# Replica wormholes and the information paradox

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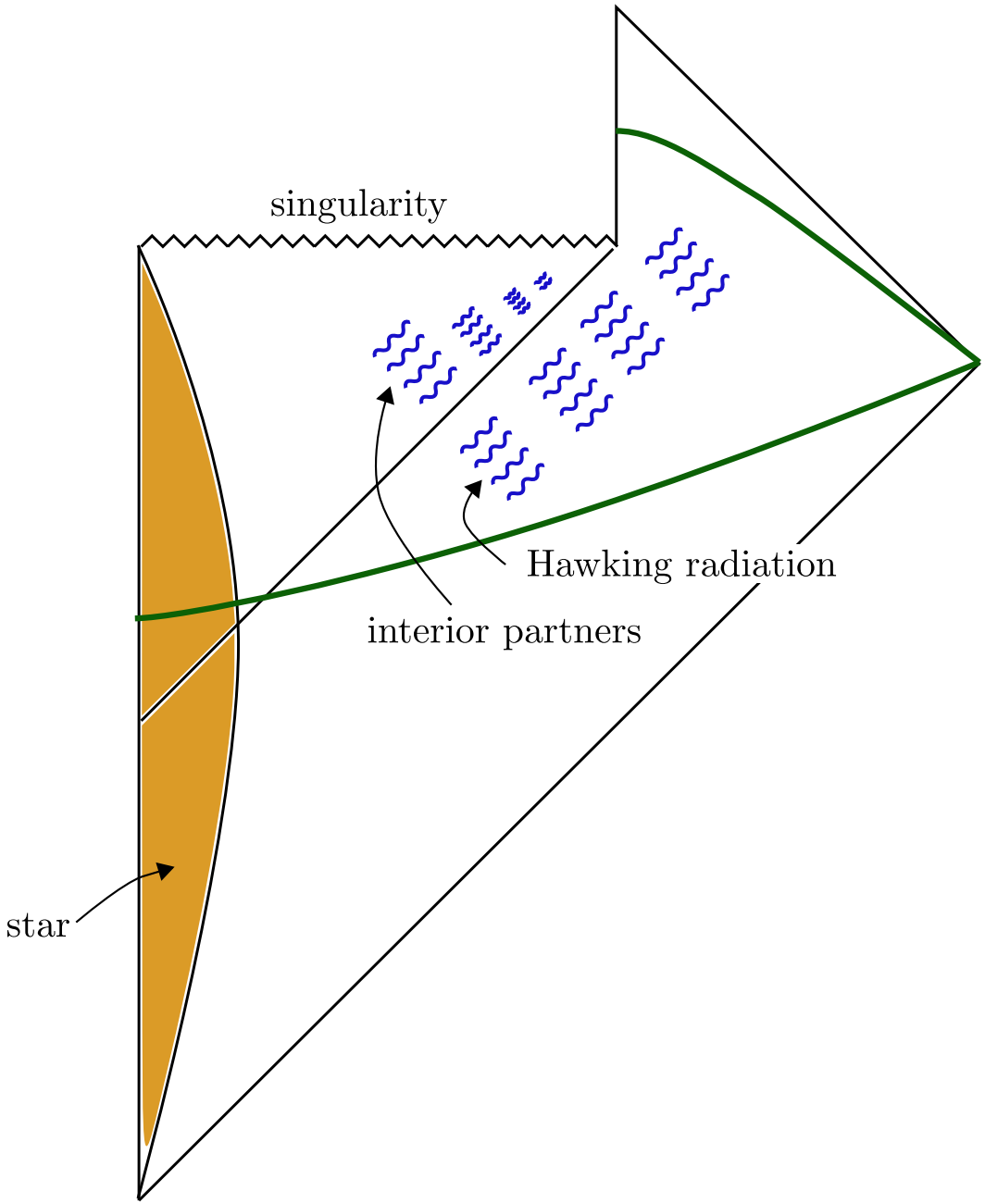
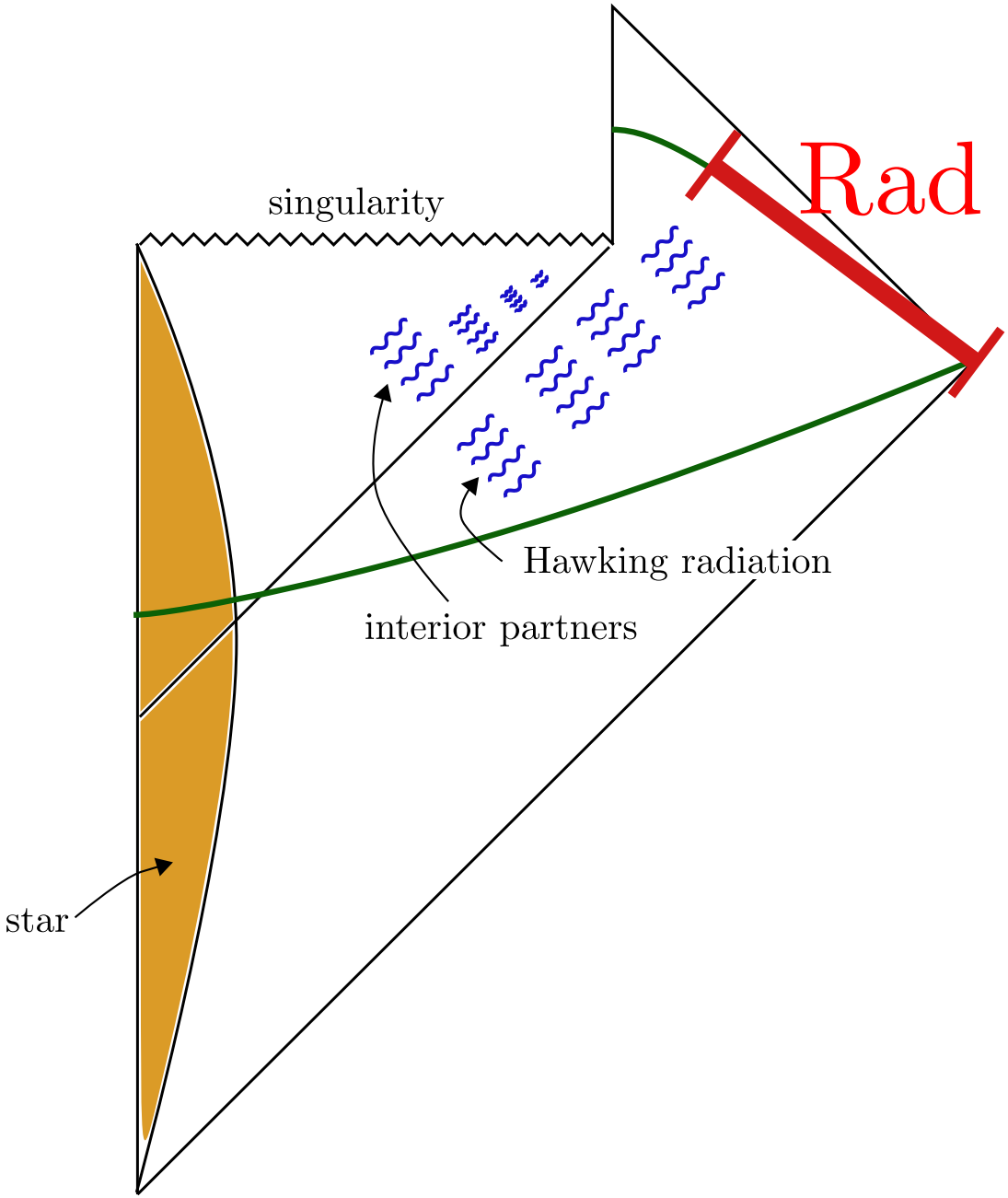
# Background:
# The Page Curve

