll am$$

Scalar FDM

$$S = \int d^4x \sqrt{-g} \left[-\frac{m_{\rm pl}^2 R}{2} + \frac{\partial_\mu \phi \partial^\mu \phi}{2} - \frac{m^2 \phi^2}{2} \right]$$

↓

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \phi) + m^2 \phi = 0$$

$$g_{\mu\nu} = a^2(\tau) \begin{pmatrix} 1+2\Phi & 0 & 0 & 0 \\ 0 & 2\Psi - 1 & 0 & 0 \\ 0 & 0 & 2\Psi - 1 & 0 \\ 0 & 0 & 0 & 2\Psi - 1 \end{pmatrix}$$

$$\phi = \sqrt{\frac{2}{m}} \Re[\psi e^{-imt}] \qquad t = \int_\tau a(\tau') d\tau'$$

$$|\partial_t \psi| \ll m|\psi| \qquad |\Phi|, |\Psi| \ll 1 \qquad \mathcal{H} \ll am$$

↓

$$\boxed{\left[i\partial_t + \frac{3}{2} H + \frac{\Delta}{2a^2 m} - \Phi m \right] \psi = 0}$$

Scalar FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} + \frac{\partial_\mu\phi\partial^\mu\phi}{2} - \frac{m^2\phi^2}{2}\right]$$

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) + m^2\phi = 0$$

$$g_{\mu\nu} = a^2(\tau)\begin{pmatrix} 1+2\Phi & 0 & 0 & 0 \\ 0 & 2\Psi-1 & 0 & 0 \\ 0 & 0 & 2\Psi-1 & 0 \\ 0 & 0 & 0 & 2\Psi-1 \end{pmatrix}$$

$$\phi = \sqrt{\frac{2}{m}}\Re[\psi e^{-imt}] \qquad t = \int_\tau a(\tau')d\tau'$$

$$|\partial_t\psi| \ll m|\psi| \qquad |\Phi|, |\Psi| \ll 1 \qquad \mathcal{H} \ll am$$

$$\boxed{\left[i\partial_t + \frac{3}{2}H + \frac{\Delta}{2a^2m} - \Phi m\right]\psi = 0} \qquad \boxed{\Delta\Phi = \frac{m|\psi|^2}{2m_{\rm pl}^2}} \qquad \boxed{\Phi = \Psi}$$

Vector FDM

Vector FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A^\mu A_\mu\right]$$

Vector FDM

$$S = \int d^4x \sqrt{-g} \left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A^\mu A_\mu \right]$$

Vector FDM

$$S = \int d^4x \sqrt{-g} \left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A^\mu A_\mu \right]$$

↓

$$\nabla_\mu F^{\mu\nu} = -m^2 A^\nu$$

Vector FDM

$$S = \int d^4x \sqrt{-g} \left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A^\mu A_\mu \right]$$

$$\downarrow$$

$$\nabla_\mu F^{\mu\nu} = -m^2 A^\nu \equiv J^\nu$$

Vector FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A^\mu A_\mu\right]$$

$$\downarrow$$

$$\nabla_\mu F^{\mu\nu} = -m^2 A^\nu \equiv J^\nu \quad \xrightarrow{\nabla_\nu}$$

Vector FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A^\mu A_\mu\right]$$

↓

$$\nabla_\mu F^{\mu\nu} = -m^2 A^\nu \equiv J^\nu \quad \xrightarrow{\nabla_\nu} \quad \nabla_\nu A^\nu = 0$$

Vector FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A^\mu A_\mu\right]$$

↓

$$\nabla_\mu F^{\mu\nu} = -m^2 A^\nu$$

$$\nabla_\nu A^\nu = 0$$

Vector FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A^\mu A_\mu\right]$$

↓

$$\Box A^\nu + m^2 A^\nu + R^\nu_\mu A^\mu = 0$$

$$\nabla_\nu A^\nu = 0$$

Vector FDM

$$S = \int d^4x \sqrt{-g} \left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A^\mu A_\mu \right]$$

