

The Breakdown of the Narrow Width Approximation and other Aspects of Timelike Processes in AdS

Lexi Costantino (UC Riverside)

2002.12335

2011.06603

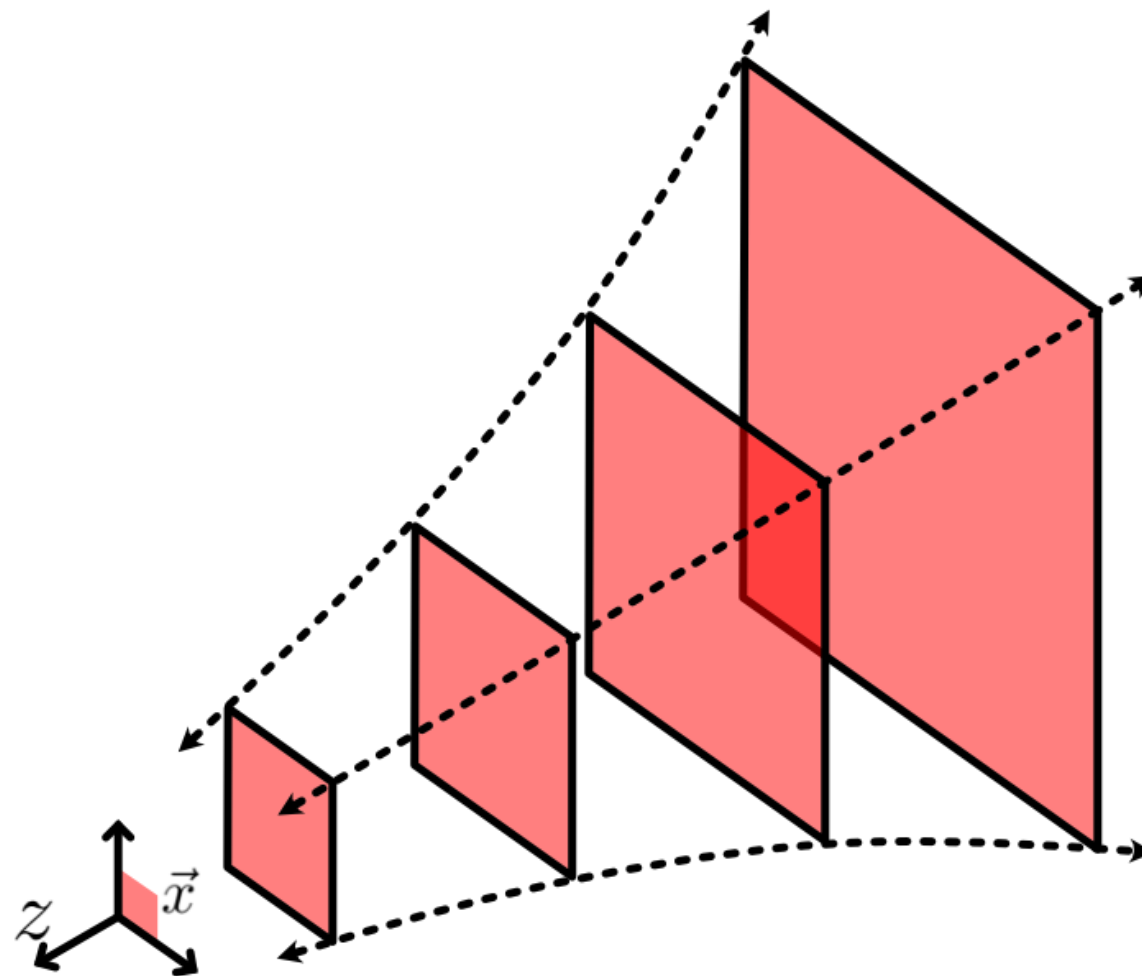
+ upcoming work

w/ Sylvain Fichtel (ICTP-SAIFR)
Flip Tanedo (UC Riverside)

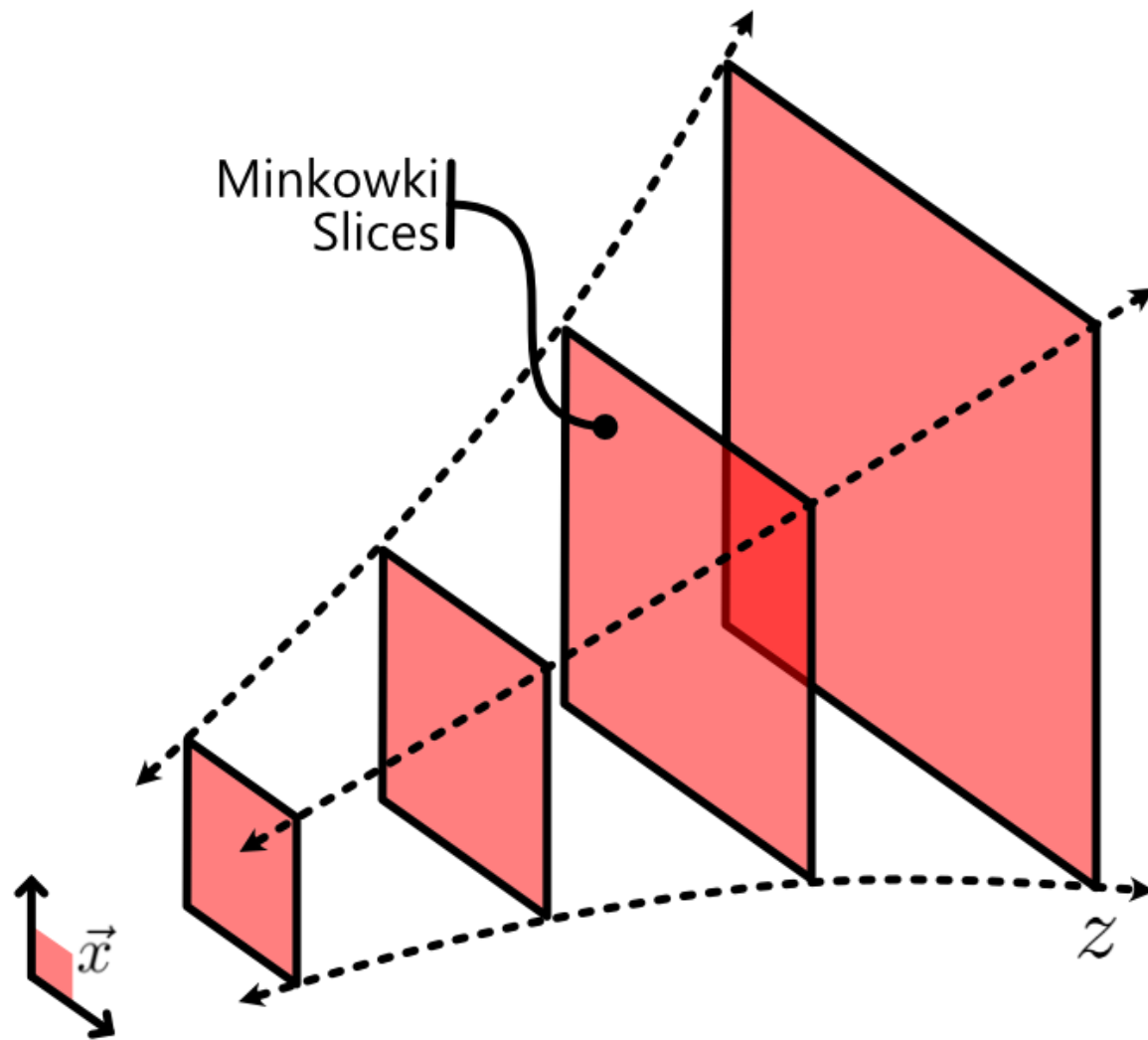


Why Anti-de Sitter spacetime?

- Dual to a conformal field theory via AdS/CFT correspondence (hep-th/9711200 by Maldacena, hep-th/9802109 by Gubser et al., hep-th/9802150 by Witten)
- Strongly coupled CFTs can produce novel phenomenon:
 - Non-integer forces, “Unparticles” (see e.g. Georgi hep-ph/0703260 and 0704.2457. See also 0804.0424 by Cacciapaglia, Marandella, Terning, and 0902.3676 by Friedland et al.)
 - Quasi-spherical cascade decays (see e.g. Csaki, Reece, Terning 0811.3001)