# The information paradox




Hawking radiation is a process of *entanglement production* between the black hole interior and the radiation.

# The information paradox

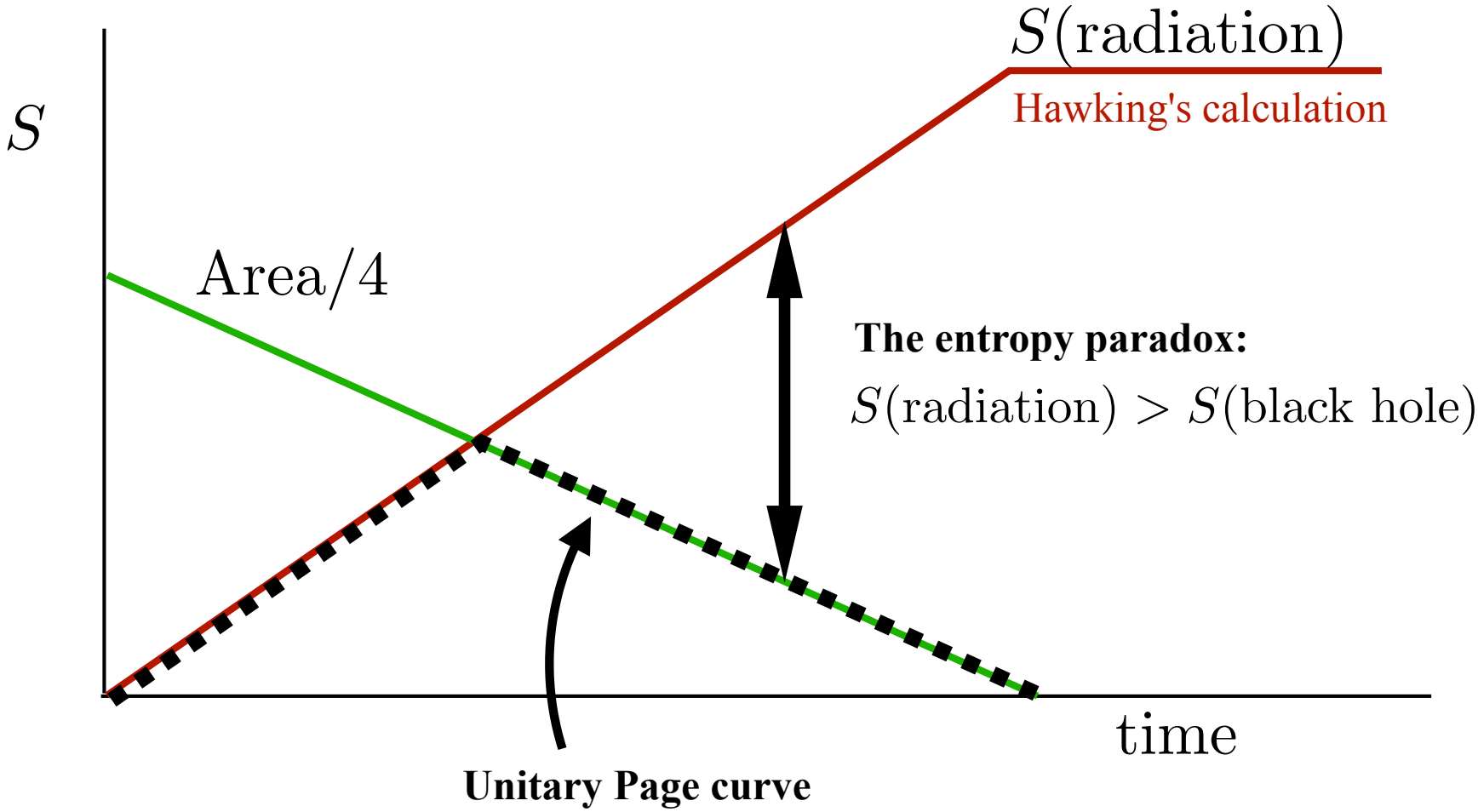



Consider the fine-grained (von Neumann) entropy of the radiation

$$S(\mathrm{Rad}) = -\mathrm{tr}\rho_R \log \rho_R$$

Fine-grained vs. coarse-grained

# The Page Curve



Hawking:

$$\rho_R = \rho_{\text{thermal}} + \text{perturbative} + \mathcal{O}(e^{-\#S})$$

$e^{-S}$ corrections to each matrix element $(\rho_R)_{mn}$ are big enough to fix the entropy

$$S(\text{rad}) = -\text{tr}\,\rho_R \log \rho_R$$

## Why is it a "paradox"?

- Local / perturbative corrections don't help
- No known *mechanism* for information escape

## Summary of new developments

$e^{-S}$ corrections to the gravitational path integral produce large corrections to the entropy.

This calculation gives a small entropy, consistent with unitary evaporation.

Nothing in this calculation requires string theory or AdS/CFT.