↓

$$\frac{\partial_\mu (\sqrt{-g} g^{\mu\alpha} \partial_\alpha A^\nu)}{\sqrt{-g}} + m^2 A^\nu + A^\mu g^{\nu\alpha} \partial_\alpha \left[\frac{\partial_\mu \sqrt{-g}}{\sqrt{-g}} \right] - \partial_\alpha A^\mu \partial_\mu g^{\nu\alpha} = 0 \qquad \nabla_\nu A^\nu = 0$$

Vector FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A^\mu A_\mu\right]$$

$$\frac{\partial_\mu(\sqrt{-g}g^{\mu\alpha}\partial_\alpha A^\nu)}{\sqrt{-g}} + m^2 A^\nu + A^\mu g^{\nu\alpha}\partial_\alpha\left[\frac{\partial_\mu\sqrt{-g}}{\sqrt{-g}}\right] - \partial_\alpha A^\mu \partial_\mu g^{\nu\alpha} = 0 \qquad \nabla_\nu A^\nu = 0$$

Vector FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A^\mu A_\mu\right]$$

↓

$$\frac{\partial_\mu(\sqrt{-g}g^{\mu\alpha}\partial_\alpha A^\nu)}{\sqrt{-g}} + m^2 A^\nu + A^\mu g^{\nu\alpha}\partial_\alpha\left[\frac{\partial_\mu\sqrt{-g}}{\sqrt{-g}}\right] - \partial_\alpha A^\mu \partial_\mu g^{\nu\alpha} = 0 \qquad \nabla_\nu A^\nu = 0$$

↓ $g_{\mu\nu} = ?$

Vector FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A^\mu A_\mu\right]$$

$$\frac{\partial_\mu(\sqrt{-g}g^{\mu\alpha}\partial_\alpha A^\nu)}{\sqrt{-g}} + m^2 A^\nu + A^\mu g^{\nu\alpha}\partial_\alpha\left[\frac{\partial_\mu\sqrt{-g}}{\sqrt{-g}}\right] - \partial_\alpha A^\mu \partial_\mu g^{\nu\alpha} = 0 \qquad \nabla_\nu A^\nu = 0$$

$$g_{\mu\nu} = a^2(\tau)\begin{pmatrix} 1+2\Phi & 0 & 0 & 0 \\ 0 & 2\Psi-1 & 0 & 0 \\ 0 & 0 & 2\Psi-1 & 0 \\ 0 & 0 & 0 & 2\Psi-1 \end{pmatrix}$$

Vector FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A^\mu A_\mu\right]$$

↓

$$\frac{\partial_\mu(\sqrt{-g}g^{\mu\alpha}\partial_\alpha A^\nu)}{\sqrt{-g}} + m^2 A^\nu + A^\mu g^{\nu\alpha}\partial_\alpha\left[\frac{\partial_\mu\sqrt{-g}}{\sqrt{-g}}\right] - \partial_\alpha A^\mu \partial_\mu g^{\nu\alpha} = 0 \qquad \nabla_\nu A^\nu = 0$$

↓

$$g_{\mu\nu} = a^2(\tau)\begin{pmatrix} 1+2\Phi & 0 & 0 & 0 \\ 0 & 2\Psi - 1 & 0 & 0 \\ 0 & 0 & 2\Psi - 1 & 0 \\ 0 & 0 & 0 & 2\Psi - 1 \end{pmatrix}$$

$$A^\mu = \sqrt{\frac{2}{m}}\Re[\mathcal{A}^\mu e^{-imt}] \qquad t = \int_\tau a(\tau')d\tau'$$