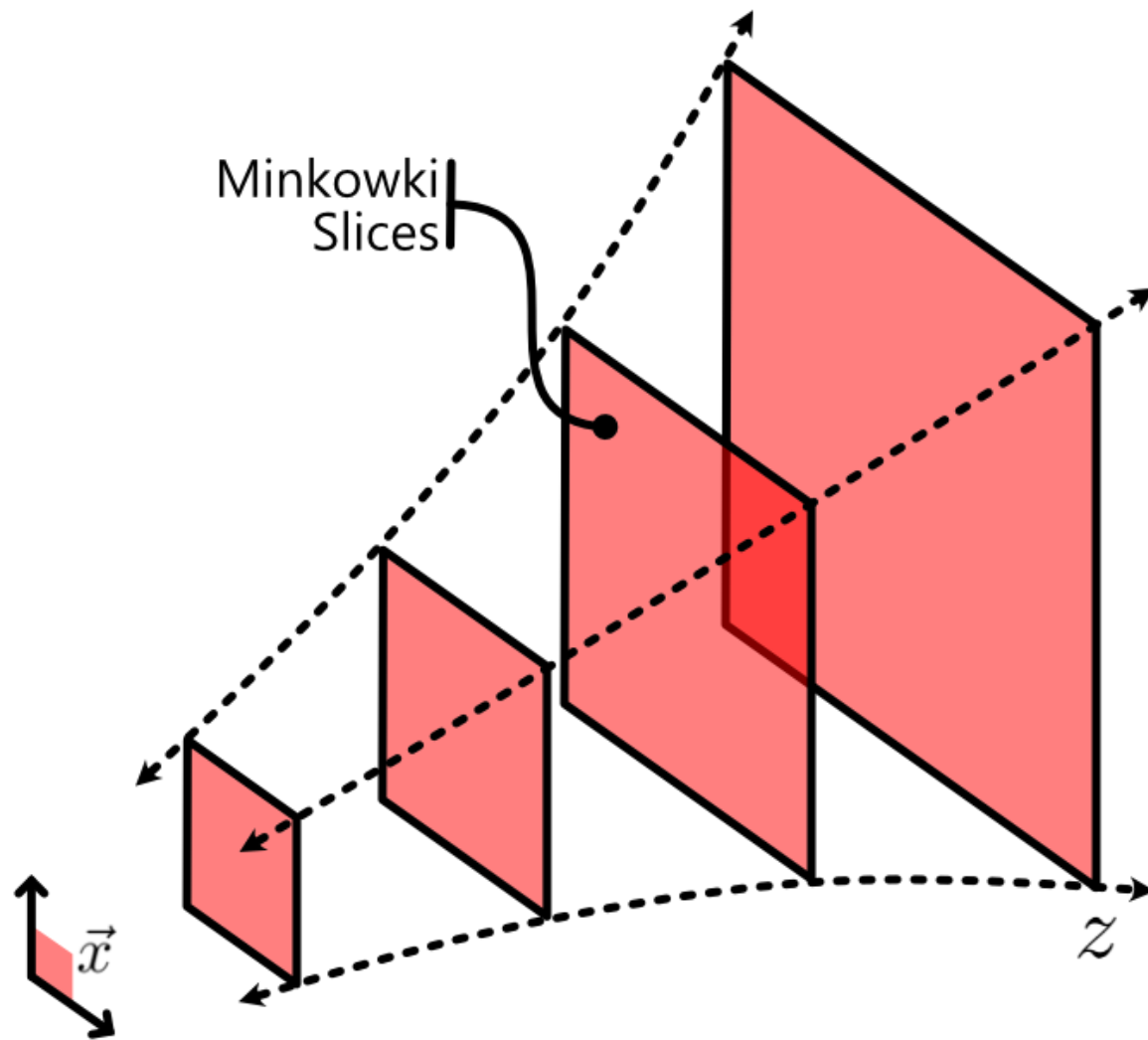
Anti-de Sitter Spacetime



$$ds^2 = \frac{1}{(kz)^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$$

\downarrow
 $(+, -, -, -)$

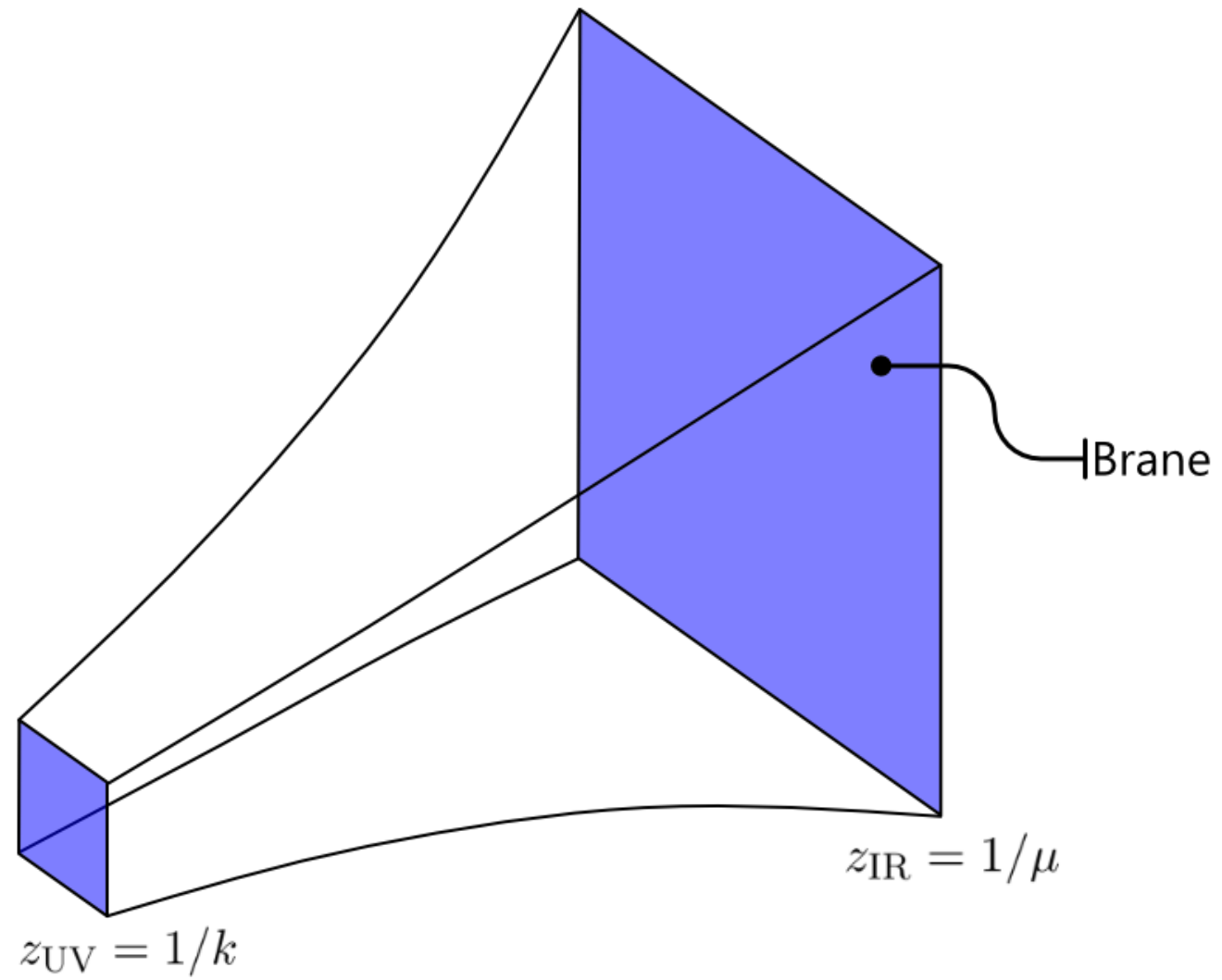
Anti-de Sitter Spacetime



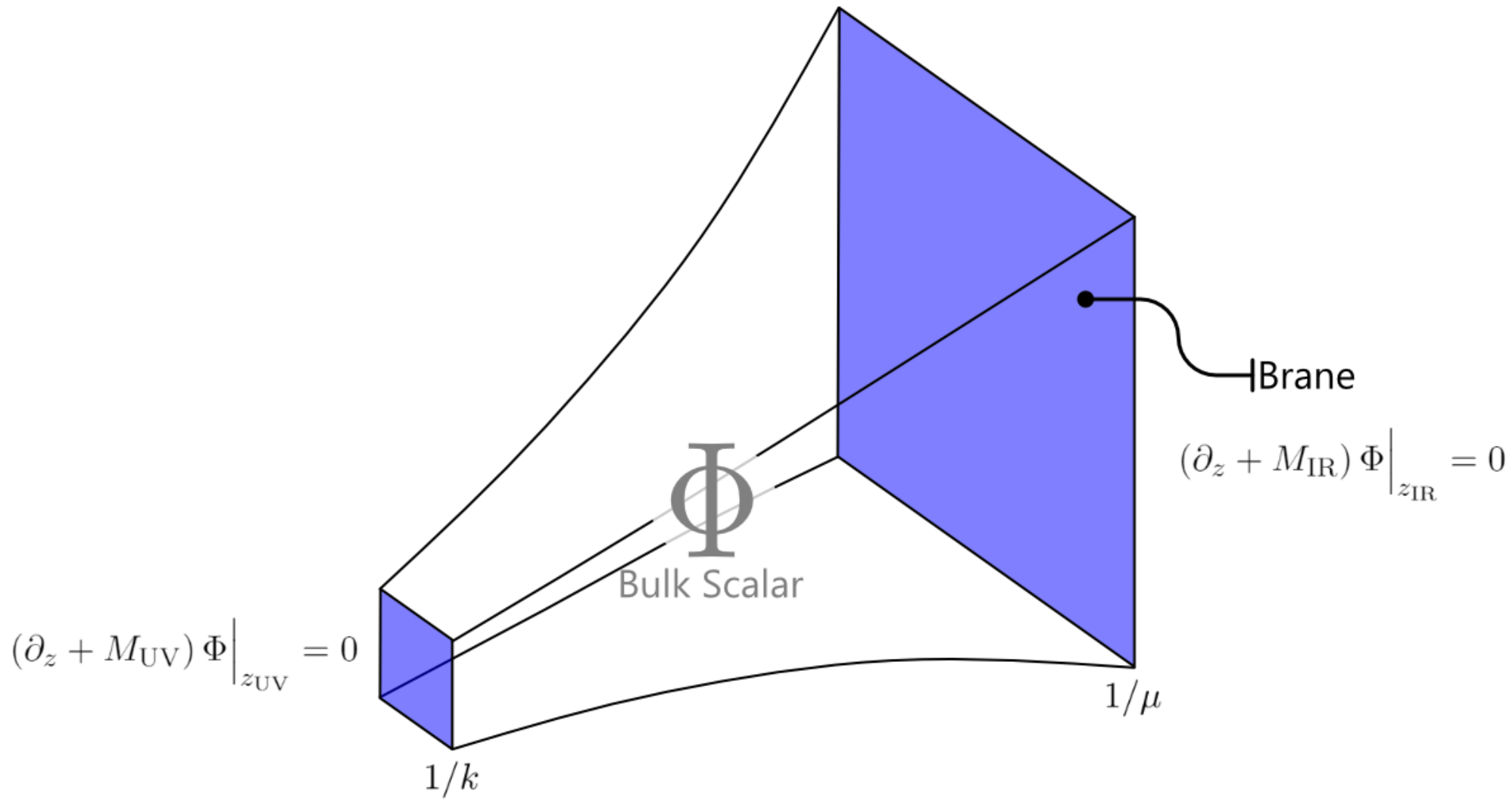
Warp Factor

$$ds^2 = \frac{1}{(kz)^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$$

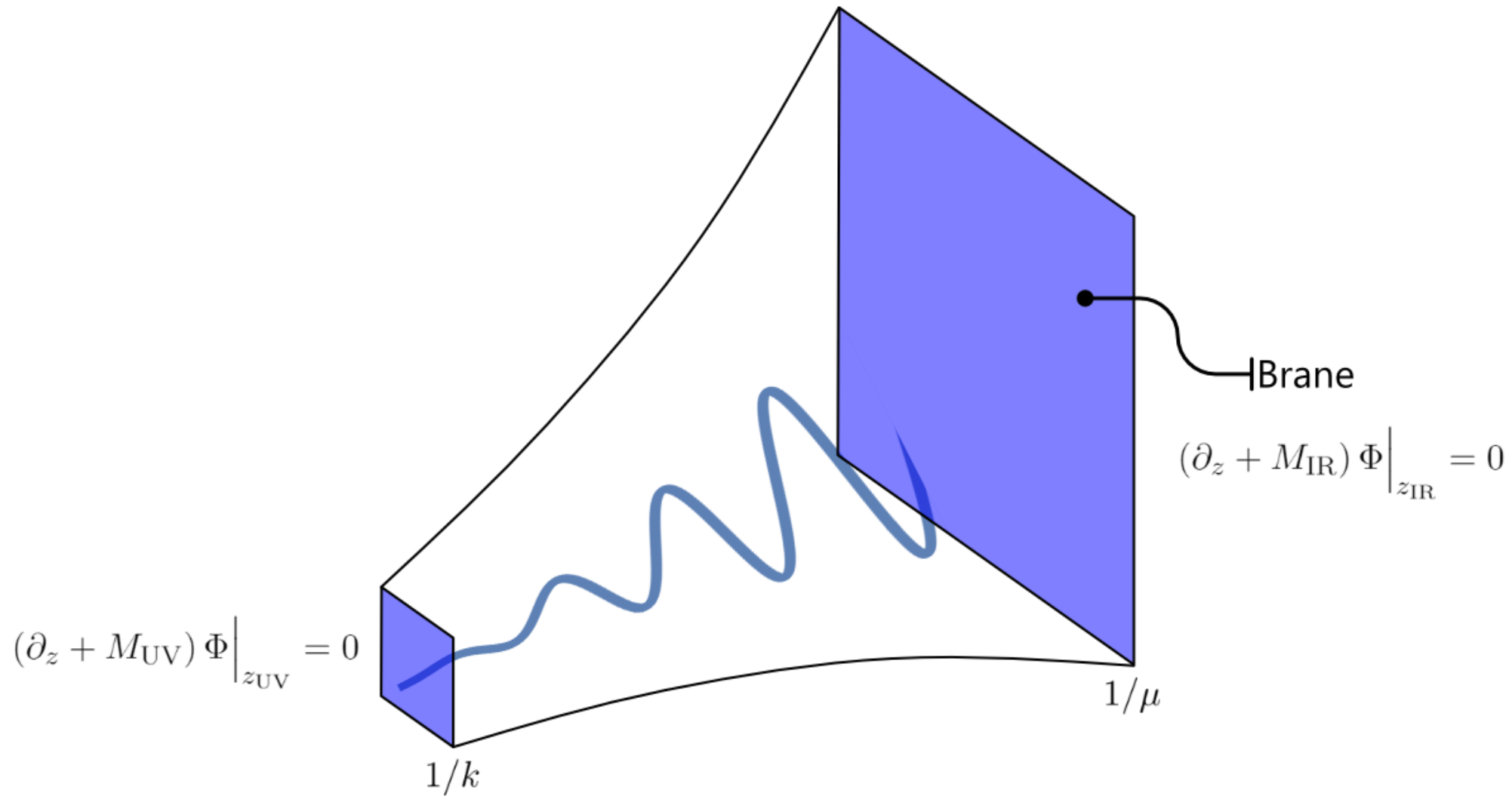
AdS with Branes



AdS with Branes



AdS with Branes

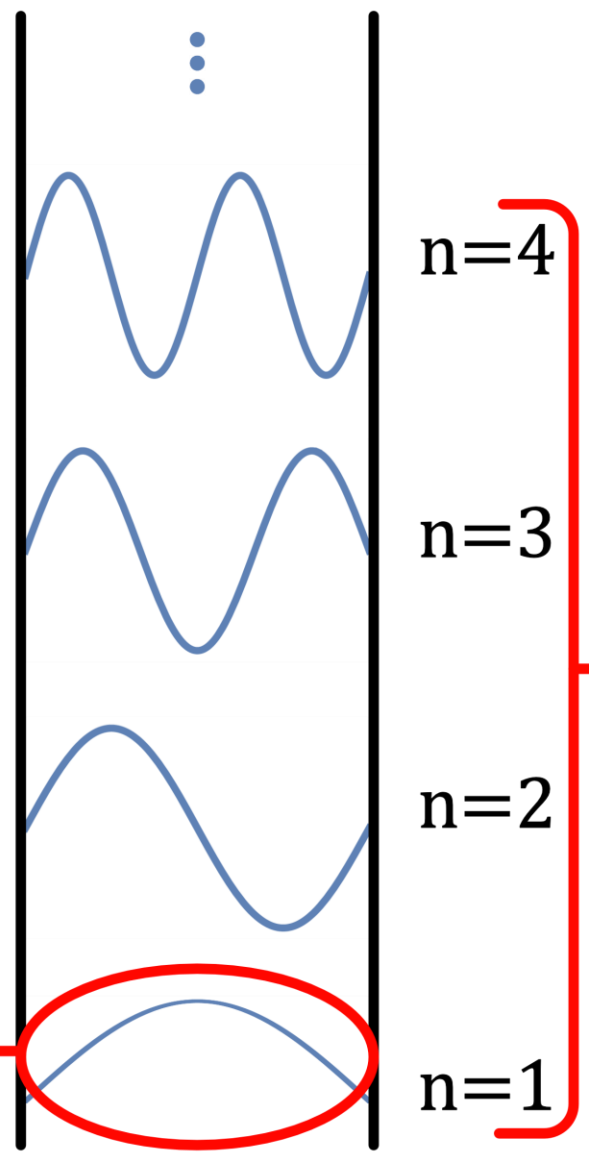


Review: Particle in a Box

$$E_n \propto n^2$$

Energy
increases
with n

Wavefunction
Profile

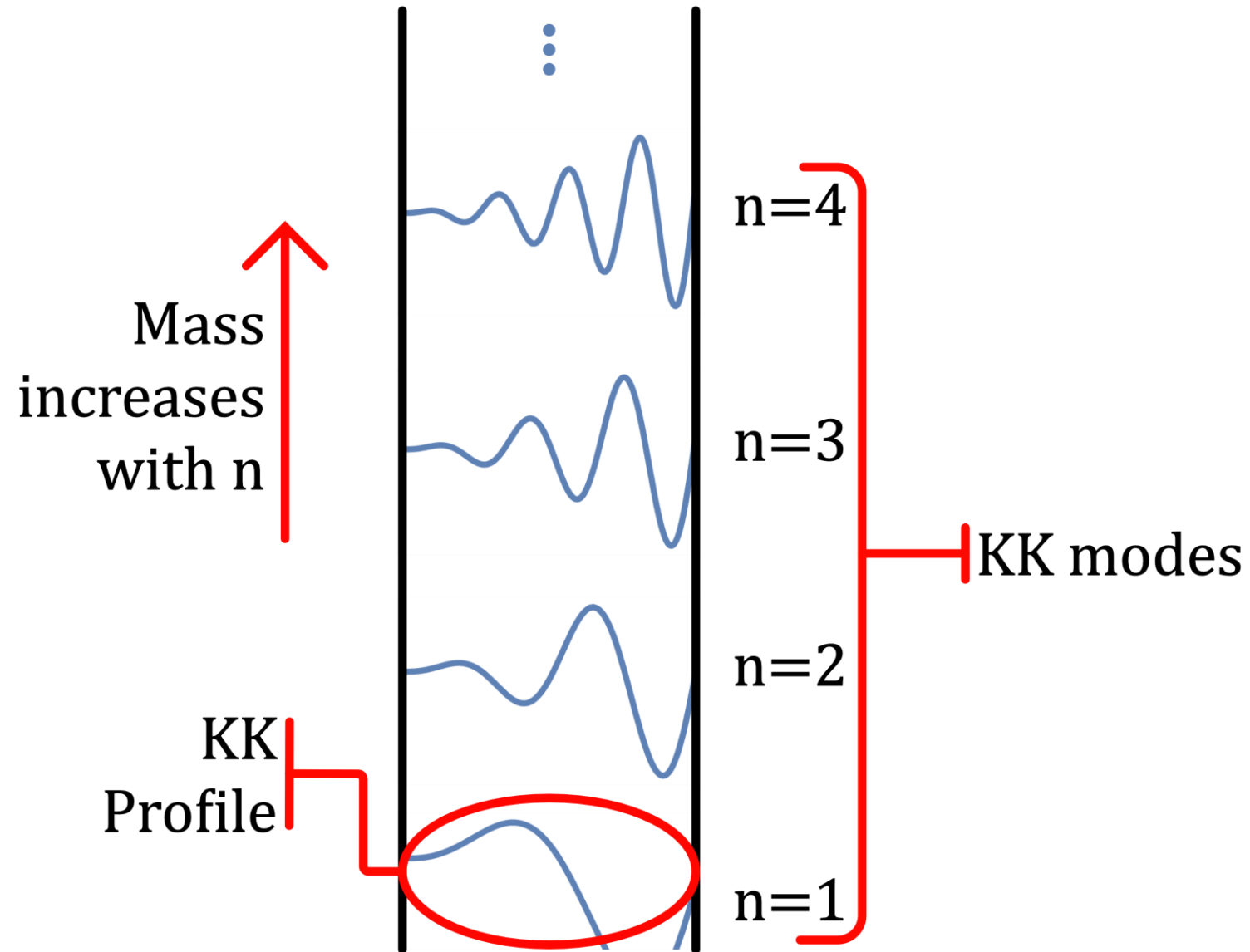


n=4
n=3
n=2
n=1

Looks like distinct
particles with
increasing mass

KK Modes

$$m_n \sim n\pi\mu$$

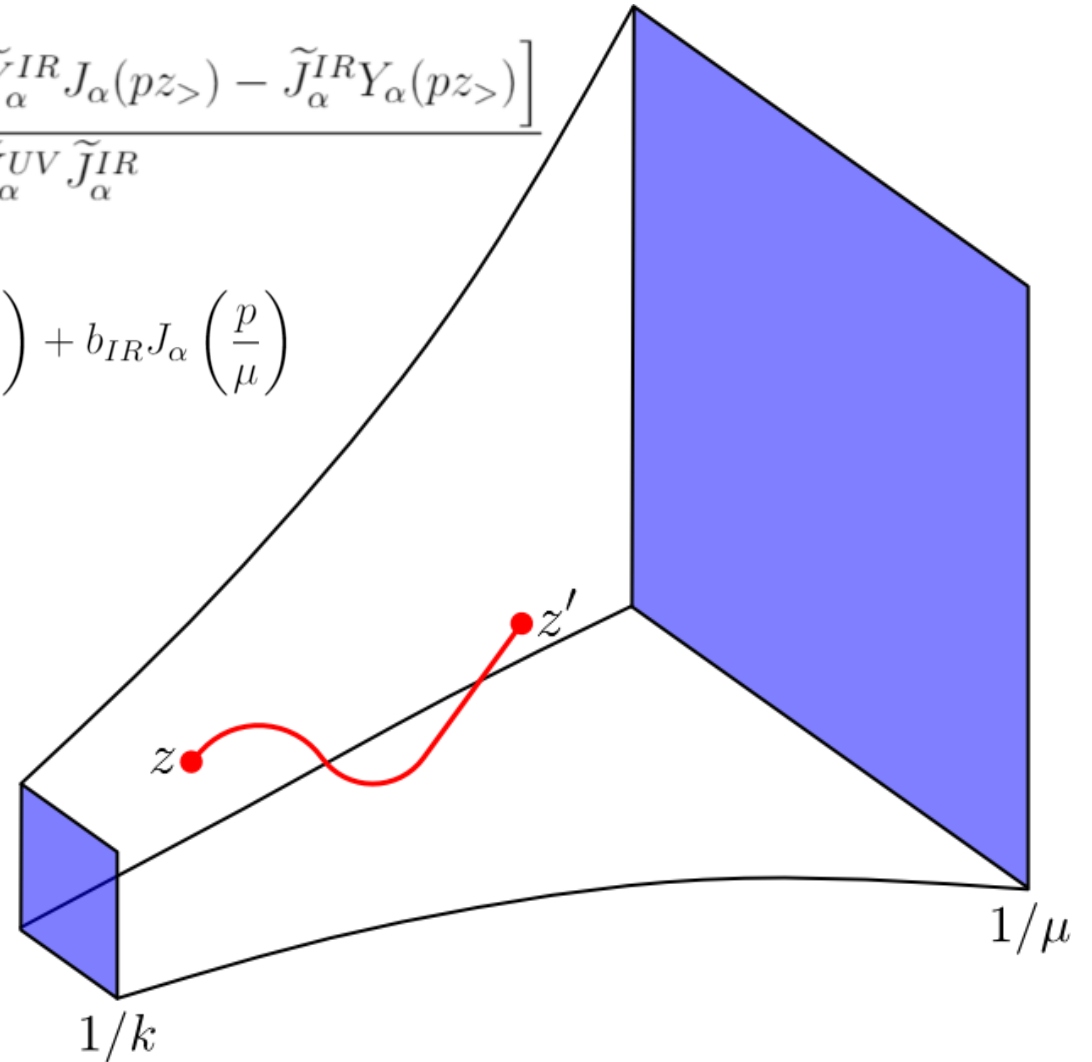


5D Propagator

$$G_p(z, z') = i \frac{\pi k^3 (zz')^2}{2} \frac{\left[\tilde{Y}_\alpha^{UV} J_\alpha(pz_{<}) - \tilde{J}_\alpha^{UV} Y_\alpha(pz_{<}) \right] \left[\tilde{Y}_\alpha^{IR} J_\alpha(pz_{>}) - \tilde{J}_\alpha^{IR} Y_\alpha(pz_{>}) \right]}{\tilde{J}_\alpha^{UV} \tilde{Y}_\alpha^{IR} - \tilde{Y}_\alpha^{UV} \tilde{J}_\alpha^{IR}}$$

$$\tilde{J}_\alpha^{UV} = \frac{p}{k} J_{\alpha-1} \left(\frac{p}{k} \right) - b_{UV} J_\alpha \left(\frac{p}{k} \right)$$

$$\tilde{J}_\alpha^{IR} = \frac{p}{\mu} J_{\alpha-1} \left(\frac{p}{\mu} \right) + b_{IR} J_\alpha \left(\frac{p}{\mu} \right)$$



5D Propagator

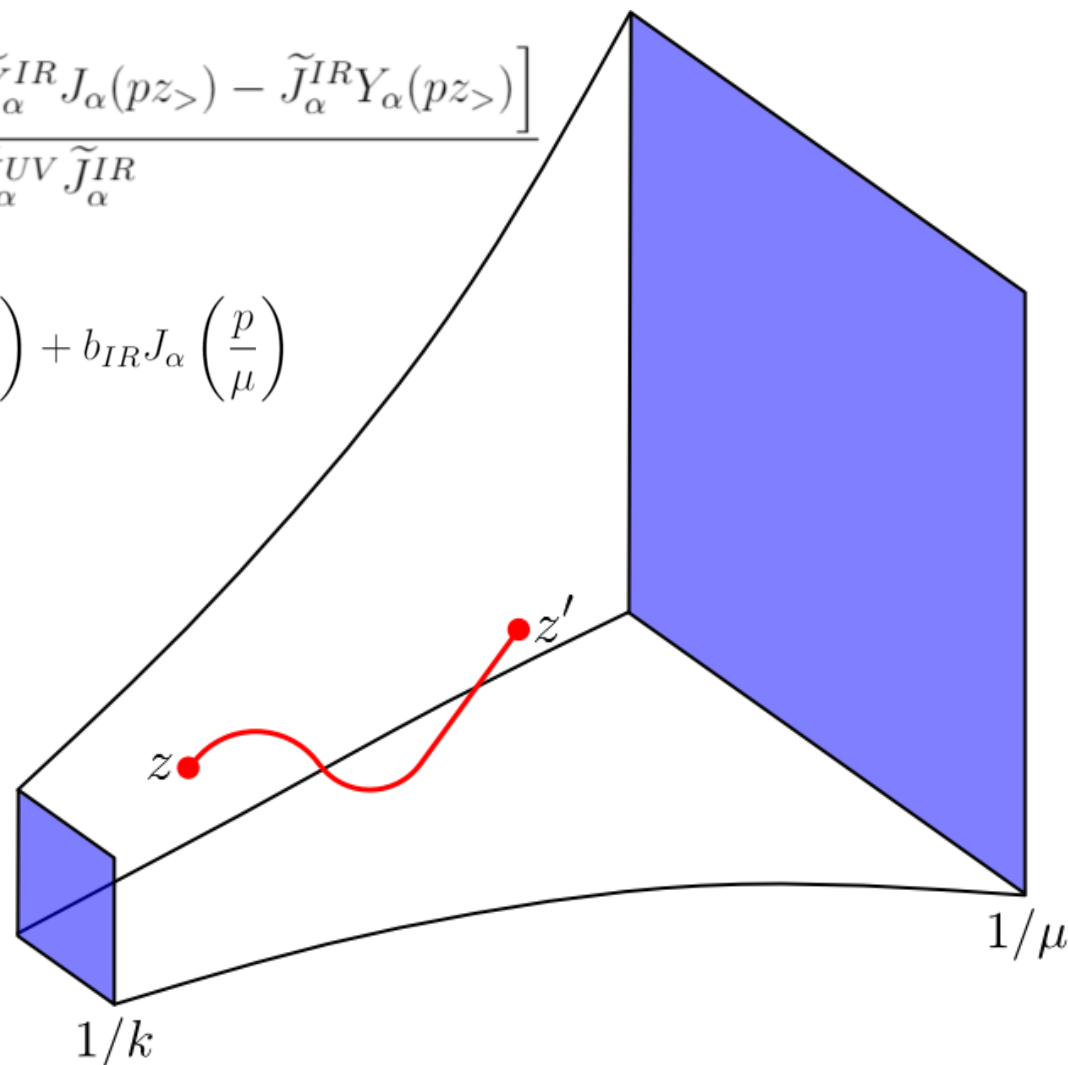
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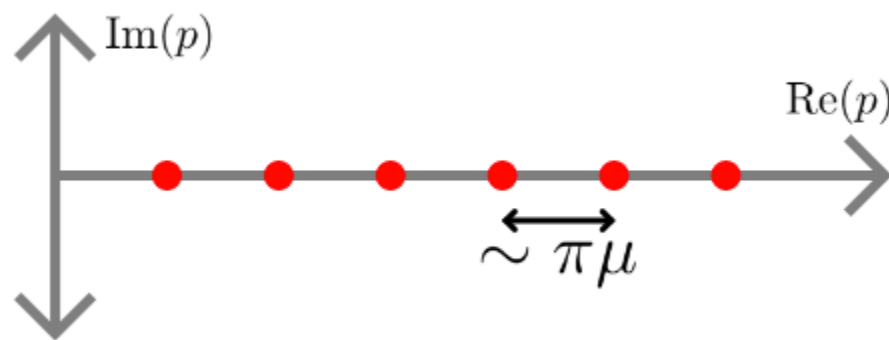
• KK profiles