***Path integral methods sidestep some of the most difficult aspects of the paradox.***

***So this addresses just one piece of the information puzzle.***

## Holographic entanglement entropy

[Ryu and Takayanagi ’06], [Hubeny, Rangamani, Takayanagi ’07], [Lewkowycz, Maldacena ’13], [Barella, Dong, Hartnoll, Martin ’13], [Faulkner, Lewkowycz, Maldacena ’13], [Engelhardt, Wall ’14], [Dong, Lewkowycz ’17]

## The “Island formula” for the radiation entropy

[Penington ’19]
[Almheiri, Engelhardt, Marolf, Maxfield ’19]
[Almheiri, Mahajan, Maldacena, Zhao ’19]

## Replica wormholes

[Almheiri, TH, Maldacena, Shaghoulian, Tajdini ’19]
[Penington, Shenker, Stanford, Yang ’19]

Conceptual review article: arXiv 2006.06872 [Almheiri, TH, Maldacena, Shaghoulian, Tajdini]

# Islands

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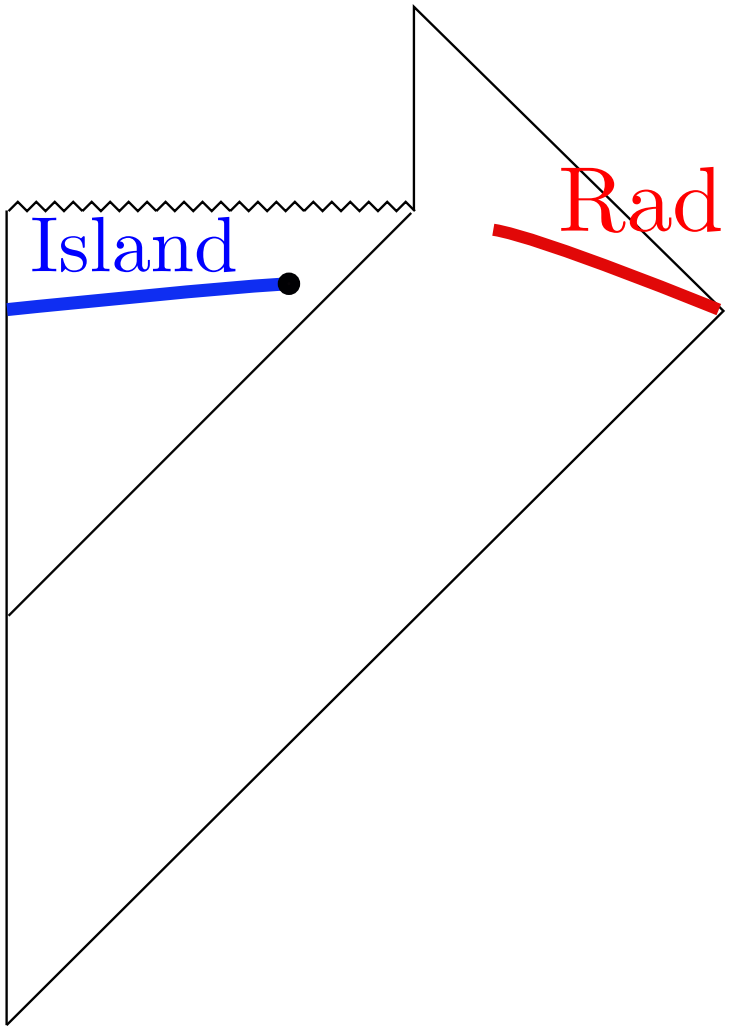

# The island formula for radiation entropy

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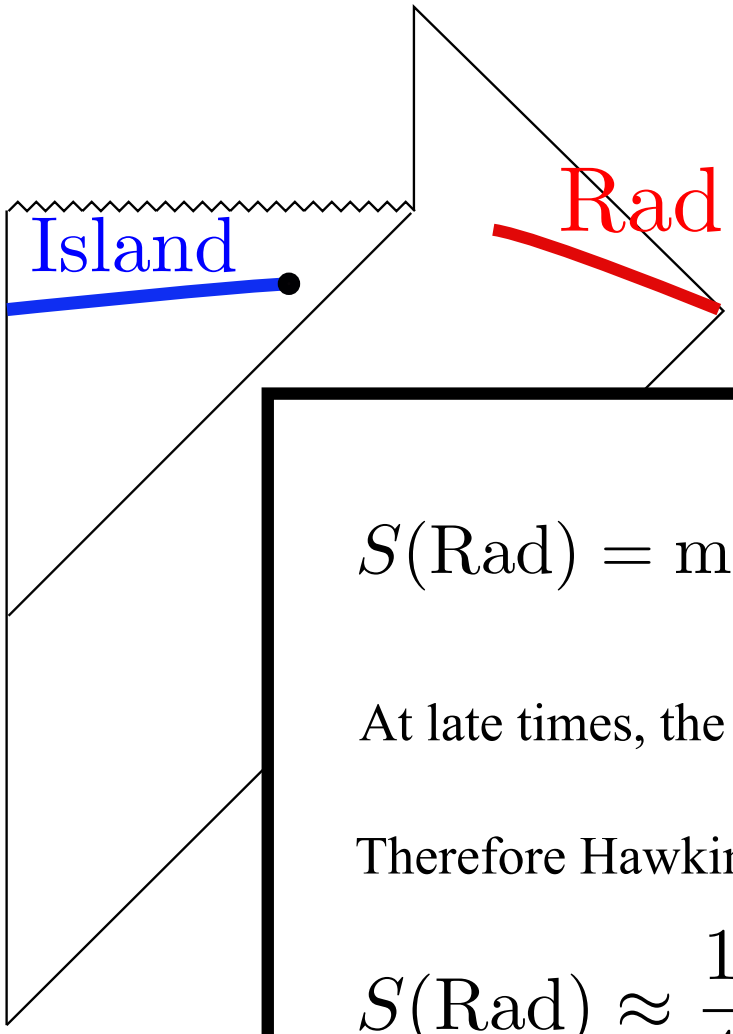



$$S(\mathrm{Rad}) = \min_I \mathrm{ext}_I \left[ \frac{\mathrm{Area}(\partial I)}{4} + S_{\mathrm{QFT}}(I \cup \mathrm{Rad}) \right]$$