↓

Vector FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A^\mu A_\mu\right]$$

$$\frac{\partial_\mu(\sqrt{-g}g^{\mu\alpha}\partial_\alpha A^\nu)}{\sqrt{-g}} + m^2 A^\nu + A^\mu g^{\nu\alpha}\partial_\alpha\left[\frac{\partial_\mu\sqrt{-g}}{\sqrt{-g}}\right] - \partial_\alpha A^\mu \partial_\mu g^{\nu\alpha} = 0 \qquad \nabla_\nu A^\nu = 0$$

$$g_{\mu\nu} = a^2(\tau)\begin{pmatrix} 1+2\Phi & 0 & 0 & 0 \\ 0 & 2\Psi-1 & 0 & 0 \\ 0 & 0 & 2\Psi-1 & 0 \\ 0 & 0 & 0 & 2\Psi-1 \end{pmatrix}$$

$$A^\mu = \sqrt{\frac{2}{m}}\Re[\mathcal{A}^\mu e^{-imt}] \qquad t = \int_\tau a(\tau')d\tau'$$

$$|\partial_t \mathcal{A}^\mu| \ll m|\mathcal{A}^\mu|$$

Vector FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A^\mu A_\mu\right]$$

↓

$$\frac{\partial_\mu(\sqrt{-g}g^{\mu\alpha}\partial_\alpha A^\nu)}{\sqrt{-g}} + m^2 A^\nu + A^\mu g^{\nu\alpha}\partial_\alpha\left[\frac{\partial_\mu\sqrt{-g}}{\sqrt{-g}}\right] - \partial_\alpha A^\mu \partial_\mu g^{\nu\alpha} = 0 \qquad \nabla_\nu A^\nu = 0$$

$$g_{\mu\nu} = a^2(\tau)\begin{pmatrix} 1+2\Phi & 0 & 0 & 0 \\ 0 & 2\Psi - 1 & 0 & 0 \\ 0 & 0 & 2\Psi - 1 & 0 \\ 0 & 0 & 0 & 2\Psi - 1 \end{pmatrix}$$

$$A^\mu = \sqrt{\frac{2}{m}}\Re[\mathcal{A}^\mu e^{-imt}] \quad t = \int_\tau a(\tau')d\tau'$$

$$|\partial_t \mathcal{A}^\mu| \ll m|\mathcal{A}^\mu| \quad |\Phi|, |\Psi| \ll 1$$

↓

Vector FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A^\mu A_\mu\right]$$

$$\frac{\partial_\mu(\sqrt{-g}g^{\mu\alpha}\partial_\alpha A^\nu)}{\sqrt{-g}} + m^2A^\nu + A^\mu g^{\nu\alpha}\partial_\alpha\left[\frac{\partial_\mu\sqrt{-g}}{\sqrt{-g}}\right] - \partial_\alpha A^\mu\partial_\mu g^{\nu\alpha} = 0 \qquad \nabla_\nu A^\nu = 0$$

$$g_{\mu\nu} = a^2(\tau)\begin{pmatrix} 1+2\Phi & 0 & 0 & 0 \\ 0 & 2\Psi-1 & 0 & 0 \\ 0 & 0 & 2\Psi-1 & 0 \\ 0 & 0 & 0 & 2\Psi-1 \end{pmatrix}$$

$$A^\mu = \sqrt{\frac{2}{m}}\Re[\mathcal{A}^\mu e^{-imt}] \qquad t = \int_\tau a(\tau')d\tau'$$

$$|\partial_t\mathcal{A}^\mu| \ll m|\mathcal{A}^\mu| \qquad |\Phi|, |\Psi| \ll 1 \qquad \mathcal{H} \ll am$$

Vector FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A^\mu A_\mu\right]$$

$$\frac{\partial_\mu(\sqrt{-g}g^{\mu\alpha}\partial_\alpha A^\nu)}{\sqrt{-g}} + m^2 A^\nu + A^\mu g^{\nu\alpha}\partial_\alpha\left[\frac{\partial_\mu\sqrt{-g}}{\sqrt{-g}}\right] - \partial_\alpha A^\mu \partial_\mu g^{\nu\alpha} = 0 \qquad \nabla_\nu A^\nu = 0$$