$$G_p(z, z') = i \sum_{n=0}^{\infty} \frac{f_n(z) f_n(z')}{p^2 - m_n^2}$$



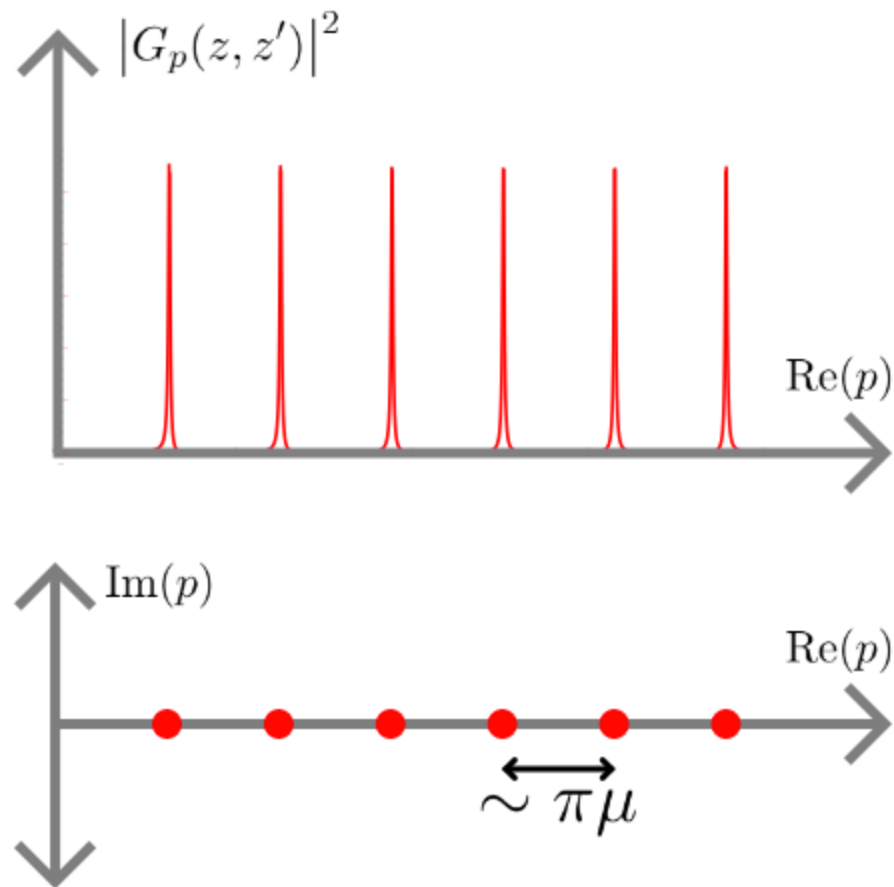
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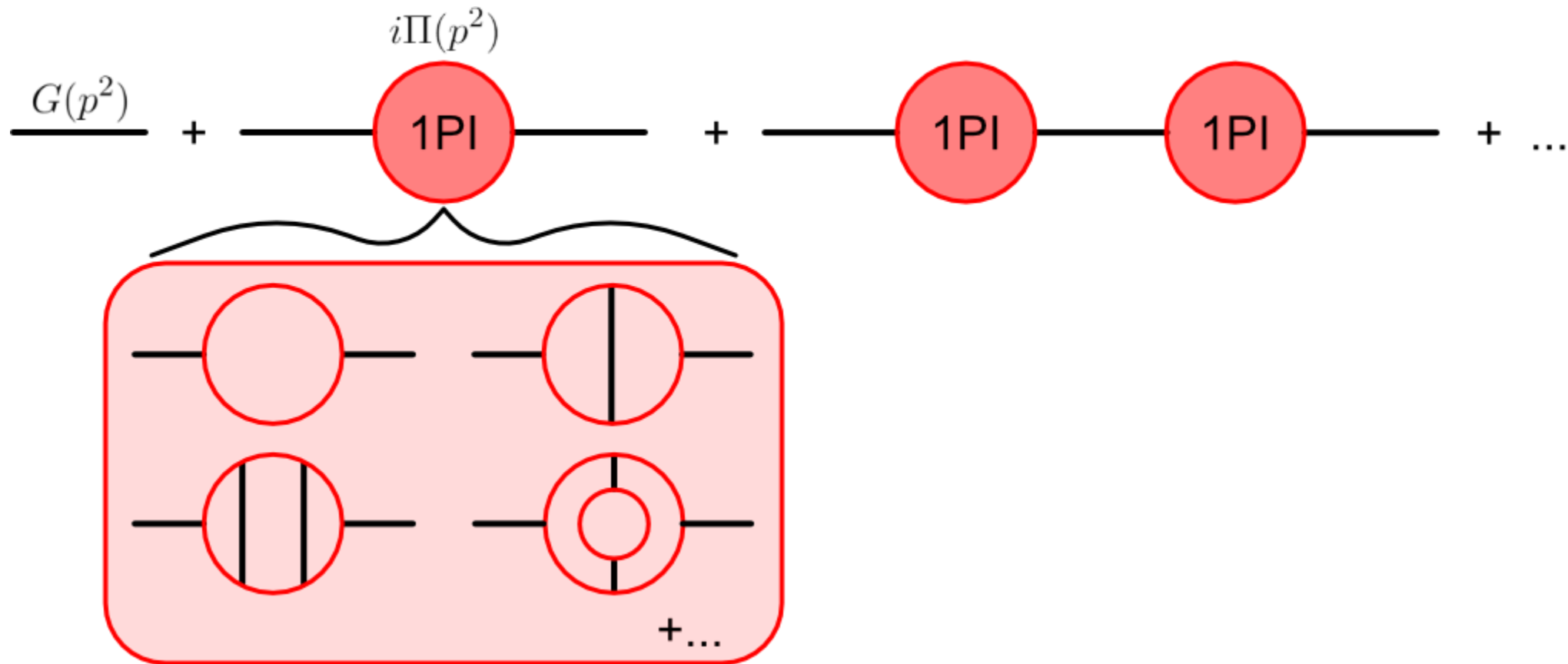
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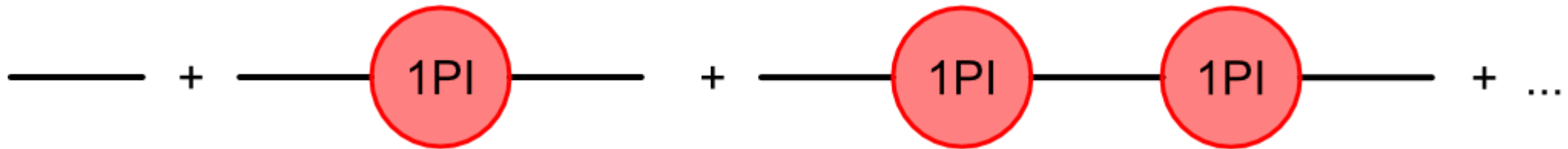


How does  impact particle propagation?

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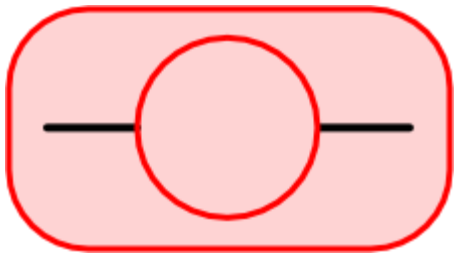
How does  impact particle propagation?



$$G_{\text{dr}}(p^2) = G + Gi\Pi G + \dots = \frac{i}{iG^{-1} + \Pi}$$

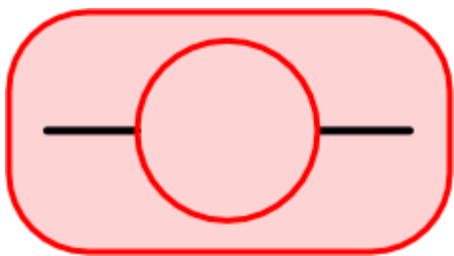
$$G_{\text{dr}}(p^2) = \frac{i}{p^2 - m_1^2 + \Pi(p^2)}$$

Resummed Propagator

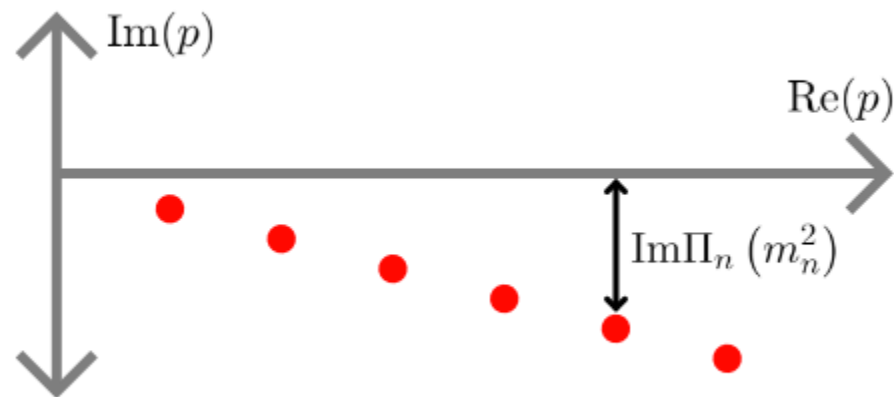


$$G_p(z, z') = i \sum_{n=0}^{\infty} \frac{f_n(z) f_n(z')}{p^2 - m_n^2 + \Pi_n(p^2)}$$

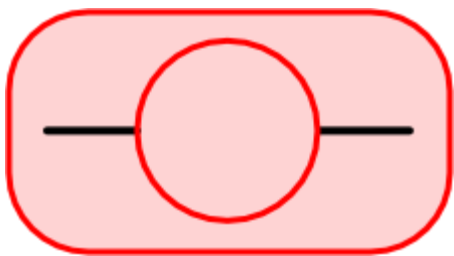
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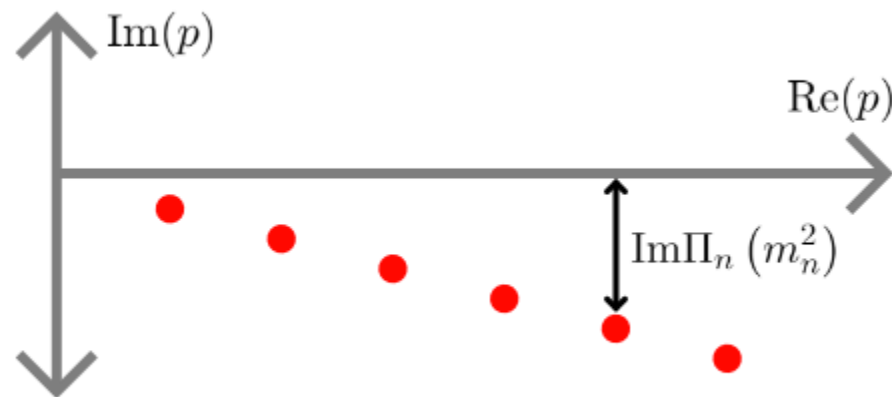
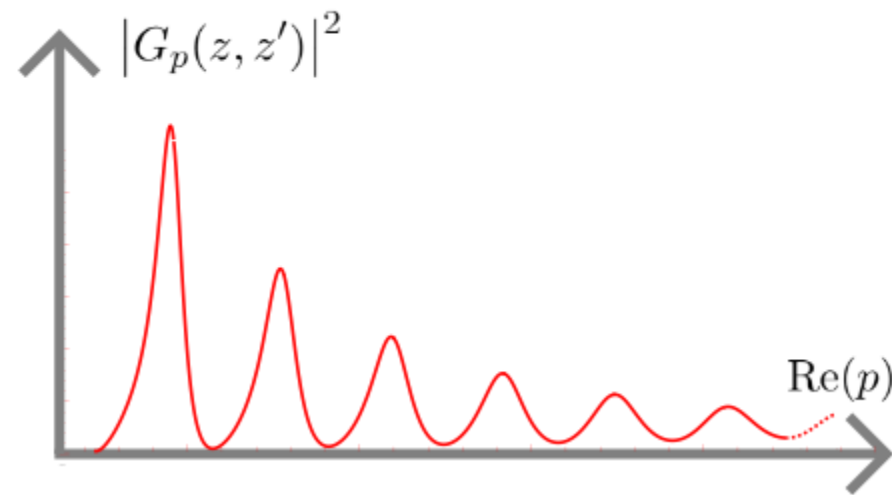
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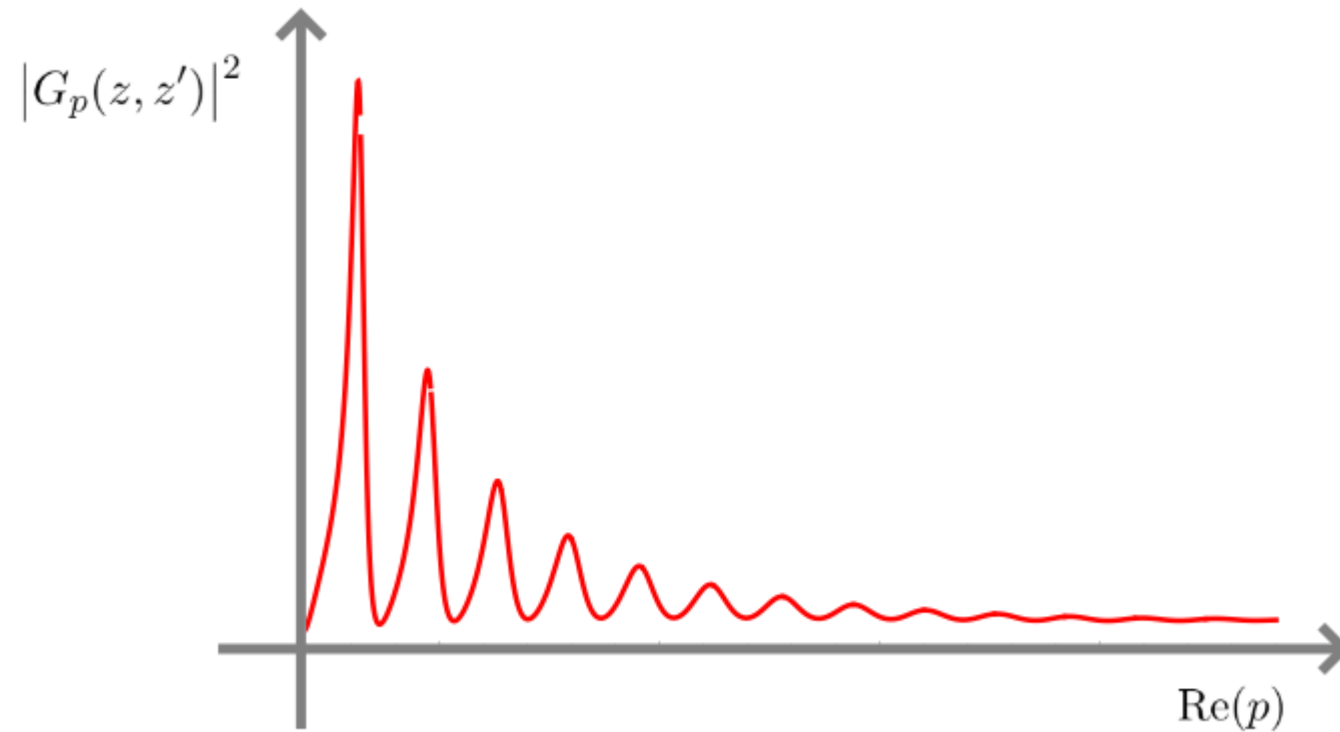
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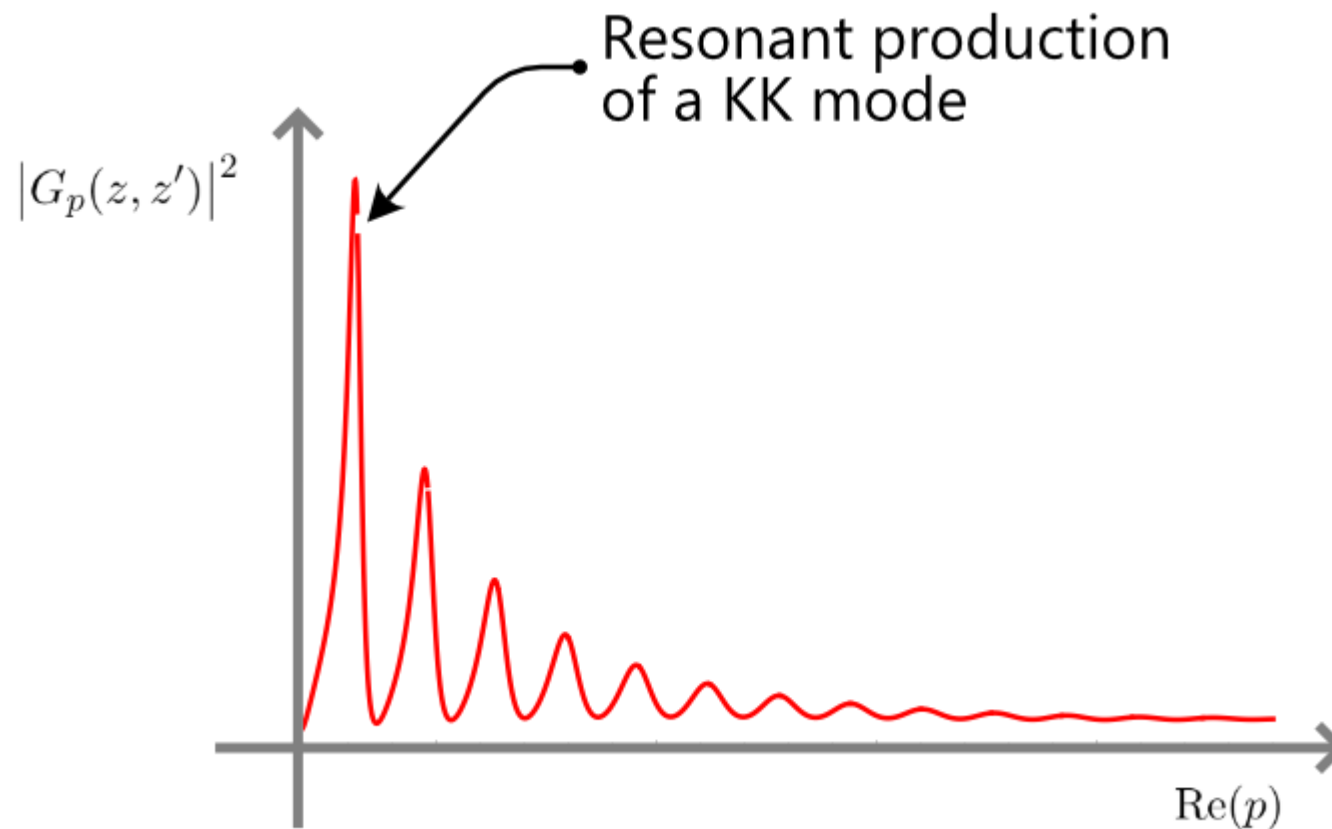
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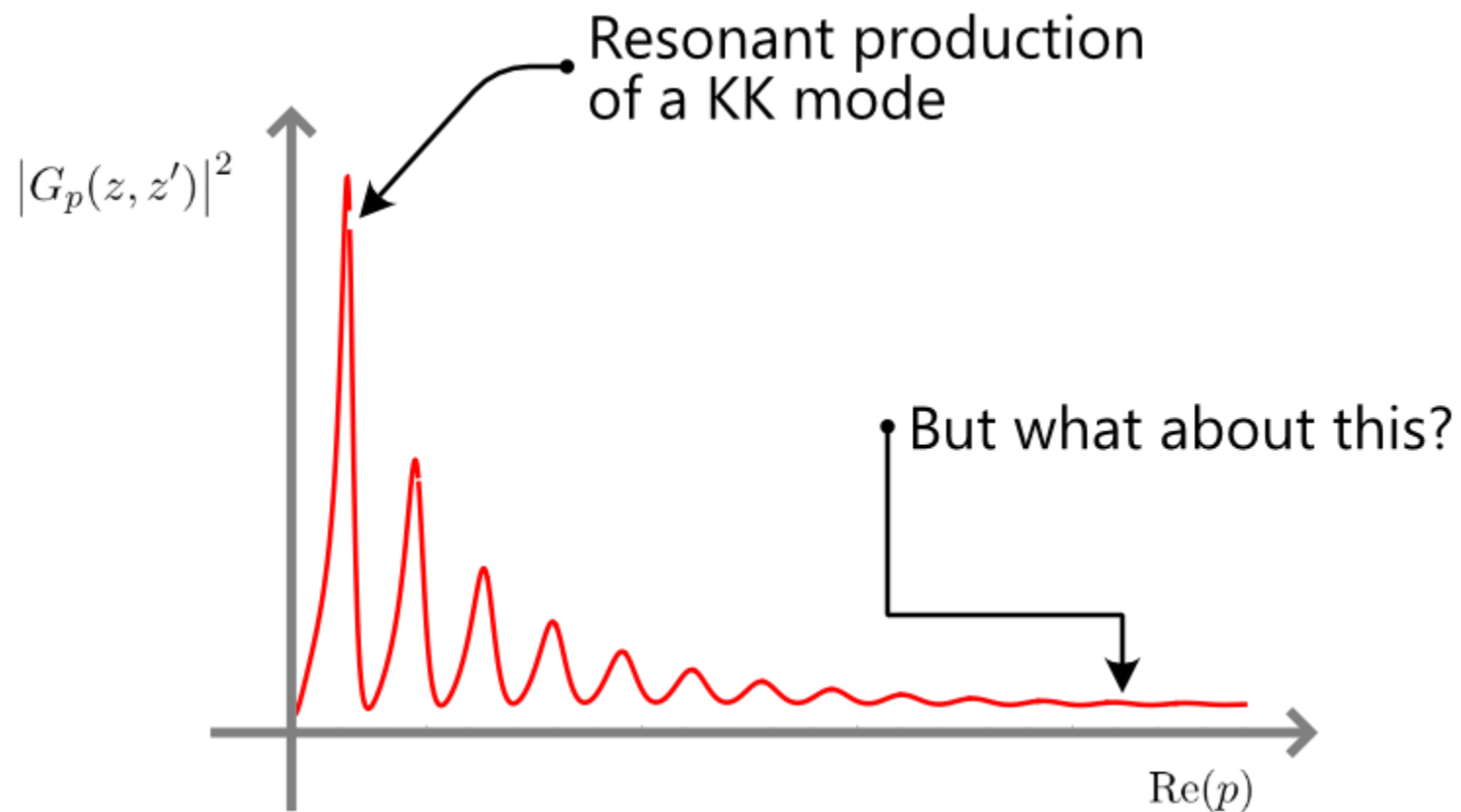
Resummed Propagator