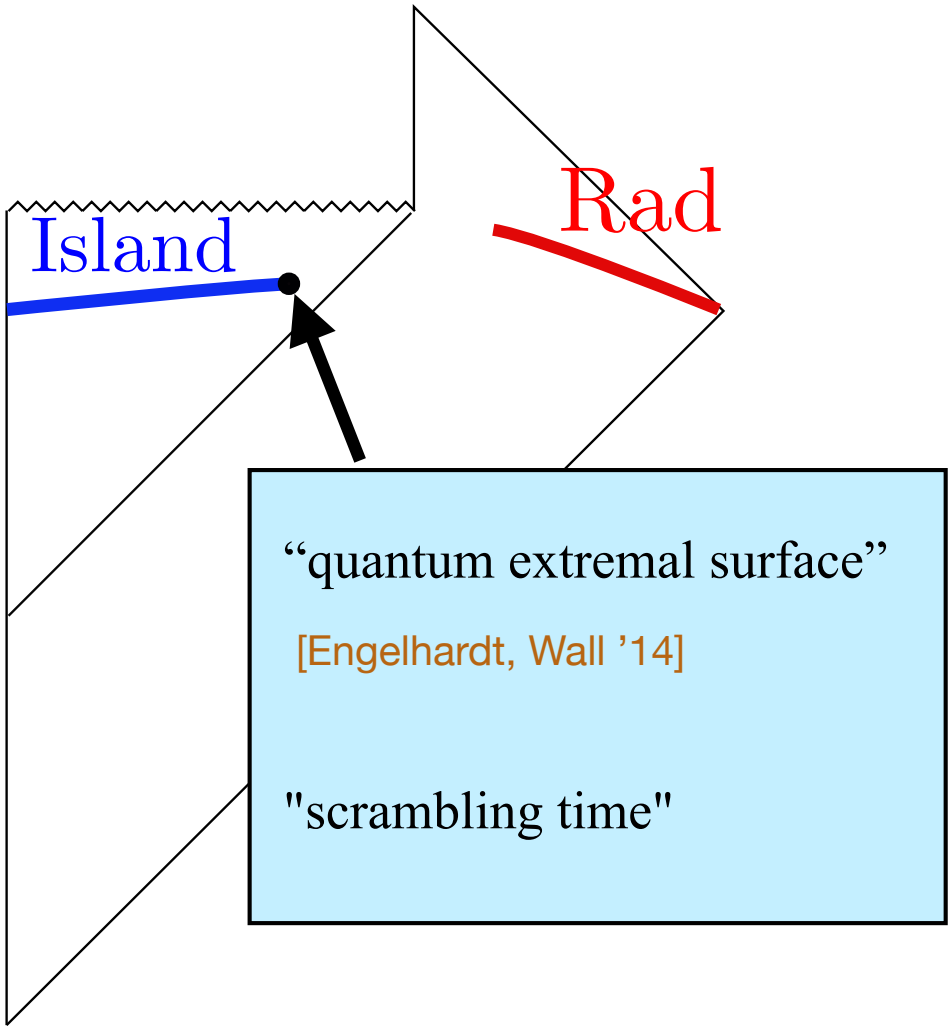
At late times, the island is approximately the full interior.

Therefore Hawking radiation does not contribute to the second term, so

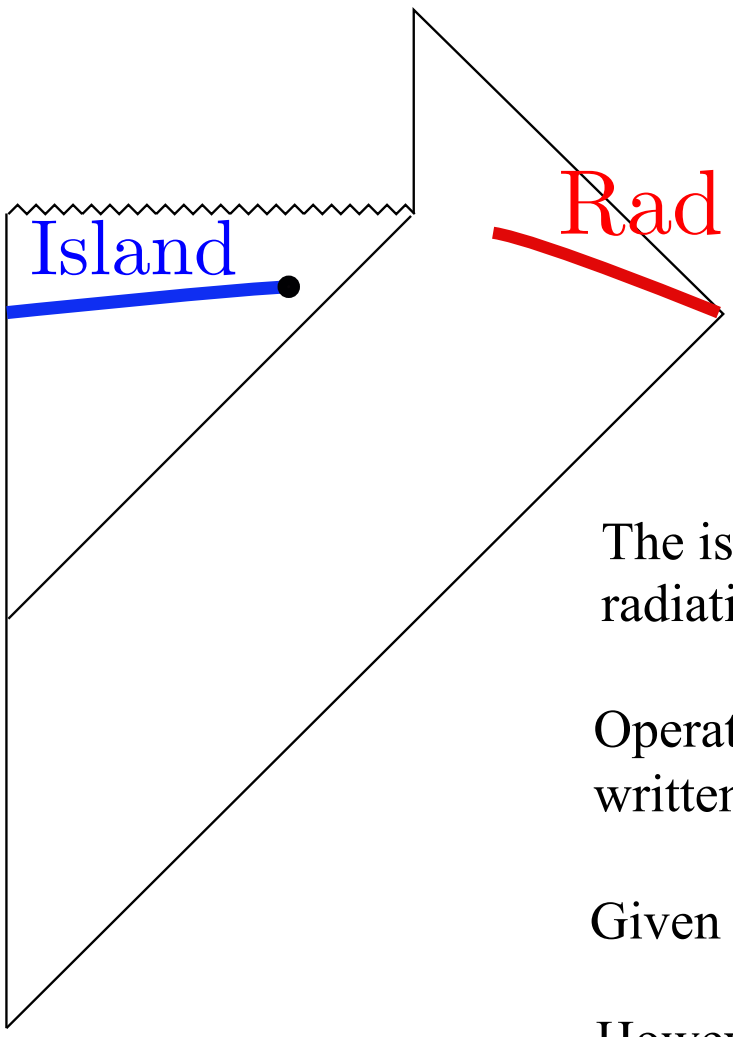
$$S(\mathrm{Rad}) \approx \frac{1}{4}\mathrm{Area}(\mathrm{horizon})$$

$$\to 0 \qquad \text{as the black hole evaporates}$$

# The island formula for radiation entropy



# Interpretation




**Entanglement wedge reconstruction:**
The island is in the "entanglement wedge" of the radiation.

[Wall '12], [Czech, Karczmarek, Nogueira, Van Raamsdonk '12], [Headrick, Hubeny, Lawrence, Rangamani '14], [Almheiri, Dong, Harlow '14], [Dong, Harlow, Wall '16]

The island is actually "encoded" in the radiation, in the sense of holographic duality.

Operators in the island $\phi(x)$ can be written as operators in `Rad` (in principle).

Given the entropy formula, this is almost a theorem in Q.I.

However, the decoding map is complicated and non-local.

[Penington, Shenker, Stanford, Yang '19], [Chen '19]

# Replica wormholes

Goal: Derive the island formula directly from the Euclidean gravitational path integral.

[Almheiri, TH, Maldacena, Shaghoulian, Tajdini ’19]
[Penington, Shenker, Stanford, Yang ’19]

Borrowing methods developed earlier in:
[Lewkowycz, Maldacena ’13]
[Barella, Dong, Hartnoll, Martin ’13]
[Faulkner, Lewkowycz, Maldacena ’13]
[Dong, Lewkowycz, Rangamani '16]
[Dong, Lewkowycz ’17]

