Cosmological Particle Production & Pairwise Spots on the CMB

Yuhsin Tsai
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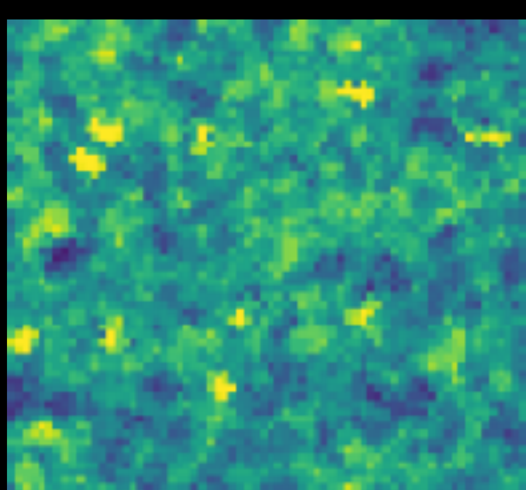


UC Davis
05/17/2021

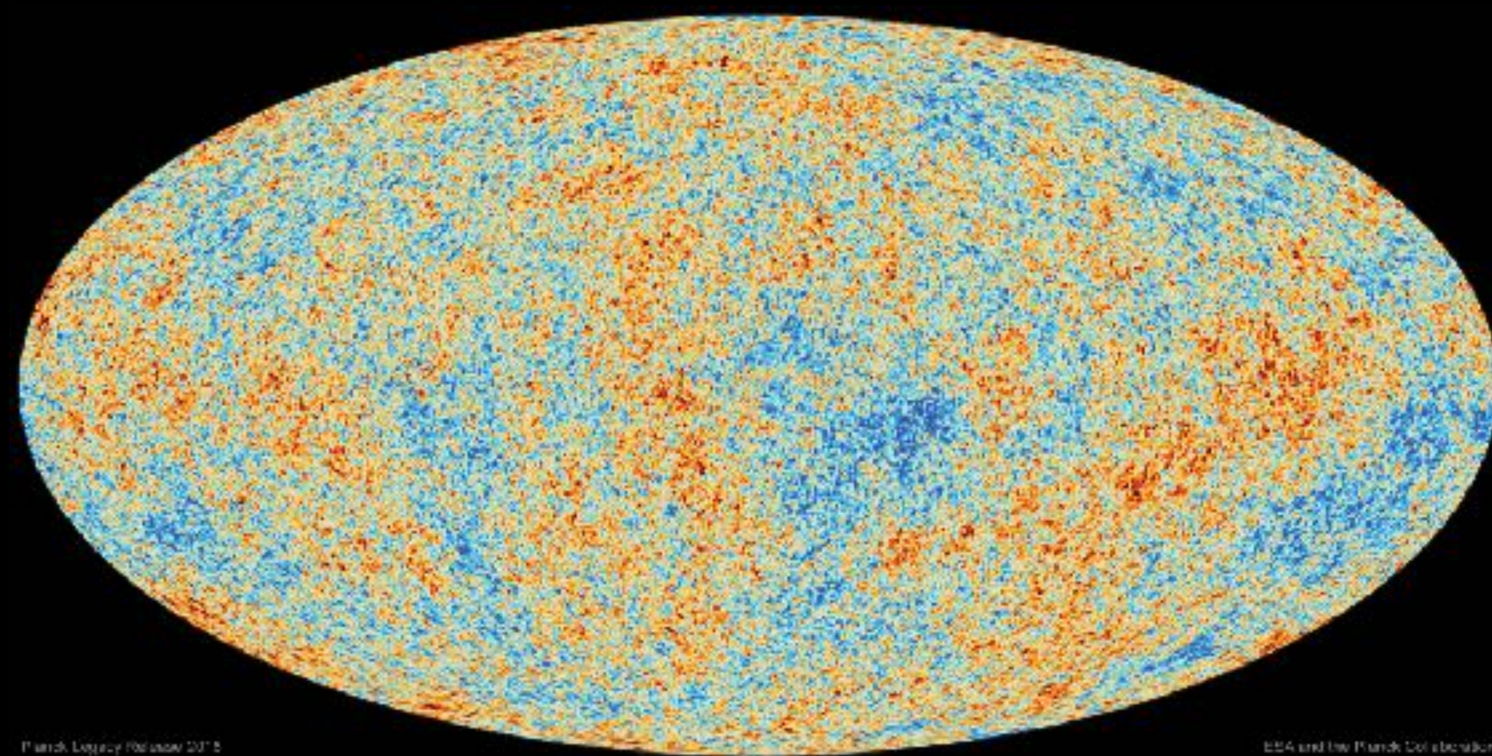
Based on an on-going work with

Jeong Han Kim
(Chungbuk National University)

Soubhik Kumar
(LBNL)

Adam Martin
(Notre Dame)

The particle detector in this talk: CMB map



- radiation with wave length ~ 1mm
- blackbody radiation with $\bar{T} \approx 2.7\,K$
- temperature anisotropy $\sigma_{\rm CMB} \approx 100\,\mu K$

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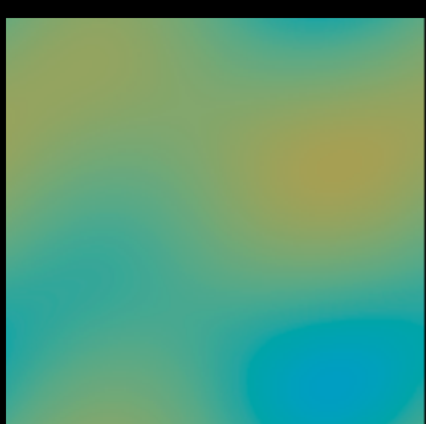
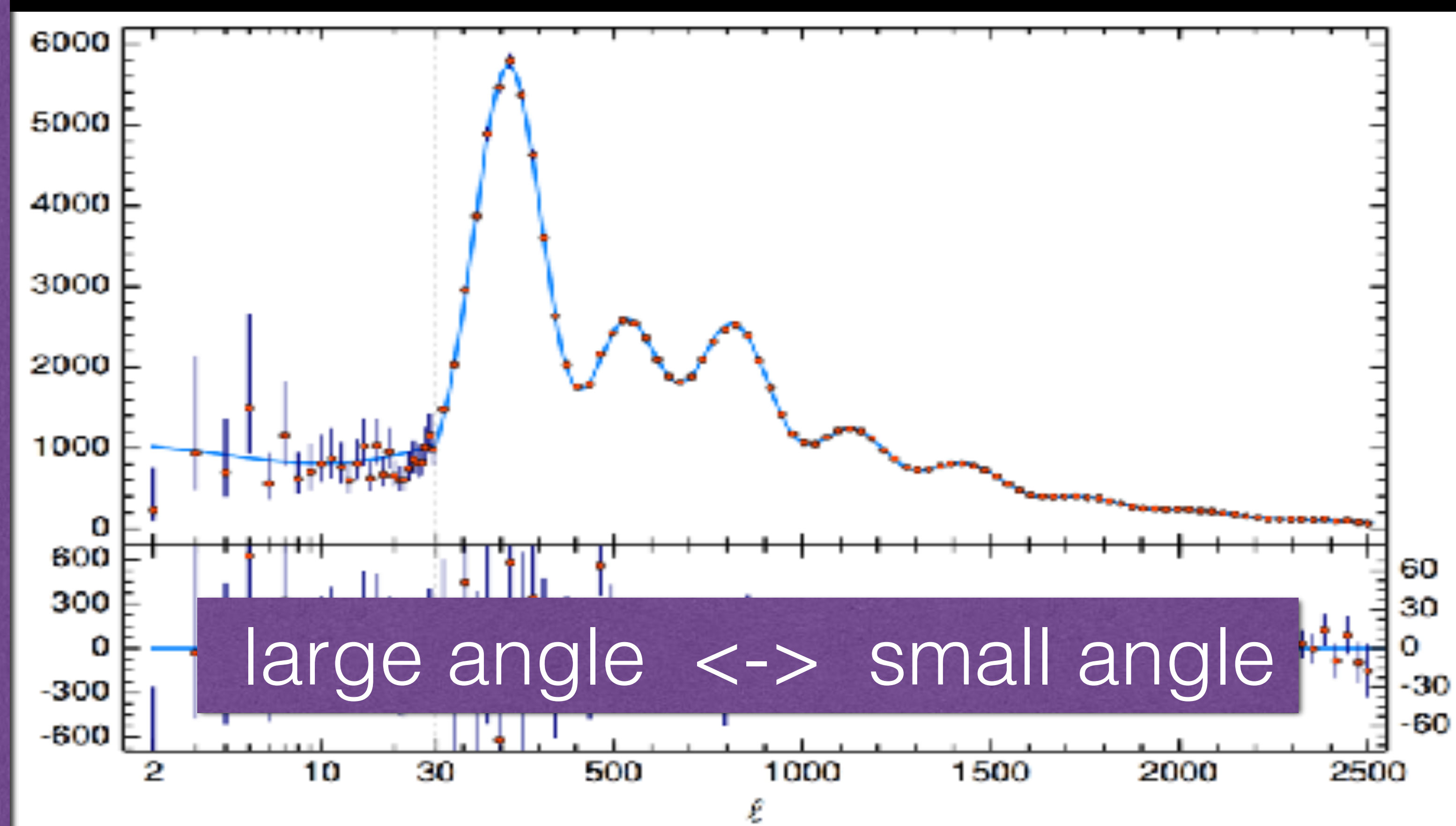
Typical CMB analysis: correlation functions

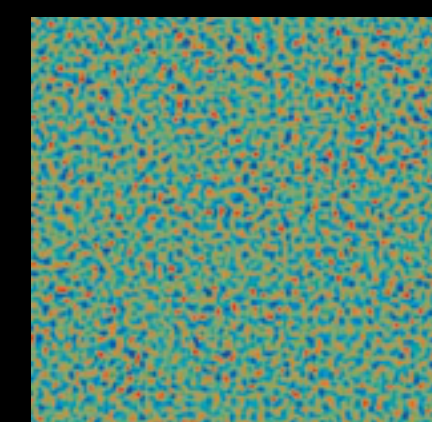
$$\langle \delta T(\theta_1,\phi_1)\delta T(\theta_2,\phi_2)\rangle \quad \langle \delta T(\theta_1,\phi_1)\delta T(\theta_2,\phi_2)\delta T(\theta_3,\phi_3)\rangle \ \ldots$$



~ (Temperature fluctuation)^2



large angle <-> small angle



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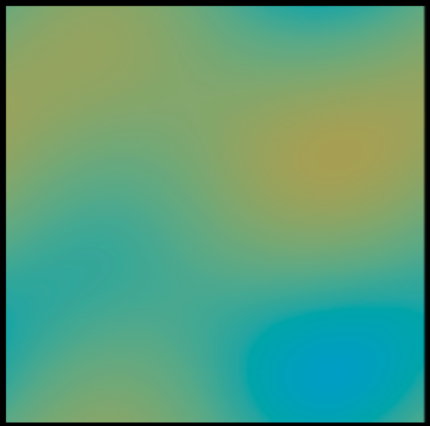
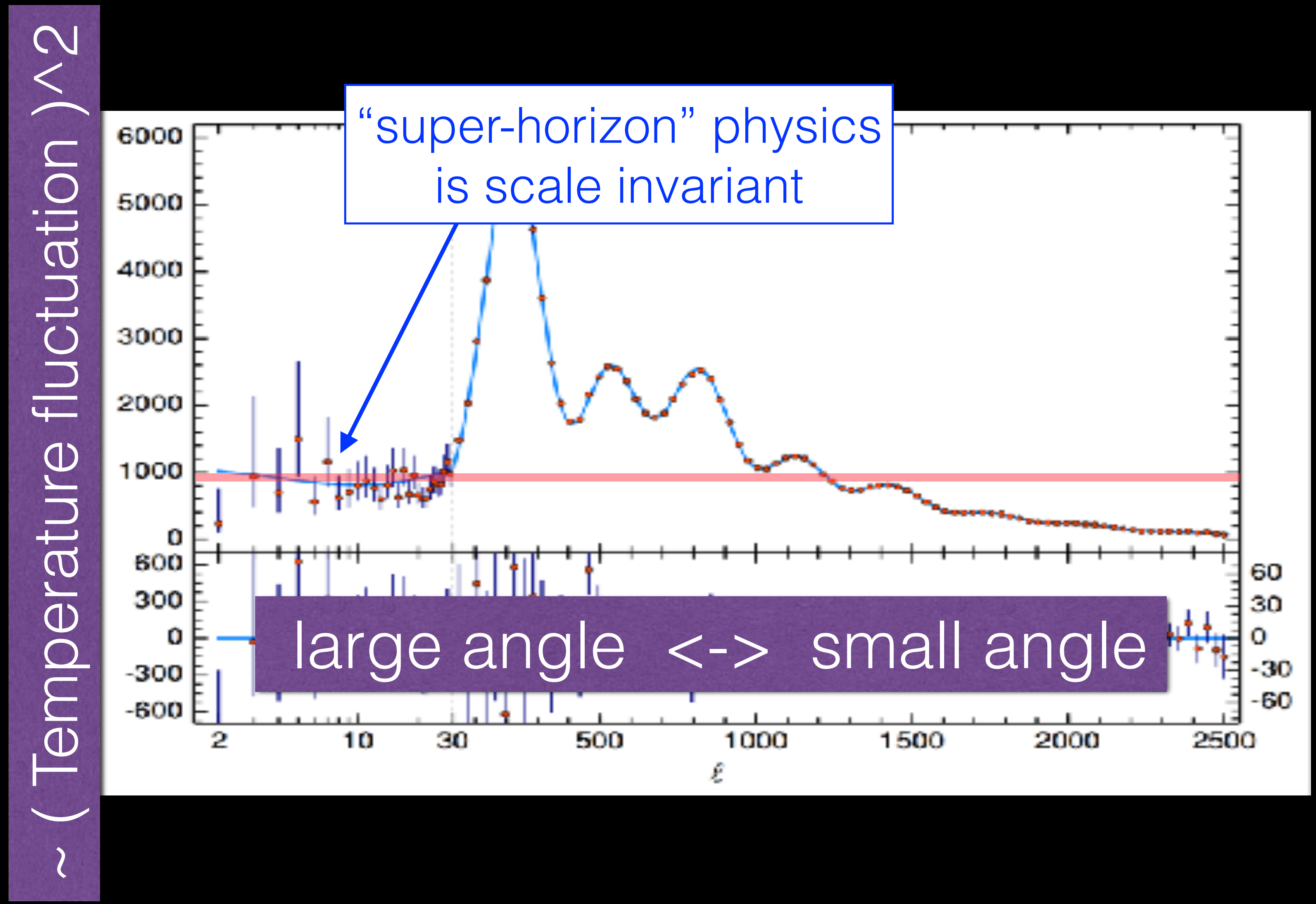
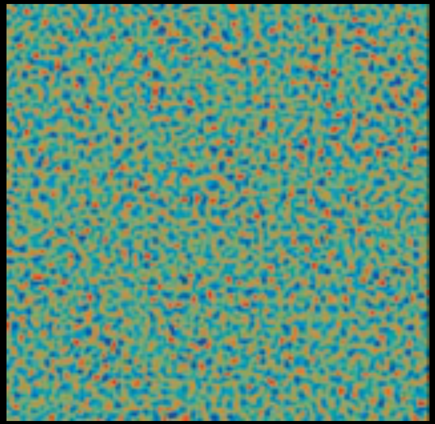
Typical CMB analysis: correlation functions

$$\langle \delta T(\theta_1, \phi_1) \delta T(\theta_2, \phi_2) \rangle \quad \langle \delta T(\theta_1, \phi_1) \delta T(\theta_2, \phi_2) \delta T(\theta_3, \phi_3) \rangle$$







It makes sense to use N-point functions. The temperature anisotropy is almost a scale-invariant (not “localized” signals)

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Motivation

- Our goal is to probe heavy particles that couple to inflaton and produced during the inflation

- In the context of "cosmological collider physics", we usually focus on signals of non-Gaussianity ($\geq$ 3-pt functions)

- The signals are usually suppressed by $\sim \exp(-\pi M/H_*)$

Question: Can we probe particles with mass $\gg H_*$?

What will be the signal to look at?

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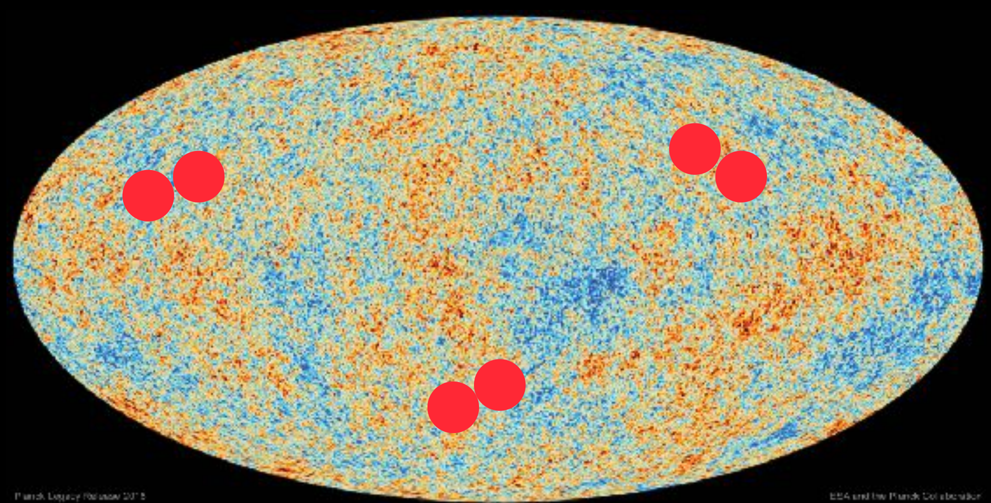
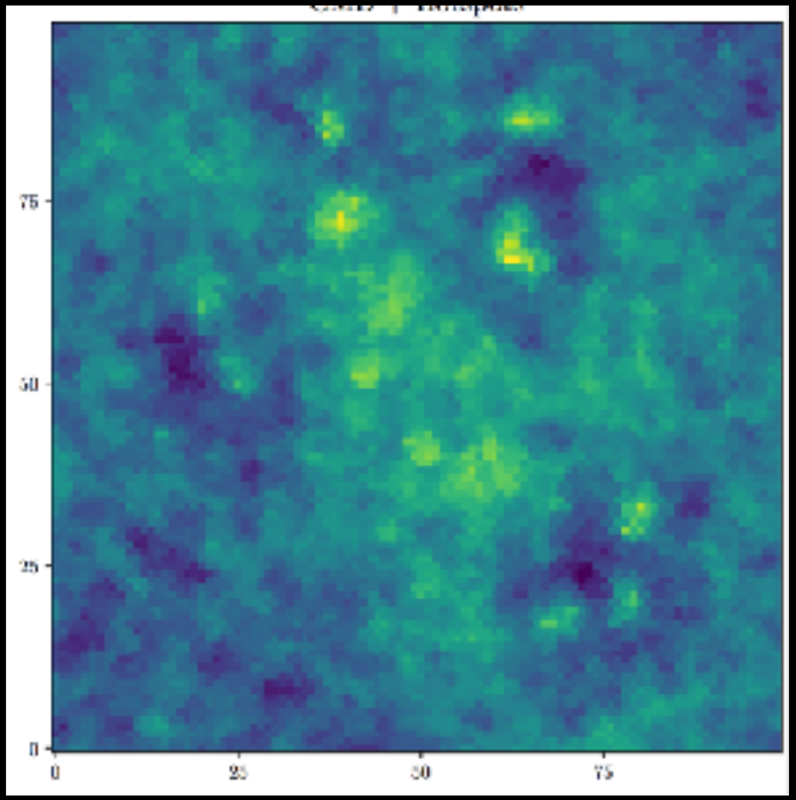
If the new particles are so heavy $\gg H_*$,

they only show up as localized signals in position space

As I will explain, in this case
we get Pairwise Spots on the CMB map



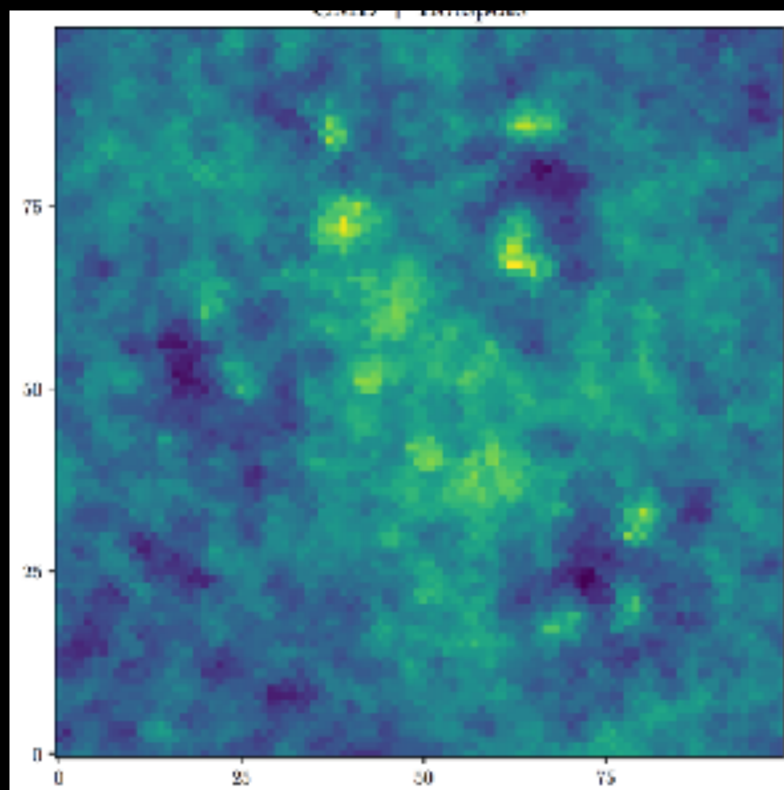
(cartoon picture)