Resummed Propagator

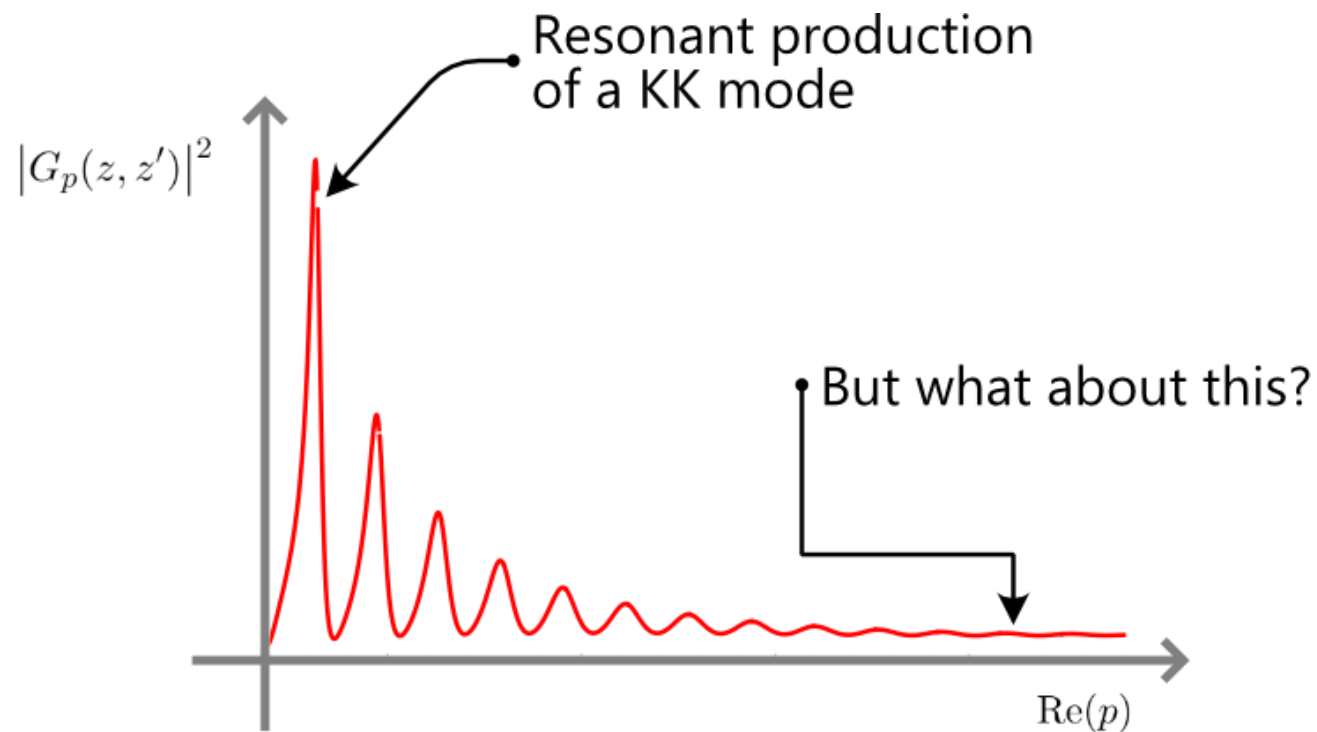


Resummed Propagator



Resummed Propagator

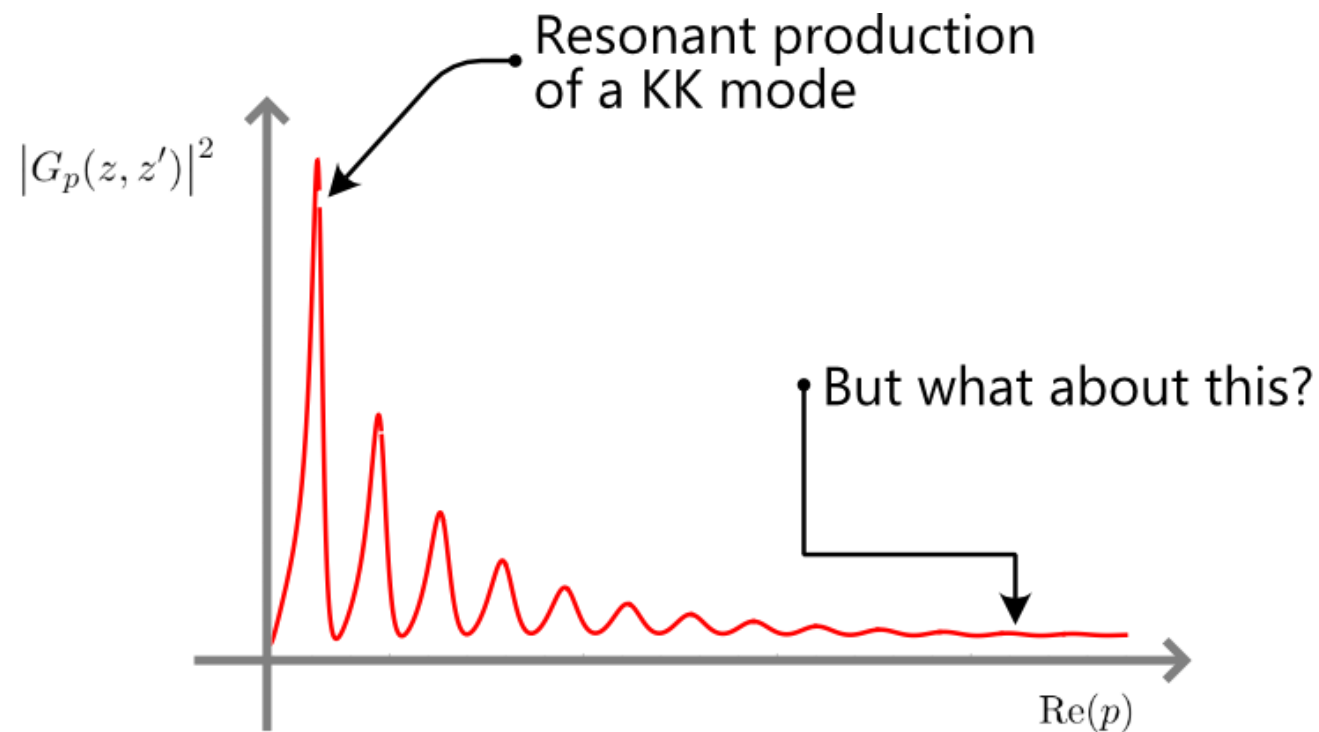
$$G_p(z, z') = i \sum_{n=0}^{\infty} \frac{f_n(z) f_n(z')}{p^2 - m_n^2 + \Pi_n(p^2)}$$



Resummed Propagator

$$G_p(z, z') = i \sum_{n=0}^{\infty} \frac{f_n(z) f_n(z')}{p^2 - m_n^2 + i\epsilon} \Pi_n(p^2)$$

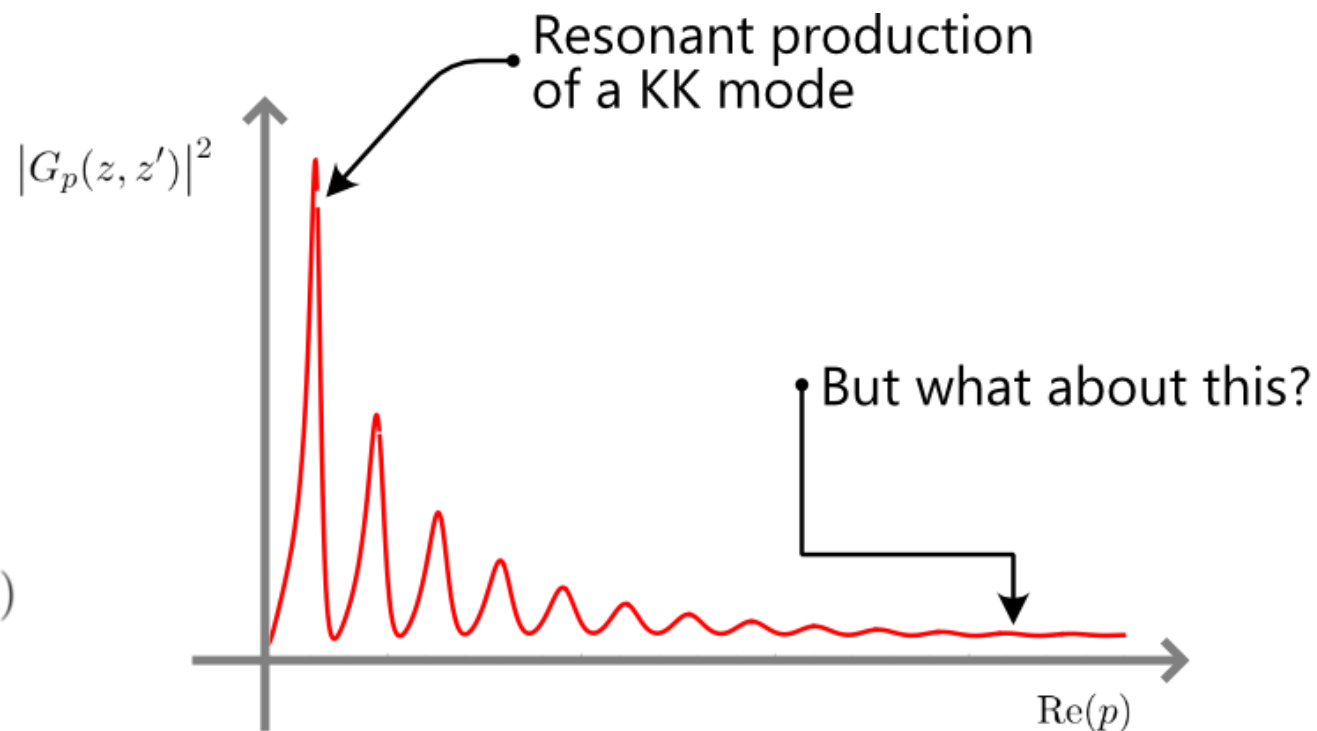
(Note: The original image has a large red 'X' over the denominator of this equation.)



Resummed Propagator

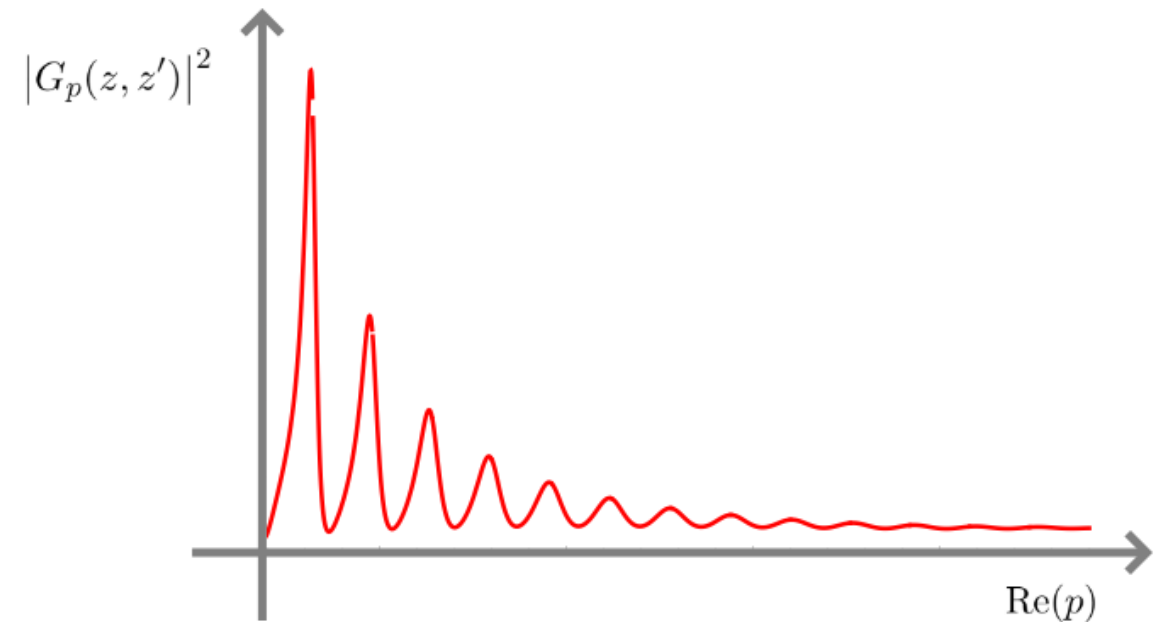
$$G_p(z, z') = i \sum_{n=0}^{\infty} \frac{f_n(z) f_n(z')}{p^2 - m_n^2 + \Pi_n(p^2)}$$

$$G_p(z, z') = i \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} f_m(z) (p^2 - m^2 + \Pi)_{mn}^{-1} f_n(z')$$



Open Questions

- Is the high energy continuum regime physically realizable?
- By treating heavy KK modes as narrow modes and good asymptotic states, could we miss something?



$$G_p(z, z') = i \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} f_m(z) (p^2 - m^2 + \Pi)_{mn}^{-1} f_n(z')$$

Outline

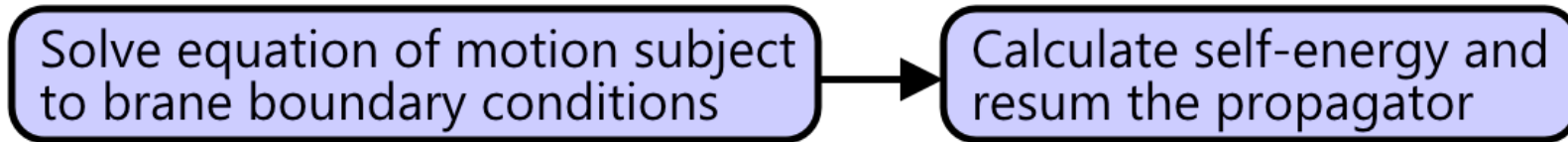
Solve equation of motion subject to brane boundary conditions

Outline

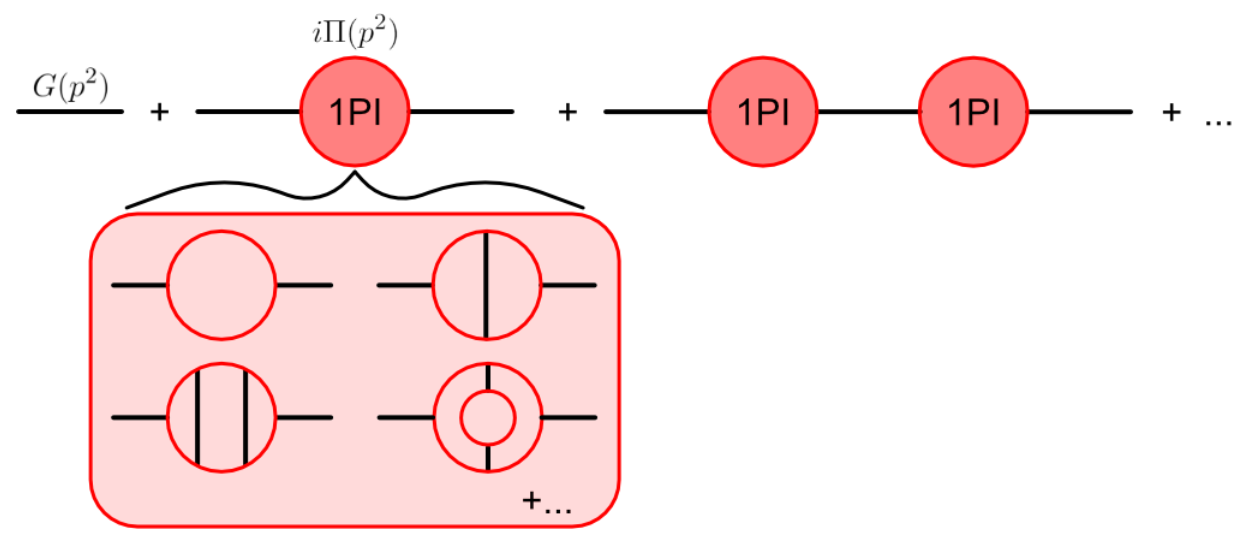
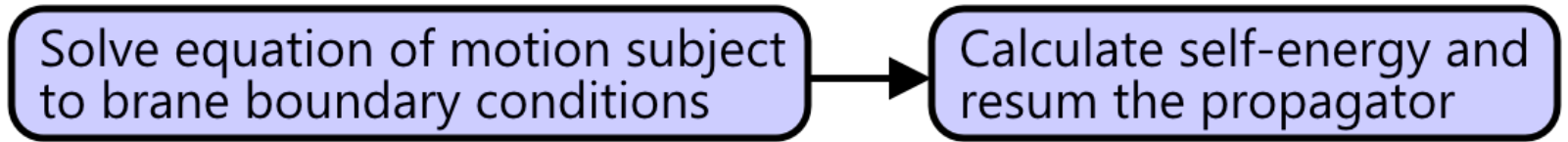
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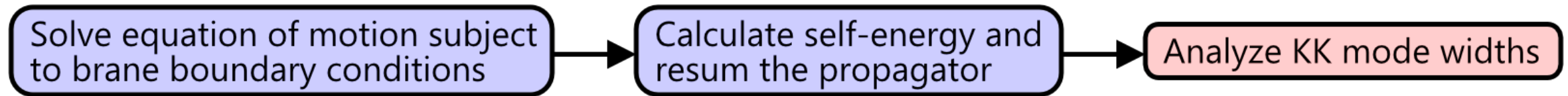
Outline



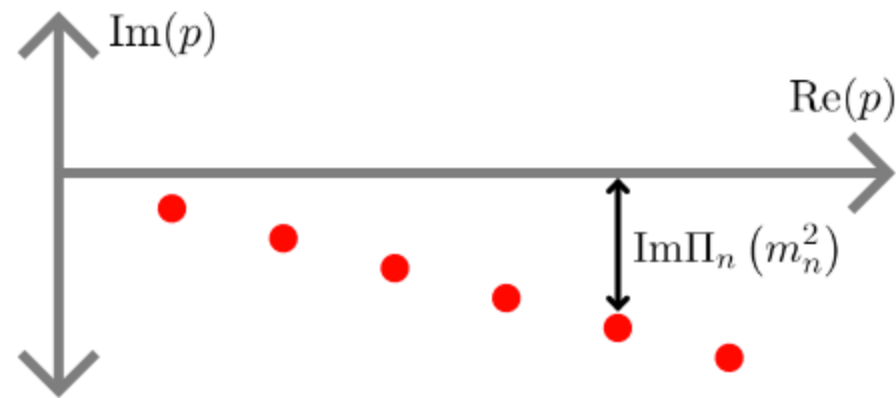
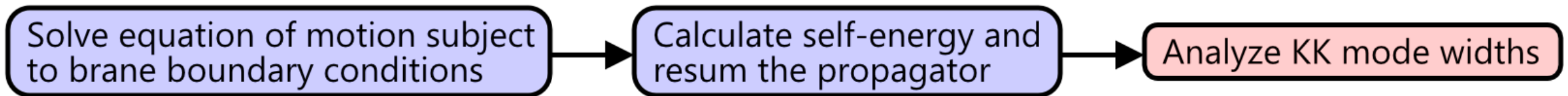
Outline