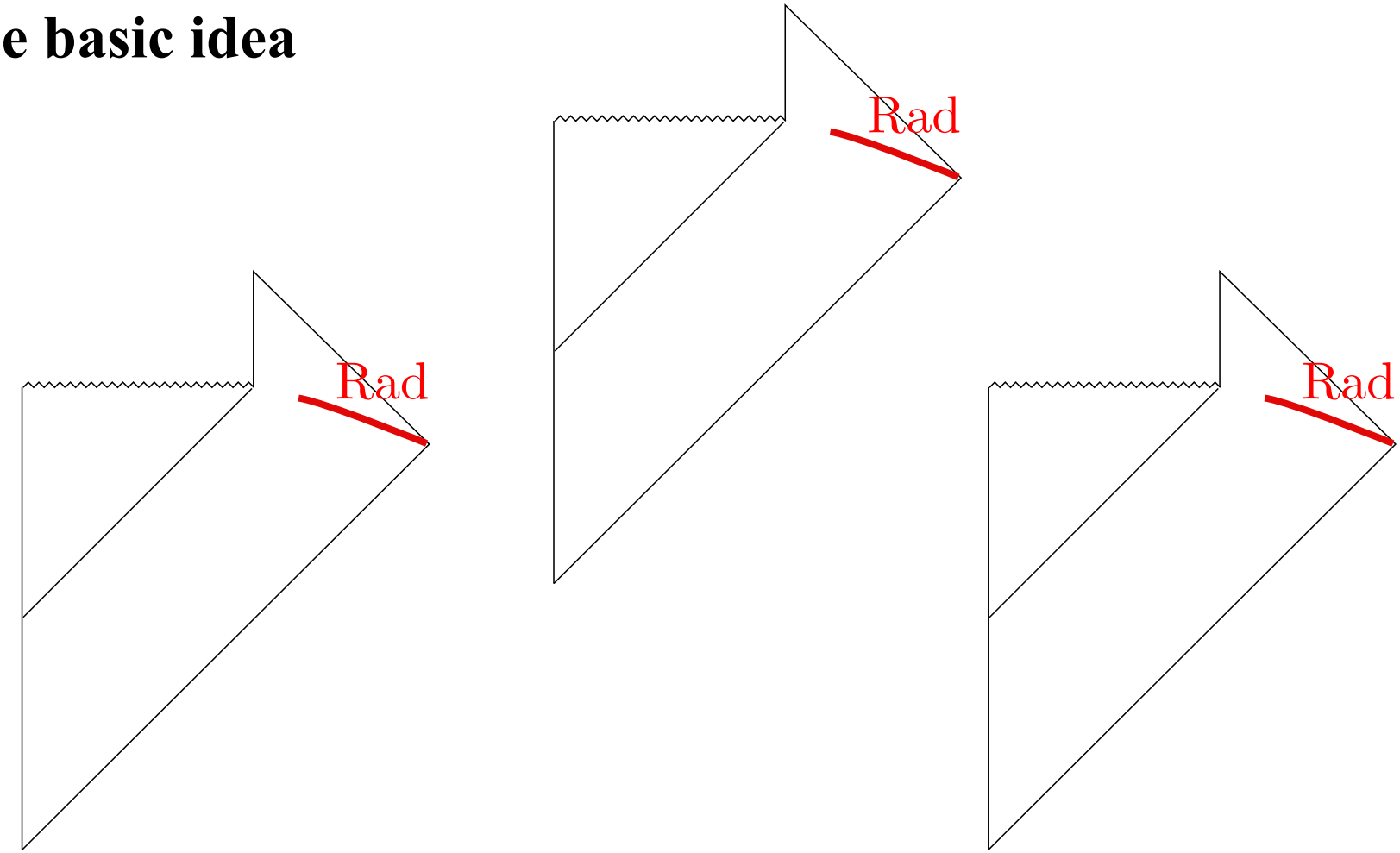
## Replica method

$$S(\rho_R) = -\mathrm{tr}\,\rho_R \log \rho_R$$

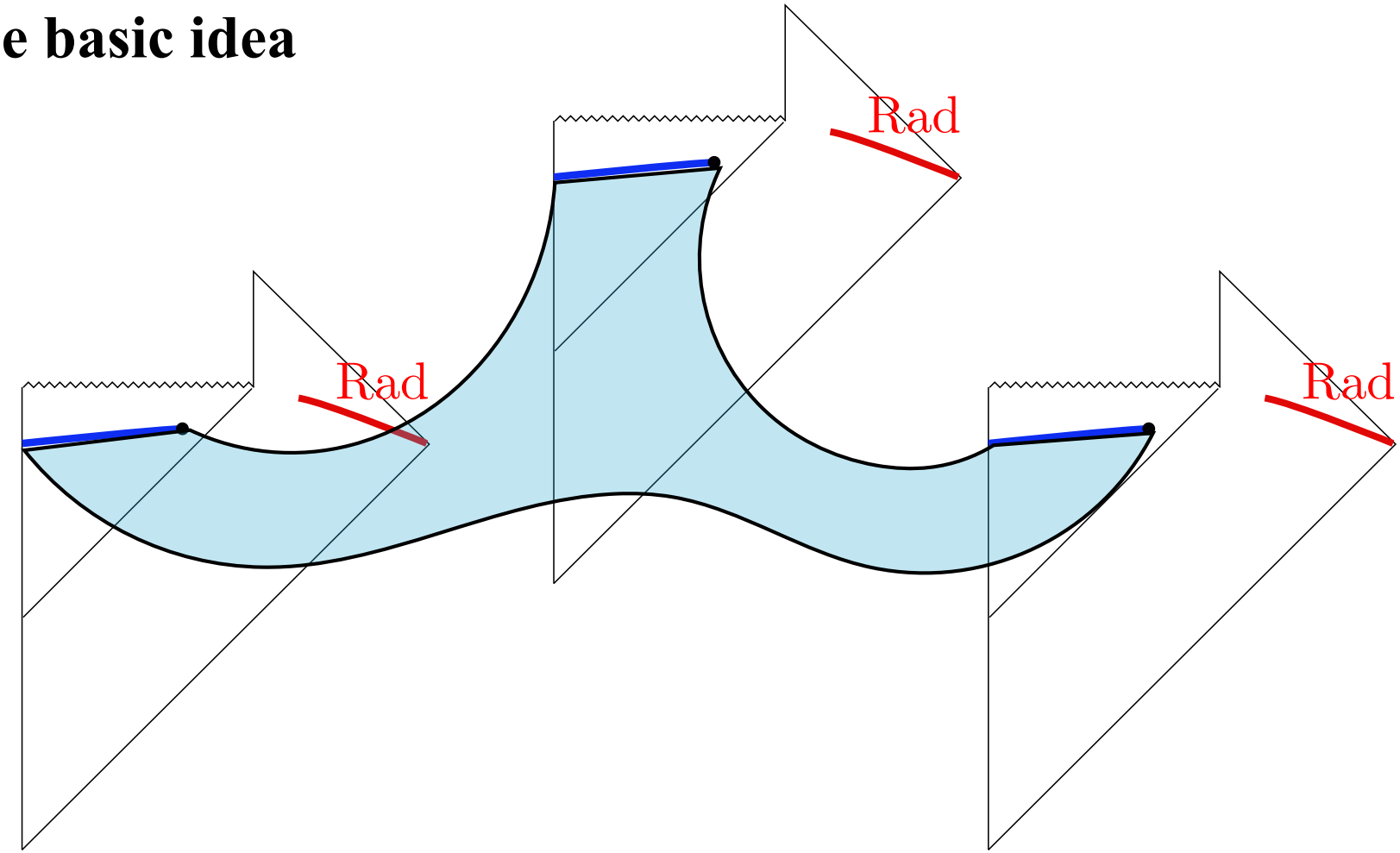
$$Z(n) = \mathrm{tr}(\rho_R)^n, \qquad n = 1, 2, 3, \ldots$$

$$S(\rho_R) = -Z'(1)$$

# The basic idea



# The basic idea




In the replica method, dynamical wormholes appear connecting the black hole interiors.