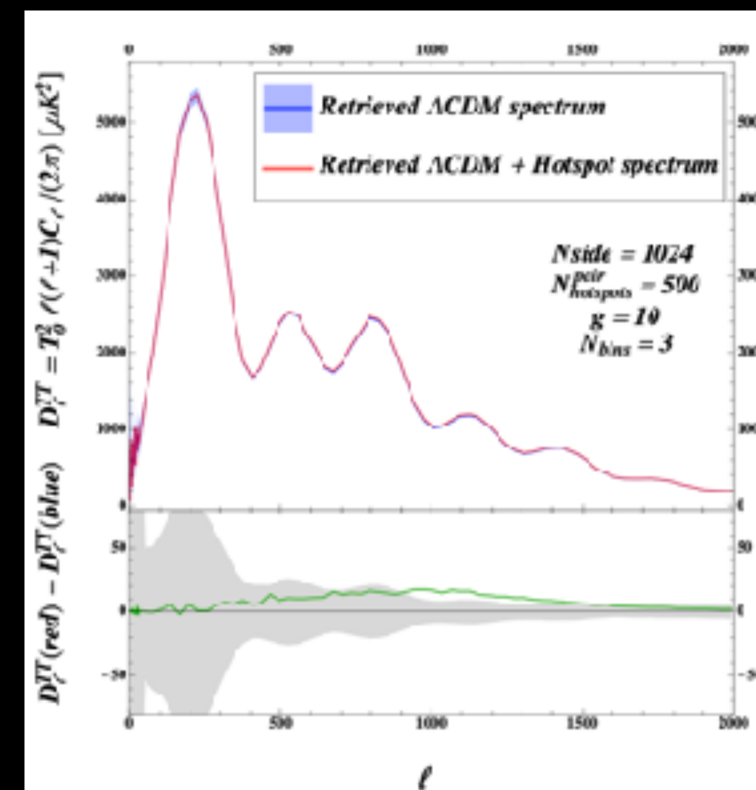


(HEALPix simulation)

Is position space search better than the N-pt function
for identifying very heavy particles?

If yes, the searching strategy will be totally different





Looking for pairwise hotspots in position space

ℓ-dependent distortion of CMB TT-spectrum

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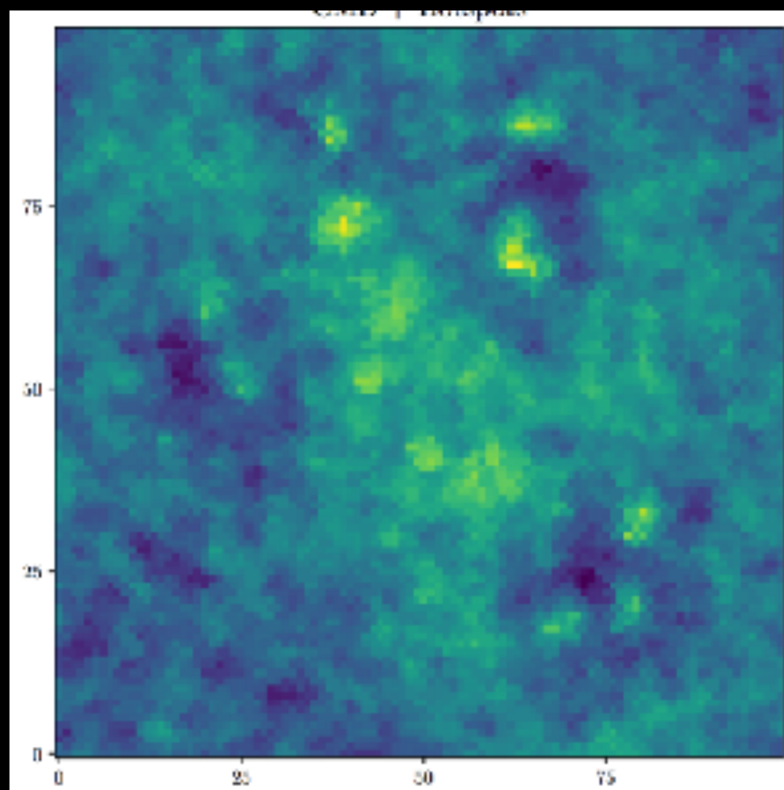
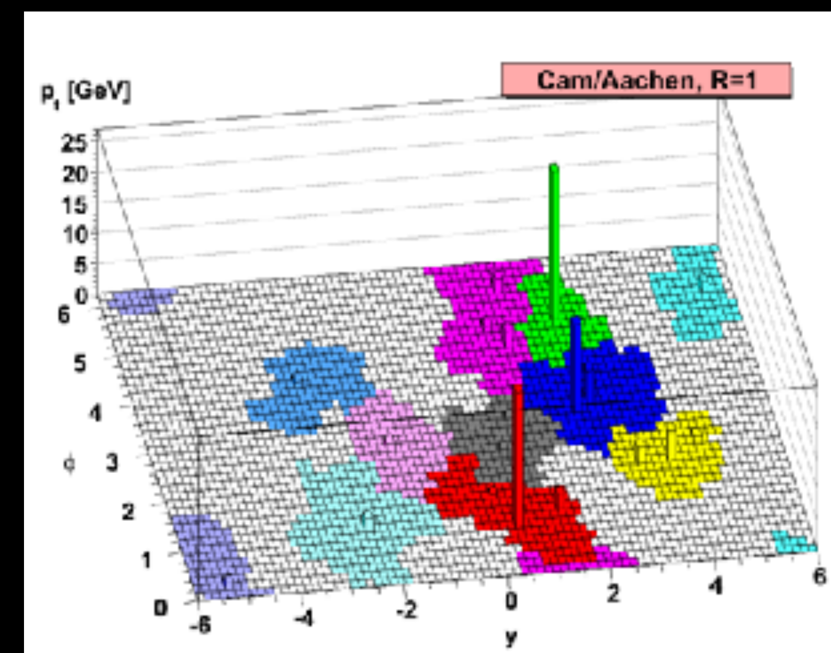
If yes, the searching strategy will be totally different



$\sim$



Looking for pairwise hotspots in position space

like studying jet substructure

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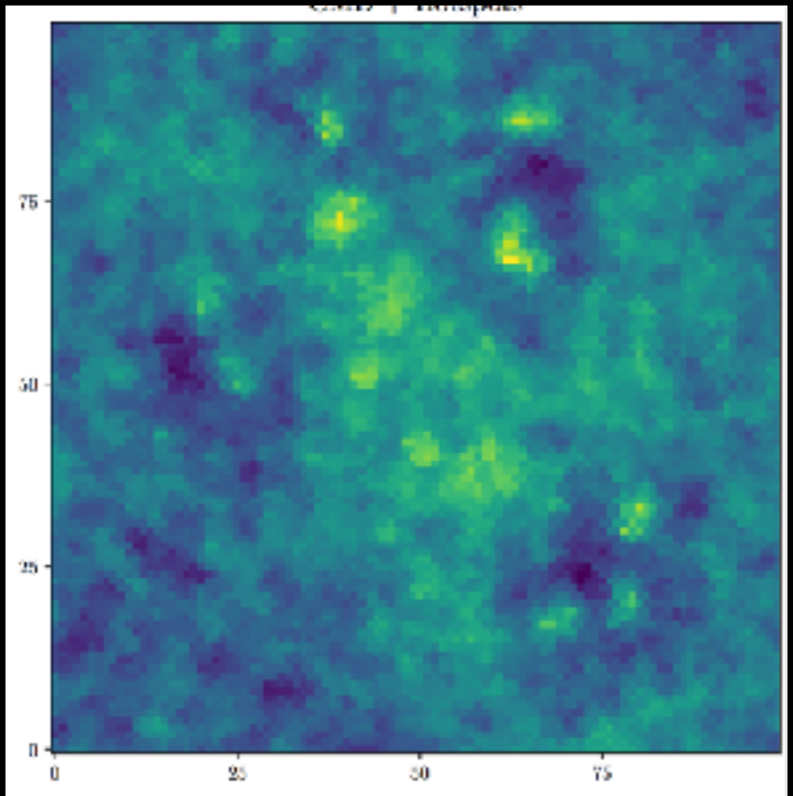
If yes, the searching strategy will be totally different



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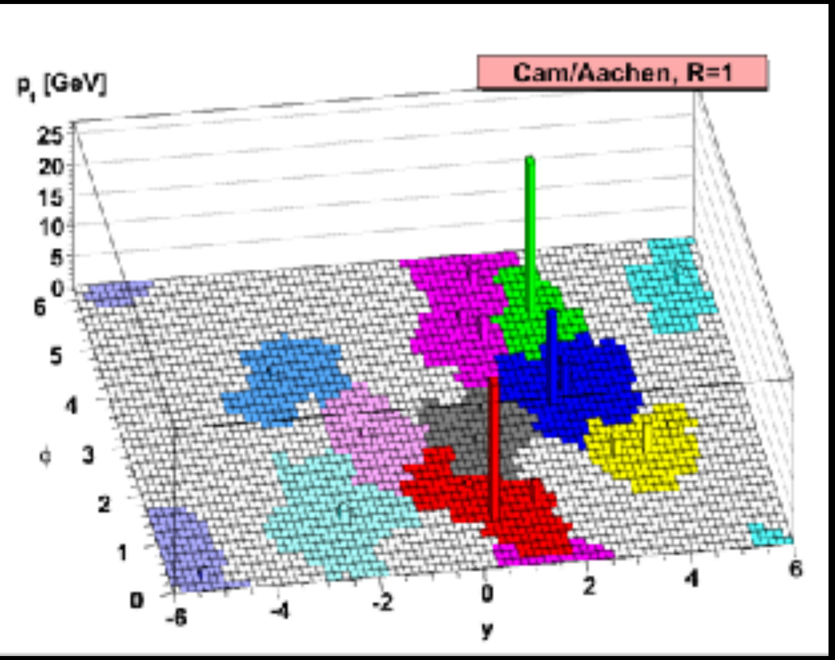


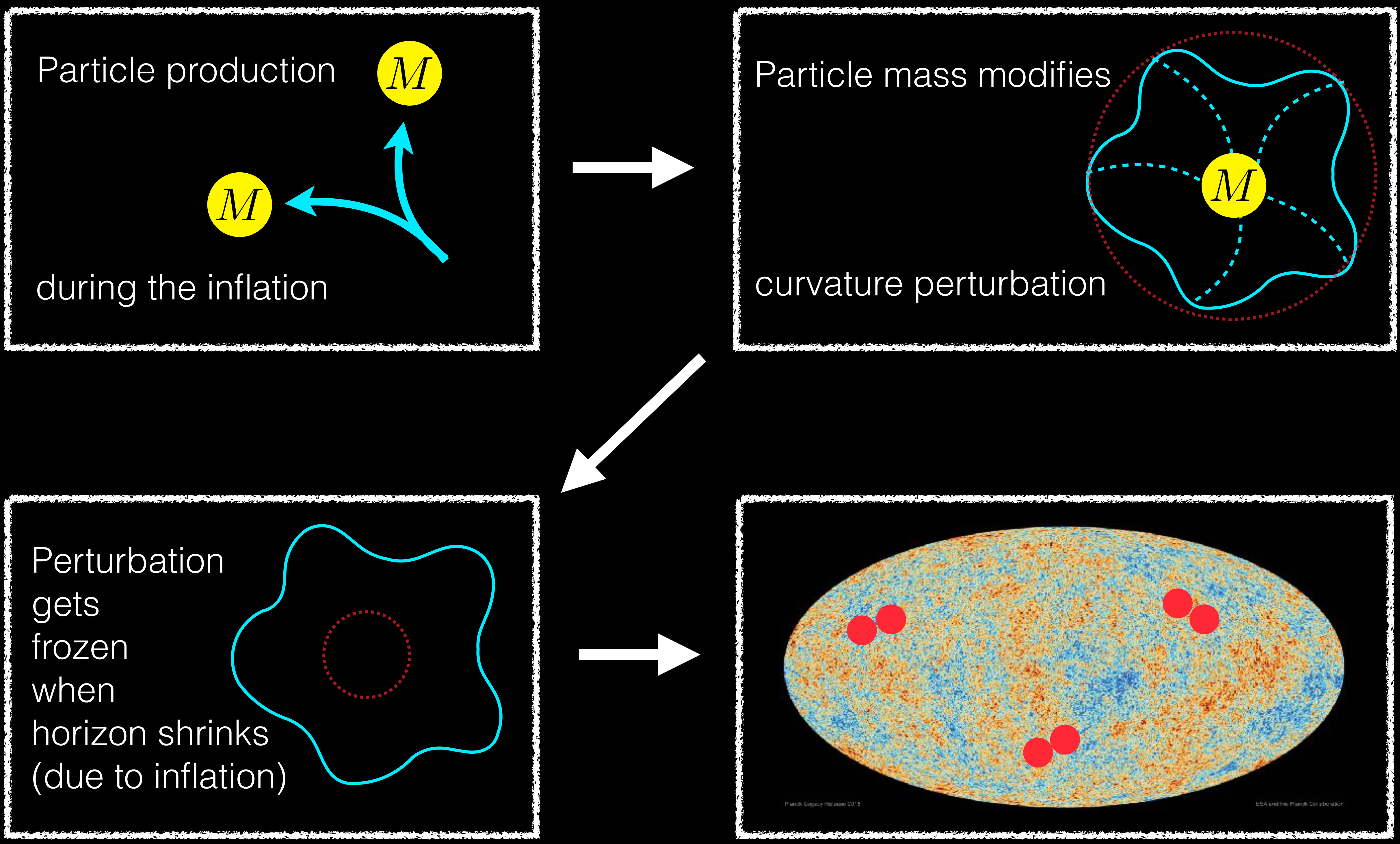
We were also motivated by Maldacena's work

A model with cosmological Bell inequalities 1508.01082

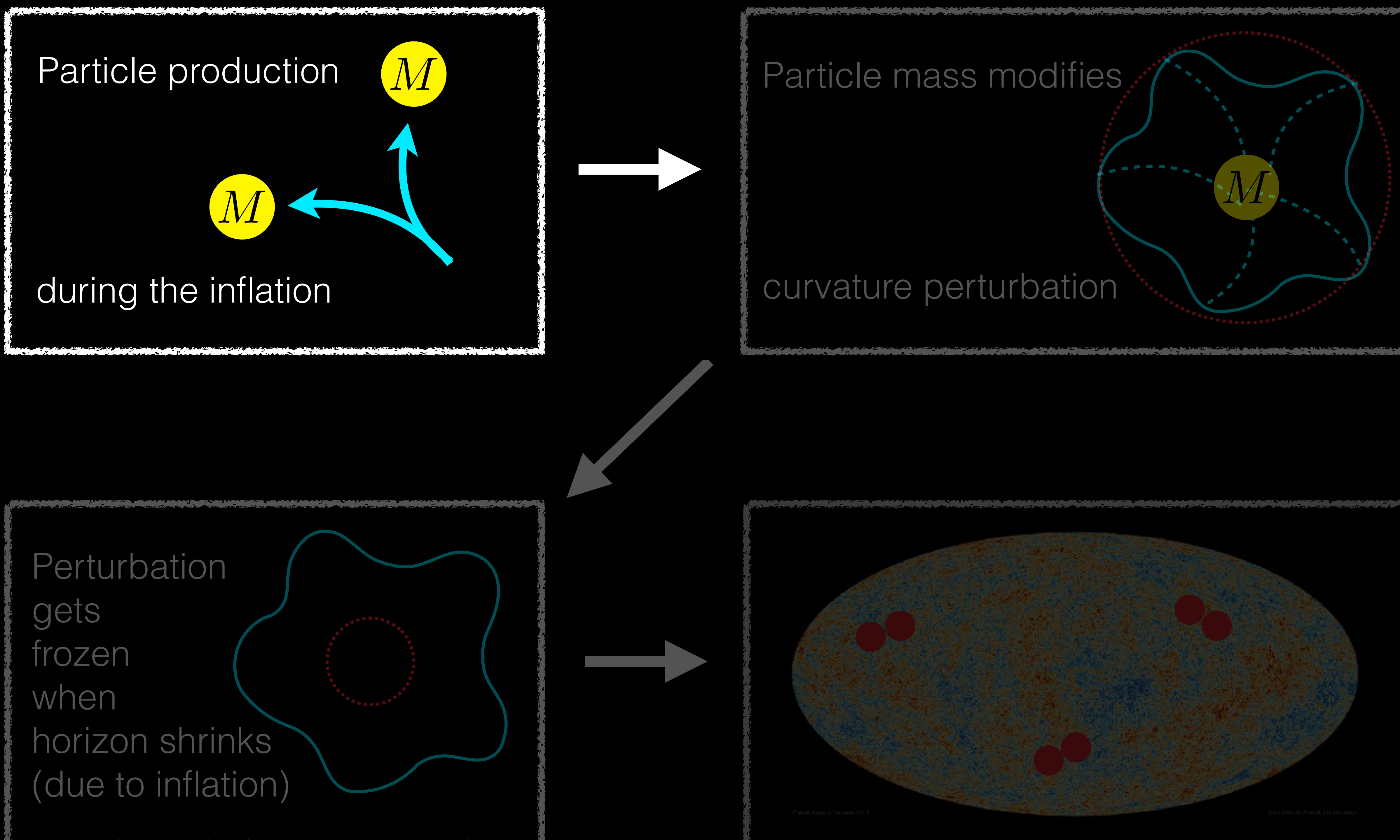
Juan Maldacena

We discuss the possibility of devising cosmological observables which violate Bell's inequalities. Such observables could be used to argue that cosmic scale features were produced by quantum mechanical effects in the very early universe. As a proof of principle, we propose a somewhat elaborate inflationary model where a Bell inequality violating observable can be constructed.