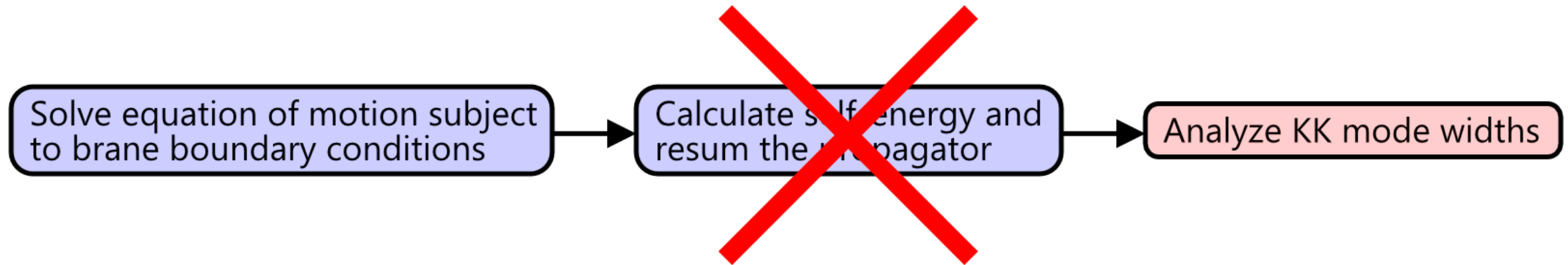
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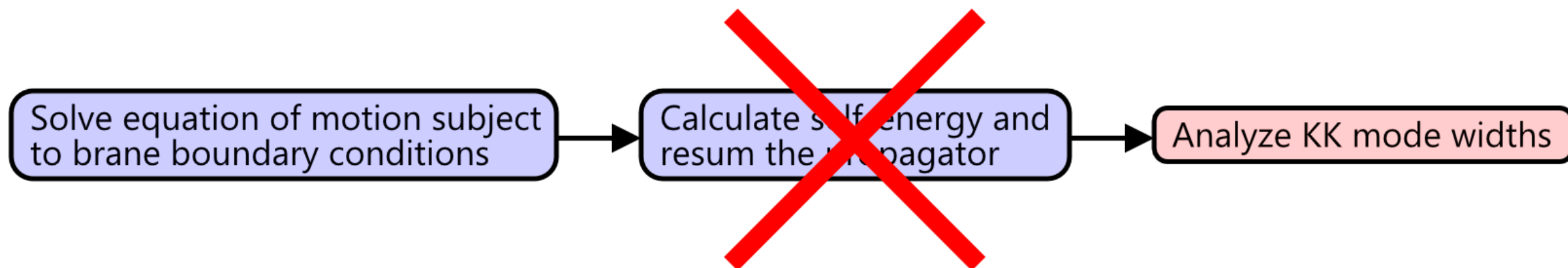
Outline



Outline

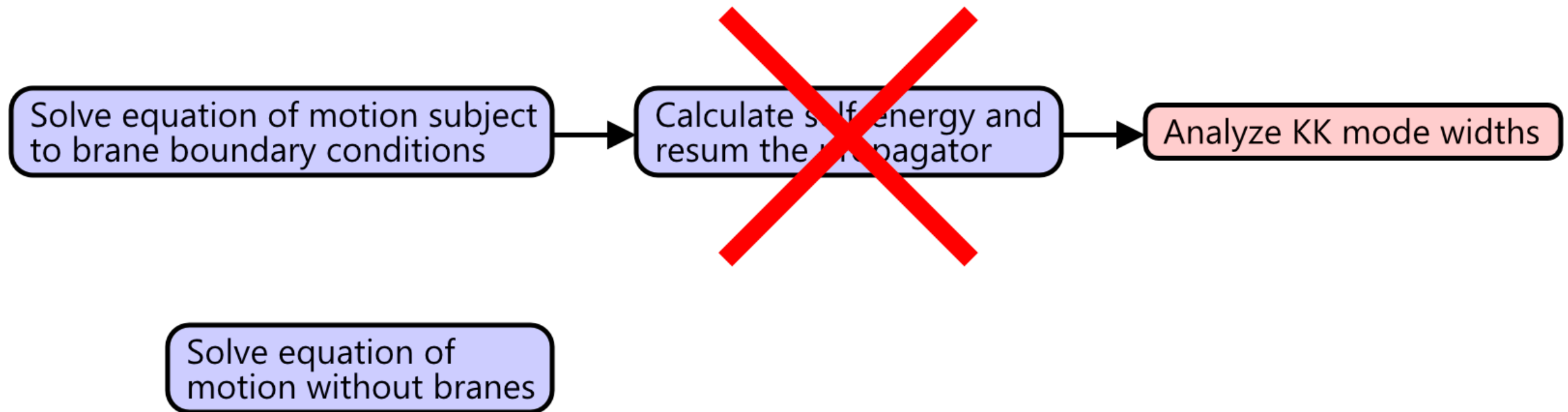


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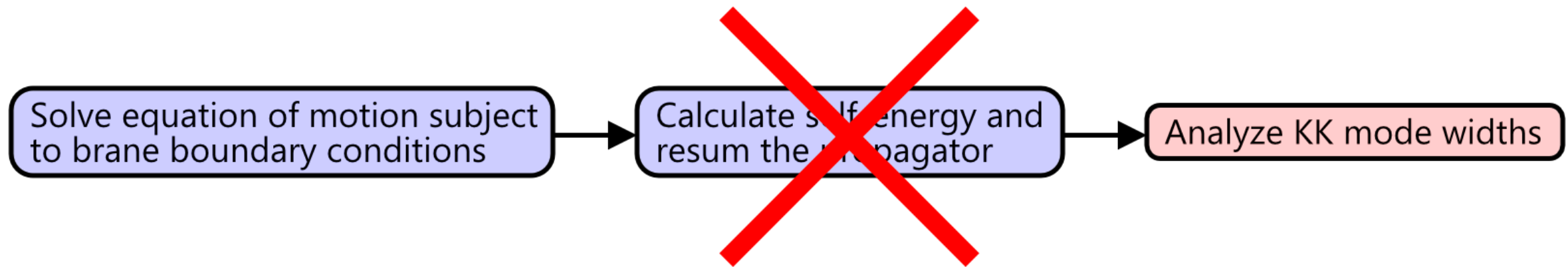


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Outline



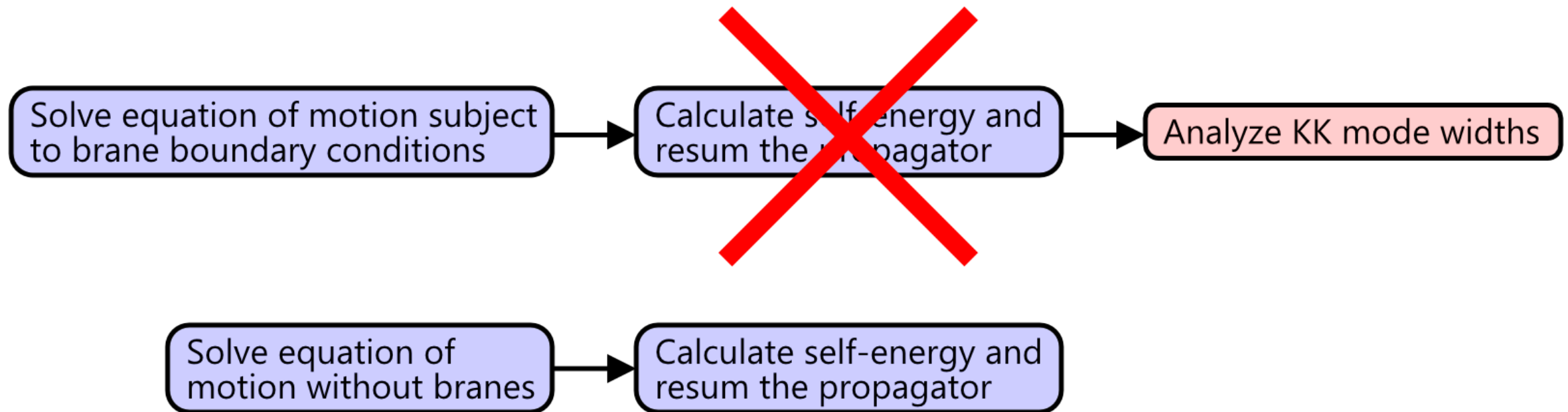
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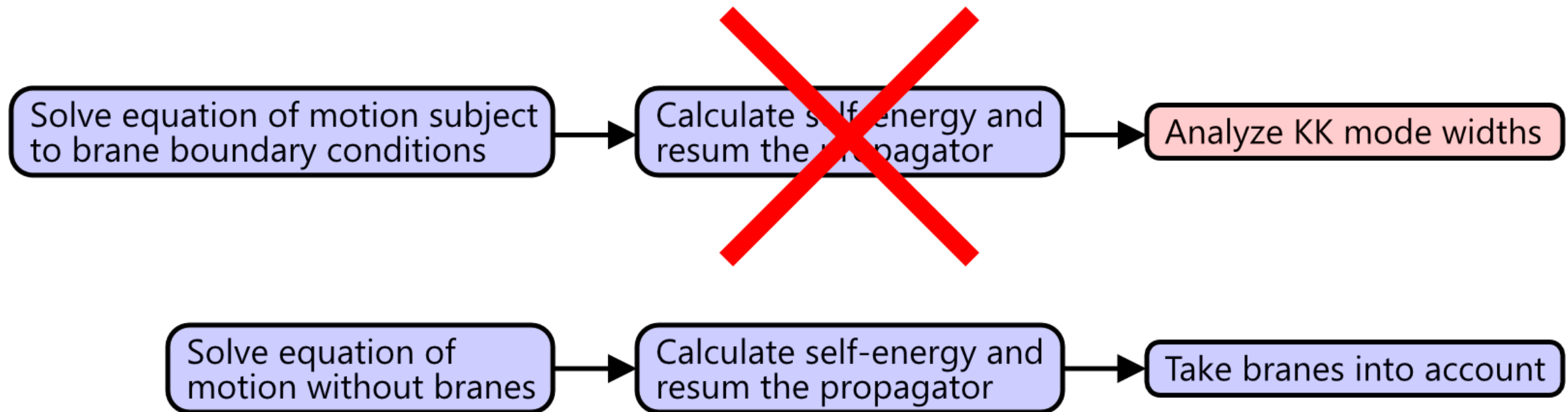
Solve equation of motion without branes

$$G_p(z, z') = \frac{\pi}{2k} (kz)^2 (kz')^2 J_\alpha(pz_{<}) H_\alpha^{(1)}(pz_{>})$$

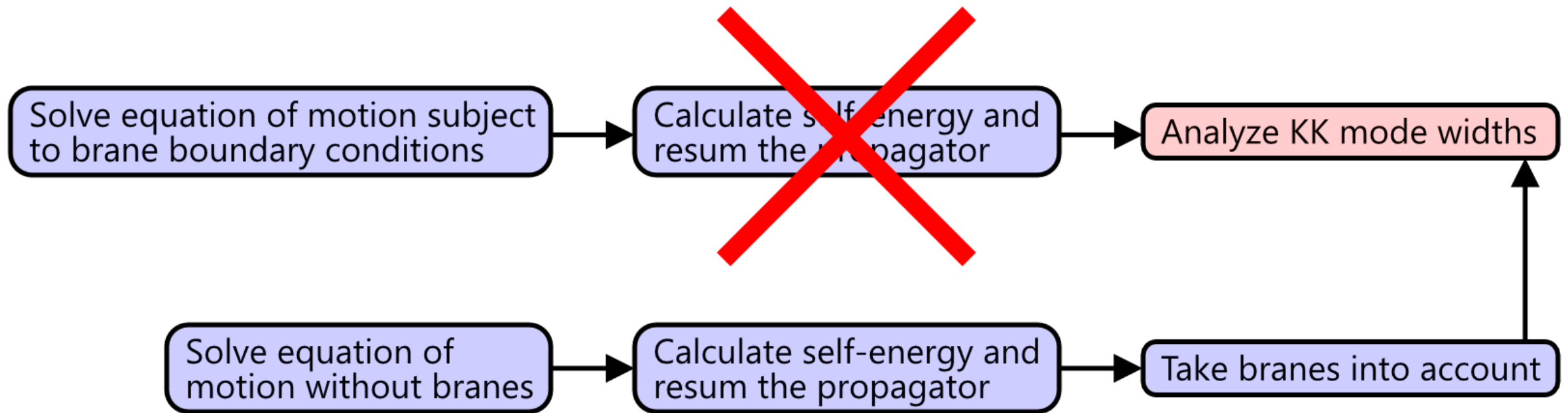
Outline



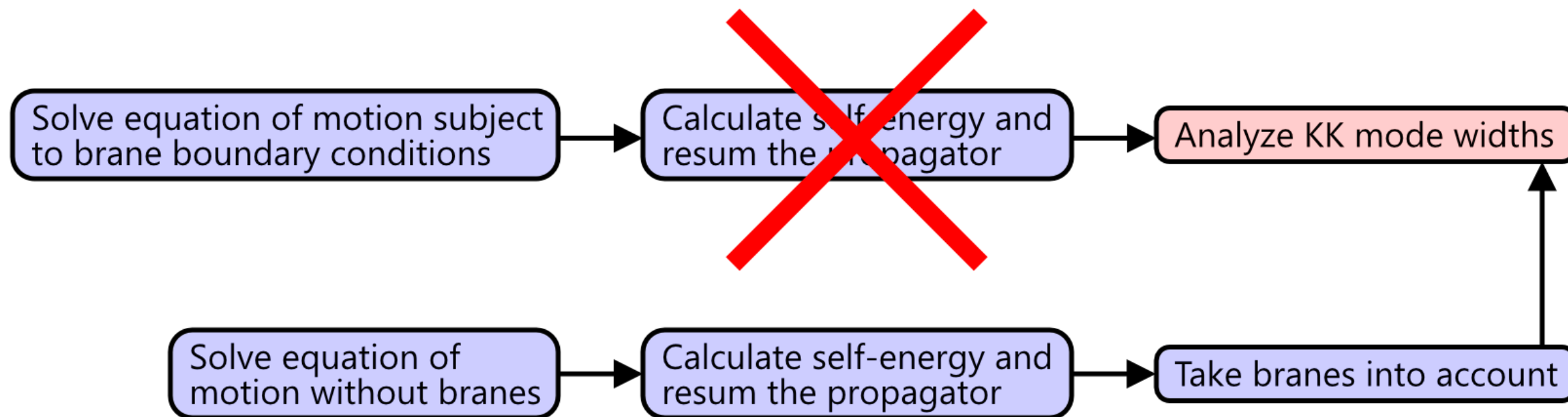
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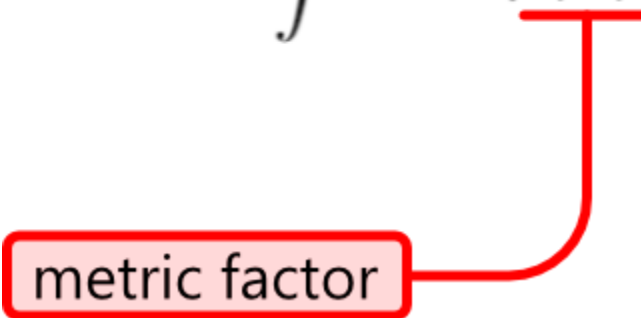
Effective Field Theory in AdS

$$S \supset \int d^5 X \sqrt{|\gamma|} (\lambda \Phi \Phi_1 \Phi_2 - \zeta \Phi \partial_M \Phi_1 \partial^M \Phi_2 + \dots)$$

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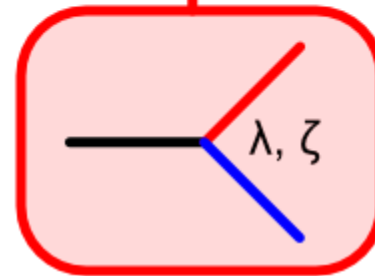
metric factor



Effective Field Theory in AdS

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metric factor



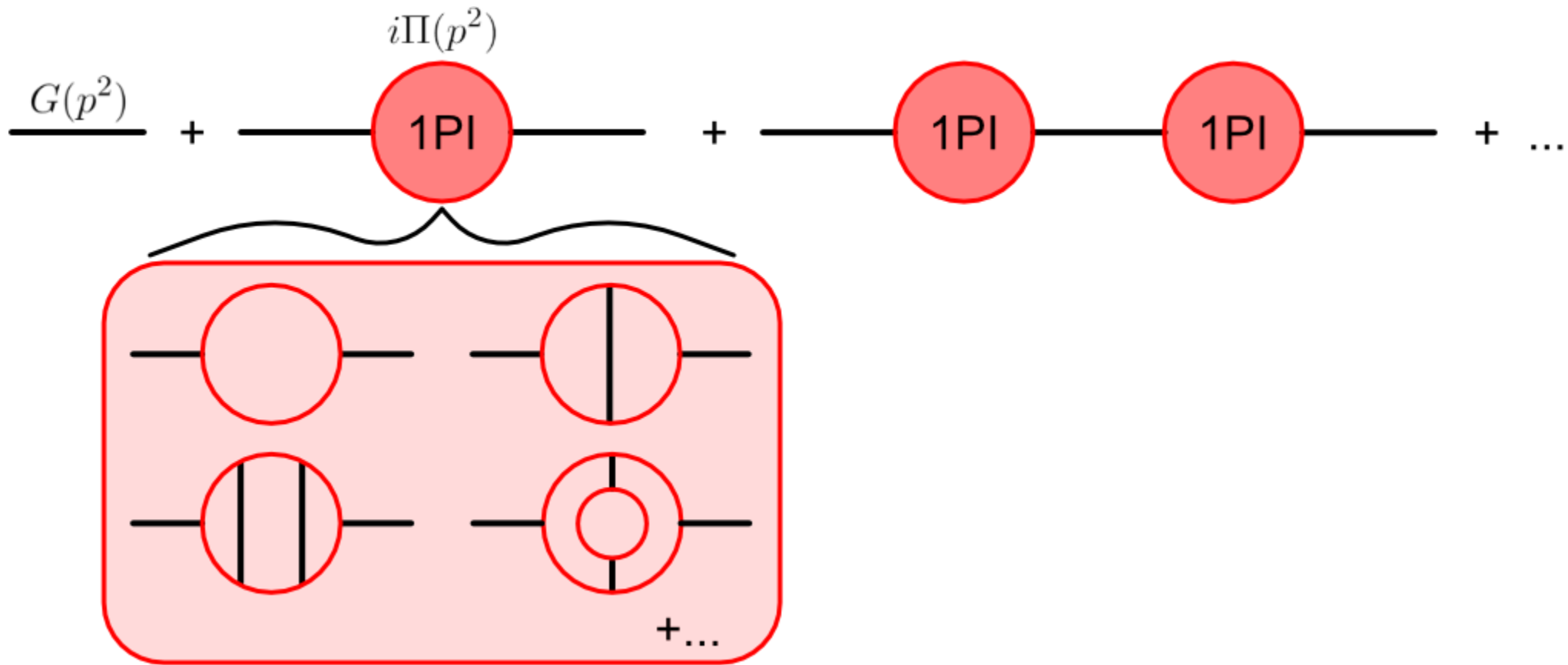
Effective Field Theory in AdS

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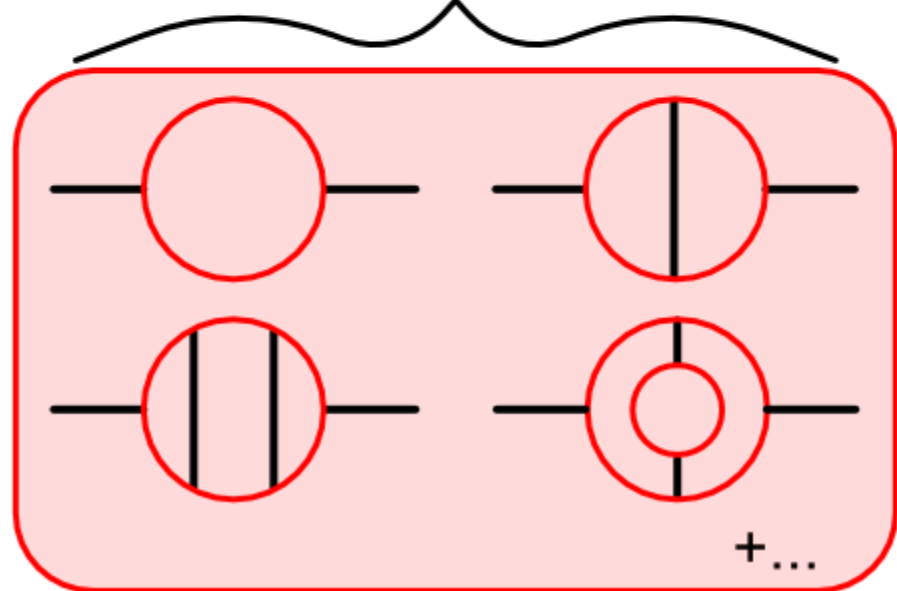
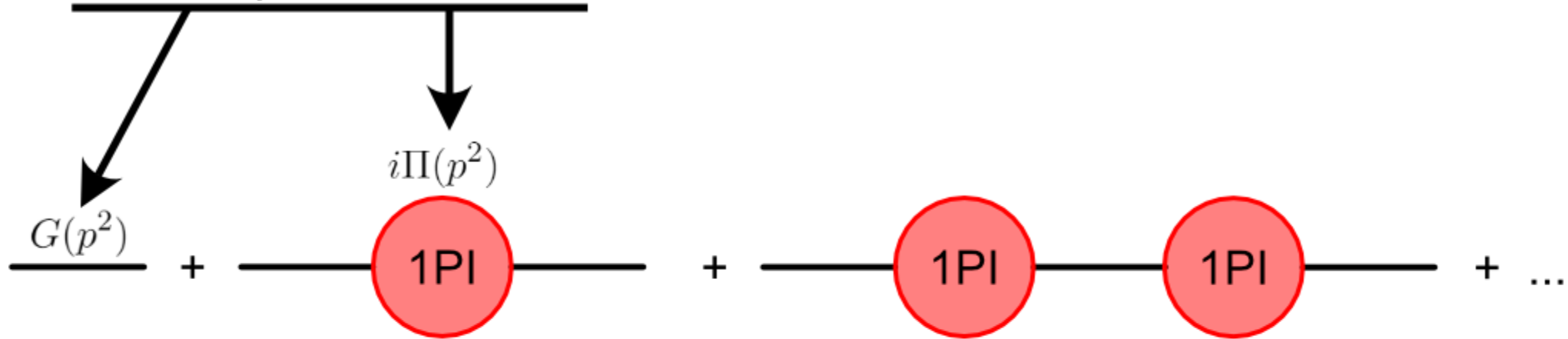
metric factor

$$\lambda \sim \sqrt{24\pi^3 \Lambda} \quad \zeta \sim \sqrt{\frac{24\pi^3}{\Lambda^3}}$$

For details on “naïve” dimensional analysis, see hep-ph/9909248 by Chacko, Luty, and Ponton. Also, see the Ponton TASI lectures.