These are complex saddles (instantons) that we can construct by explicit solution of the EOM for gravity+matter in certain simple cases.

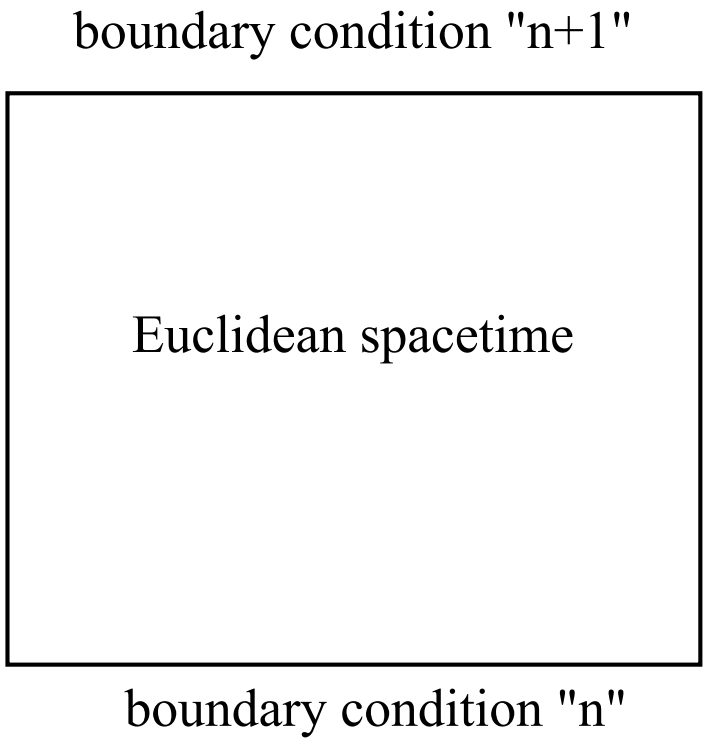
In the replica limit, these saddles leave an imprint on certain observables, including the von Neumann entropy.

## Replica Calculation

$$Z(n) = \mathrm{tr}(\rho_R)^n$$

## Recall the path integral calculation of a transition amplitude:

$\langle n+1|n\rangle =$



## Replica Calculation

$$Z(n) = \mathrm{tr}(\rho_R)^n$$

## *n=1* replica in Euclidean signature

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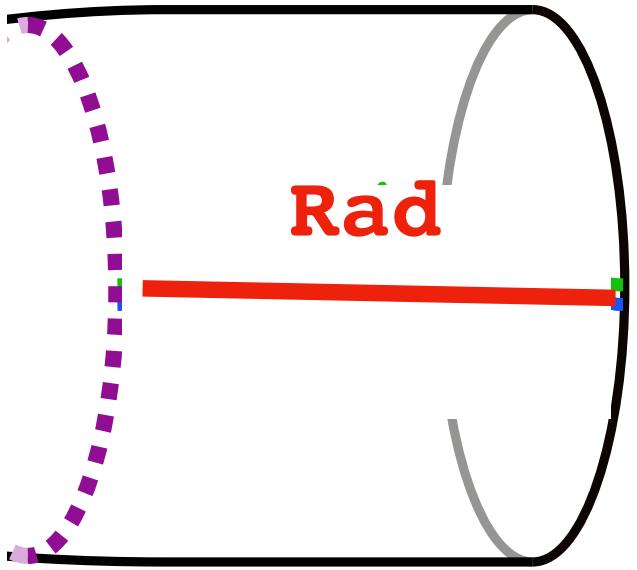
## *n=1* replica in Euclidean signature

$$\mathrm{tr}\,\rho_R \;=$$




black hole region

asymptotic region; gravity frozen

## *n=2* replicas

$$\mathrm{tr}\,(\rho_R)^2 =$$

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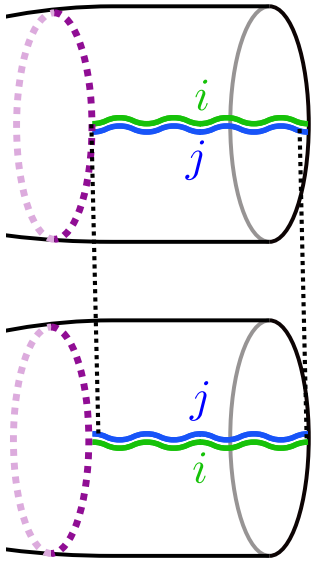
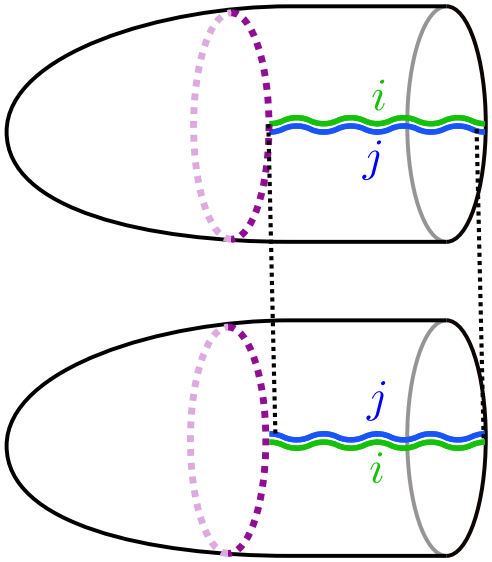



Hawking saddle

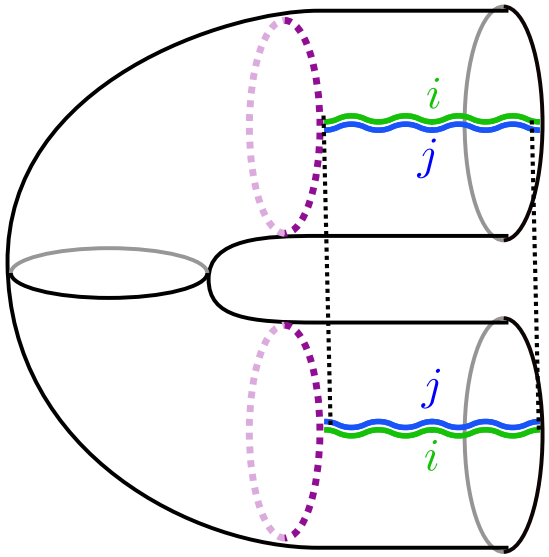
## *n=2* replicas

$$\mathrm{tr}\,(\rho_R)^2 =$$




Hawking saddle

$+$




replica wormhole

$+ \quad \cdots$

$$= \quad e^{-S_2^{\mathrm{Hawking}}} \quad + \quad e^{-S_2^{\mathrm{Wormhole}}} \quad + \quad \cdots$$