$$g_{\mu\nu} = a^2(\tau)\begin{pmatrix} 1+2\Phi & 0 & 0 & 0 \\ 0 & 2\Psi - 1 & 0 & 0 \\ 0 & 0 & 2\Psi - 1 & 0 \\ 0 & 0 & 0 & 2\Psi - 1 \end{pmatrix}$$

$$A^\mu = \sqrt{\frac{2}{m}}\Re[\mathcal{A}^\mu e^{-imt}] \qquad t = \int_\tau a(\tau')d\tau'$$

$$|\partial_t \mathcal{A}^\mu| \ll m|\mathcal{A}^\mu| \qquad |\Phi|, |\Psi| \ll 1 \qquad \mathcal{H} \ll am$$

$$\boxed{\left[i\partial_t + \frac{3}{2}H + \frac{\Delta}{2a^2 m} - \Phi m\right]\mathcal{A}^i = 0}$$

Vector FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A^\mu A_\mu\right]$$

$$\frac{\partial_\mu(\sqrt{-g}g^{\mu\alpha}\partial_\alpha A^\nu)}{\sqrt{-g}} + m^2 A^\nu + A^\mu g^{\nu\alpha}\partial_\alpha\left[\frac{\partial_\mu\sqrt{-g}}{\sqrt{-g}}\right] - \partial_\alpha A^\mu \partial_\mu g^{\nu\alpha} = 0 \qquad \nabla_\nu A^\nu = 0$$

$$g_{\mu\nu} = a^2(\tau)\begin{pmatrix} 1+2\Phi & 0 & 0 & 0 \\ 0 & 2\Psi - 1 & 0 & 0 \\ 0 & 0 & 2\Psi - 1 & 0 \\ 0 & 0 & 0 & 2\Psi - 1 \end{pmatrix}$$

$$A^\mu = \sqrt{\frac{2}{m}}\Re[\mathcal{A}^\mu e^{-imt}] \qquad t = \int_\tau a(\tau')d\tau'$$

$$|\partial_t \mathcal{A}^\mu| \ll m|\mathcal{A}^\mu| \qquad |\Phi|, |\Psi| \ll 1 \qquad \mathcal{H} \ll am$$

$$\left[i\partial_t + \frac{3}{2}H + \frac{\Delta}{2a^2m} - \Phi m\right]\mathcal{A}^i = 0$$

$$iam\mathcal{A}^0 = \partial_j \mathcal{A}^j$$

Vector FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A^\mu A_\mu\right]$$

$$\frac{\partial_\mu(\sqrt{-g}g^{\mu\alpha}\partial_\alpha A^\nu)}{\sqrt{-g}} + m^2 A^\nu + A^\mu g^{\nu\alpha}\partial_\alpha\left[\frac{\partial_\mu\sqrt{-g}}{\sqrt{-g}}\right] - \partial_\alpha A^\mu \partial_\mu g^{\nu\alpha} = 0 \qquad \nabla_\nu A^\nu = 0$$

$$g_{\mu\nu} = a^2(\tau)\begin{pmatrix} 1+2\Phi & 0 & 0 & 0 \\ 0 & 2\Psi-1 & 0 & 0 \\ 0 & 0 & 2\Psi-1 & 0 \\ 0 & 0 & 0 & 2\Psi-1 \end{pmatrix}$$

$$A^\mu = \sqrt{\frac{2}{m}}\Re[\mathcal{A}^\mu e^{-imt}] \qquad t = \int_\tau a(\tau')d\tau'$$

$$|\partial_t \mathcal{A}^\mu| \ll m|\mathcal{A}^\mu| \qquad |\Phi|, |\Psi| \ll 1 \qquad \mathcal{H} \ll am$$

$$\left[i\partial_t + \frac{3}{2}H + \frac{\Delta}{2a^2m} - \Phi m\right]\mathcal{A}^i = 0 \qquad iam\mathcal{A}^0 = \partial_j\mathcal{A}^j \qquad \Delta\Phi = \frac{m\sum_j |\mathcal{A}^j|^2}{2m_{\rm pl}^2} \qquad \Phi = \Psi$$