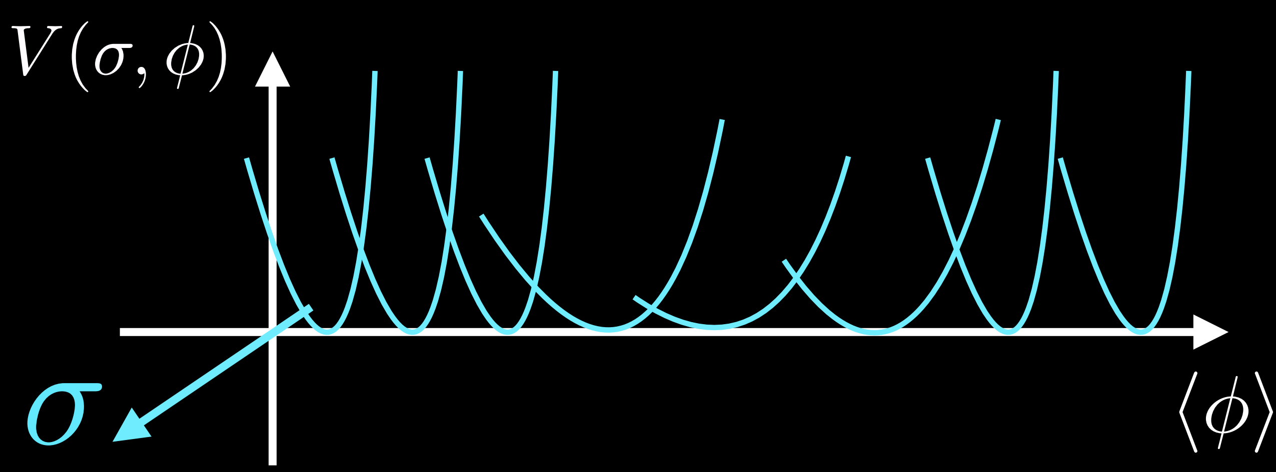
Production of the pairwise spots on the CMB

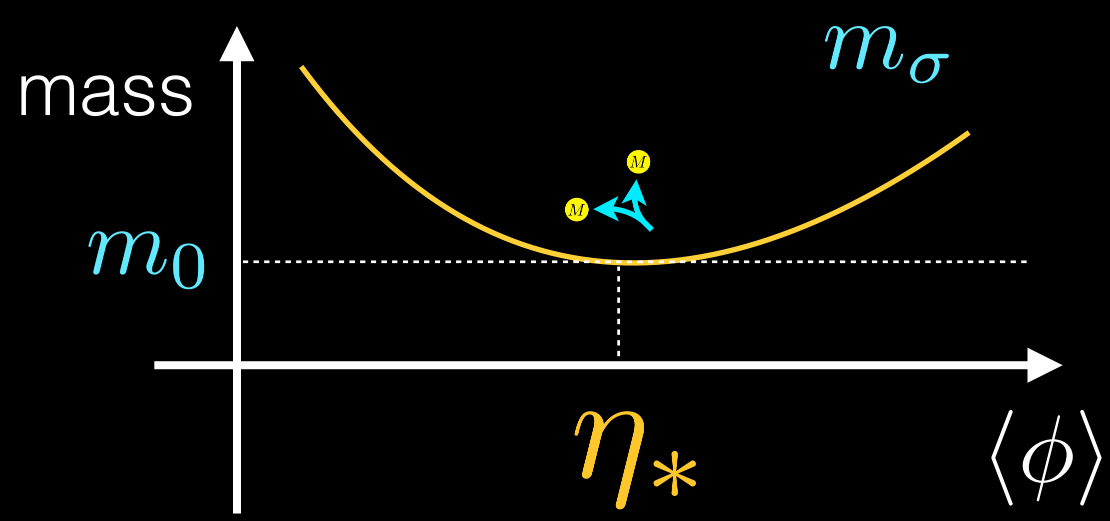


Step I : the non-adiabatic particle production



Consider a scalar particle σ that carries a mass depending on the inflaton-VEV





- Sigma mass is typically heavy (comparing to Hubble scale)
- mass takes its minimum value at time η_*
- Sigma can be produced from the inflaton energy around η_*

A toy model example

$$V(\phi,\sigma) = V_{\rm inf}(\phi) + \frac{1}{2}\left(M_0^2 + (g\phi - M)^2\right)\sigma^2 \quad \text{with} \quad M \sim g\phi \gg M_0$$

Minimum mass when $g\phi \sim M$

$$M_{\rm eff}^2 \equiv M_0^2 + (g\phi - M)^2 \approx M_0^2 \ll M^2$$

(also see a similar setup in Flauger et al. (2017),
and Muchmeyer et al. (2019) for the N-point function study)

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e.o.m. during the inflation

$$\sigma'' - \frac{2}{\eta}\sigma' + \left(k^2 + \frac{M^2(\eta)}{H^2\eta^2}\right)\sigma = 0$$

$$u = \sigma/\eta$$

$$u'' + \left(k^2 + \frac{M^2(\eta)/H^2 - 2}{\eta^2}\right)u \equiv u'' + \omega(\eta)^2 u = 0$$

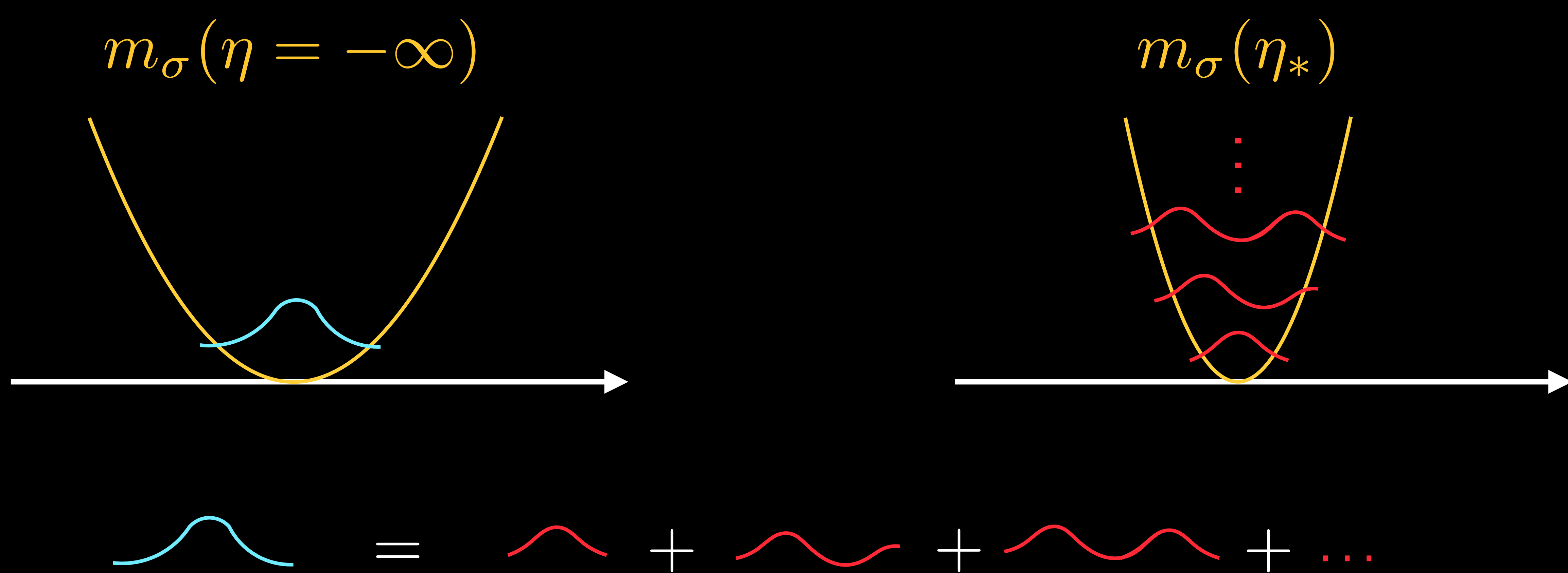
simple harmonic oscillator
with time-dependent frequency

$$\omega(\eta)^2 = k^2 + \frac{M^2(\eta)}{\eta^2}$$

How to calculate the particle production?

- σ is produced from the kinetic energy of inflaton
- cannot calculate the production as in collider experiments. Inflaton & sigma are time-dependent fields
- calculate the number of non-adiabatic particle production from Bogolyubov transformation

Particle production from time-variant vacuum



when promoting field into an operator, initial raising/lowering, operators will be a combination of later raising/lowering operators

$$
\begin{aligned}
\hat{u}(\eta, \mathbf{x}) &= \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left[\hat{a}_{\mathbf{k}} \mathcal{I}_k(\eta)\, e^{i\mathbf{k}\cdot\mathbf{x}} + \hat{a}^\dagger_{\mathbf{k}} \mathcal{I}^*_k(\eta)\, e^{-i\mathbf{k}\cdot\mathbf{x}} \right] \\
&= \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left[\hat{b}_{\mathbf{k}} \mathcal{F}_k(\eta)\, e^{i\mathbf{k}\cdot\mathbf{x}} + \hat{b}^\dagger_{\mathbf{k}} \mathcal{F}^*_k(\eta)\, e^{-i\mathbf{k}\cdot\mathbf{x}} \right]
\end{aligned}
$$

$\mathcal{I}$, $\mathcal{F}$ are the initial & final wave functions

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Bogolyubov Transformation

Relation between the raising/lowering operators defined in the initial and final vacua

$$\hat{b}_k = \alpha_k\,\hat{a}_k + \beta_k^*\,\hat{a}_k^\dagger, \qquad \hat{b}_k^\dagger = \beta_k\,\hat{a}_k + \alpha_k^*\,\hat{a}_k^\dagger\,,$$

Number density of particles in the "final vacua" (in Heisenberg's picture)

$$\begin{aligned}
{}_{\rm univ}\langle 0|\hat{N}_k|0\rangle_{\rm univ} &= {}_{\rm univ}\langle 0|\hat{b}_k^\dagger\hat{b}_k|0\rangle_{\rm univ} \\
&= {}_{\rm univ}\langle 0|(\beta_k\,\hat{a}_k + \alpha_k^*\,\hat{a}_k^\dagger)(\alpha_k\,\hat{a}_k + \beta_k^*\,\hat{a}_k^\dagger)|0\rangle_{\rm univ} \\
&= |\beta_k|^2\delta(0)\,.
\end{aligned}$$

$$n \equiv \int d^3\mathbf{k}\,n_k = \int d^3\mathbf{k}\,|\beta_k|^2$$

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Number of σ pairs in the CMB last scattering surface (with a thickness)

$$N_{\sigma\,\mathrm{pairs}} = \frac{1}{2\pi^2}\left(\frac{g\dot{\phi}}{H_*^2}\right)^{3/2} e^{-\frac{\pi(M_0^2-2H_*^2)}{|g\dot{\phi}|}} \left(\frac{k_*}{k_{\mathrm{CMB}}}\right)^3 \left(\frac{\Delta\eta_{rec}}{\eta_{rec}}\right)$$

looks like a thermal production suppressed by the Sigma mass

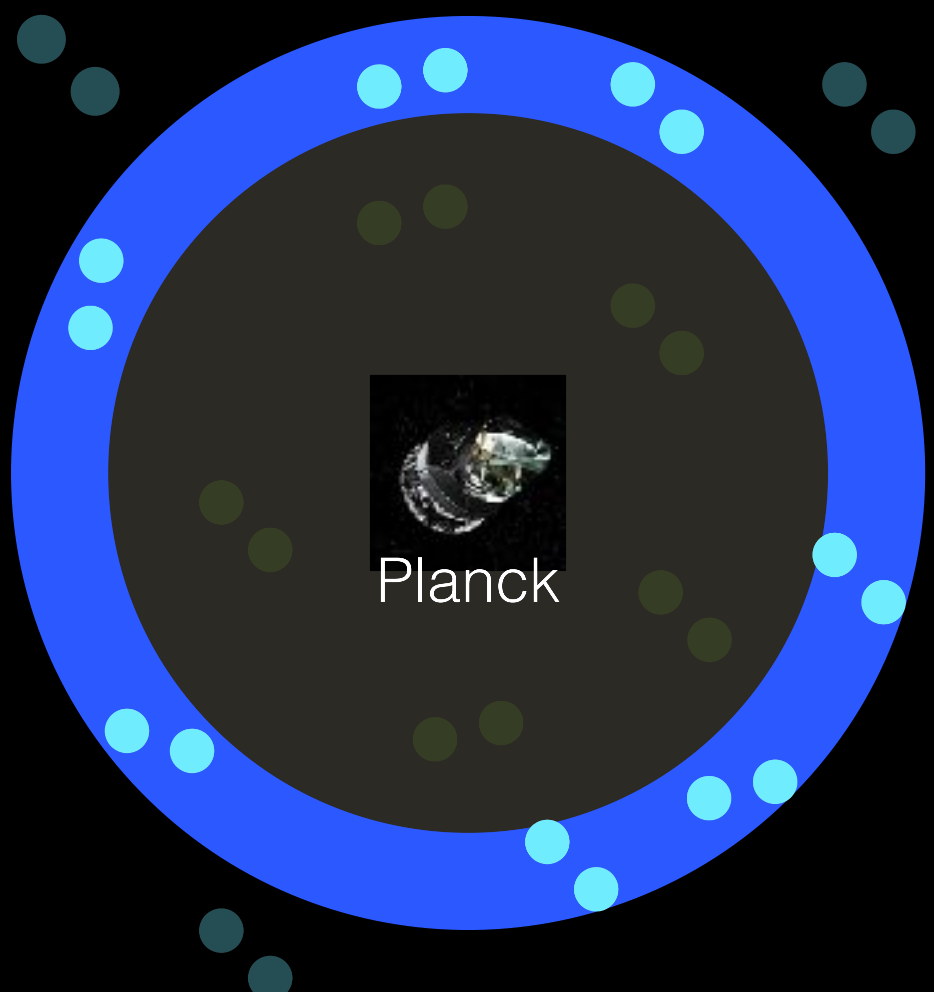
$\sqrt{\dot{\phi}} \approx 60 H_*$ from CMB measurement, ~ kinetic energy of inflaton

CMB horizon with finite thickness

$$\frac{\Delta\eta_{rec}}{\eta_{rec}} \approx 0.04$$

Number of σ pairs in the CMB last scattering surface (with a thickness)

$$N_{\sigma\,\text{pairs}} = \frac{1}{2\pi^2}\left(\frac{g\dot{\phi}}{H_*^2}\right)^{3/2} e^{-\frac{\pi(M_0^2-2H_*^2)}{|g\dot{\phi}|}}\left(\frac{k_*}{k_{\text{CMB}}}\right)^3\left(\frac{\Delta\eta_{rec}}{\eta_{rec}}\right)$$



As a reminder, $M_{\text{eff}}^2 \approx M_0^2 + g^2\phi'^2(\eta-\eta_*)^2$

If $g = 5$ $M_0 = 5.5\sqrt{\dot{\phi}} \approx 330 H_*$

and the spot size (η_*) is similar to a pixel of chopping CMB into 1000^2 pieces

$$N_{\sigma\,\text{pairs}} \sim 10^3$$

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Back-reaction constraints

Need to make sure the field of heavy particle do not affect inflaton's slow-roll e.o.m. $3H_*\dot{\phi} \approx -\frac{\partial V_\phi}{\partial \phi}$

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Since $\frac{\partial V}{\partial \phi} = \frac{\partial V_\phi}{\partial \phi} + g(g\phi - M)\sigma^2$

this requires $g(g\phi - M)\sigma^2 \sim gM_\sigma\sigma^2 \sim g\, n_\sigma \ll H_*\dot{\phi}$

and an upper bound $N_{\sigma\,\mathrm{pairs}} < 10^8$ for the same spot size that gives < 1% of the correction

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Radiative correction => assume a UV completion (e.g. SUSY) takes care of that (see e.g., Flauger et al. (2016))

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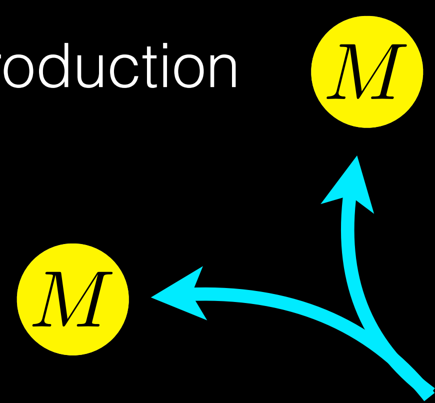
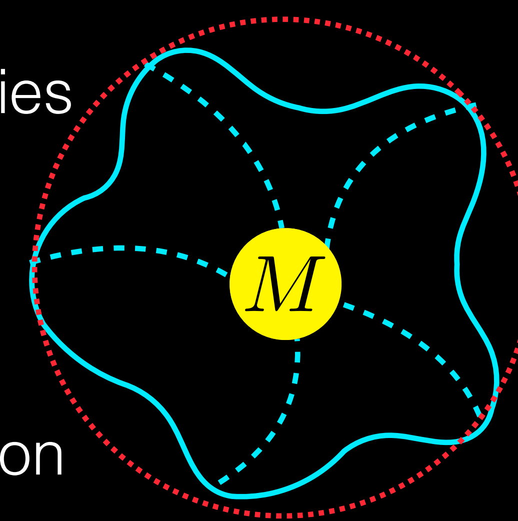
Step II: mass modifies curvature perturbation

Particle production



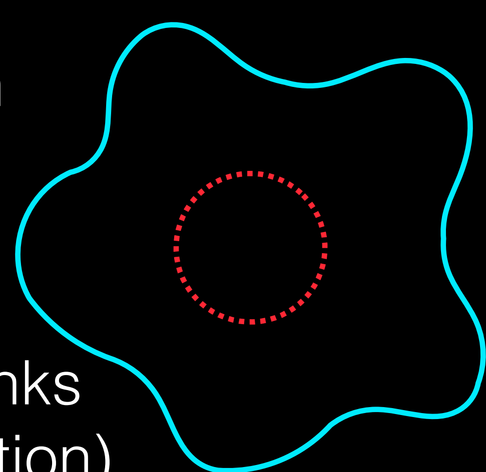
during the inflation

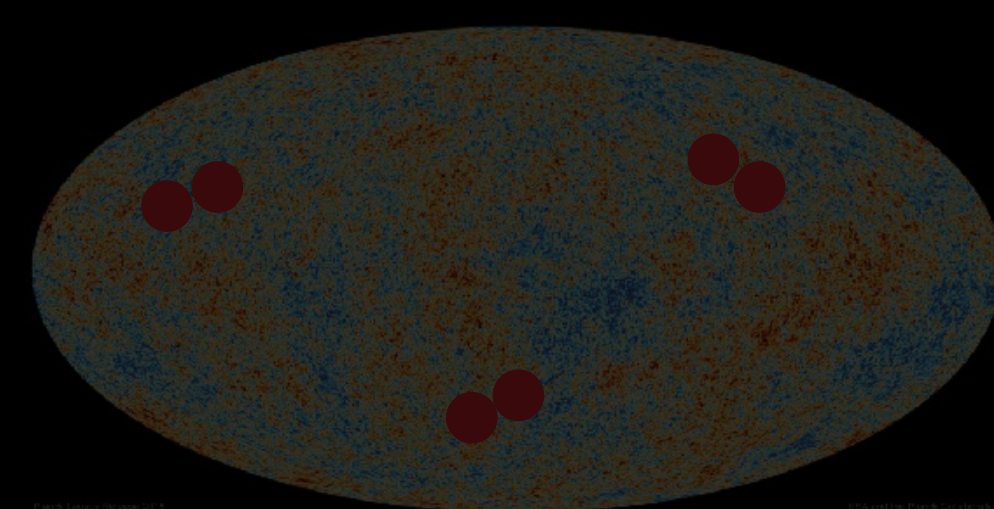
Particle mass modifies



curvature perturbation

Perturbation
gets
frozen
when
horizon shrinks
(due to inflation)