Also depends on z, z'




Better way: Schwinger-Dyson equation for the 2-point function

$$\mathcal{D}_X G_{\text{dr}}(X, X') - \int d^4 Y \Pi(X, Y) G_{\text{dr}}(Y, X') = -i\delta^4(X - X')$$

Schwinger-Dyson equation for the 2-point function

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$\partial_X^2 + m^2$ 

Schwinger-Dyson equation for the 2-point function

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$\partial_X^2 + m^2$

$$G_{\text{dr}}(X, X') = G_0(X, X') + G_1(X, X') + \dots$$

Schwinger-Dyson equation for the 2-point function

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$$G_{\text{dr}}(X, X') = G_0(X, X') + G_1(X, X') + \dots$$

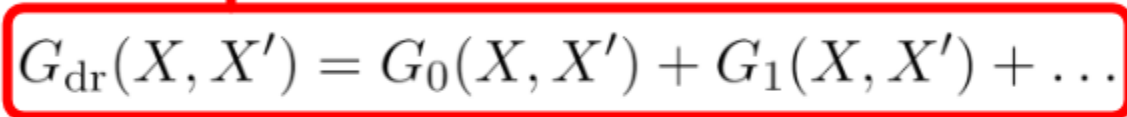
Regroup terms

$$\mathcal{D}_X G_0(X, X') = -i\delta^4(X - X')$$

$$\mathcal{D}_X G_1(X, X') = \int d^4Y \Pi(X, Y) G_0(Y, X')$$

Schwinger-Dyson equation for the 2-point function

$$\mathcal{D}_X G_{\text{dr}}(X, X') - \int d^4Y \Pi(X, Y) G_{\text{dr}}(Y, X') = -i\delta^4(X - X')$$


$$G_{\text{dr}}(X, X') = G_0(X, X') + G_1(X, X') + \dots$$

$$\mathcal{D}_X G_0(X, X') = -i\delta^4(X - X') \quad \xrightarrow[\text{Transform}]{\text{Fourier}} \quad \mathcal{D}G_0 \sim -i$$

$$\mathcal{D}_X G_1(X, X') = \int d^4Y \Pi(X, Y) G_0(Y, X') \quad \xrightarrow[\text{Transform}]{\text{Fourier}} \quad \mathcal{D}G_1 \sim \Pi G_0$$

Schwinger-Dyson equation for the 2-point function

$$\mathcal{D}_X G_{\text{dr}}(X, X') - \int d^4Y \Pi(X, Y) G_{\text{dr}}(Y, X') = -i\delta^4(X - X')$$

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$$(iG_0\mathcal{D}) G_1 \sim \underline{G_1 \sim G_0 i \Pi G_0}$$

2-point Schwinger-Dyson equation in AdS₅

$$\sqrt{|\gamma|} \mathcal{D}_{p;z} G(p^2; z, z') - \int du \Pi(p^2; z, u) G(p^2; u, z') = -i\delta(z - z')$$

2-point Schwinger-Dyson equation in AdS₅

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$$pz \gg |\alpha^2 - \frac{1}{4}|$$

$$G_{\text{dr}}(p^2; z, z') \Big|_{pz \gg |\alpha^2 - \frac{1}{4}|} \propto e^{ipz - S(z)}$$

2-point Schwinger-Dyson equation in AdS₅

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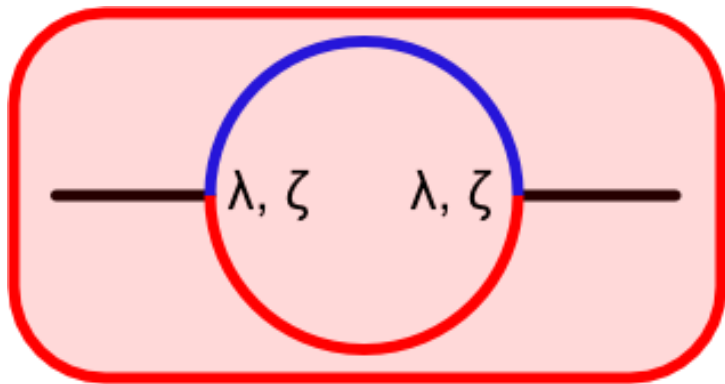
$$pz \gg |\alpha^2 - \frac{1}{4}|$$

$$G_{\text{dr}}(p^2; z, z') \Big|_{pz \gg |\alpha^2 - \frac{1}{4}|} \propto e^{ipz - S(z)}$$

If we know the function $S(z)$, then we know the resummed propagator!

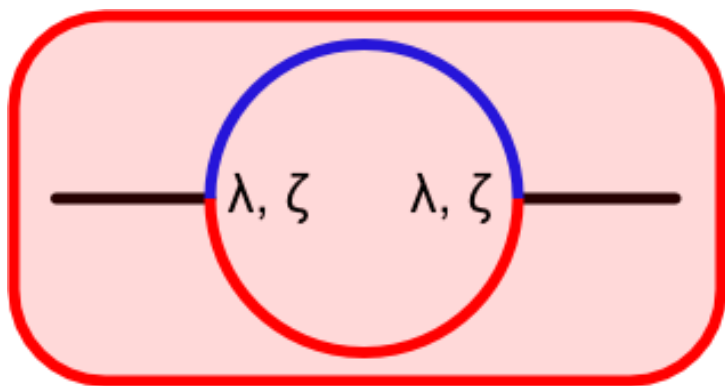
WKB Analysis

$$G_{\text{dr}} \sim e^{-S(z>)}$$



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$$S_{\lambda\lambda}(z) \approx 3 \cdot 10^{-5} \frac{\lambda^2}{k} \log(pz) \quad \left. \vphantom{S_{\lambda\lambda}(z)} \right\} \text{Renormalizable Interaction}$$

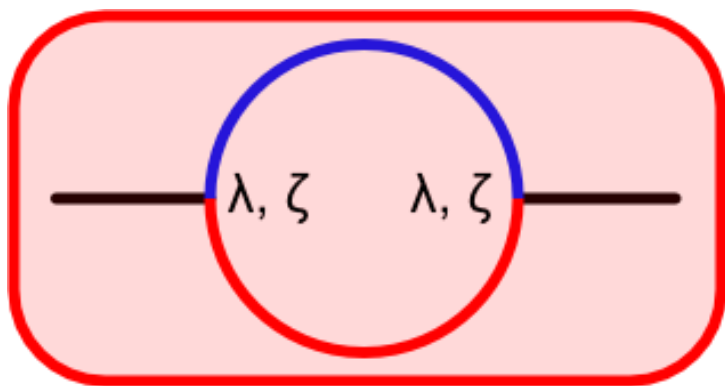
$$S_{\lambda\zeta}(z) \approx 1 \cdot 10^{-5} \lambda\zeta k (pz)^2$$

$$S_{\zeta\zeta}(z) \approx 5 \cdot 10^{-7} \zeta^2 k^3 (pz)^4$$

Higher Dim. Operators

WKB Analysis

$$G_{\text{dr}} \sim e^{-S(z)}$$



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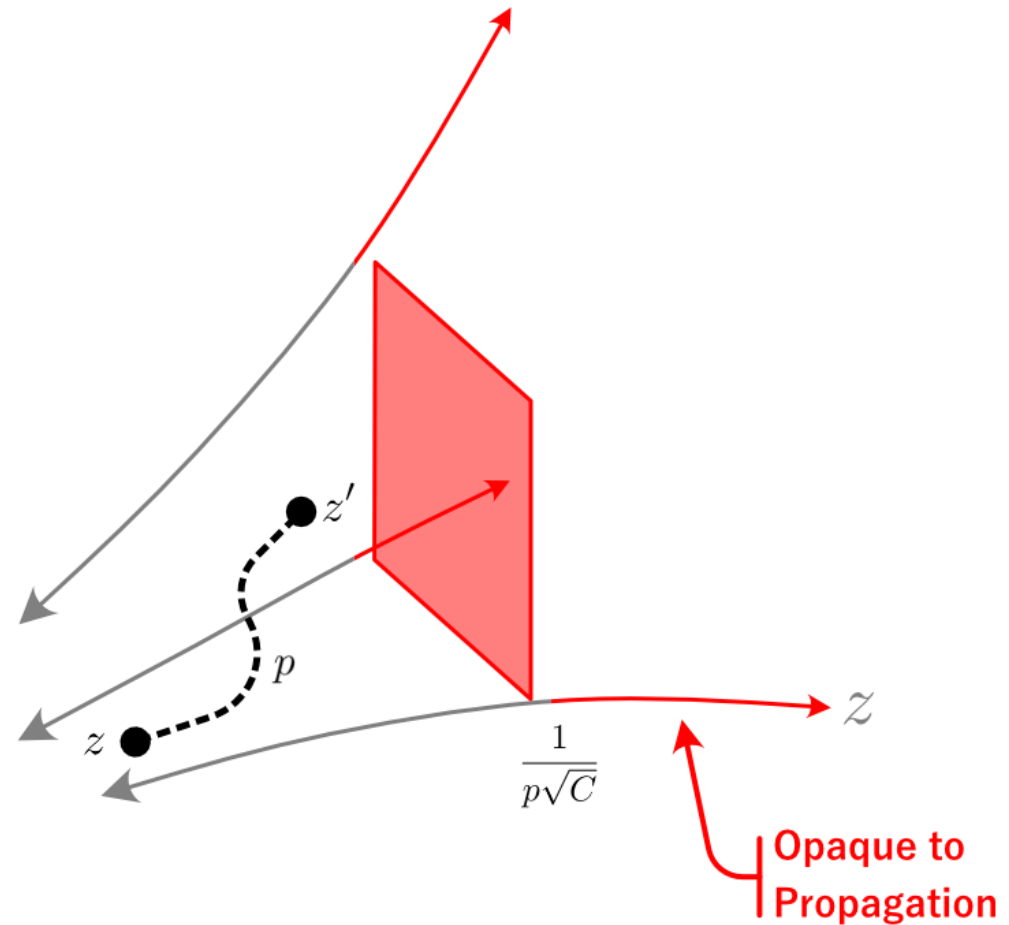
$$S_{\zeta\zeta}(z) \approx 5 \cdot 10^{-7} \zeta^2 k^3 (pz)^4$$

$$G_{\text{dr}}(p; z, z') \propto e^{ipz} e^{-C(pz)^2}$$

$$C = 10^{-5} \lambda \zeta k$$

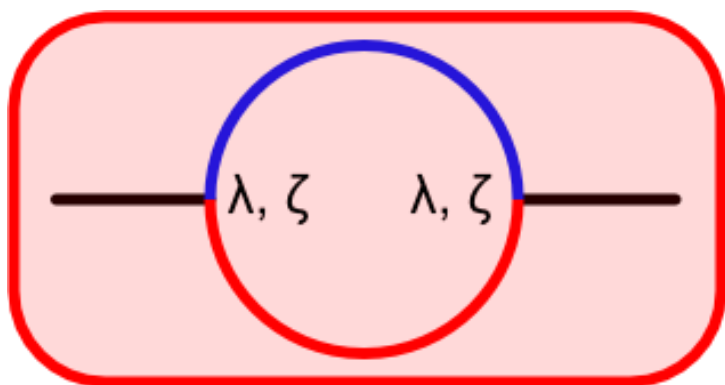
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WKB Analysis

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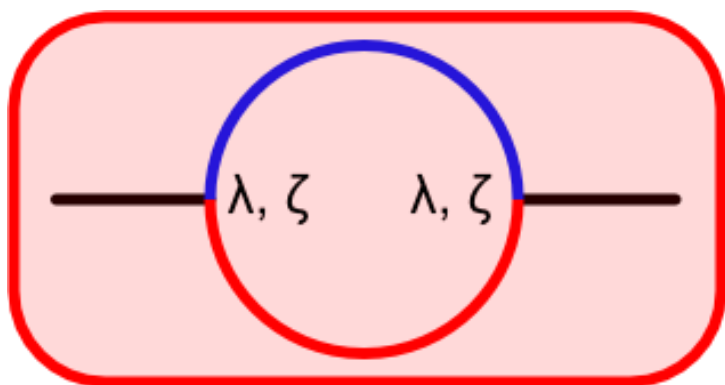
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$$S_{\zeta\zeta}(z) \approx 5 \cdot 10^{-7} \zeta^2 k^3 (pz)^4$$

Same order when EFT breaks down

Is $S \sim \mathcal{O}(1)$ possible in the EFT?

$$S_{\lambda\zeta}(z) \approx 7 \cdot 10^{-3} \frac{k}{\Lambda} (pz)^2$$

$$S_{\zeta\zeta}(z) \approx 4 \cdot 10^{-4} \frac{k^3}{\Lambda^3} (pz)^4$$

Is $S \sim \mathcal{O}(1)$ possible in the EFT?

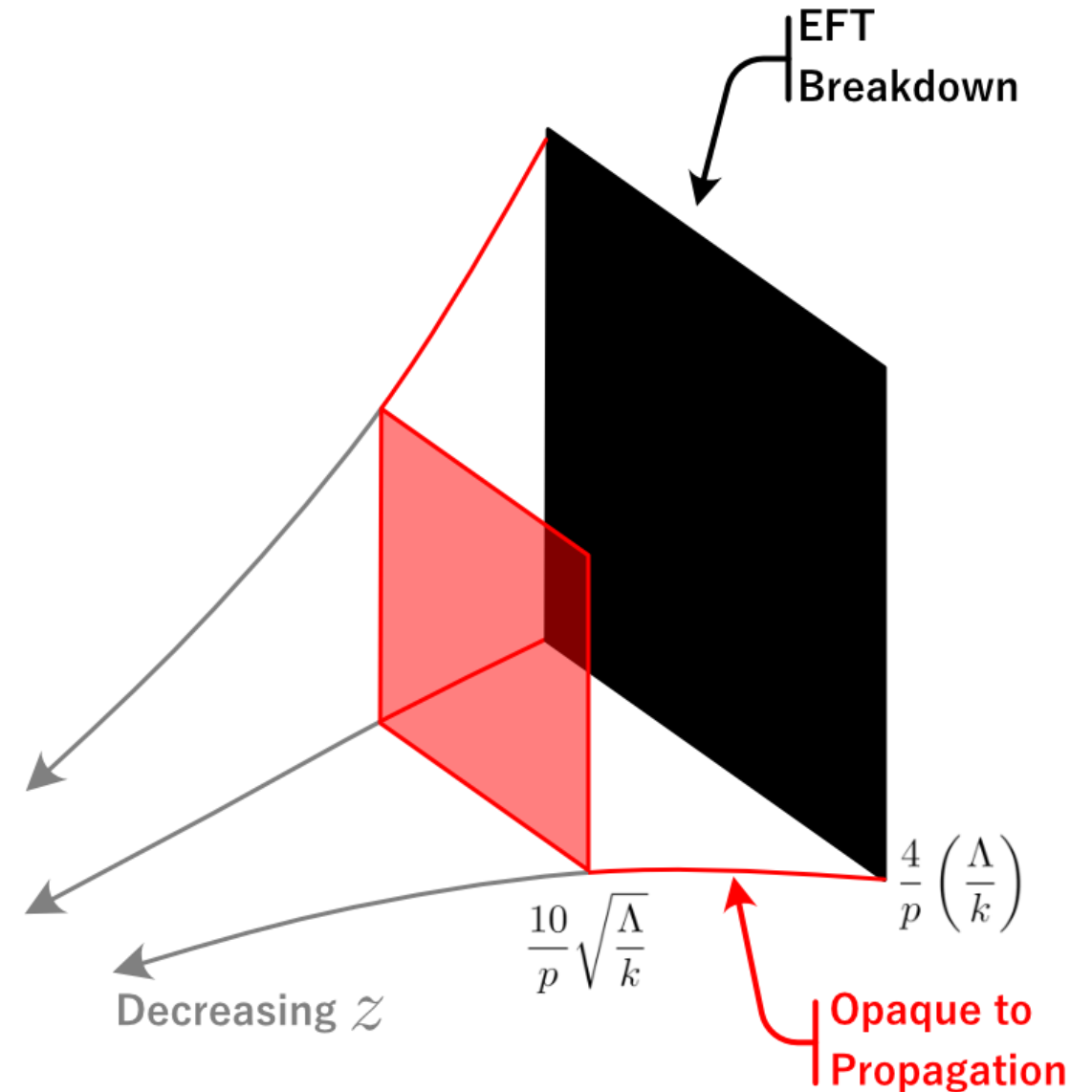
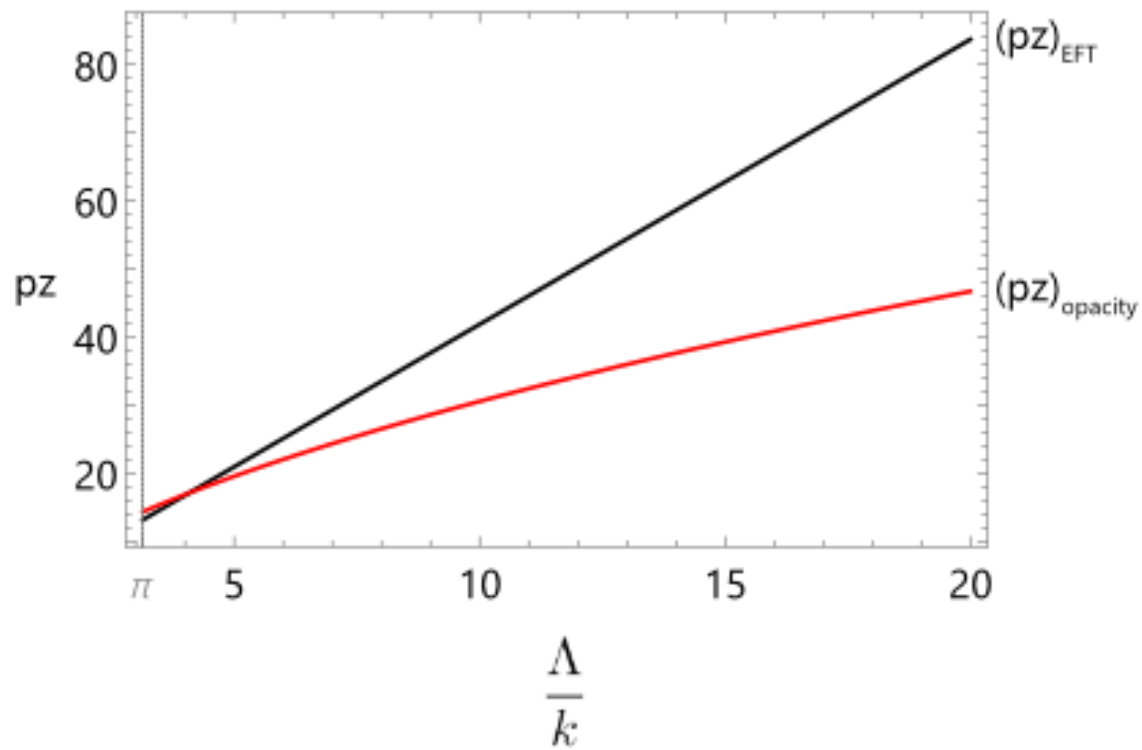
$$S_{\lambda\zeta}(z) \approx 7 \cdot 10^{-3} \frac{k}{\Lambda} (pz)^2 \xrightarrow{S_{\lambda\zeta} \sim \mathcal{O}(1)} (pz)_{\text{opacity}} \sim 10 \sqrt{\frac{\Lambda}{k}}$$

$$S_{\zeta\zeta}(z) \approx 4 \cdot 10^{-4} \frac{k^3}{\Lambda^3} (pz)^4$$

Is $S \sim \mathcal{O}(1)$ possible in the EFT?