Vector FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A^\mu A_\mu\right]$$

$$\frac{\partial_\mu(\sqrt{-g}g^{\mu\alpha}\partial_\alpha A^\nu)}{\sqrt{-g}} + m^2 A^\nu + A^\mu g^{\nu\alpha}\partial_\alpha\left[\frac{\partial_\mu\sqrt{-g}}{\sqrt{-g}}\right] - \partial_\alpha A^\mu \partial_\mu g^{\nu\alpha} = 0 \qquad \nabla_\nu A^\nu = 0$$

$$g_{\mu\nu} = a^2(\tau)\begin{pmatrix} 1+2\Phi & V_1 & V_2 & V_3 \\ V_1 & 2\Psi-1 & 0 & 0 \\ V_2 & 0 & 2\Psi-1 & 0 \\ V_3 & 0 & 0 & 2\Psi-1 \end{pmatrix}$$

Vector FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A^\mu A_\mu\right]$$

$$\downarrow$$

$$\frac{\partial_\mu(\sqrt{-g}g^{\mu\alpha}\partial_\alpha A^\nu)}{\sqrt{-g}} + m^2A^\nu + A^\mu g^{\nu\alpha}\partial_\alpha\left[\frac{\partial_\mu\sqrt{-g}}{\sqrt{-g}}\right] - \partial_\alpha A^\mu\partial_\mu g^{\nu\alpha} = 0 \qquad \nabla_\nu A^\nu = 0$$

$$\downarrow$$

$$g_{\mu\nu} = a^2(\tau)\begin{pmatrix} 1+2\Phi & V_1 & V_2 & V_3 \\ V_1 & 2\Psi-1 & 0 & 0 \\ V_2 & 0 & 2\Psi-1 & 0 \\ V_3 & 0 & 0 & 2\Psi-1 \end{pmatrix}$$

$$A^\mu = \sqrt{\frac{2}{m}}\Re[\mathcal{A}^\mu e^{-imt}] \qquad t = \int_\tau a(\tau')d\tau'$$

$$\downarrow$$

Vector FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A^\mu A_\mu\right]$$

↓

$$\frac{\partial_\mu(\sqrt{-g}g^{\mu\alpha}\partial_\alpha A^\nu)}{\sqrt{-g}} + m^2 A^\nu + A^\mu g^{\nu\alpha}\partial_\alpha\left[\frac{\partial_\mu\sqrt{-g}}{\sqrt{-g}}\right] - \partial_\alpha A^\mu \partial_\mu g^{\nu\alpha} = 0 \qquad \nabla_\nu A^\nu = 0$$

↓

$$g_{\mu\nu} = a^2(\tau)\begin{pmatrix} 1+2\Phi & V_1 & V_2 & V_3 \\ V_1 & 2\Psi - 1 & 0 & 0 \\ V_2 & 0 & 2\Psi - 1 & 0 \\ V_3 & 0 & 0 & 2\Psi - 1 \end{pmatrix}$$

$$A^\mu = \sqrt{\frac{2}{m}}\Re[\mathcal{A}^\mu e^{-imt}] \qquad t = \int_\tau a(\tau')d\tau'$$

$$|\partial_t \mathcal{A}^\mu| \ll m|\mathcal{A}^\mu| \qquad |\Phi|, |\Psi|, |V_i| \ll 1 \qquad \mathcal{H} \ll am$$

Vector FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A^\mu A_\mu\right]$$

$$\frac{\partial_\mu(\sqrt{-g}g^{\mu\alpha}\partial_\alpha A^\nu)}{\sqrt{-g}} + m^2 A^\nu + A^\mu g^{\nu\alpha}\partial_\alpha\left[\frac{\partial_\mu\sqrt{-g}}{\sqrt{-g}}\right] - \partial_\alpha A^\mu \partial_\mu g^{\nu\alpha} = 0 \qquad \nabla_\nu A^\nu = 0$$

