On-Shell Constructions of the Non-linear Sigma Model

Based on 1904.12859, 1911.08490 and 2009.00008, collaborations with Ian Low and Laurentiu Rodina

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Outline

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1. The local constructions
2. The soft bootstrap
3. The double copy

The EFT for NGBs

NLSM: EFT that describes NGB's

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- In the amplitudes:

$$\mathcal{M}_{n,L} = \frac{1}{f^{n+2L-2}} \left(\text{Series of } \frac{p}{\Lambda}\right).$$

The local constructions

The coset construction:

Callan, Coleman, Wess and Zumino, 1969

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where $\xi = \exp(i\pi^a \mathrm{X}^a/f)$

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$$d_\mu \to h d_\mu h^\dagger, E_\mu \to h E_\mu h^\dagger - i h \partial_\mu h^\dagger$$

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- The QCD chiral Lagrangian: $\mathrm{SU}(N) \times \mathrm{SU}(N)/\mathrm{SU}(N)$, $N = 2, 3$
- The standard model: $\mathrm{SO}(4)/\mathrm{SO}(3)$
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We only consider representations R that can be embedded into a symmetry coset: "Closure condition"

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Universality in pNGB Higgs!

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Otherwise, Adler's zero no longer holds: $\mathcal{M}(\tau p) = \mathcal{O}(\tau^0)$

Kampf, Novotny, Shifman, Trnka, 1910.04766

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- Unveil hidden structures, e.g. double copy structures

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Constructing all amplitudes without $\mathcal{L}$ or Feynman diagrams

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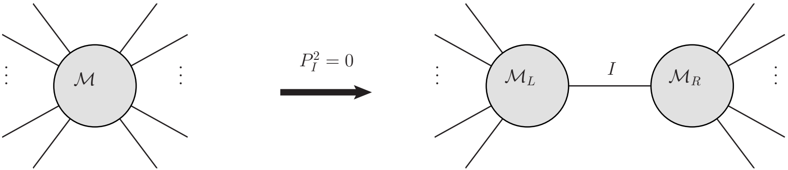

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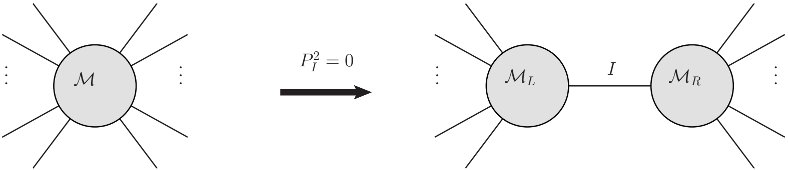

Contact terms? Need more constraints:

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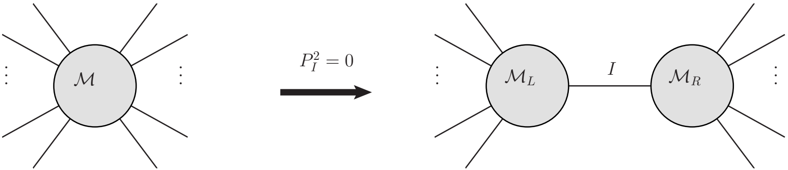

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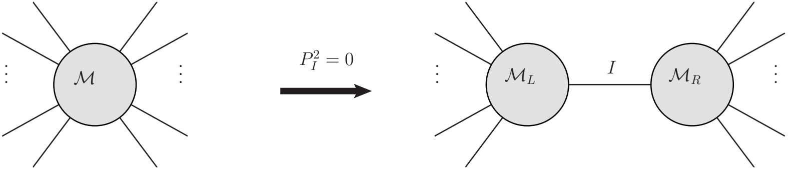