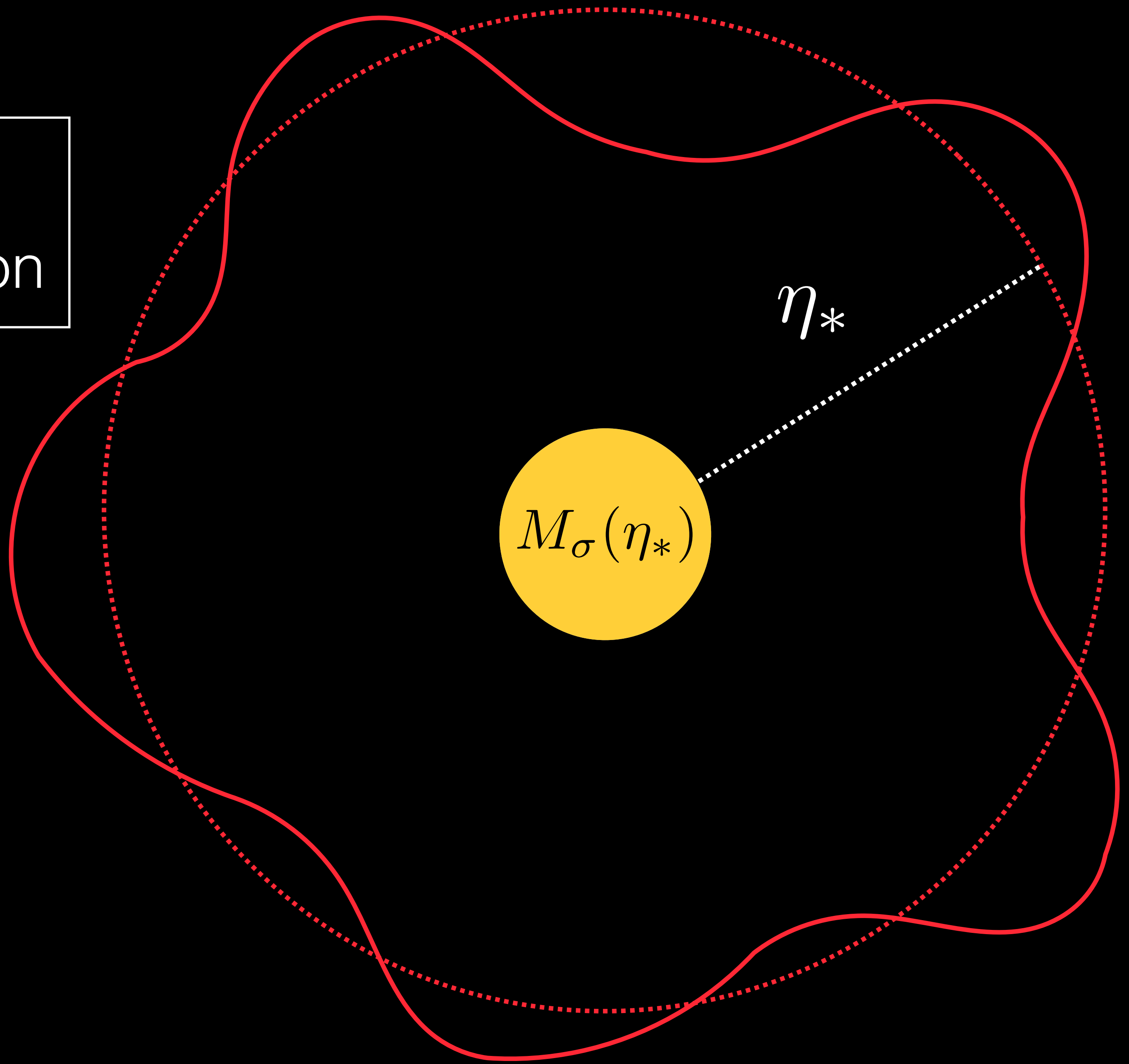


particle mass modifies curvature perturbation in the horizon

Curvature perturbation ~ (energy perturbation)/4 - gravity perturbation

for radiation in Conformal Newtonian gauge

horizon size ~ $|\eta_*|$
at particle production



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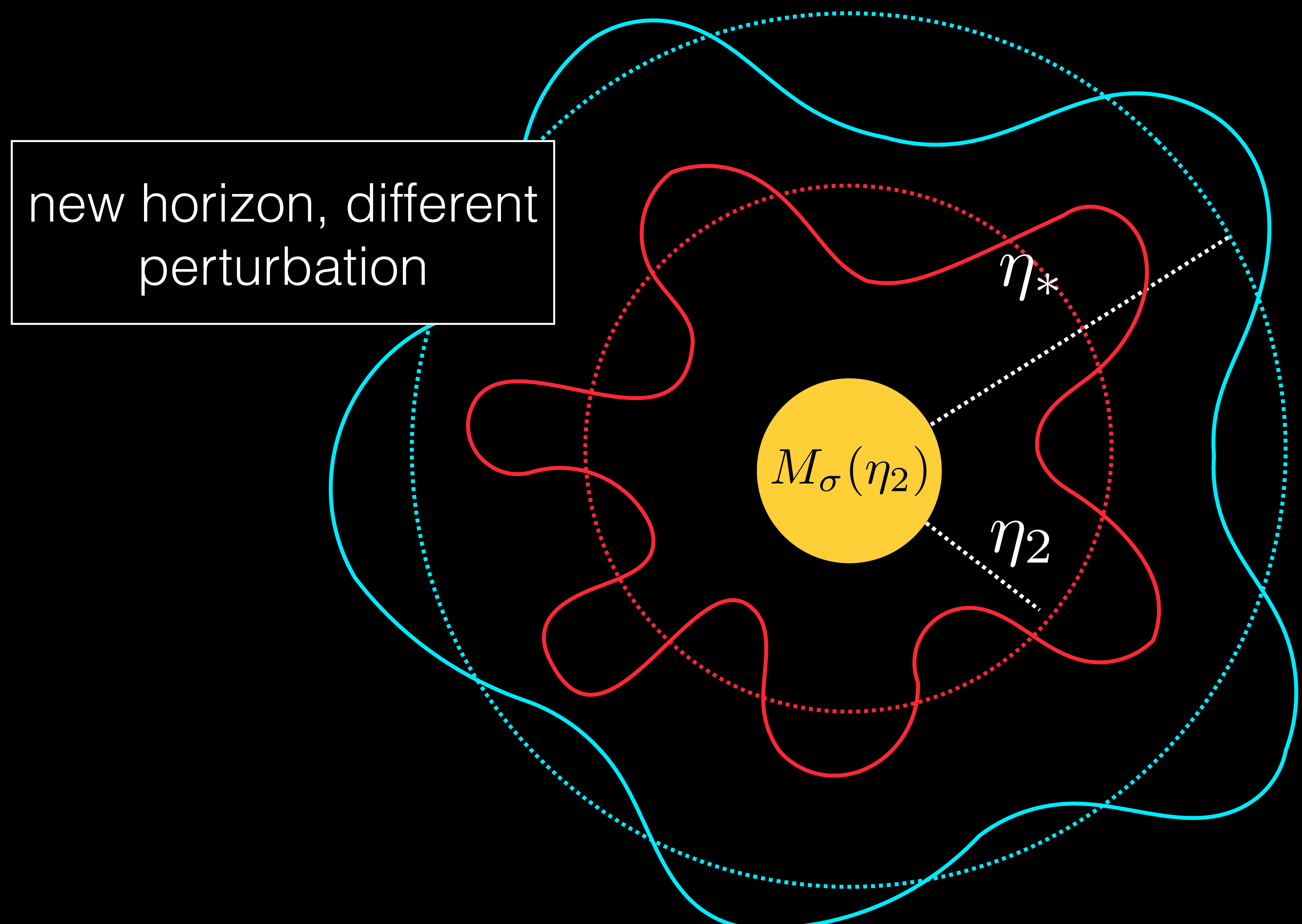
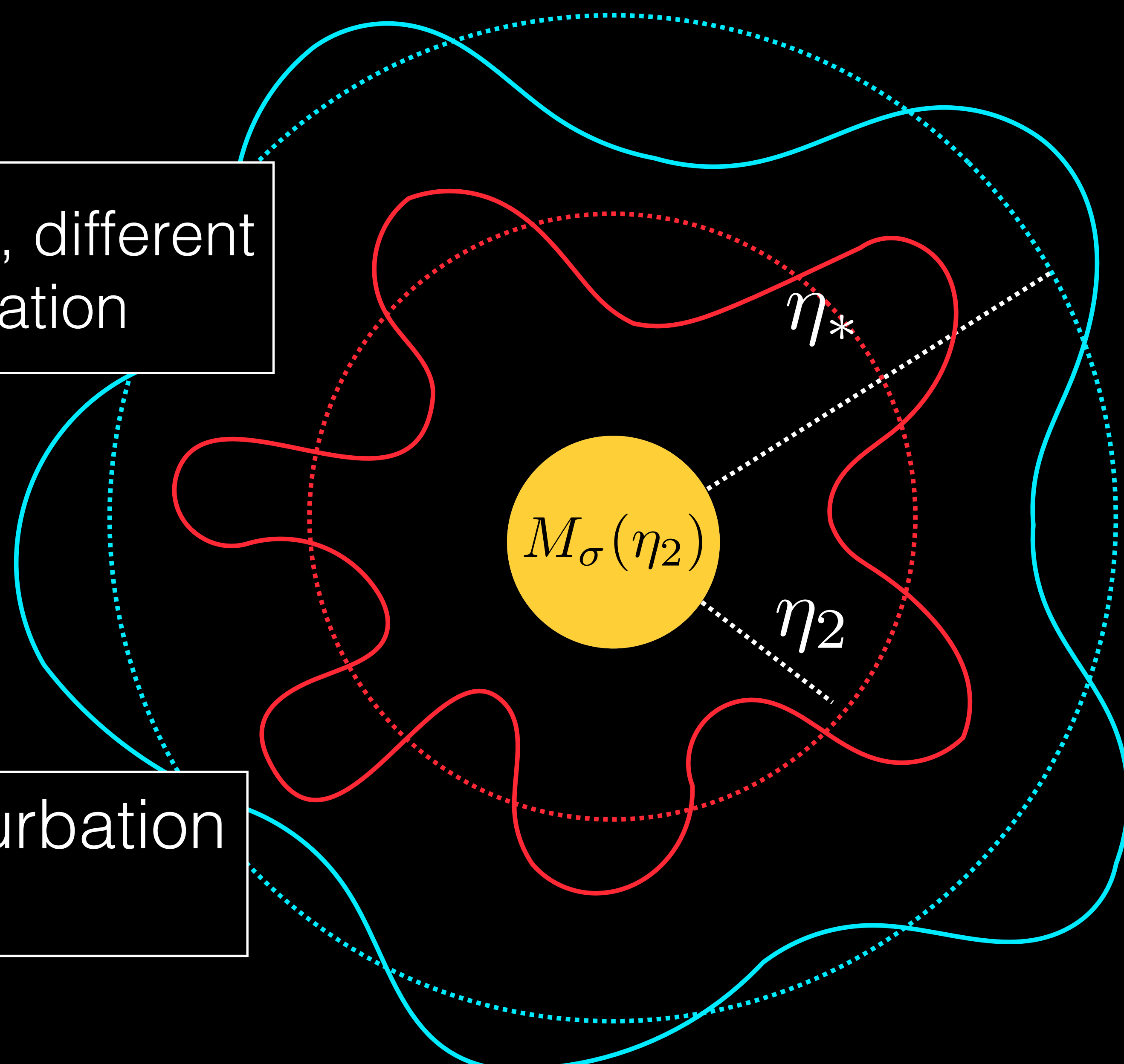
Once perturbation in the old horizon is frozen, **NEW** particle mass Modifies the perturbation in the **NEW** horizon



Once perturbation in the old horizon is frozen, **NEW** particle mass
Modifies the perturbation in the **NEW** horizon

new horizon, different perturbation



radius-dependent perturbation with $r < |\eta_*|$

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Curvature perturbation in position space

Produced heavy particles backreact on spacetime

Maldacena (1508.01082)

Fialkov et. al. (0911.2100)

$$S_\sigma = \int dt \sqrt{-g_{00}}\, M_{\text{eff}} \supset \int d\eta\, \partial_\eta \zeta\, \frac{M_{\text{eff}}(\eta)}{H}$$

comoving curvature perturbation

Curvature perturbation in position space

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comoving curvature perturbation

Give rise to a non-trivial one-point function

$$\langle \zeta_k \rangle = -i \int_{\eta_*}^{0} d\eta \, \langle 0 | \zeta_k(\eta_0) \partial_\eta \zeta_k(\eta) | 0 \rangle \, \frac{M_{\rm eff}(\eta)}{H} + c.c.$$

given by the inflaton fluctuation

Curvature perturbation in position space

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comoving curvature perturbation

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given by the inflaton fluctuation

Profile in the position space

$$\langle\zeta_k\rangle = \left[\frac{M_{\rm eff}(|\eta|=r)}{2\sqrt{2\epsilon}M_{pl}}\right]\frac{H}{2\pi\sqrt{2\epsilon}M_{pl}} \qquad r\le|\eta_*| \qquad (=0\,,\;\; r>|\eta_*|)$$

Curvature perturbation in position space

The resulting curvature profile in $r \leq |\eta_*|$ from the spot center,

Adiabatic fluctuation $\langle \zeta_{ad} \rangle = \sqrt{A_s} \sim 10^{-5}$

$$\langle \zeta_\sigma \rangle = \left[\frac{g}{2} \log \left(\frac{|\eta_*|}{r} \right) \right] \frac{H}{2\pi\sqrt{2\epsilon} M_{pl}} \sim \left[\frac{g}{2} \log \left(\frac{|\eta_*|}{r} \right) \right] \langle \zeta_{ad} \rangle$$

Spot size $\sim |\eta_*|$ and the coupling g controls the spot temperature over CMB fluctuations

Curvature perturbation in position space

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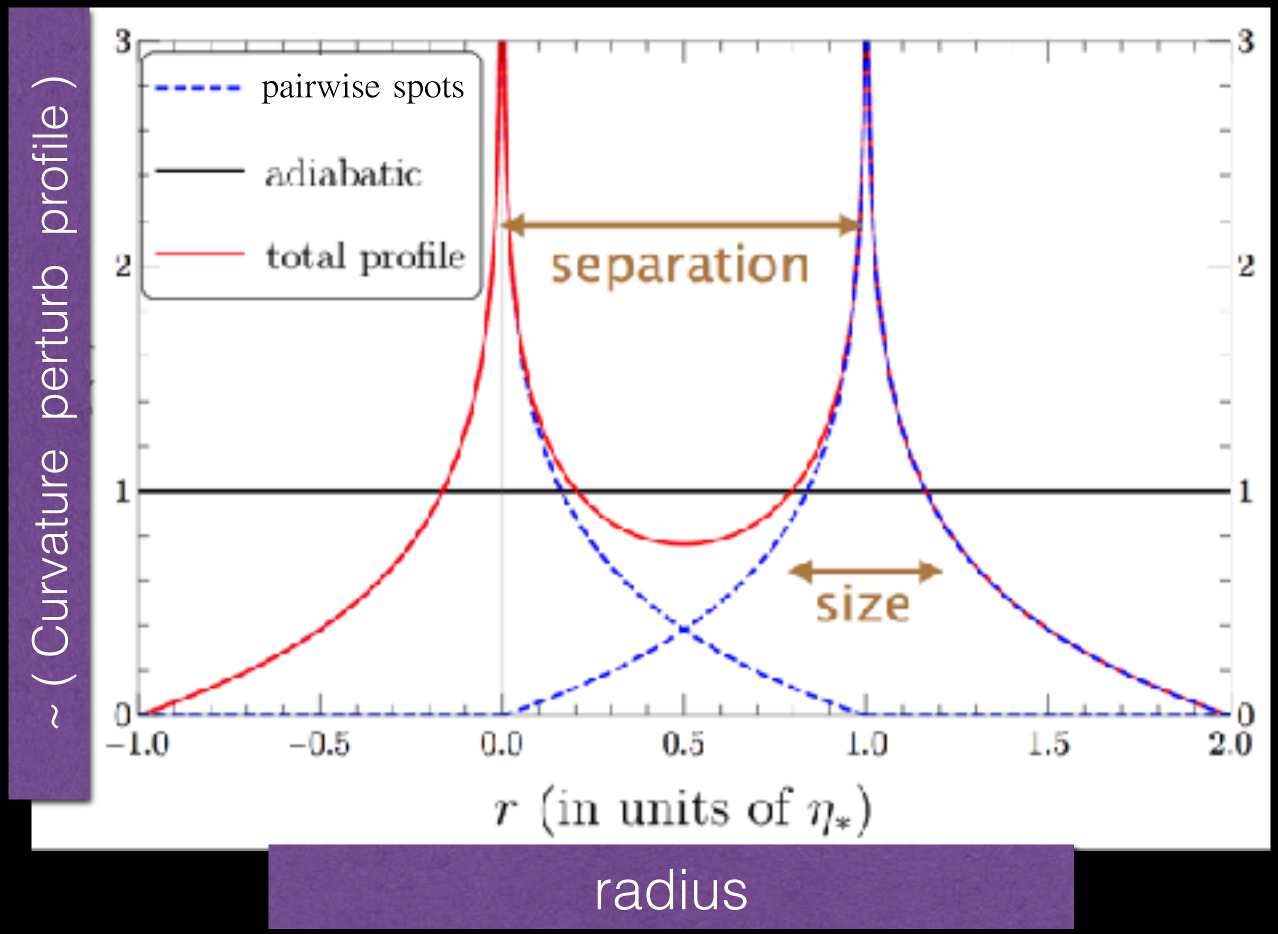
Spot size $\sim |\eta_*|$ and the coupling g controls the spot temperature over CMB fluctuations

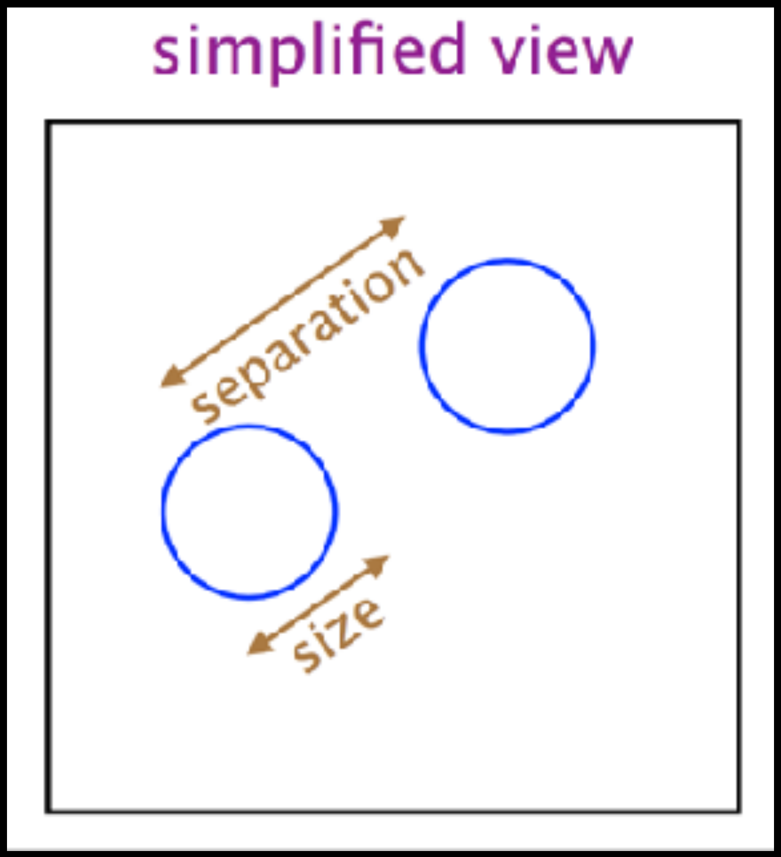
$$\phi - \phi_* = \dot{\phi}(t - t_*) = -\frac{\dot{\phi}}{H_*} \log \left(\frac{\eta}{\eta_*} \right)$$

log comes from the exponential growth of scale factor during the inflation

Heavy particles are produced in pairs:

momentum conservation





Particles are produced non-relativistically and have separation $\leq |\eta_*|$

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Hot or Cold spots?