$$g_{\mu\nu} = a^2(\tau)\begin{pmatrix} 1+2\Phi & V_1 & V_2 & V_3 \\ V_1 & 2\Psi - 1 & 0 & 0 \\ V_2 & 0 & 2\Psi - 1 & 0 \\ V_3 & 0 & 0 & 2\Psi - 1 \end{pmatrix}$$

$$A^\mu = \sqrt{\frac{2}{m}}\Re[\mathcal{A}^\mu e^{-imt}] \qquad t = \int_\tau a(\tau')d\tau'$$

$$|\partial_t \mathcal{A}^\mu| \ll m|\mathcal{A}^\mu| \qquad |\Phi|, |\Psi|, |V_i| \ll 1 \qquad \mathcal{H} \ll am$$

$$\left[i\partial_t + \frac{3}{2}H + \frac{\Delta}{2a^2m} - \Phi m\right]\mathcal{A}^i = 0$$

$$iam\mathcal{A}^0 = \partial_j \mathcal{A}^j$$

$$\Delta\Phi = \frac{m\sum_j |\mathcal{A}^j|^2}{2m_{\rm pl}^2}$$

$$\Phi = \Psi \qquad V_i = 0$$

Vector FDM

$$S = \int d^4x \sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A^\mu A_\mu\right]$$

$$A^\mu = \sqrt{\frac{2}{m}}\Re[\mathcal{A}^\mu e^{-imt}]$$

Vector FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A^\mu A_\mu\right]$$

$$A^\mu = \sqrt{\frac{2}{m}}\Re[\mathcal{A}^\mu e^{-imt}]$$

$$\boxed{\left[i\partial_t + \frac{3}{2}H + \frac{\Delta}{2a^2m} - \Phi m\right]\mathcal{A}^i = 0}$$

$$\boxed{\Delta\Phi = \frac{m\sum_j |\mathcal{A}^j|^2}{2m_{\rm pl}^2}}$$

Vector FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A^\mu A_\mu\right]$$

$$A^\mu = \sqrt{\frac{2}{m}}\Re[\mathcal{A}^\mu e^{-imt}]$$

$$\boxed{\left[i\partial_t + \frac{3}{2}H + \frac{\Delta}{2a^2m} - \Phi m\right]\mathcal{A}^i = 0}$$

$$\boxed{\Delta\Phi = \frac{m\sum_j |\mathcal{A}^j|^2}{2m_{\rm pl}^2}}$$

Vector FDM

Vector FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A^\mu A_\mu\right]$$

$$A^\mu = \sqrt{\frac{2}{m}}\Re[\mathcal{A}^\mu e^{-imt}]$$

$$\boxed{\left[i\partial_t + \frac{3}{2}H + \frac{\Delta}{2a^2m} - \Phi m\right]\mathcal{A}^i = 0}$$

$$\boxed{\Delta\Phi = \frac{m\sum_j |\mathcal{A}^j|^2}{2m_{\rm pl}^2}}$$

$$\text{Vector FDM} \Longleftrightarrow$$

Vector FDM

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A^\mu A_\mu\right]$$

$$A^\mu = \sqrt{\frac{2}{m}}\Re[\mathcal{A}^\mu e^{-imt}]$$

$$\boxed{\left[i\partial_t + \frac{3}{2}H + \frac{\Delta}{2a^2m} - \Phi m\right]\mathcal{A}^i = 0}$$

$$\boxed{\Delta\Phi = \frac{m\sum_j |\mathcal{A}^j|^2}{2m_{\rm pl}^2}}$$

$$\text{Vector FDM} \Longleftrightarrow 3\times\text{Scalar FDM}$$

Vector FDM

$$S = \int d^4x \sqrt{-g} \left[-\frac{m_{\rm pl}^2 R}{2} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A^\mu A_\mu \right]$$

$$A^\mu = \sqrt{\frac{2}{m}} \Re[\mathcal{A}^\mu e^{-imt}]$$

$$\boxed{\left[i\partial_t + \frac{3}{2} H + \frac{\Delta}{2a^2 m} - \Phi m \right] \mathcal{A}^i = 0}$$