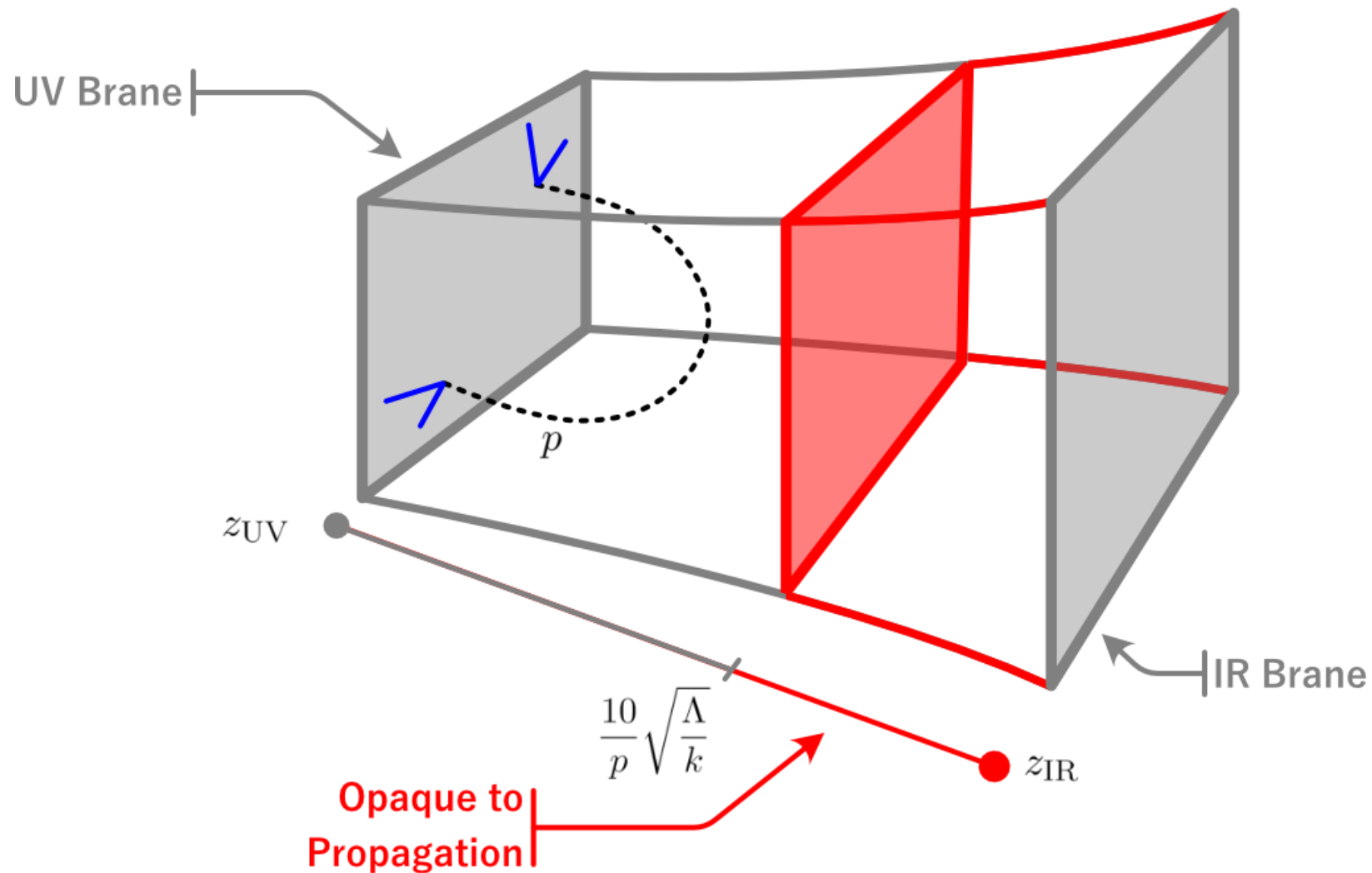
$$\begin{array}{l} S_{\lambda\zeta}(z) \approx 7 \cdot 10^{-3} \frac{k}{\Lambda} (pz)^2 \\ S_{\zeta\zeta}(z) \approx 4 \cdot 10^{-4} \frac{k^3}{\Lambda^3} (pz)^4 \end{array} \quad \begin{array}{l} (pz)_{\text{opacity}} \sim 10 \sqrt{\frac{\Lambda}{k}} \\ S_{\lambda\zeta} \sim S_{\zeta\zeta} \\ (pz)_{\text{EFT}} \sim \frac{4\Lambda}{k} \end{array}$$

Is $S \sim \mathcal{O}(1)$ possible in the EFT?

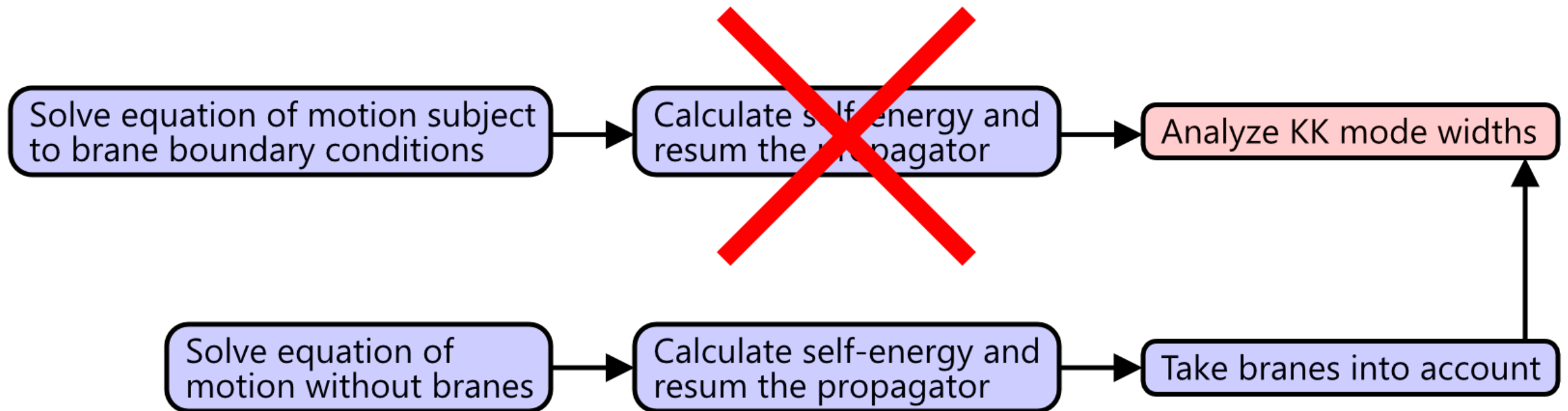


See also hep-th/0012148, where Arkani-Hamed et al. hint at a large z censorship property.

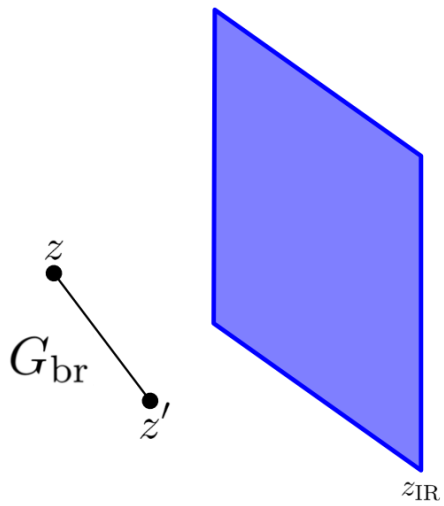
Propagation to the IR brane can be suppressed!



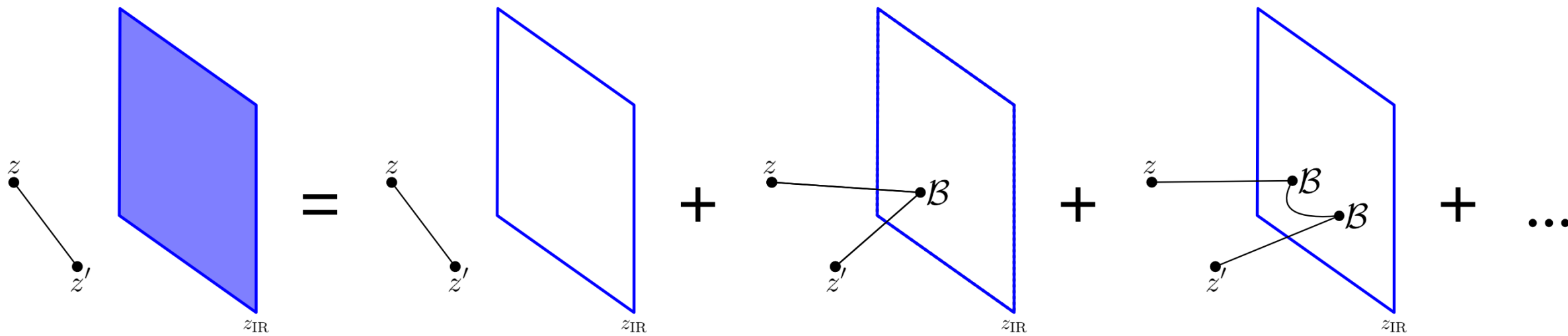
Outline



$$\sqrt{\gamma} \mathcal{D}_{p;z} G(p^2; z, z') = -i\delta(z - z')$$
$$\mathcal{B}(G_{\text{br}}) \Big|_{z=z_{\text{IR}}} = 0$$

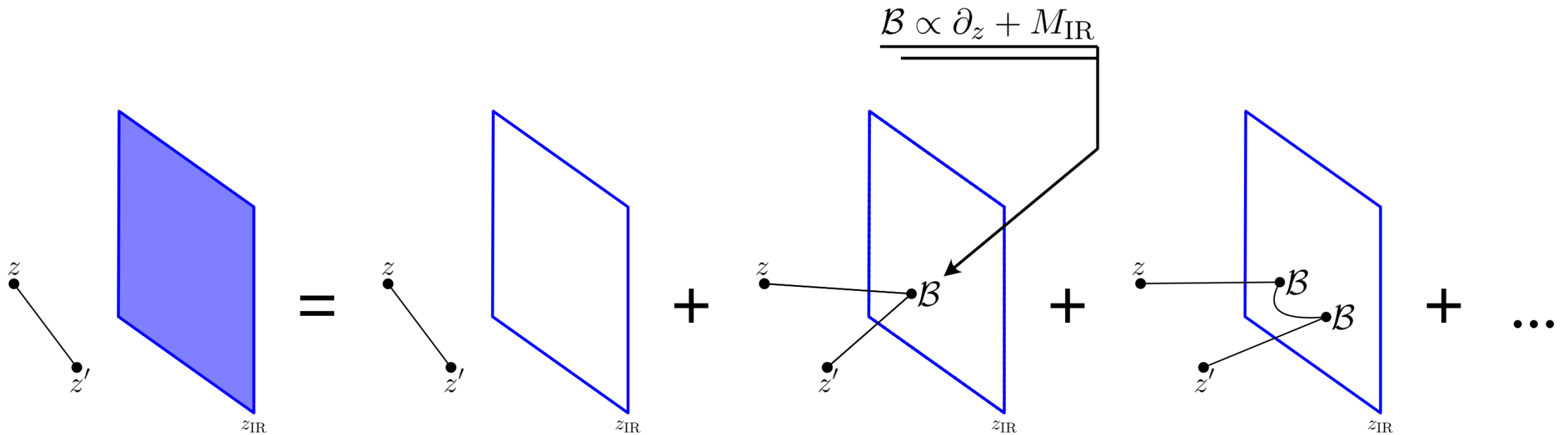


Details will appear in an upcoming paper
by AC and Sylvain Fichet, 2106.xxxxx :-)

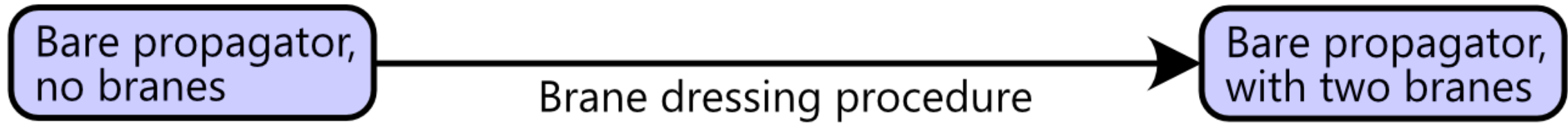


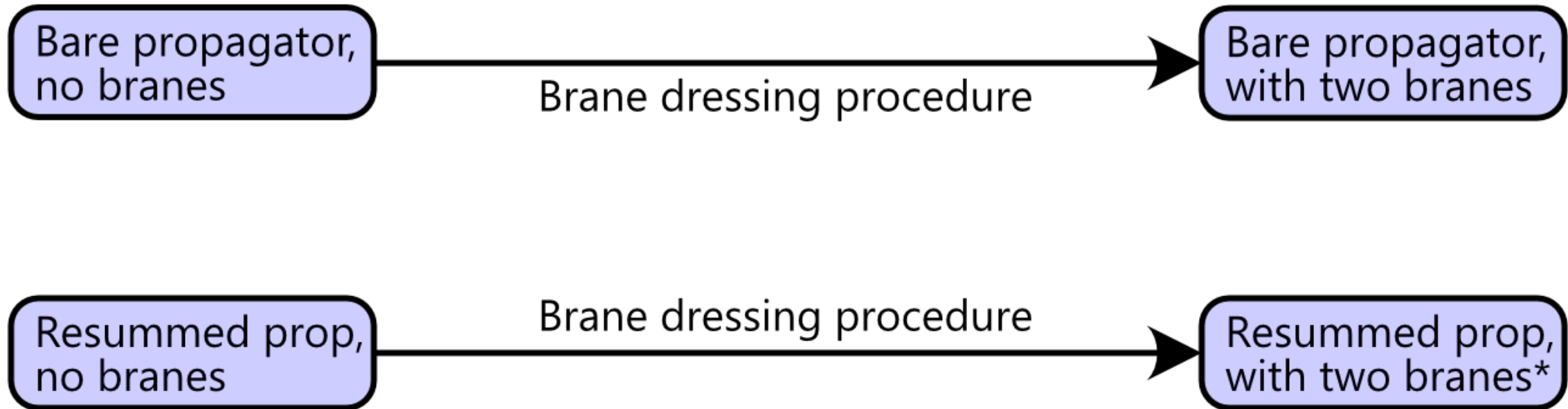
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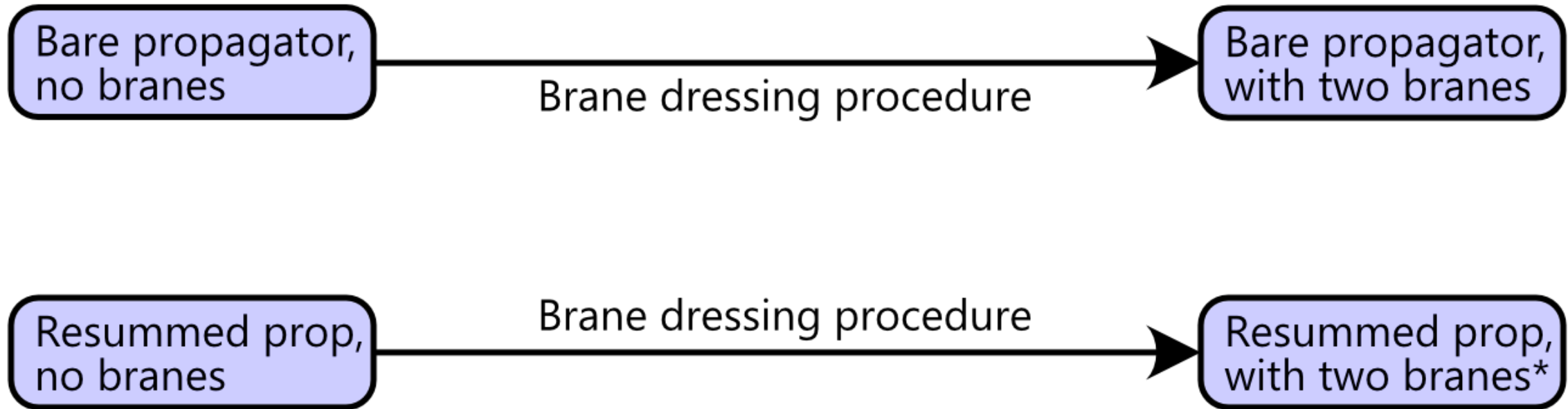
$$G_p(z, z') = \frac{\pi}{2k} (kz)^2 (kz')^2 J_\alpha(pz_{<}) H_\alpha^{(1)}(pz_{>}) \quad G_p(z, z') = i \frac{\pi k^3 (zz')^2}{2} \frac{\left[\tilde{Y}_\alpha^{UV} J_\alpha(pz_{<}) - \tilde{J}_\alpha^{UV} Y_\alpha(pz_{<}) \right] \left[\tilde{Y}_\alpha^{IR} J_\alpha(pz_{>}) - \tilde{J}_\alpha^{IR} Y_\alpha(pz_{>}) \right]}{\tilde{J}_\alpha^{UV} \tilde{Y}_\alpha^{IR} - \tilde{Y}_\alpha^{UV} \tilde{J}_\alpha^{IR}}$$