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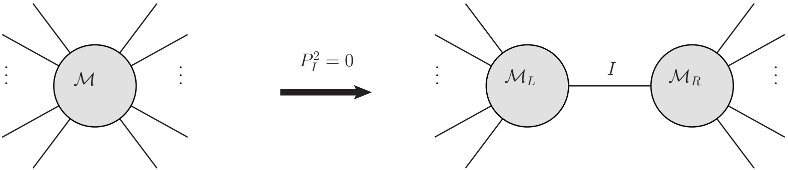

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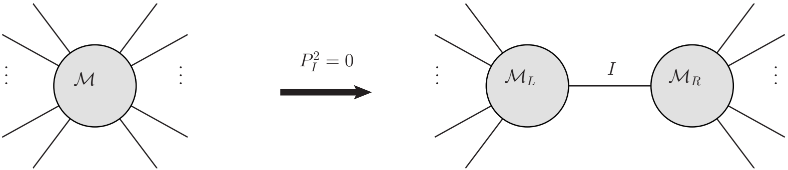

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- Extra constraints of IR/UV, amplitude relations...

The machinery: recursion relations

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$$\text{Res}_{z=z_I} \frac{\hat{\mathcal{M}}_n(z)}{z} = -\frac{\hat{\mathcal{M}}_L(z_I)\hat{\mathcal{M}}_R(z_I)}{z_I} \text{Res}_{z=z_I} \frac{1}{\hat{P}_I^2(z)} .$$

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- Recursion relation

$$\oint \frac{dz}{z} \hat{\mathcal{M}}_n(z) = 0$$

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Works fine for gravity and gauge theory because of gauge invariance

NLSM starts at $\mathcal{O}(p^2) \rightarrow \mathcal{O}(z^2)$

Need to incorporate the shift symmetry!

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Independence of $a_i \leftrightarrow$ allowed theory space

Cheung, Kampf, Novotny, Shen, Trnka, 1509.03309, 1611.03137;
Elvang, Hadjiantonis, Jones, Paranjape, 1806.06079

The machinery: flavor ordering

The leading order Lagrangian:

$$\mathcal{L}^{(2)} = \frac{f^2}{2} d^a_\mu d^{a\mu} = \frac{1}{2}\partial_\mu \pi^a \left[\frac{\sin^2\sqrt{\mathcal{T}}}{\mathcal{T}}\right]_{ab} \partial^\mu \pi^b,$$

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Single trace flavor-ordering:

$$\mathcal{M}_n^{a_1 a_2 \cdots a_n} = \sum_\sigma \mathrm{tr}\left(\mathrm{X}^{a_{\sigma(1)}} \mathrm{X}^{a_{\sigma(2)}} \cdots \mathrm{X}^{a_{\sigma(n)}}\right) M_n(\sigma)$$

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- Works for a general R of H
- Continue to work at $\mathcal{O}(p^4)$ with single/double traces

Factorization of flavors?

We would like to construct the ordered amplitudes

$$M_\sigma = -\sum_{I,\pm} \frac{1}{P_I^2} \frac{\hat{M}^{(I)}_{\{\sigma_L, I\}}(z_I^\pm)\hat{M}^{(I)}_{\{\sigma_R, -I\}}(z_I^\pm)}{K_n(z_I^\pm)(1 - z_I^\pm/z_I^\mp)} ,$$

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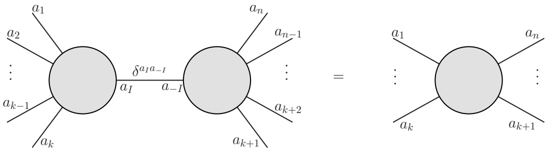
Constraint on the flavor factor:

$$C^{a_1 a_2 \cdots a_n} = C^{a_1 a_2 \cdots a_k a_I} C^{a_I a_{k+1} \cdots a_n},$$



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Can be done for the adjoint of $H = \mathrm{U}(N)$ ($\mathrm{SU}(N)$) in the trace basis: we have $H \times H/H \approx H$,

$$\mathrm{tr}\left(\cdots X^a \cdots\right) \to \mathrm{tr}\left(\cdots T^a \cdots\right), \quad \left(T^a\right)_{bc}\left(T^a\right)_{de} = \delta^{be}\delta^{cd}.$$

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However, the amplitudes up to $\mathcal{O}(p^4)$ in the trace basis are universal!