Suppressed by topology

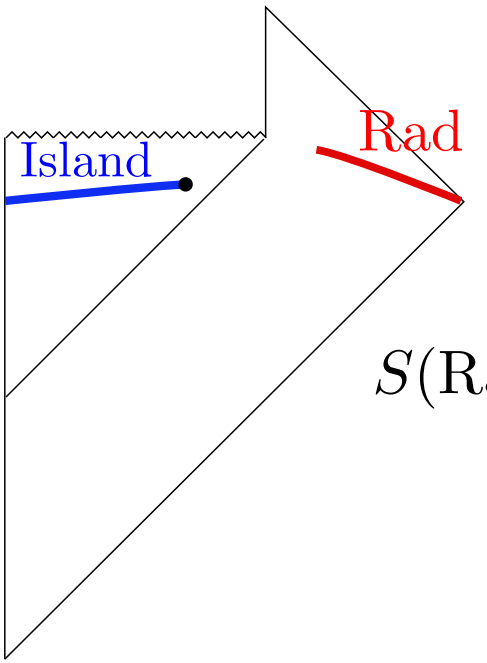
Suppressed by large entanglement of radiation with interior

Replica wormholes are n-copy solutions of the gravitational EOM, sourced by the 1-loop matter stress tensor on the replica manifold.

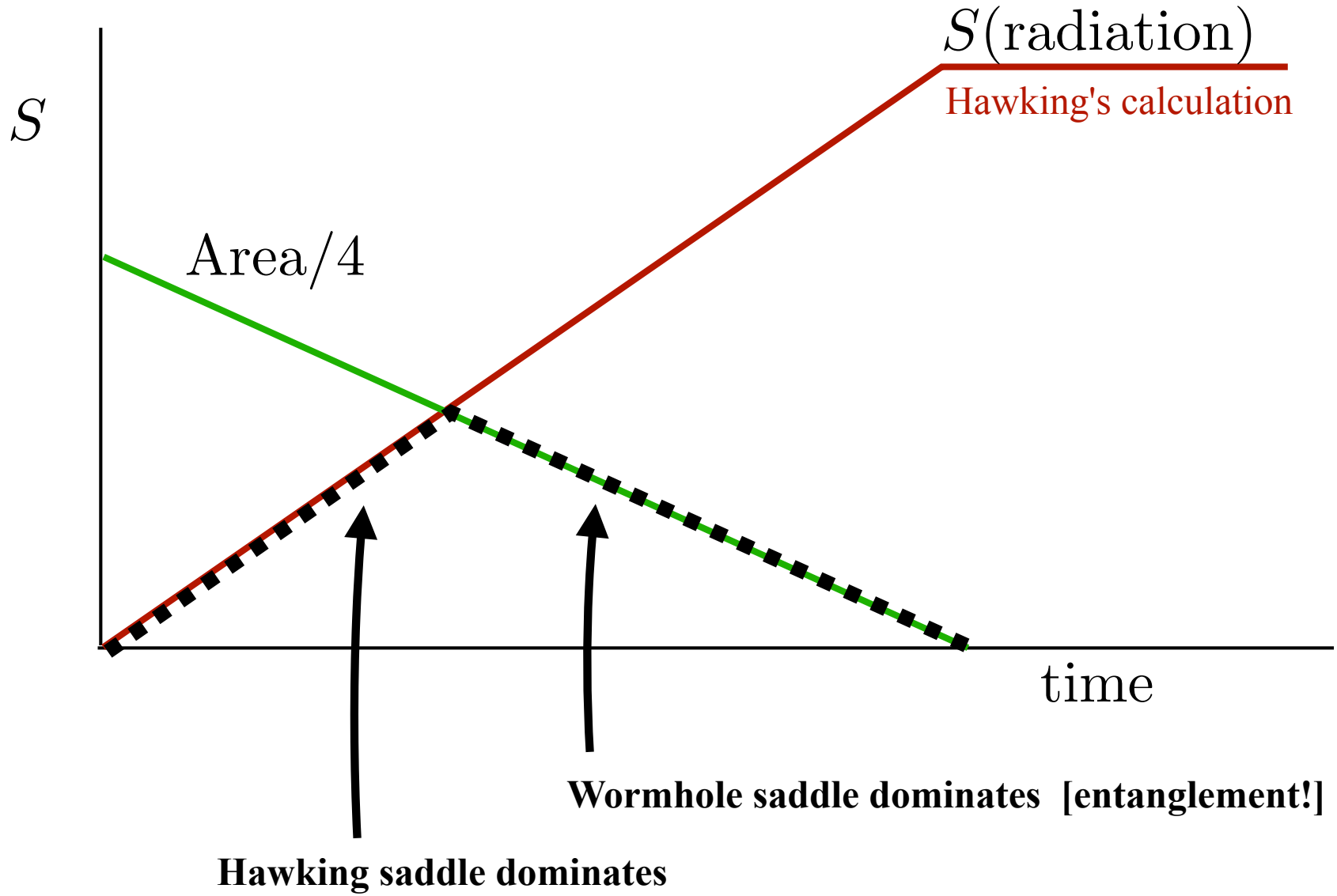
## von Neumann entropy

$$S(\rho_R) = -Z'(1)$$

The mouth of the wormhole becomes the island as we take *n* to 1, and the path integral gives the island formula:




$$S(\mathrm{Rad}) = \min_I \mathrm{ext}_I \left[ \frac{\mathrm{Area}(\partial I)}{4} + S_{\mathrm{QFT}}(I \cup \mathrm{Rad}) \right]$$


$S$(radiation)
Hawking's calculation
$S$
Area/4
time
Wormhole saddle dominates [entanglement!]
Hawking saddle dominates

# Remarks

## Does this show that black hole evaporation is unitary?

No, it does not. But it is evidence in this direction.

$$S(\text{Rad}) = -\text{tr}\rho_R \log \rho_R$$

We used the Euclidean path integral to calculate this without calculating $\rho_R$