Perturbation enters in the radiation-dominant & matter-dominant era has temperature fluctuation

$$\left.\frac{\delta T}{T}\right|_{\rm CMB,\,RD} = -\frac{1}{3}\langle\zeta_\sigma\rangle \qquad \left.\frac{\delta T}{T}\right|_{\rm CMB,\,MD} = -\frac{1}{5}\langle\zeta_\sigma\rangle$$

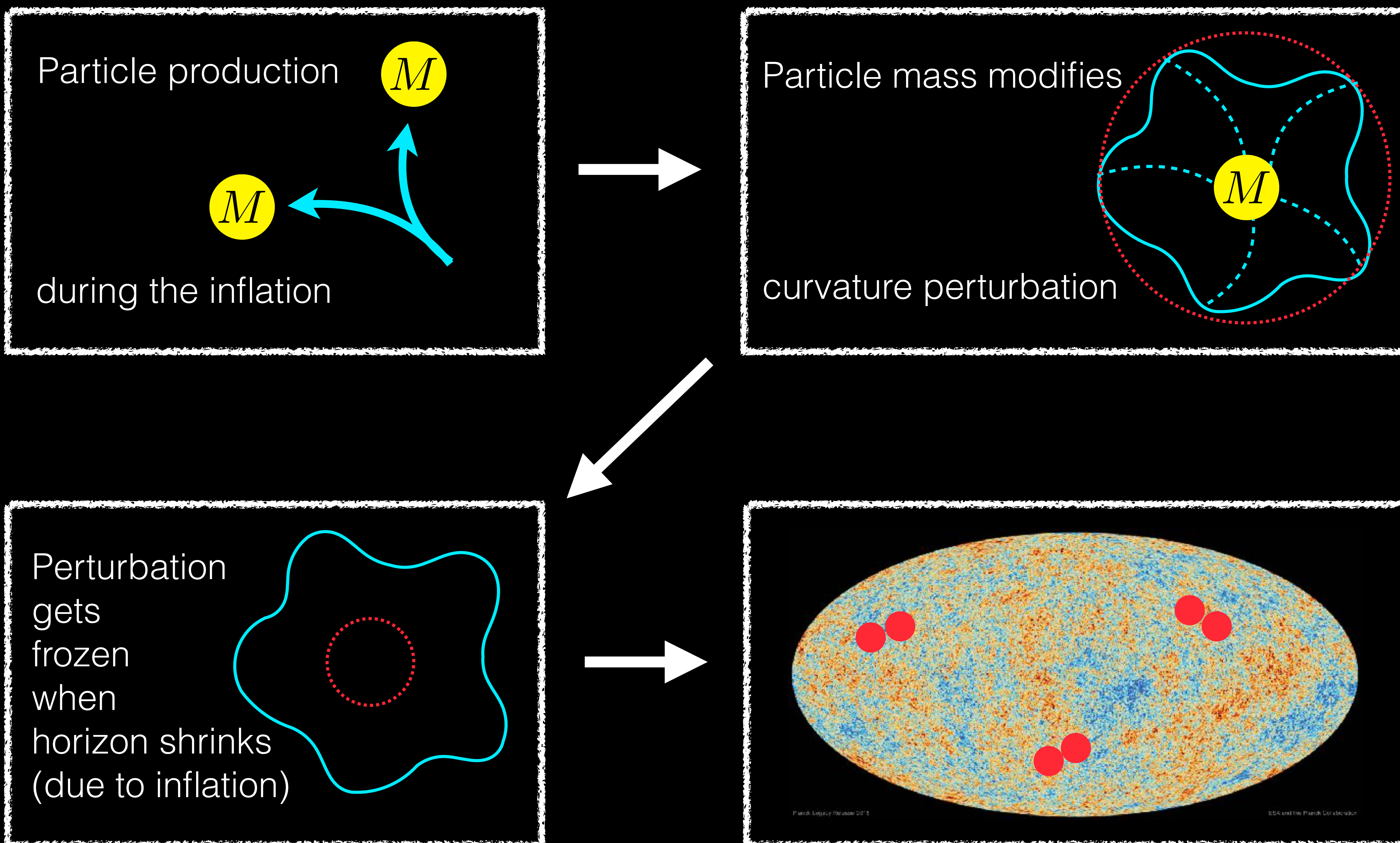
The minus sign comes from the gravity potential (Sachs-Wolfe), makes pairwise spots **COLD** before entering the horizon

However, we find that the baryon acoustic oscillation (sub-horizon phys) converts the signal into **HOT** spots and suppresses the fluctuation

We are currently calculating the sub-horizon evolution of δT_{sig}

see also Fialkov et al. (2009) for the pre-inflationary particle study

Step III: localized signals on the CMB



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For the following discussion,

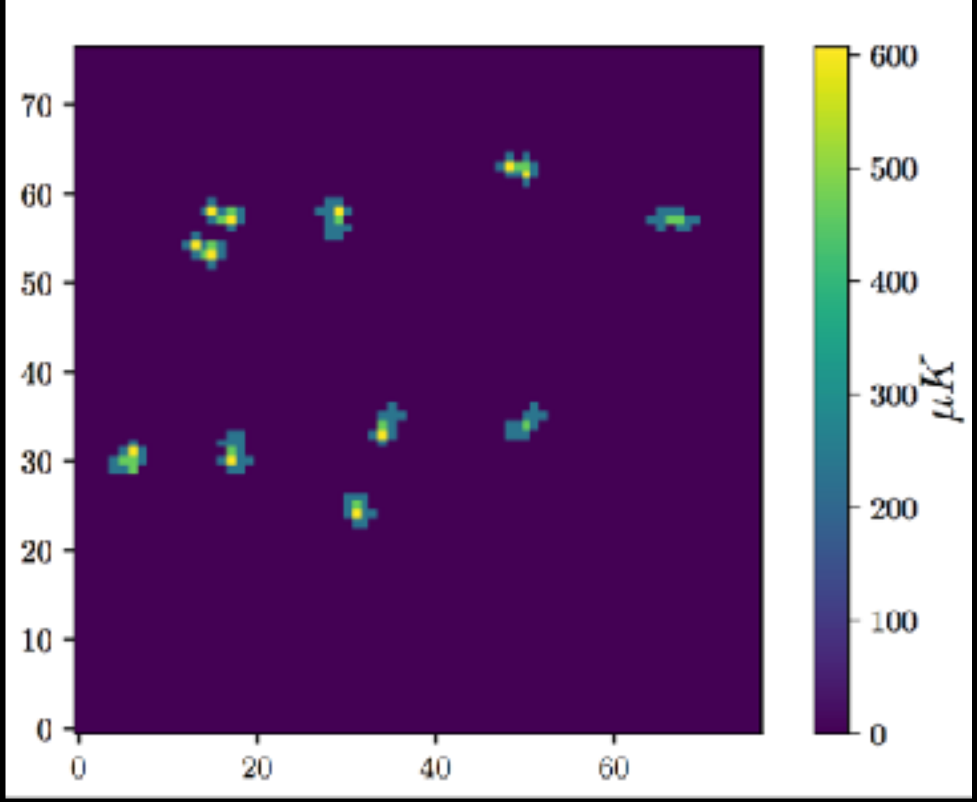
I will ignore contribution from sub-horizon physics

to temperature perturbation

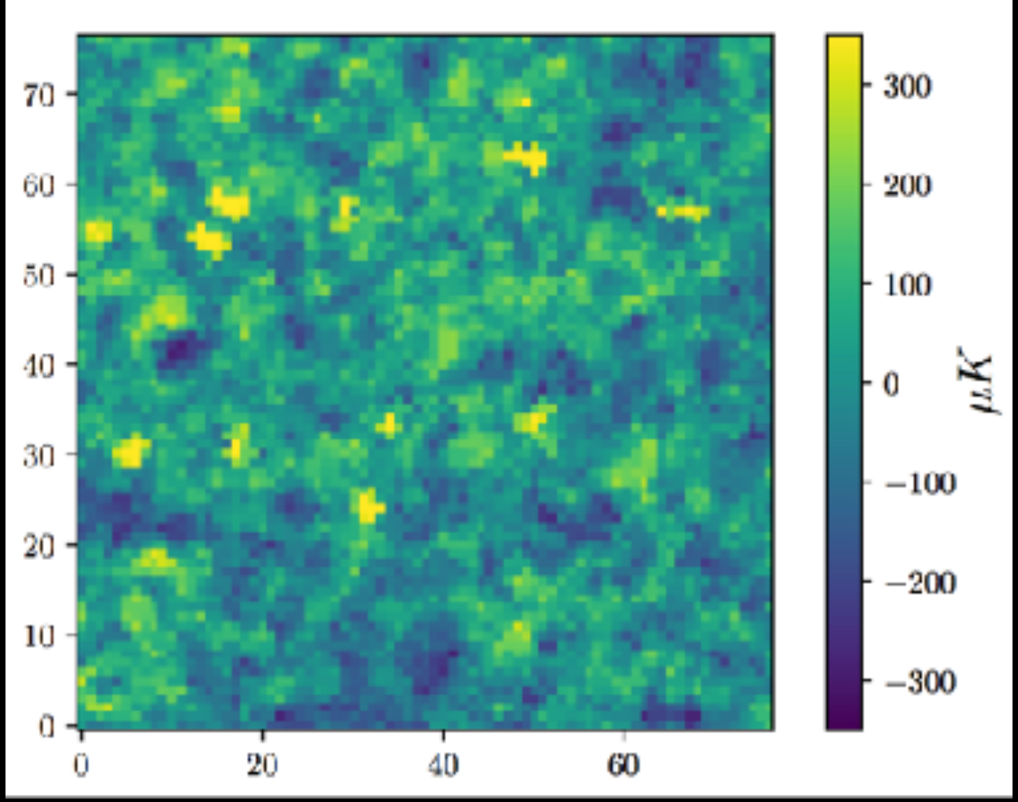
Simulate pairwise spot signals

- We use HEALPix to generate fake CMB image that follows the temperature fluctuation of the best fitted LCDM model
- for signal events, we add pairwise hotspots with a given temperature profile, pixel size, and separation between two spots

signal



signal + fake CMB



$$g = 30$$

only for illustration

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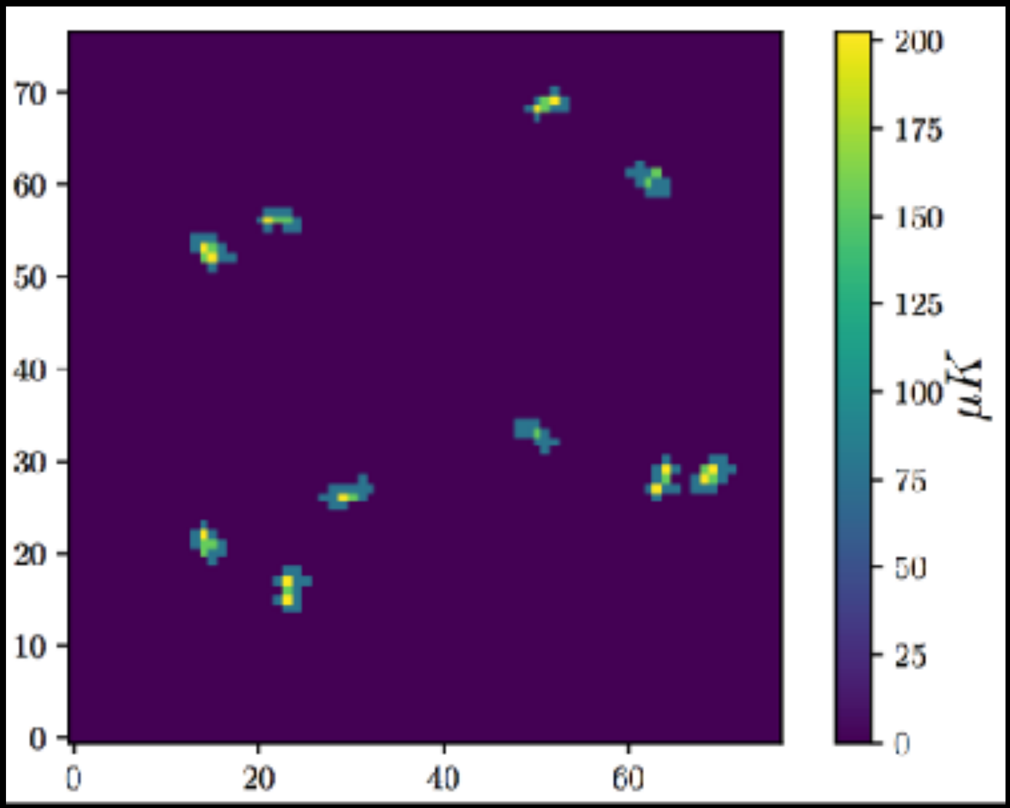
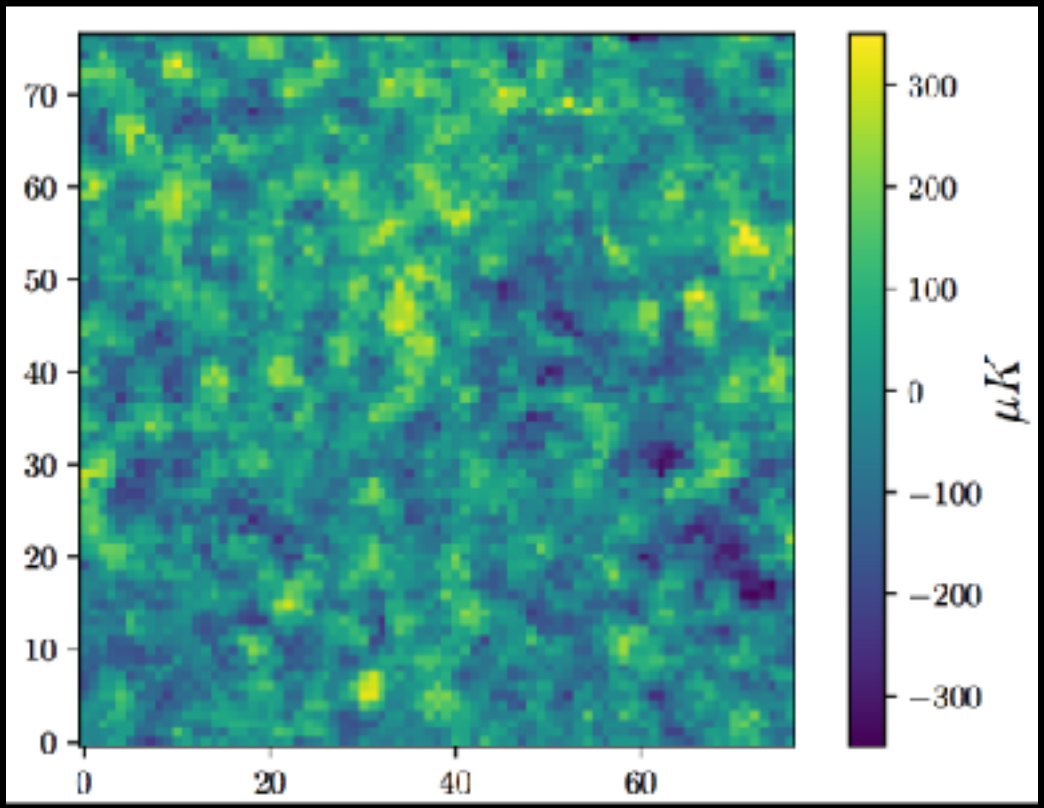
signal



signal + fake CMB



$$g = 10$$

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Identify signal on the CMB map

Different types of backgrounds to consider:

- instrumental noise
- fore-ground from compact objects (stars, galaxies,…)
- primordial fluctuation background (indistinguishable)

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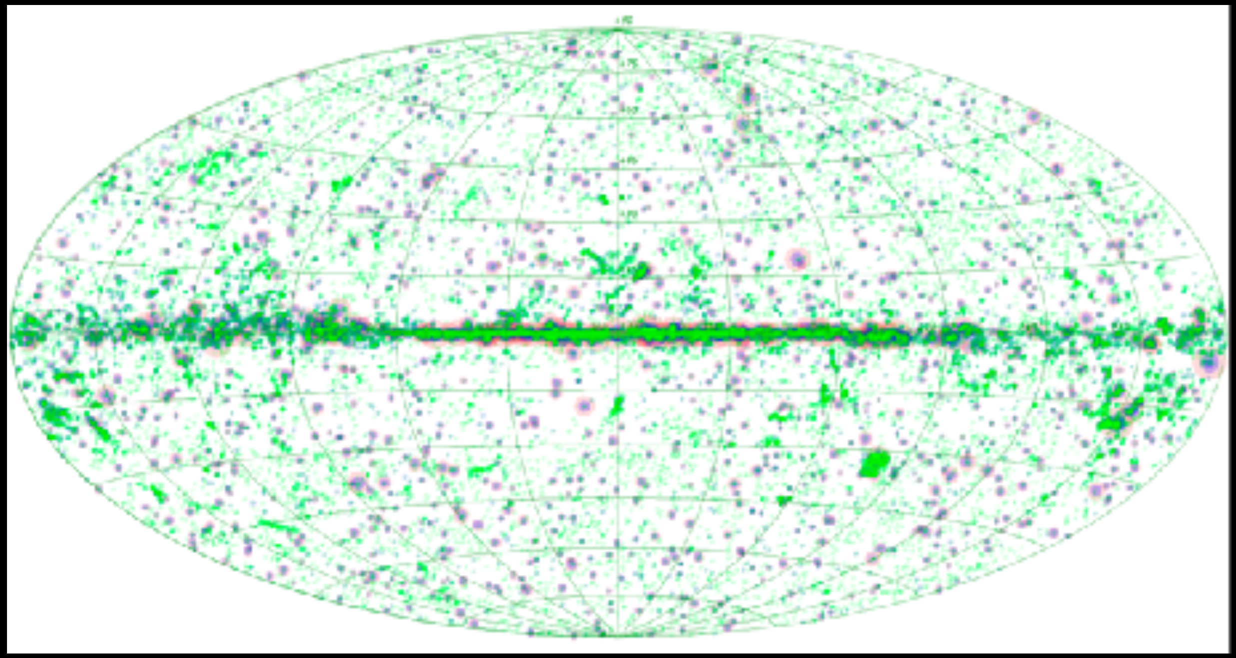

"may" veto the background by correlating Planck's maps in 9 frequency bands (need more study)

Planck 2013 results. XXVIII

Fig. 1. Sky distribution of the PCCS sources at three different channels: 30 GHz (pink circles); 143 GHz (magenta circles); and 857 GHz (green circles). The dimension of the circles is related to the brightness of the sources and the beam size of each channel. The figure is a full-sky Aitoff projection with the Galactic equator horizontal; longitude increases to the left with the Galactic centre in the centre of the map.