$$\boxed{\Delta\Phi = \frac{m \sum_j |\mathcal{A}^j|^2}{2m_{\rm pl}^2}}$$

Dynamical eqns.
Use for:
1.) Solitonic cores
2.) FDM simulations

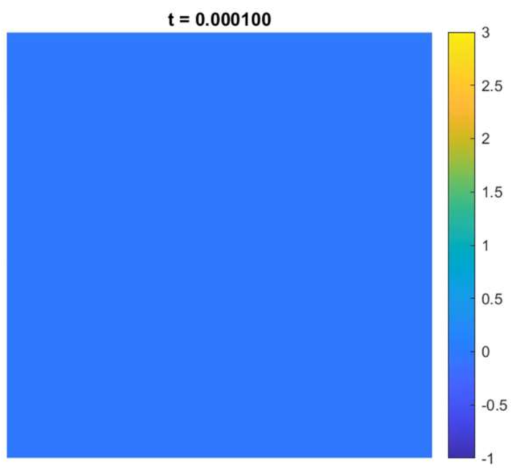
Vector FDM $\Longleftrightarrow$ 3×Scalar FDM

Vector FDM Simulations

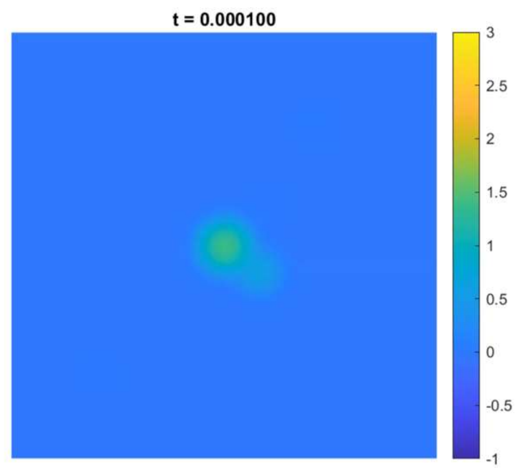
$$\boxed{\left[i\partial_t + \frac{3}{2}H + \frac{\Delta}{2a^2m} - \Phi m\right]\mathcal{A}^i = 0}$$

$$\boxed{\Delta\Phi = \frac{m\sum_j |\mathcal{A}^j|^2}{2m_{\rm pl}^2}}$$

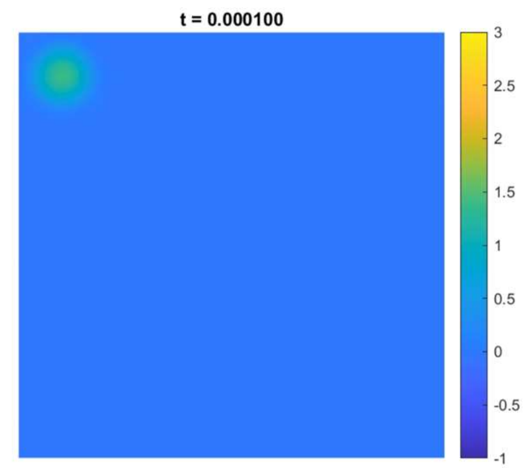
set $H = 0$



$\ln(|\mathcal{A}^x|^2)$



$\ln(|\mathcal{A}^y|^2)$



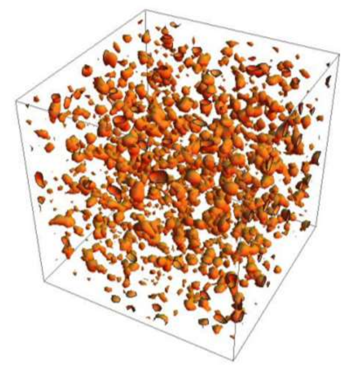
$\ln(|\mathcal{A}^z|^2)$

Work in progress

Conclusions

Reheating at the end of inflation:

- very rich dynamics



Future plans:

- more realistic models (e.g. gauge fields)

- observational signatures
  -expansion history, relics, GWs