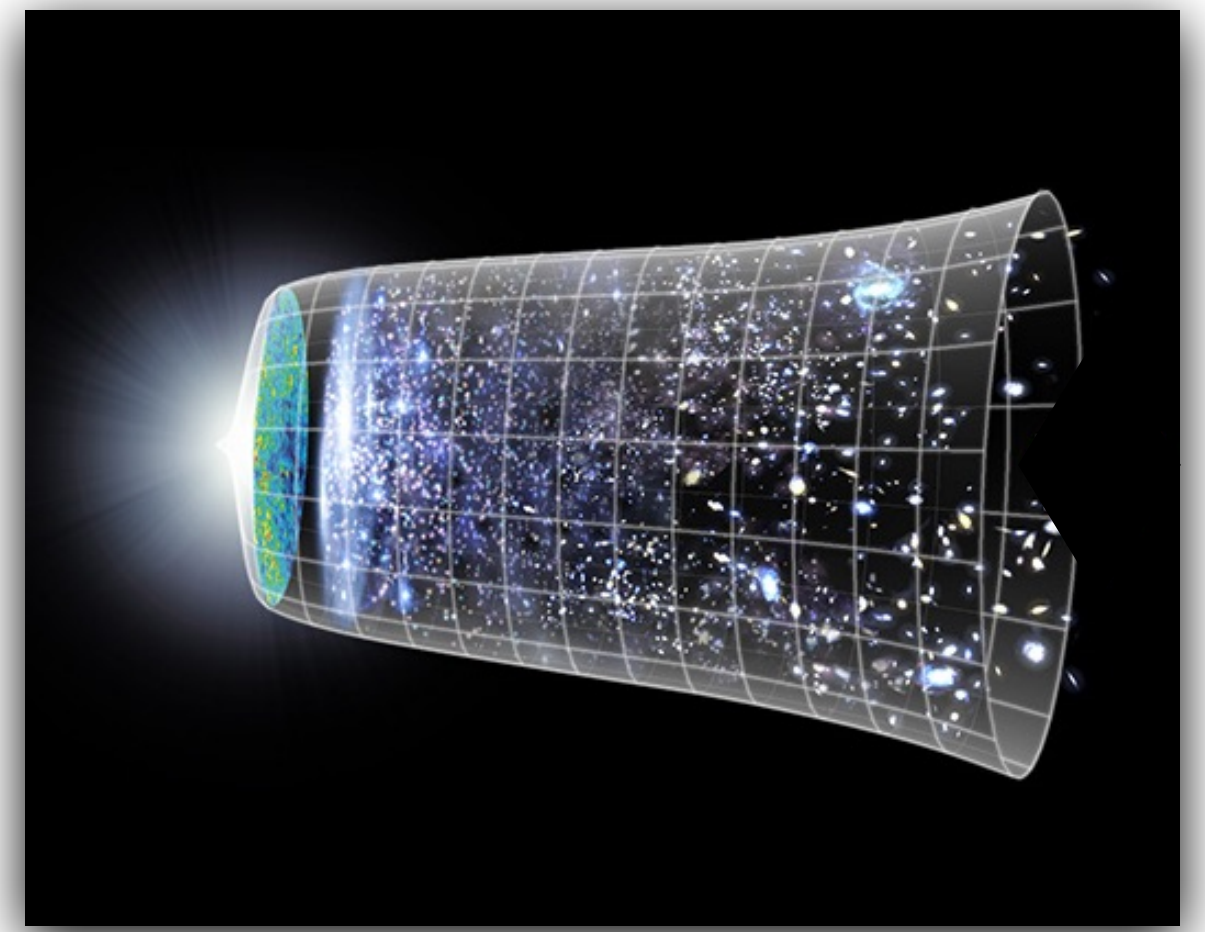
or even showing that it exists!

This is on the same footing as the Euclidean calculation of the black hole entropy by Gibbons and Hawking,

$$S(\text{black hole}) = \frac{\text{Area}}{4}$$

which we believe gives the right answer but does not exhibit the microstates.

# Quantum cosmology revisited?



Cosmology also has horizons, large entropy, etc. Does it have islands?

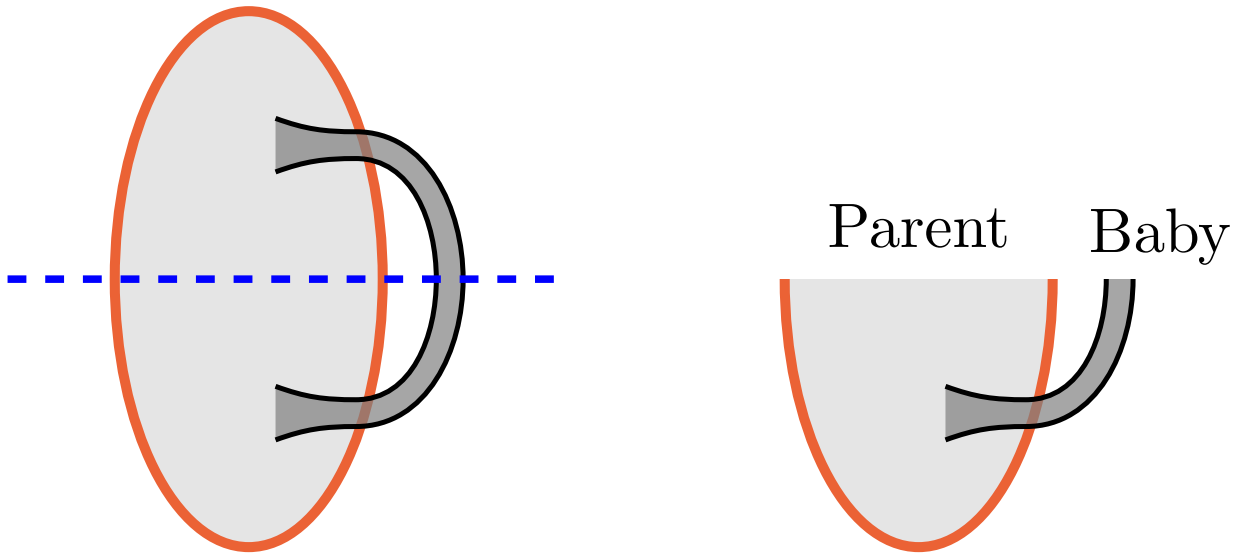
Maybe. The interpretation is unclear.

[Anous, Kruthoff, Mahajan ’20], [Chen, Gorbenko, Maldacena ’20], [TH, Jiang, Shaghoulian ’20], [Balasubramanian, Kar, Ugajin ’20], [Van Raamsdonk ’20]
etc.

# Do wormholes violate quantum mechanics?

In other situations, wormholes seem to violate some basic properties of quantum mechanics.

"Factorization paradoxes"




[Old work by Coleman, Giddings, Strominger, etc.]
[Saad Shenker Stanford ’19]
[Marolf, Maxfield ’20]
etc.

## Conclusion

The ordinary rules of the Euclidean path integral can be used to calculate the entropy of Hawking radiation.

The result agrees with unitarity, and *quantitatively* matches earlier predictions.

But it also highlights the remaining puzzles of how to interpret the gravitational path integral.

# Thank you