Identify signal on the CMB map

Different types of backgrounds to consider:

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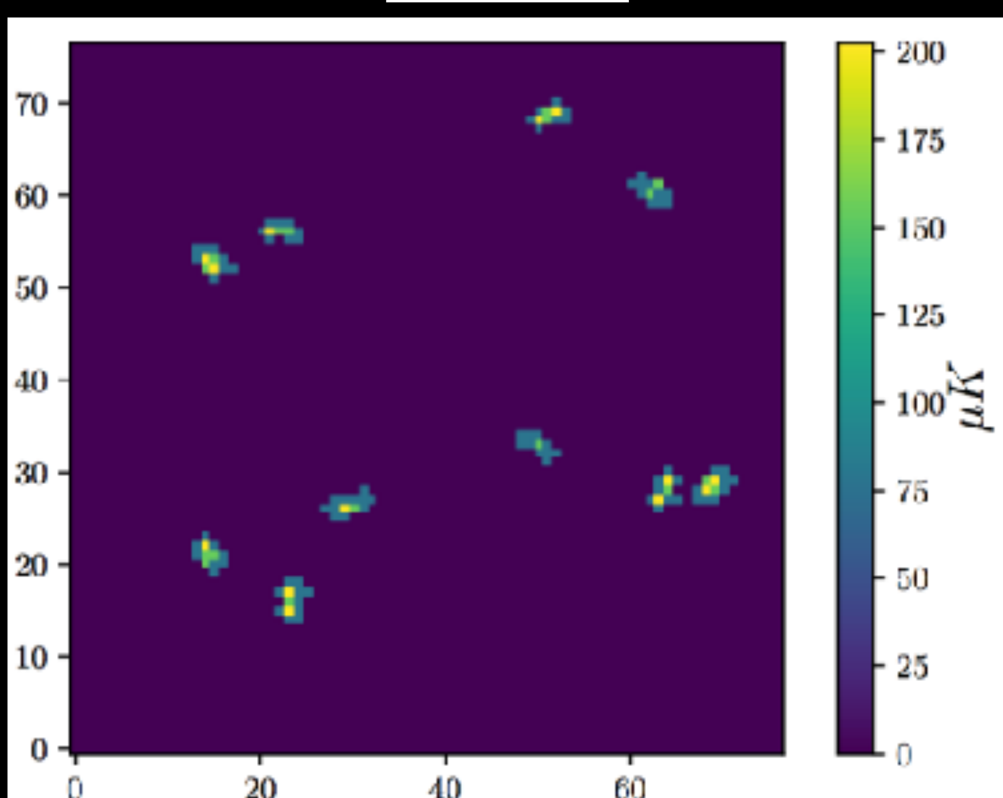
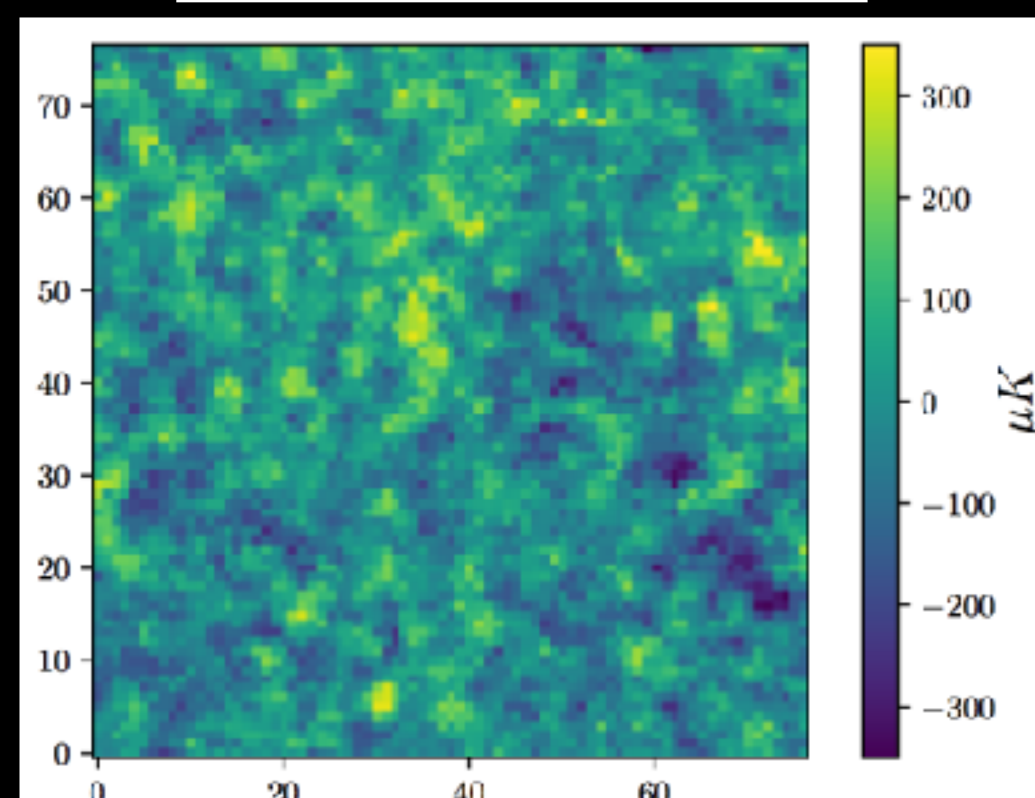
signal



signal + fake CMB



$$g = 10$$

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Here we only consider background
from primordial temperature fluctuations

Assume a perfect CMB measurement
with zero noise and perfect foreground subtraction

We are interested in the “irreducible” background
from primordial quantum fluctuations

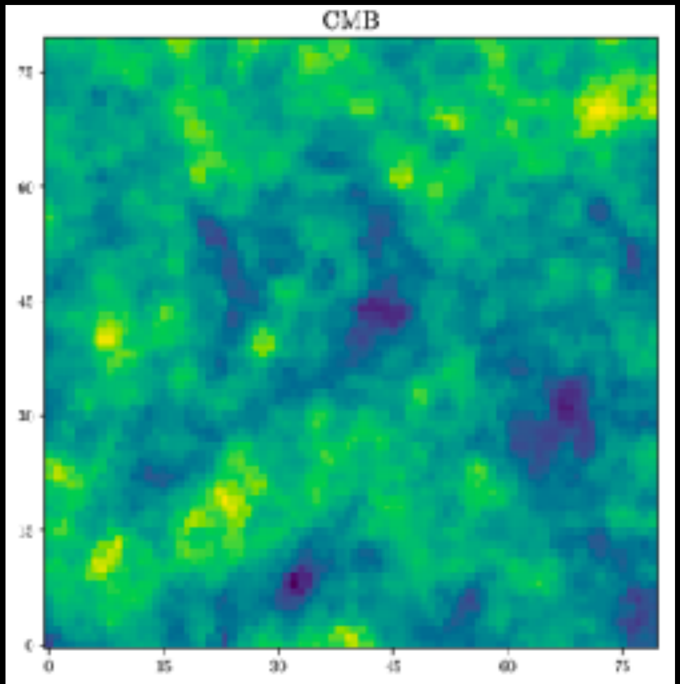
Sounds easy!

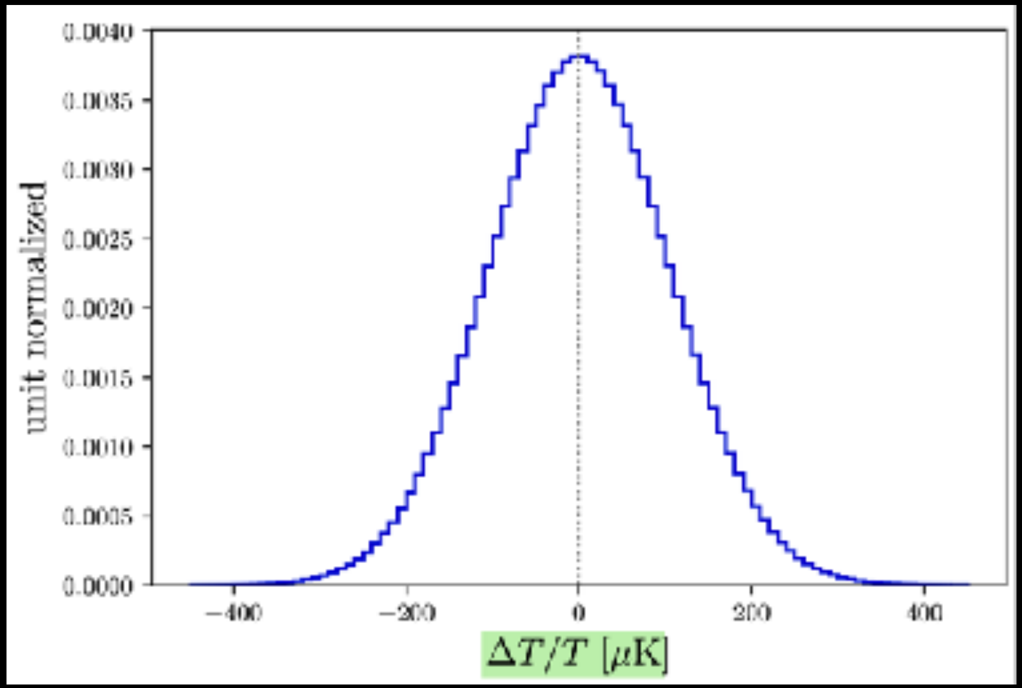
Since our signals are very hot (or very cold),

simply apply a minimal temperature cut to veto the background!

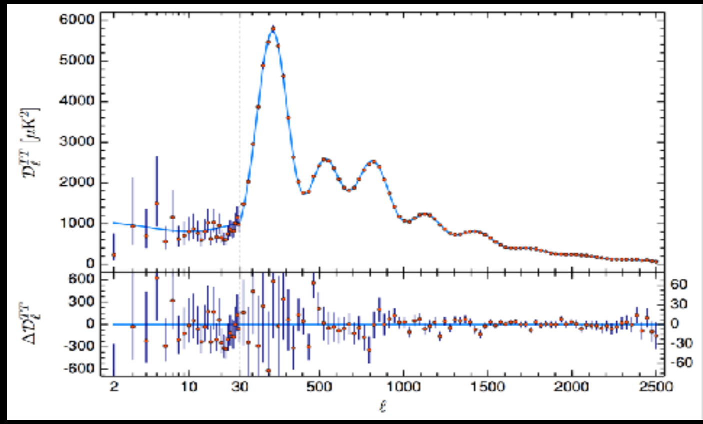
Well… primordial T-fluctuation can be very hot (or cold)

Our signal $\delta T_{\rm sig} \approx \frac{g}{2} 27\,\mu K$





Standard deviation of CMB fluctuation $\approx 100\,\mu$K



This is 4x larger than the primordial $\delta T \approx 27\mu$K of each l-mode in the CMB spectrum

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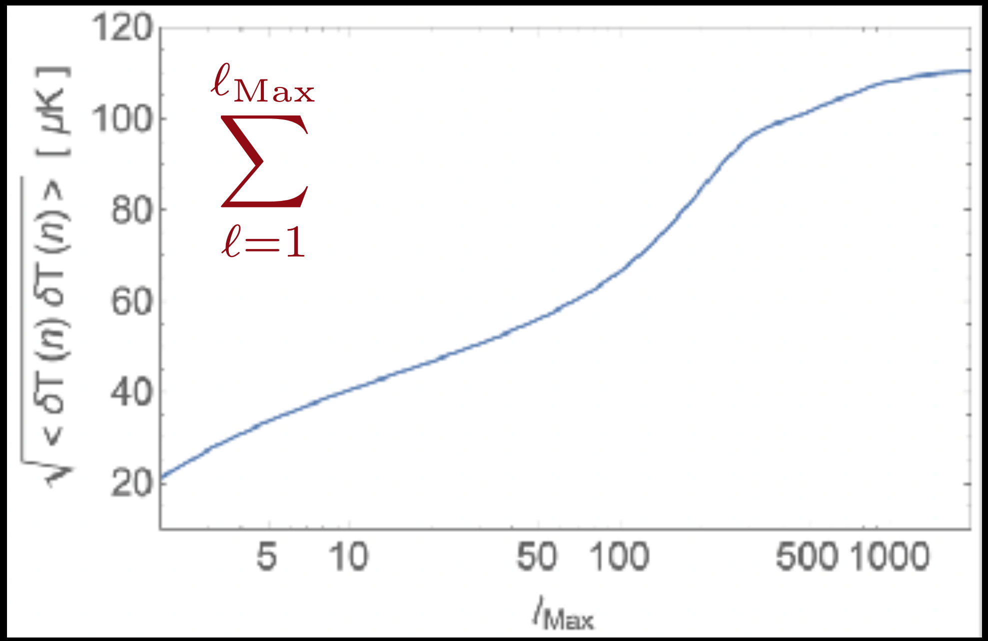
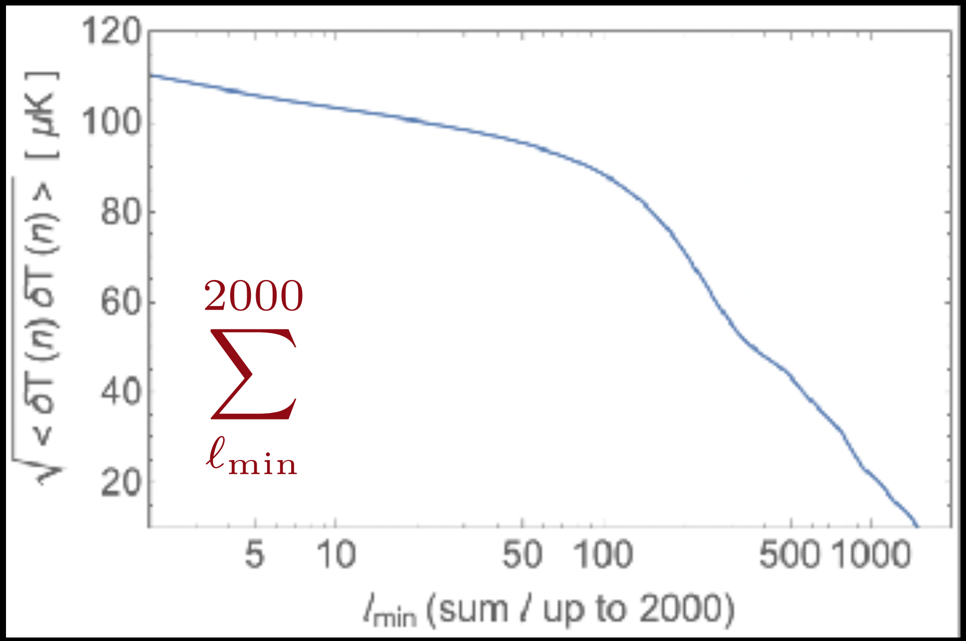
Well… primordial T-fluctuation can be very hot (or cold)

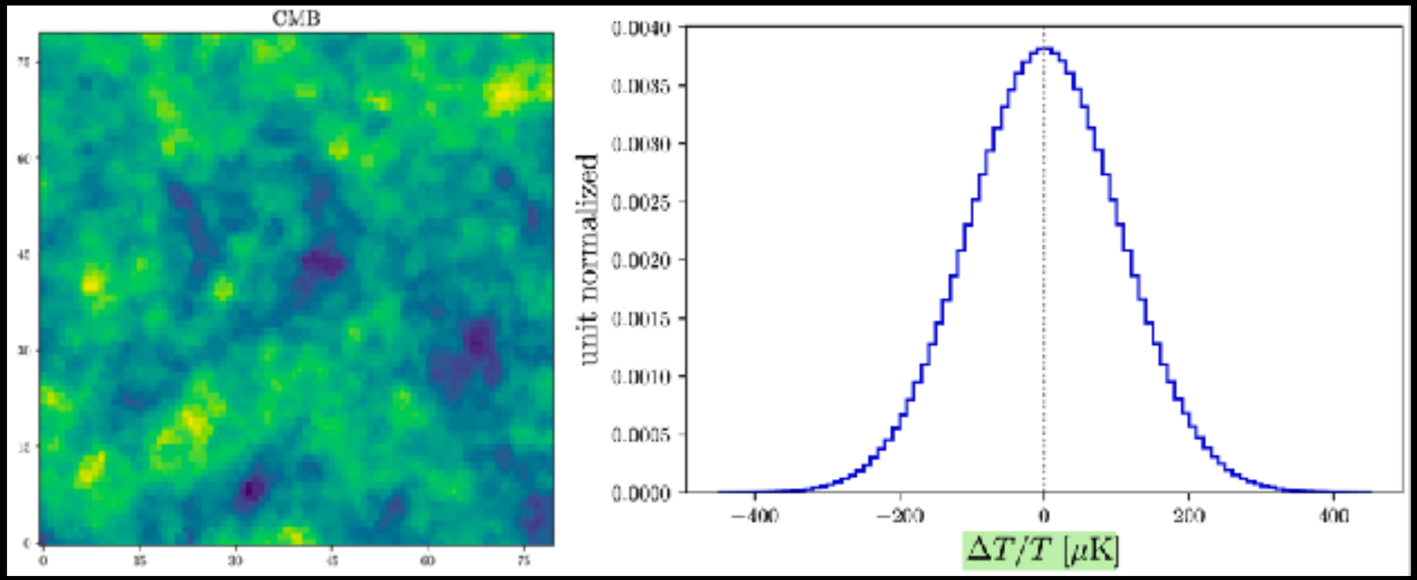
The large temperature fluctuation in position space comes from the sum of fluctuations with different wavelengths

$$\delta T^2\Big|_{\mathrm{CMB}} = \langle \delta T(\hat{\theta})\delta T(\hat{\theta})\rangle = \sum_{\ell} \frac{2\ell+1}{4\pi} C_\ell^{TT}$$



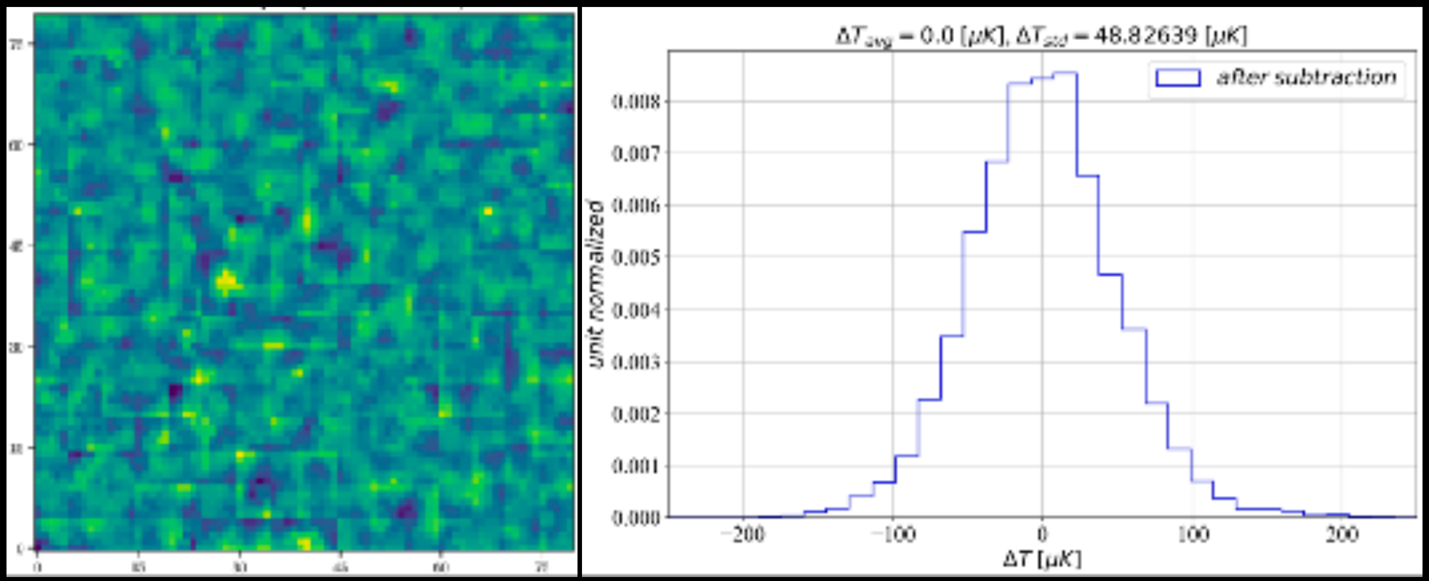


Subtract background from lower l-modes



Standard deviation $\approx 100\,\mu\mathrm{K}$

subtract avg T from each patch with area ~ l=500 mode



Standard deviation $\approx 50\,\mu\mathrm{K}$

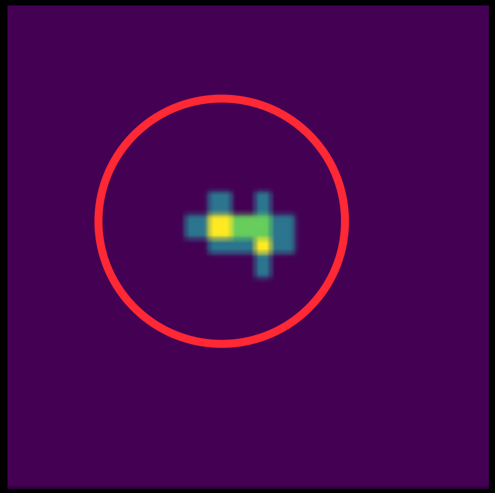
The subtraction plus additional δT cut does veto most of the fake signals

Isolation cut!