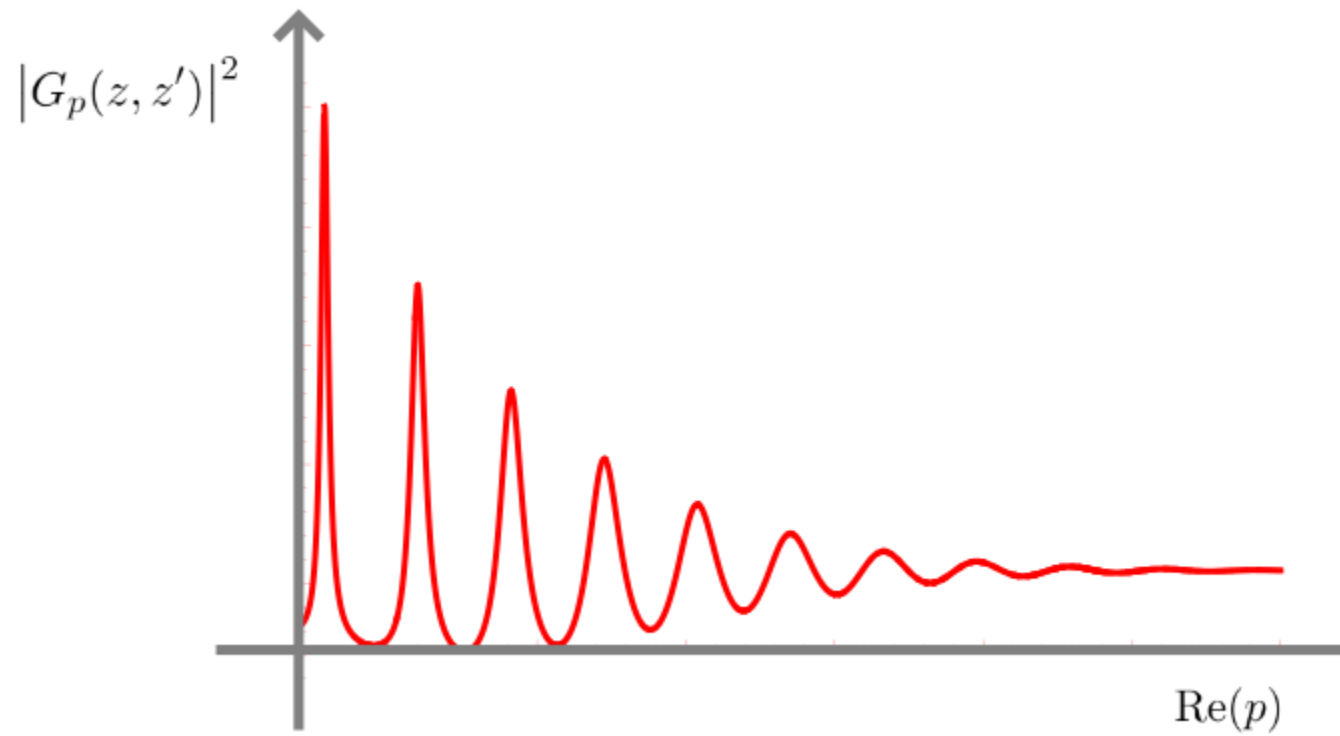
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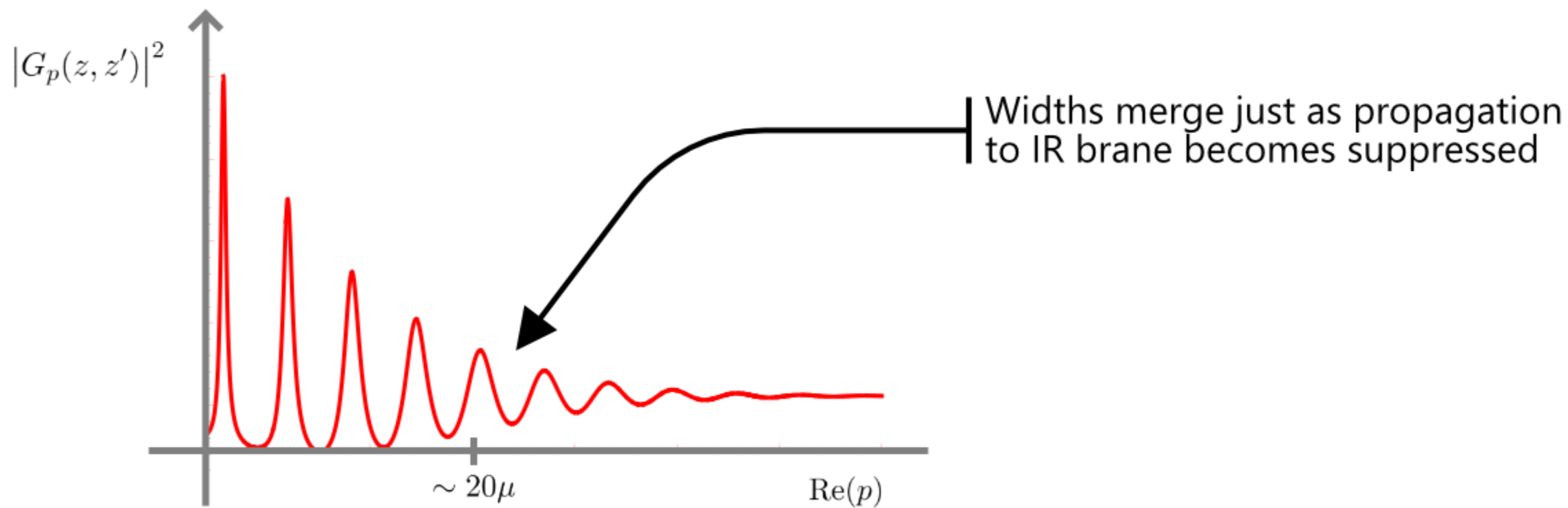


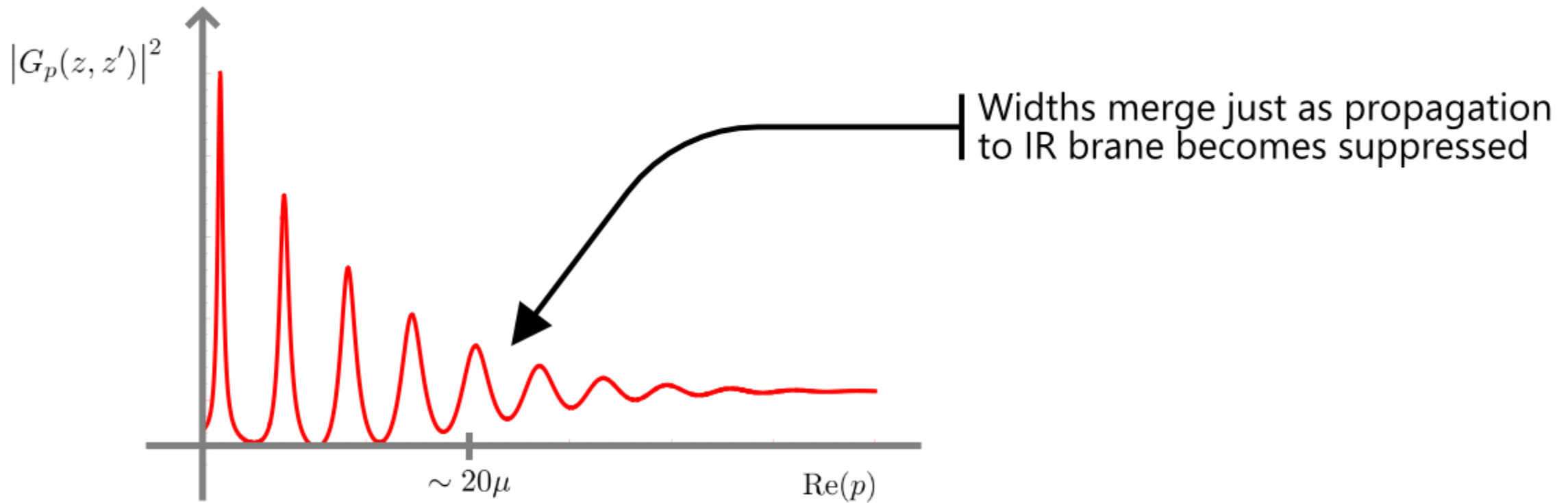




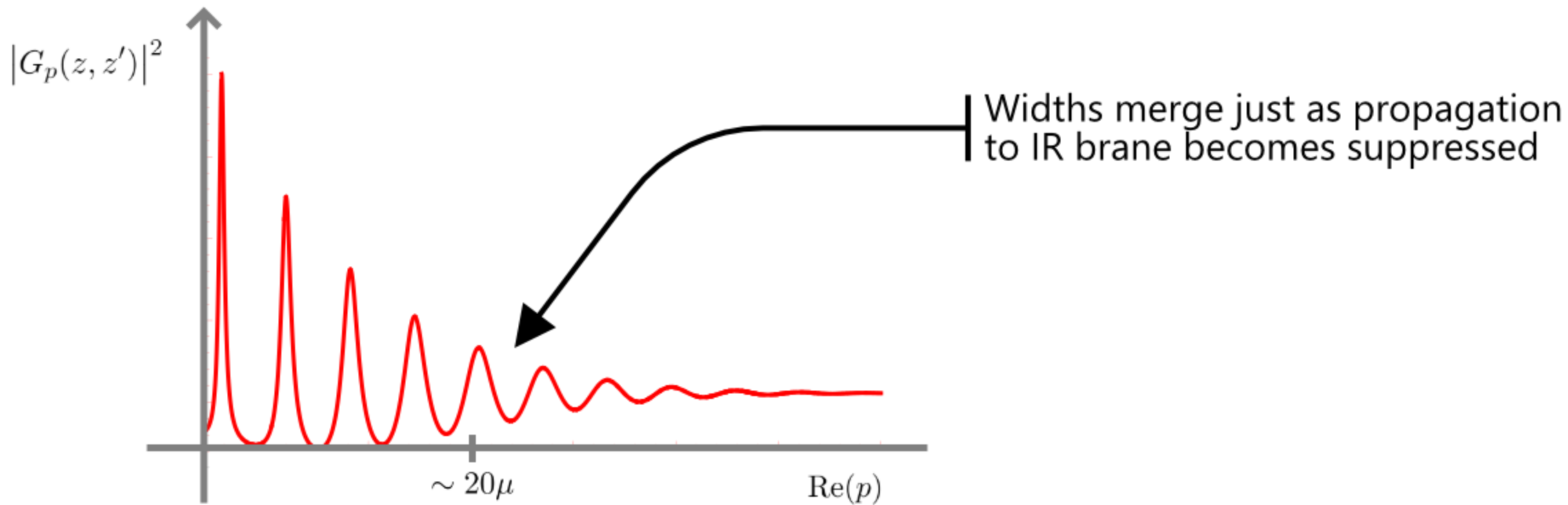
*self-energy was calculated without branes, and this isn't updated by the brane dressing procedure



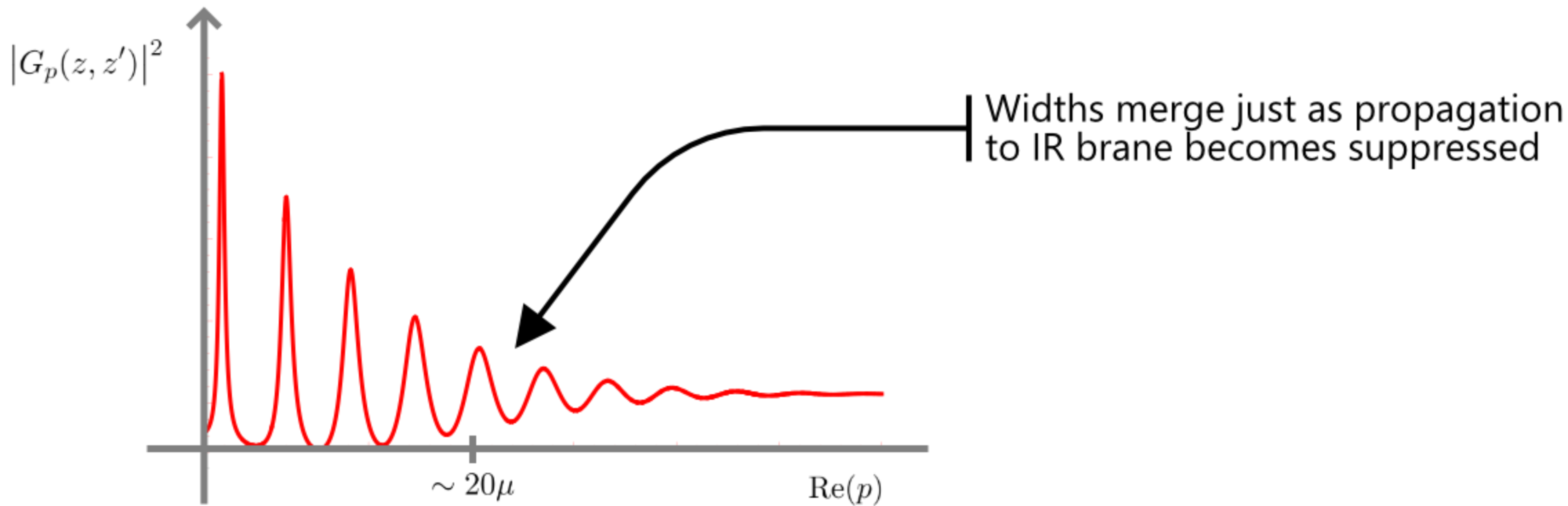




$$G_p(z, z') = i \sum_{n=0}^{\infty} \frac{f_n(z) f_n(z')}{p^2 - m_n^2 + \Pi_n(p^2)}$$



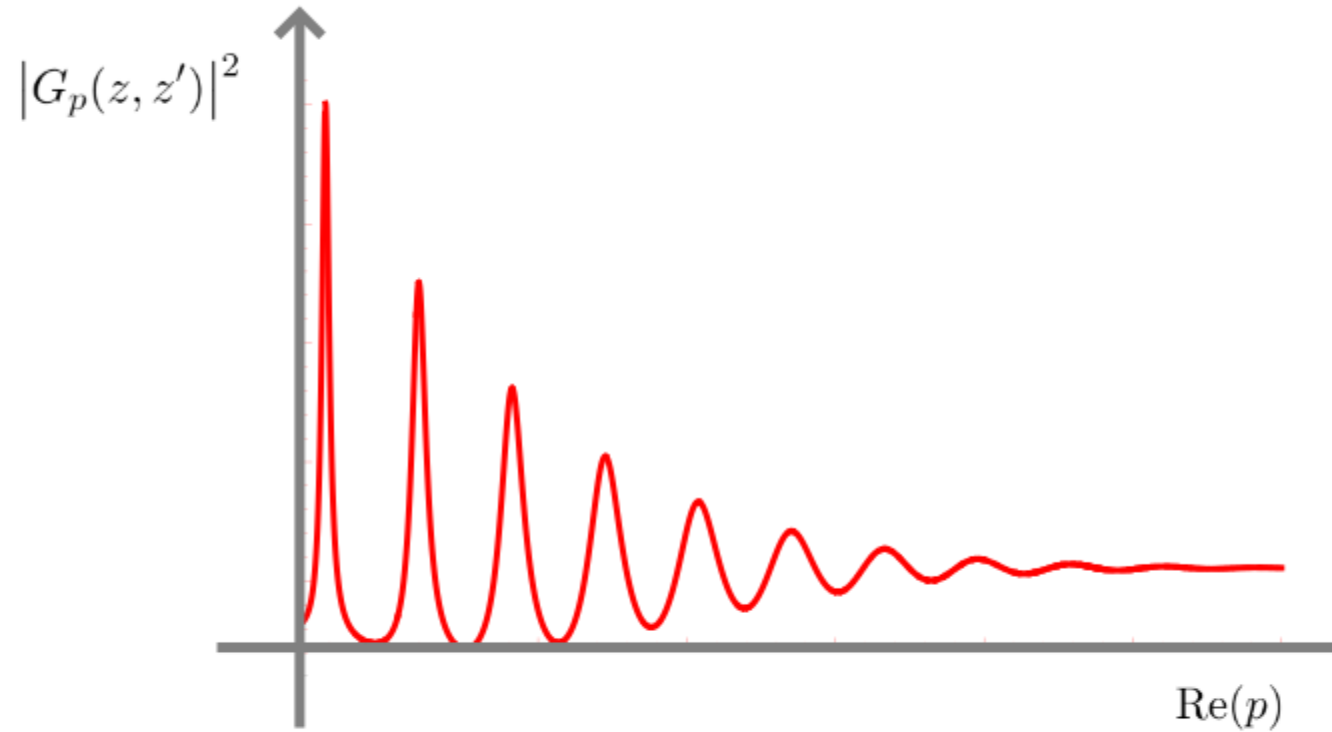
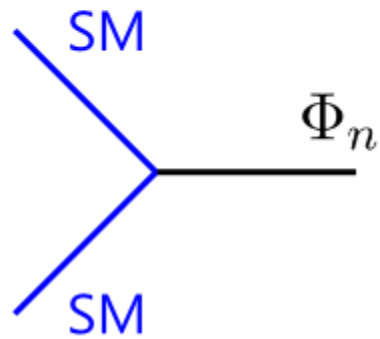
~~$$G_p(z, z') = i \sum_{n=0}^{\infty} \frac{f_n(z) f_n(z')}{p^2 - m_n^2 + \Pi_n(p^2)}$$~~



~~$$G_p(z, z') = i \sum_{n=0}^{\infty} \frac{f_n(z) f_n(z')}{p^2 - m_n^2 + \Pi_n(p^2)}$$~~

$$G_p(z, z') = i \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} f_m(z) (p^2 - m^2 + \Pi)_{mn}^{-1} f_n(z')$$

There is a really nice paper by Cacciapaglia et al. on interference from nondiagonal width matrices: 0906.3417

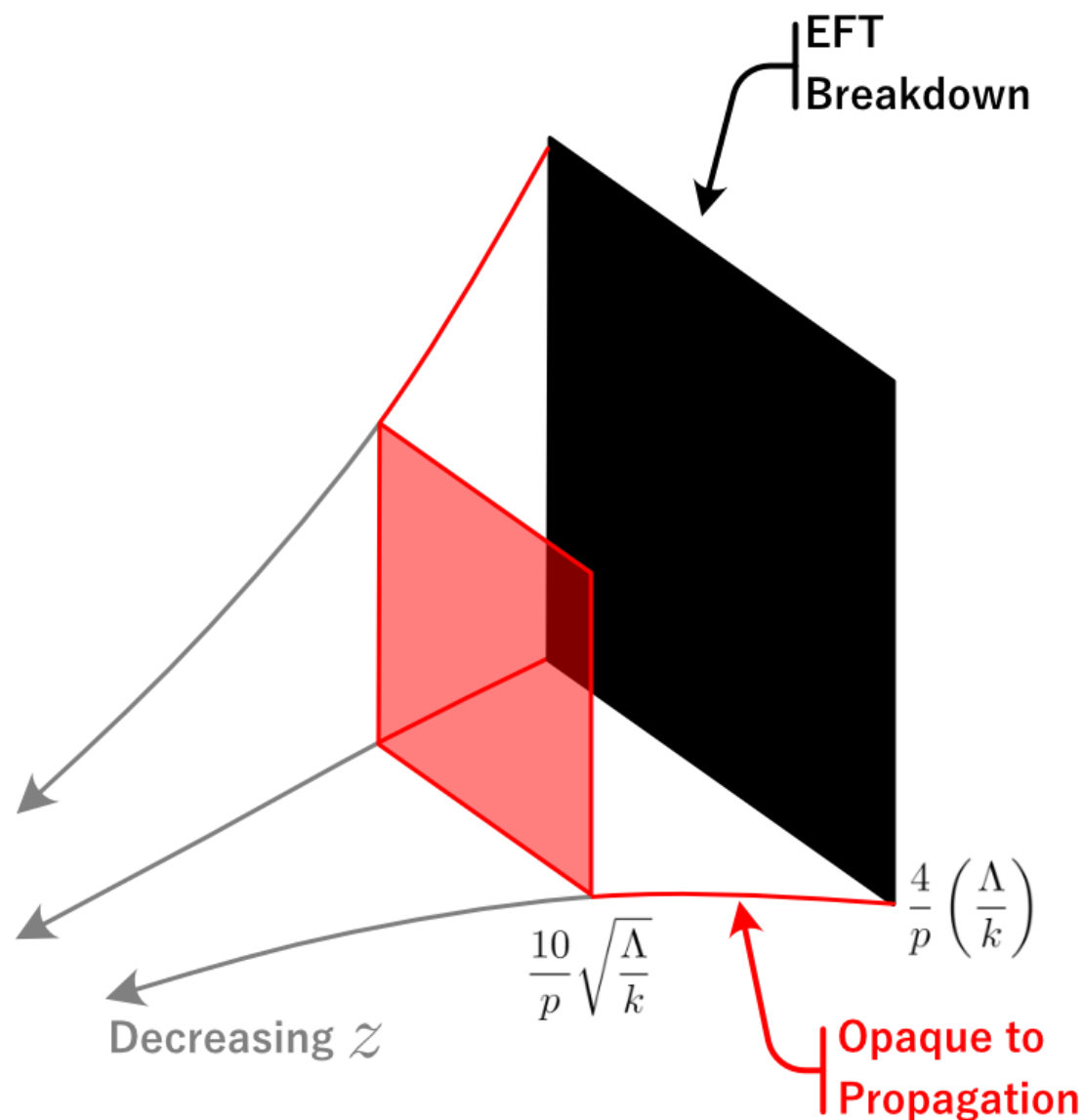


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Summary

- When interactions are present, propagation into the IR region can be suppressed at high energies.
- Region of EFT breakdown is censored by this suppression
- Widths mix and merge to form a continuum at energies when the IR brane becomes opaque to propagation



Thanks!