Since our signals show up in pairs,

we can draw a "cone" around each signal candidate

and require 2 spots inside the cone

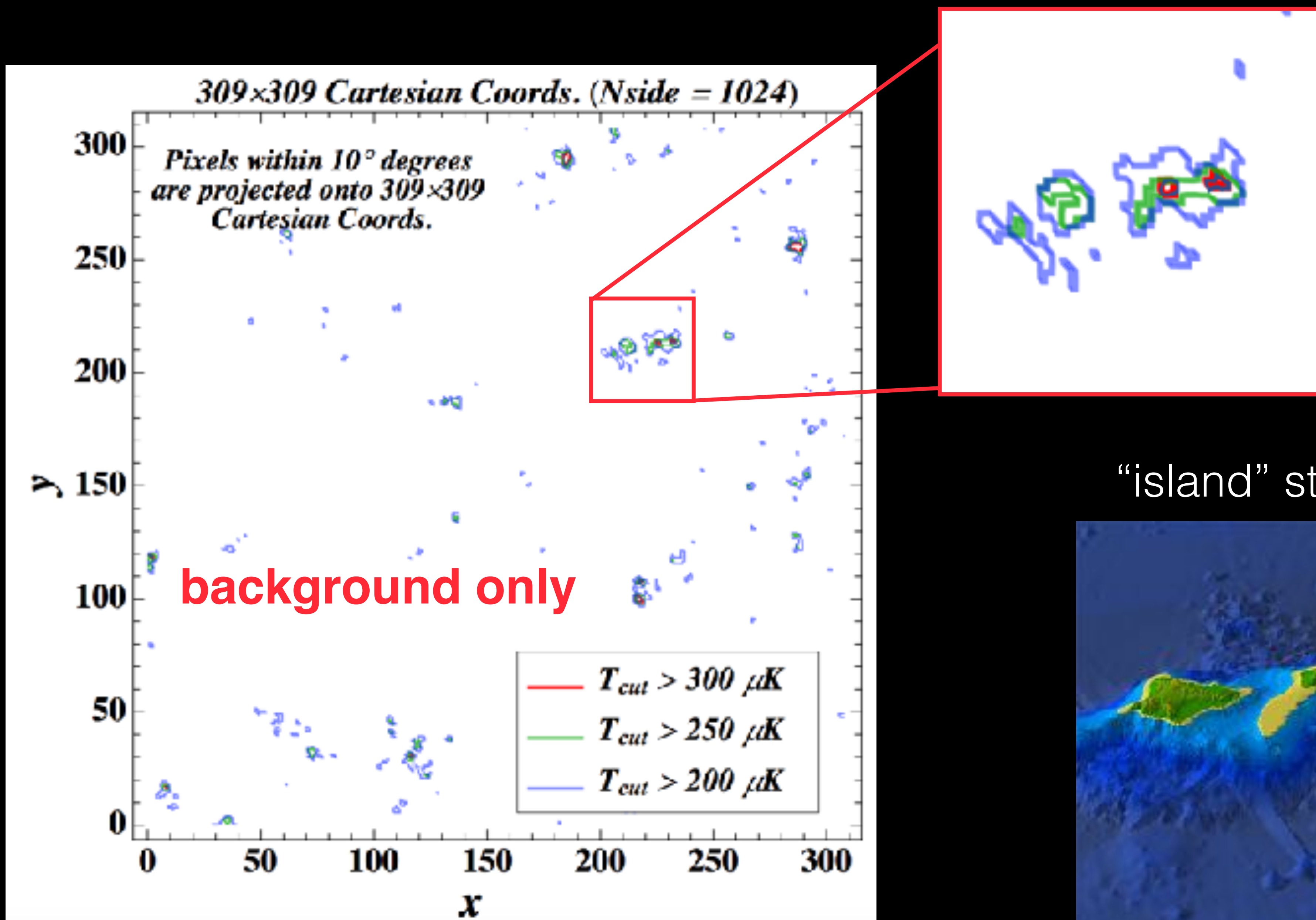


=> This indeed further reduces fake signals.

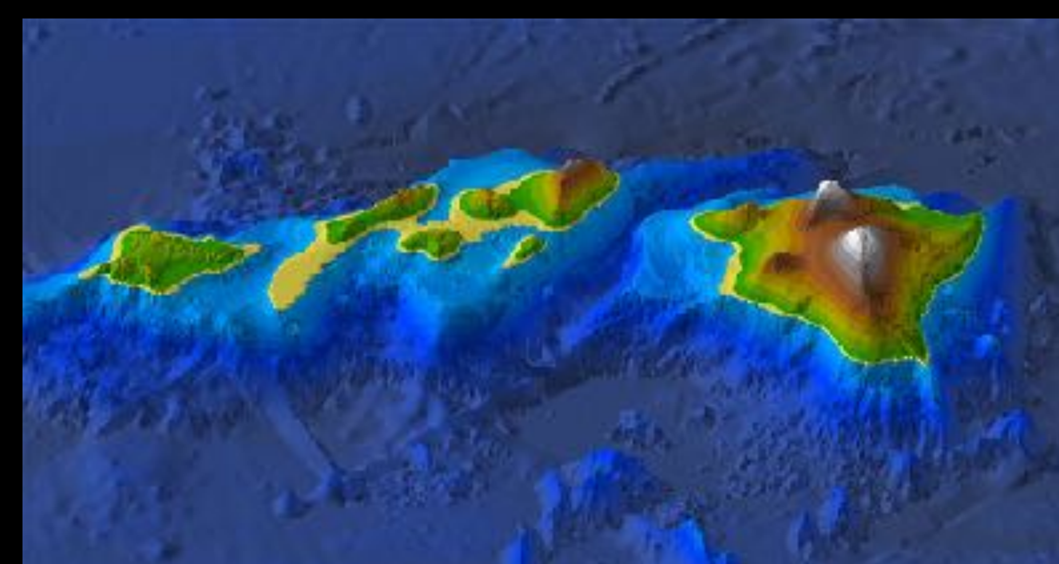
BUT it doesn't work as well as we thought

The very hot (or very cold) CMB spots

like to show up close to each other in position space



"island" structure in position space



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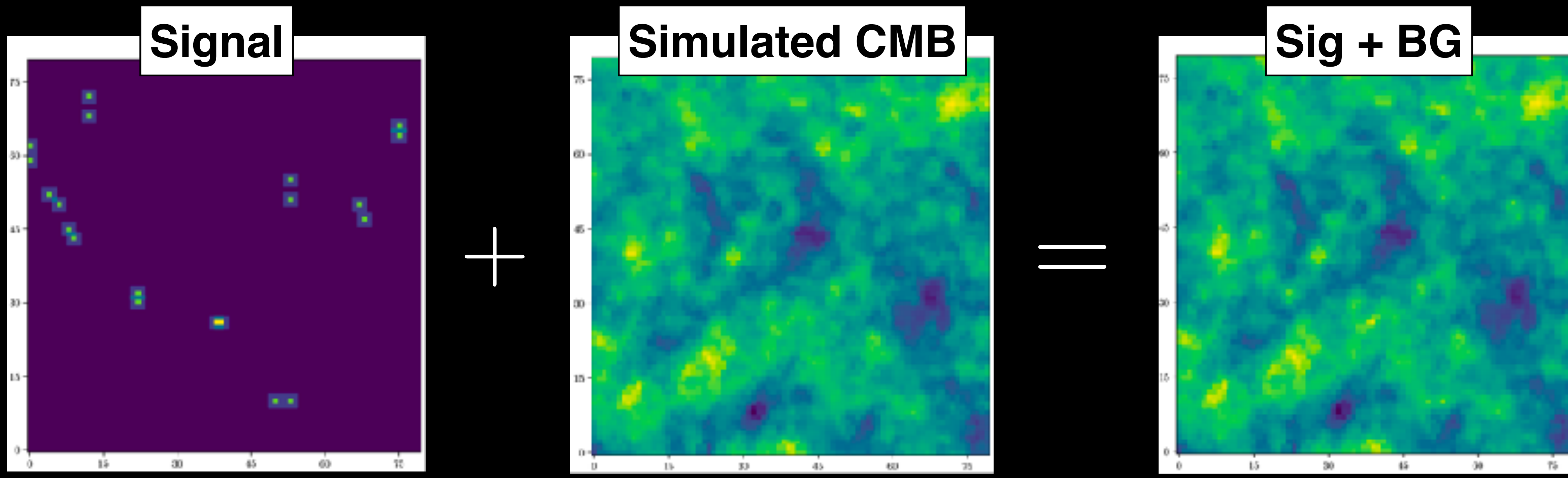
These are the procedures we currently use

$\delta T_{\rm sig} \approx 68\,\mu{\rm K}$



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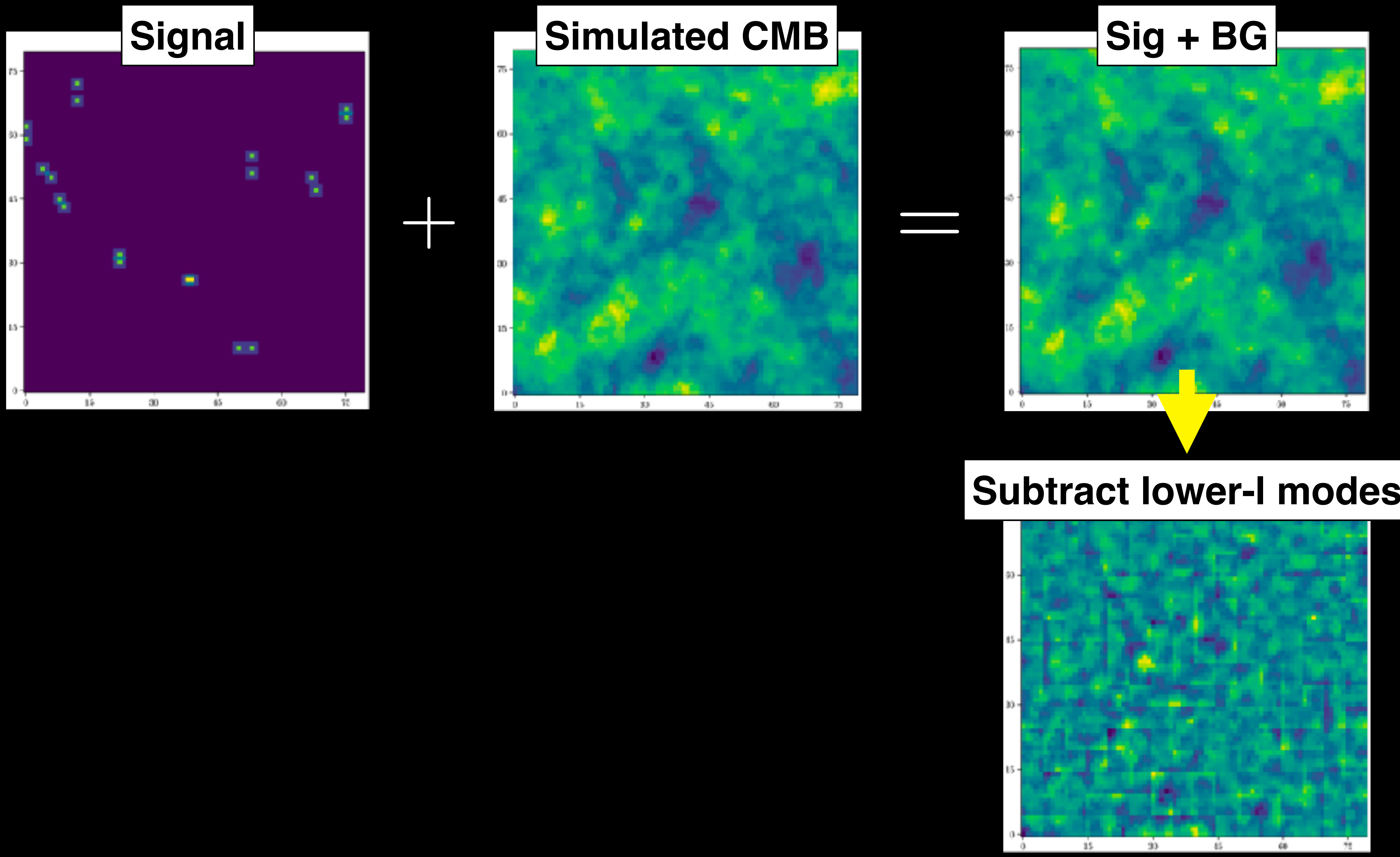

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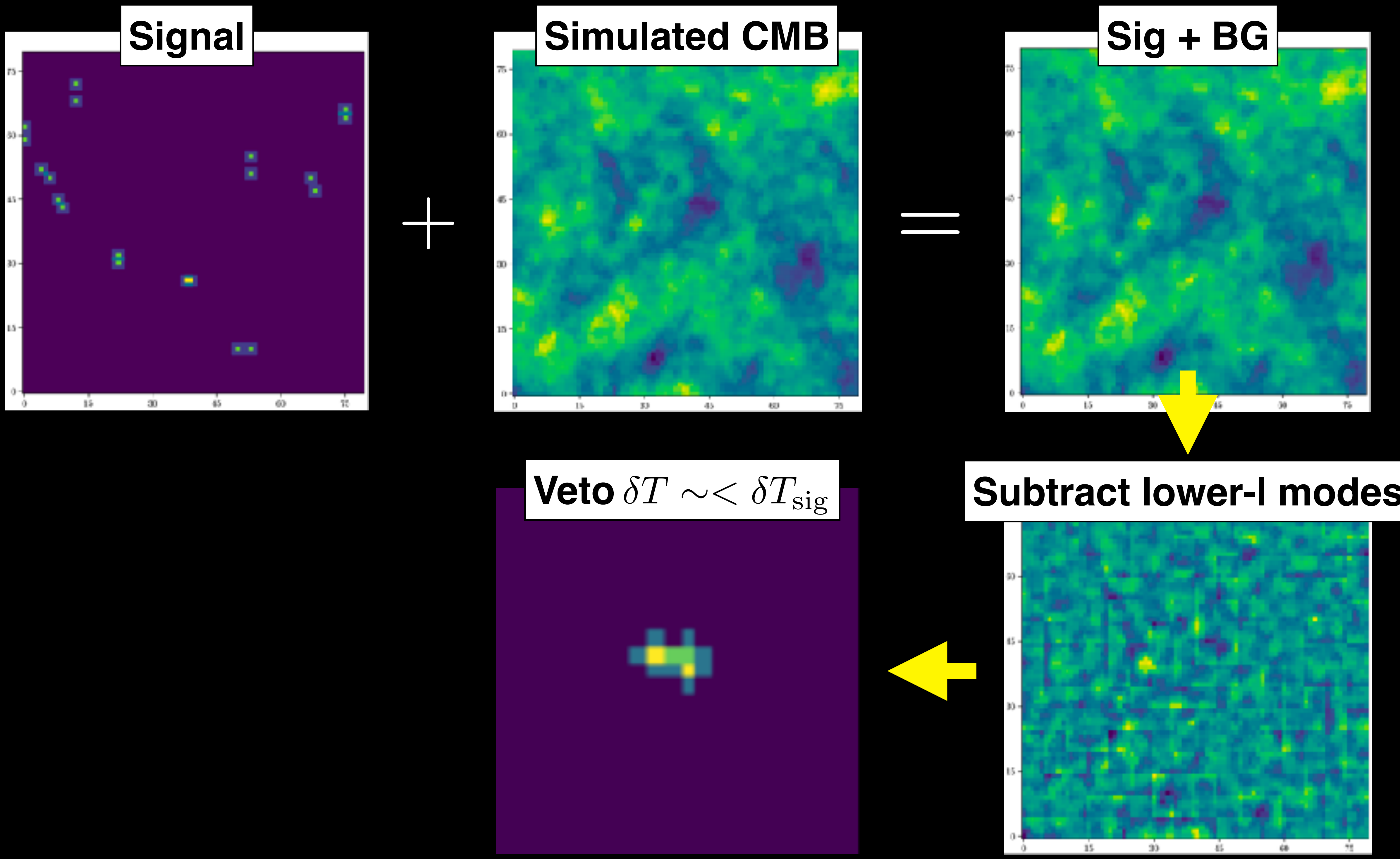

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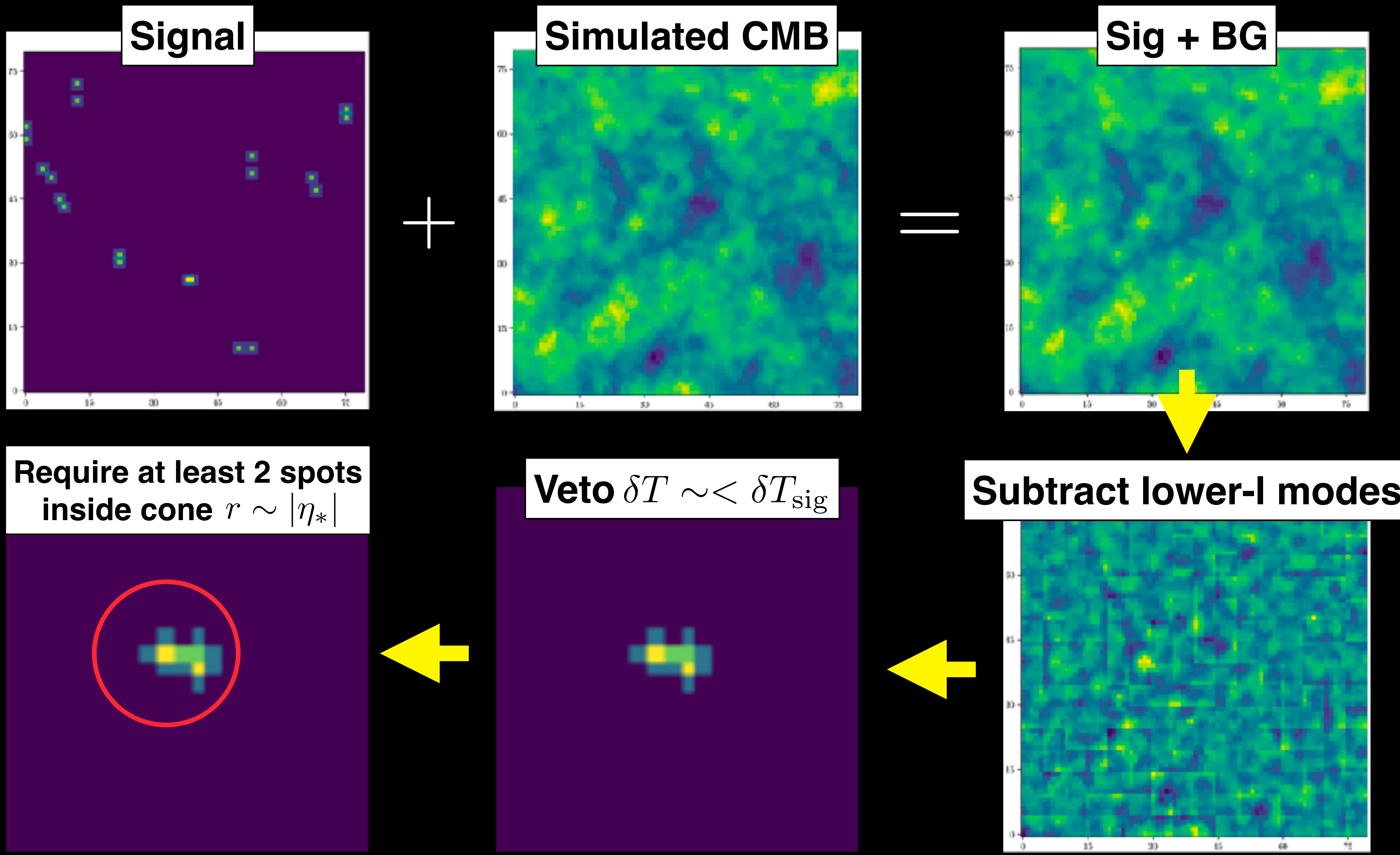

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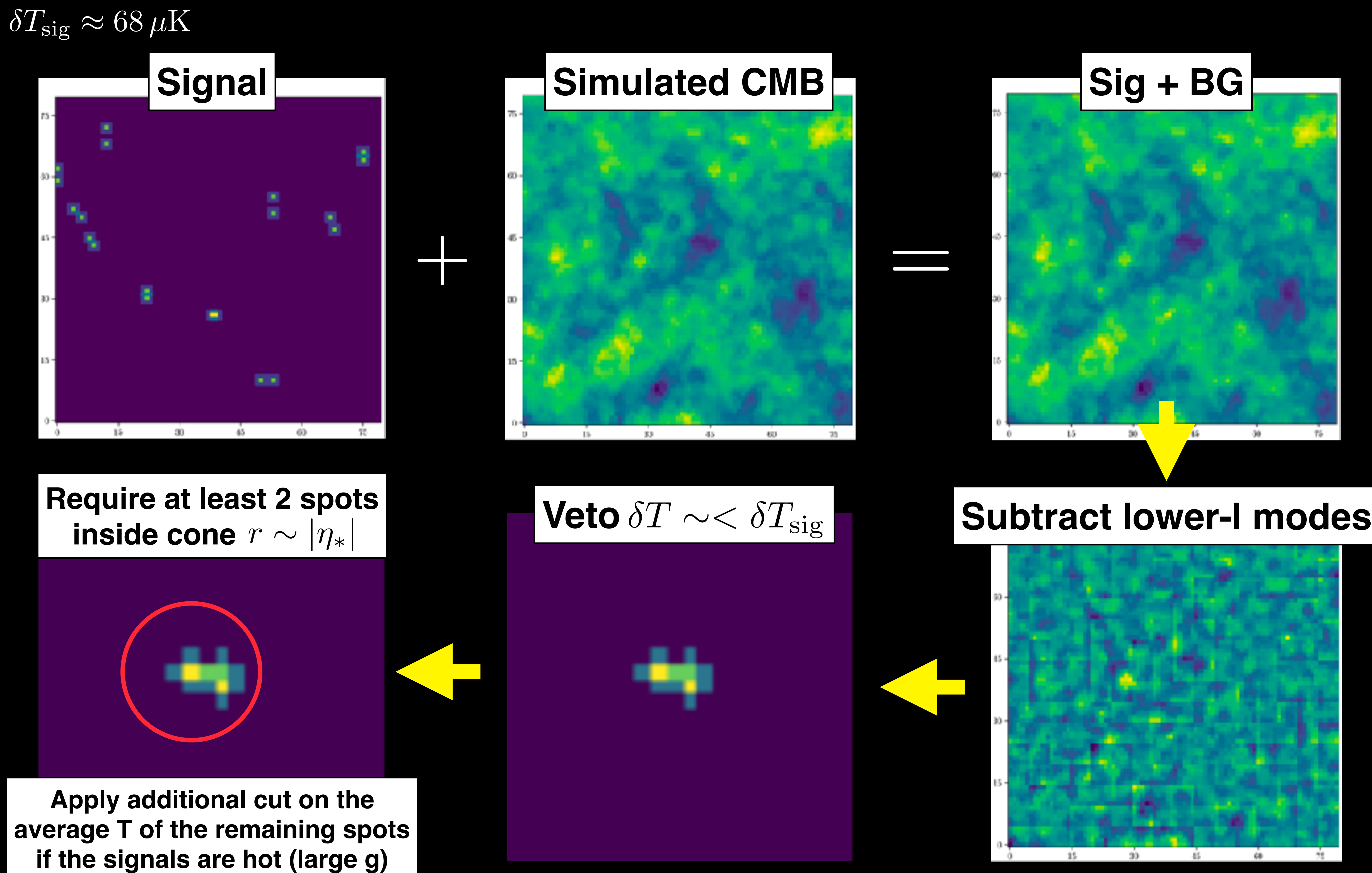

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Number of pairwise hotspots for 2 sigma excess

For signal hotspots with angular size around $\ell \approx 2500$ modes

$\delta T_{\rm sig}$ (μK)	135	95	68
Cuts (+ at least two spots inside the cone)	$\delta T_{\rm cut} = 180\,\mu$K $T_{\rm HS}^{avg} > 260\,\mu$K	$\delta T_{\rm cut} = 120\,\mu$K $T_{\rm HS}^{avg} > 170\,\mu$K	$\delta T_{\rm cut} = 70\,\mu$K $T_{\rm HS}^{avg} > 70\,\mu$K
Signal number for 2σ excess	90	560	1200

preliminary

Cut the sky into pixel number $N_{\rm pixel} \approx 10^7 = \ell^2_{\rm max}$, from simulation of $2 \cdot 10^4$ CMB maps

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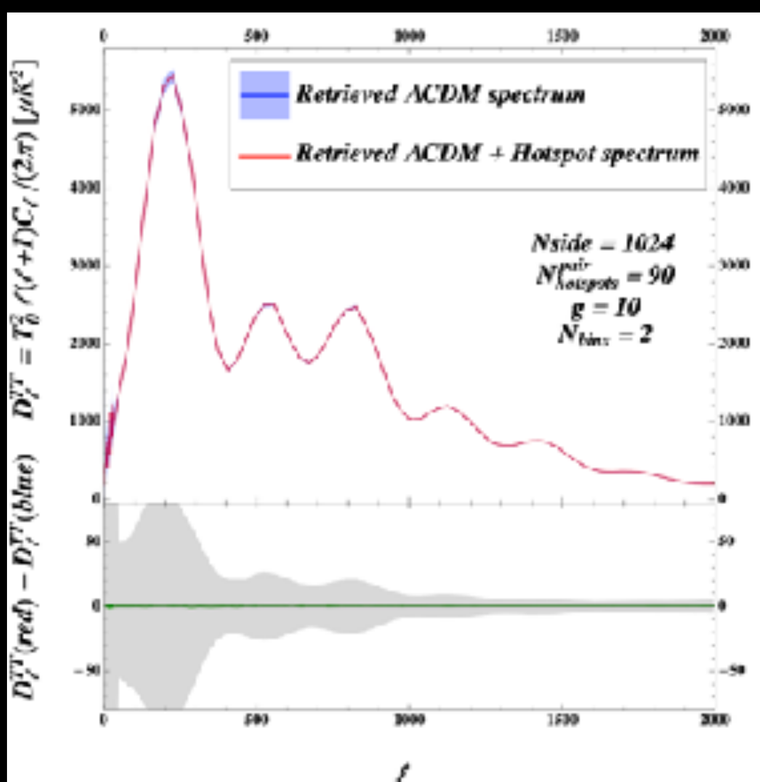
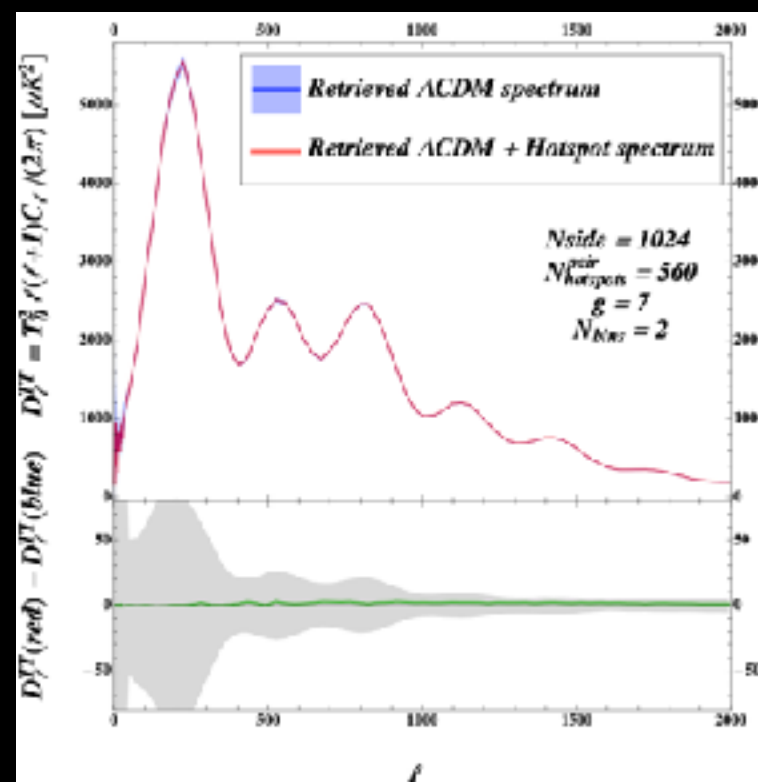
The corresponding C_ℓ^{TT} distortion

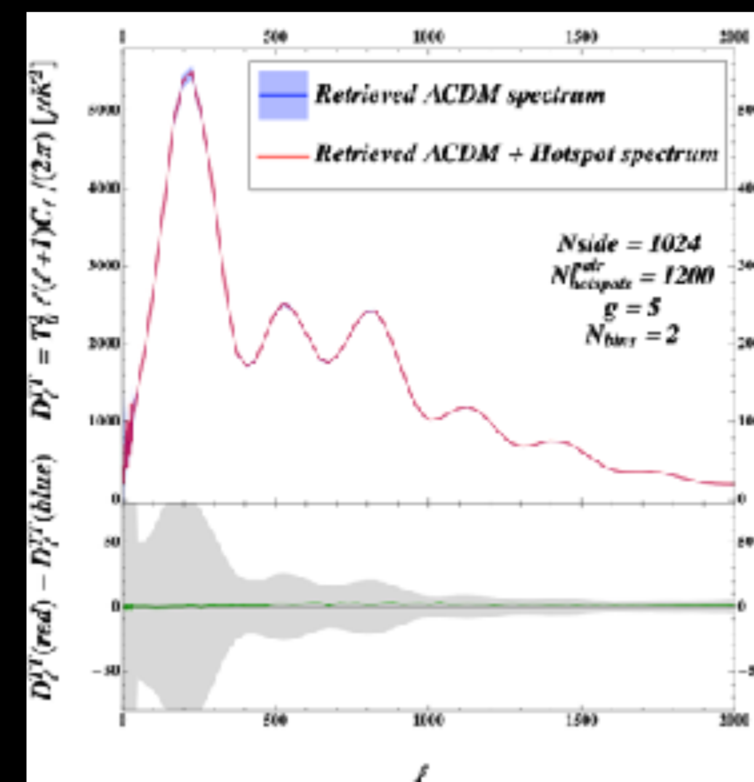
$\delta T_{\mathrm{sig}} = 135\mu\mathrm{K},\ N_{\mathrm{sig}} = 90$



$\delta T_{\mathrm{sig}} = 95\mu\mathrm{K},\ N_{\mathrm{sig}} = 560$



$\delta T_{\mathrm{sig}} = 68\mu\mathrm{K},\ N_{\mathrm{sig}} = 1200$



All the reduced chi2 are much less than 2sigma

The cut & count search provides a better probe of the signal for these δT_{sig}'s

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Bounds on the heavy particle mass

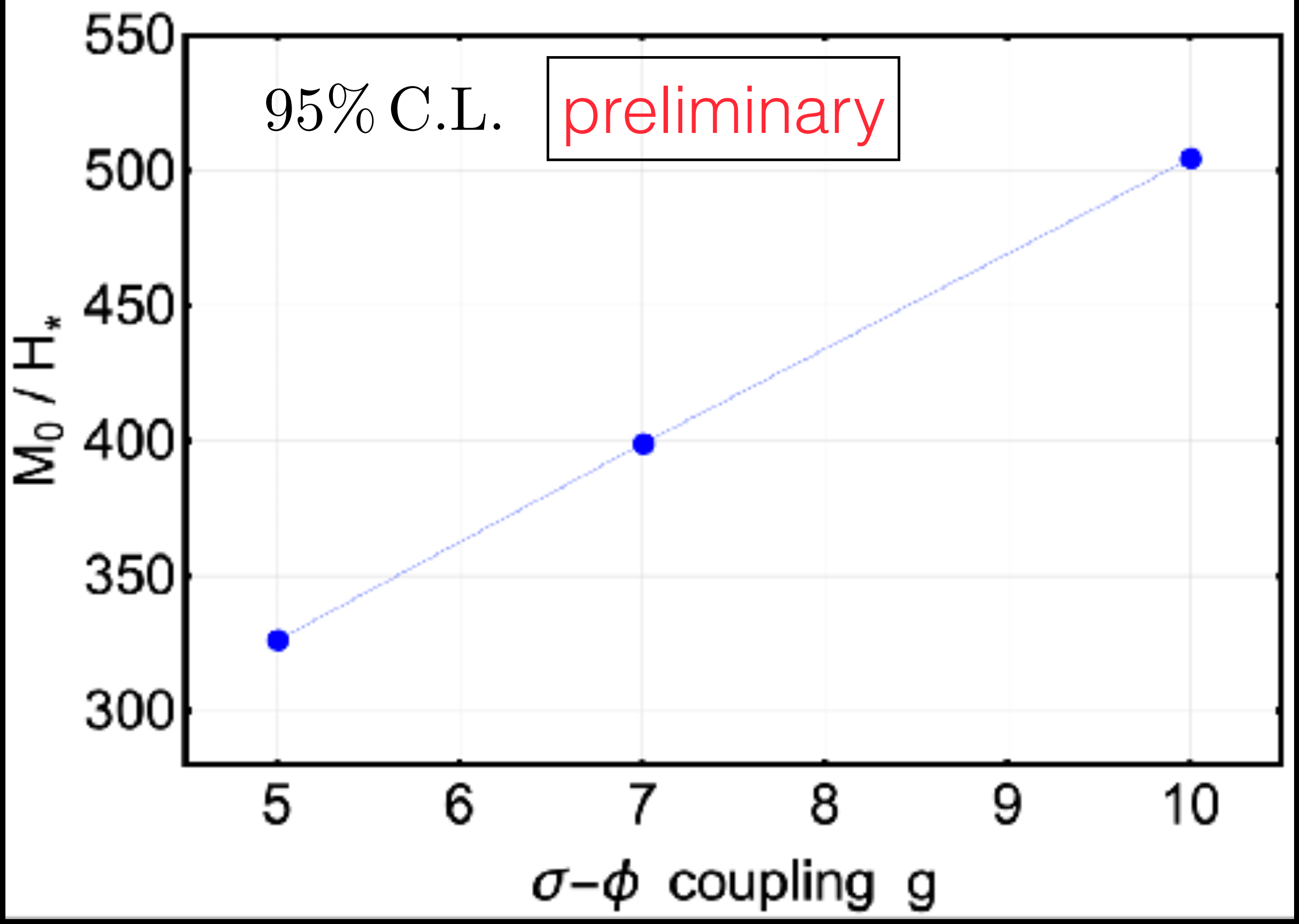
Before including corrections from sub-horizon physics

$$\delta T_{\rm sig} \approx \frac{g}{2}\, 27\,\mu K \qquad N_{\sigma\,\rm pairs} = \frac{1}{2\pi^2}\left(\frac{g\dot\phi}{H_*^2}\right)^{3/2} e^{-\frac{\pi(M_0^2-2H_*^2)}{|g\dot\phi|}} \left(\frac{k_*}{k_{\rm CMB}}\right)^3 \left(\frac{\Delta\eta_{rec}}{\eta_{rec}}\right)$$

$$M_{\rm eff}^2 \approx M_0^2 + g^2\phi'^2(\eta-\eta_*)^2$$

Lower bounds on the bare mass M_0 of σ.

Sensitive to mass ~ 100x of Hubble



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Conclusion

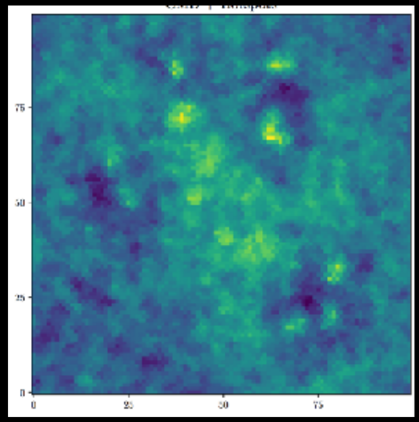
Production of heavy particles with inflaton-dependent mass generate pairwise spots on the CMB map

Can use both "position space" and "N-point function" studies to dig out the signal

More things to explore:

- different heavy field potential can generate different CMB signals
- improving search by wavelets or deep learning technique?
- pairwise clumps in Large Scale Structure? CMB lensing, cosmic shear, etc?

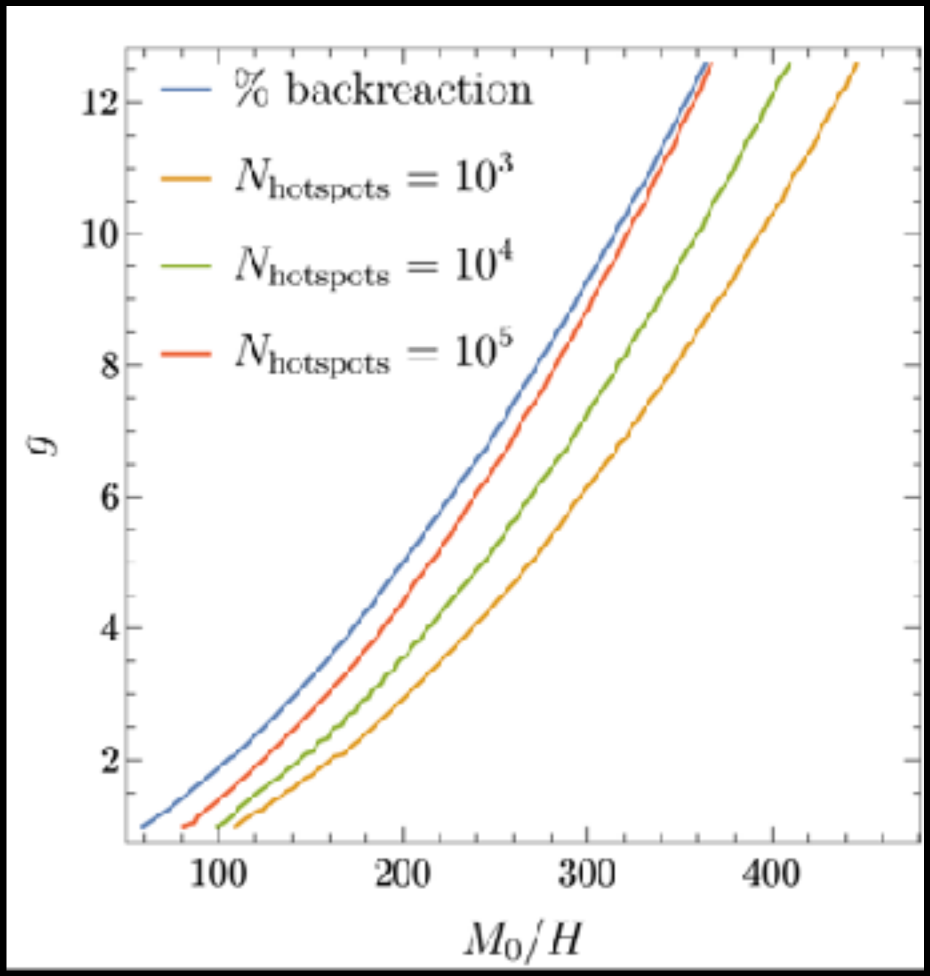
Thank you!



Backup Slides

Number of σ pairs in the CMB last scattering surface (with a thickness)

$$N_{\sigma\,\mathrm{pairs}} = \frac{1}{2\pi^2}\left(\frac{g\dot{\phi}}{H_*^2}\right)^{3/2} e^{-\frac{\pi(M_0^2-2H_*^2)}{|g\dot{\phi}|}} \left(\frac{k_*}{k_{\mathrm{CMB}}}\right)^3 \left(\frac{\Delta\eta_{rec}}{\eta_{rec}}\right)$$



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More details on the Sigma production

Expand σ mass around inflaton value at η_* (min- m_σ for particle production)

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\tau^2} + (\kappa^2 + \tau^2)u = 0$$

$$\tau = \gamma(\eta - \eta_*) \quad \kappa^2 = \frac{k^2}{\gamma^2} + \frac{M_0^2 - 2}{\eta_*^2 \gamma^2} \quad \gamma^4 = \frac{g^2 \phi'^2}{\eta_*^2}$$

The solution is a combination of parabolic cylinder functions

$$u = i\sqrt{\sigma} W(-\frac{\kappa^2}{2}, +\sqrt{2}\tau) + \frac{1}{\sqrt{\sigma}} W(-\frac{\kappa^2}{2}, -\sqrt{2}\tau) \quad \sigma = \sqrt{1 + e^{-\pi\kappa^2}} - e^{-\pi\kappa^2/2}$$

have chosen the initial condition that the solution gives a positive frequency function at initial time

$$u \sim e^{-i\frac{1}{2}\tau^2}$$

$$\tau \to -\infty$$

More details on the Sigma production

$$u = i\sqrt{\sigma}W(-\frac{\kappa^2}{2}, +\sqrt{2}\tau) + \frac{1}{\sqrt{\sigma}}W(-\frac{\kappa^2}{2}, -\sqrt{2}\tau)$$

$$u \sim e^{-i\frac{1}{2}\tau^2}$$

$$\tau \to -\infty$$

However, at the late time, the solution contains a negative frequency mode

$$\tau \to +\infty \qquad u = \frac{2^{1/4}}{\sqrt{\tau}}\left[\left(\frac{i\sigma}{2} - \frac{i}{2\sigma}\right)e^{+\frac{i}{2}\tau^2} + \left(\frac{i\sigma}{2} + \frac{i}{2\sigma}\right)e^{-\frac{i}{2}\tau^2}\right]$$

$$\beta \qquad \alpha \qquad |\alpha|^2 - |\beta|^2 = 1$$

Therefore, Sigma is produced with a number density

$$n = \int \mathrm{d}^3k\, |\beta|^2$$