$\mathcal{O}(p^2)$

Soft blocks:

- Correct mass dimension and little group scaling
- Local
- Satisfy symmetry constraints: ordering, Adler's zero...

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Flavor factor: $\mathrm{tr}(X^{a_1}X^{a_2})\mathrm{tr}(X^{a_3}X^{a_4}) = \delta^{a_1 a_2}\delta^{a_3 a_4}$.

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Result:

- $\mathcal{S}_4^{(2)}(1,2|3,4)$ generates the $\mathcal{O}(p^2)$ pair basis amplitudes $M(12|34|56|\cdots)$
- "Mixed ordering" does not work

Low, **ZY**, 1904.12859.

Pair basis for **N** of SO(N)

The leading order Lagrangian:

$$\mathcal{L}^{(2)} = \frac{f^2}{2} d^a_\mu d^{a\mu} = \frac{1}{2}\partial_\mu \pi^a \left[\frac{\sin^2\sqrt{\mathcal{T}}}{\mathcal{T}}\right]_{ab} \partial^\mu \pi^b,$$

where $(\mathcal{T})_{ab} = \frac{1}{f^2} T^i_{ac} T^i_{db} \pi^c \pi^d$

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The pair basis:

$$\mathcal{M}_n^{a_1\cdots a_n} = \sum_{\dot\alpha}\left(\prod_{j=1}^{n/2}\delta^{a_{\dot\alpha(2j-1)}a_{\dot\alpha(2j)}}\right) \times M_n(\dot\alpha(1),\dot\alpha(2)|\dot\alpha(3),\dot\alpha(4)|\cdots|\dot\alpha(2n-1),\dot\alpha(2n)),$$

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Factorization of the flavor factor:

$$C^{a_1 a_2\cdots a_n} = C^{a_1 a_2\cdots a_k a_I}C^{a_I a_{k+1}\cdots a_n},$$

Automatic!

$\mathcal{O}(p^4)$

To build the Lagrangian, we need traces of d_μ, with ∇_μ acting on them so that

$$\nabla_\mu d_\nu = \partial_\mu d_\nu + i[E_\mu, d_\nu].$$

$\mathcal{O}(p^4)$

The most general Lagrangian

$$\mathcal{L}^{\mathrm{NLSM}} = \mathcal{L}^{(2)} + \frac{f^2}{\Lambda^2}\left(\sum_{i=1}^{4} C_i O_i + C_- O_{\mathrm{wzw}}\right) + \mathcal{O}\left(\frac{1}{\Lambda^4}\right),$$

with

$$\begin{aligned} O_1 &= [\mathrm{tr}(d_\mu d^\mu)]^2, \quad O_2 = [\mathrm{tr}(d_\mu d_\nu)]^2, \\ O_3 &= \mathrm{tr}([d_\mu, d_\nu]^2), \quad O_4 = \mathrm{tr}(\{d_\mu, d_\nu\}^2), \\ S_{\mathrm{wzw}} &\propto \int d^5y\ \varepsilon^{\alpha\beta\gamma\delta\epsilon}\mathrm{tr}(d_\alpha d_\beta d_\gamma d_\delta d_\epsilon) = \int d^4x\ O_{\mathrm{wzw}} \end{aligned}$$

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To get this:

- Total derivatives
- Symmetry, e.g. $\nabla_{[\mu} d_{\nu]} = 0$, $[\nabla_\mu, \nabla_\nu] = [d_\mu, d_\nu]$
- Equation of motion

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The on-shell way:

- Total derivatives $\to$ total momentum conservation
- Symmetry $\to$ orderings, Adler's zero
- Equation of motion $\to$ on-shell condition

$\mathcal{O}(p^4)$ soft bootstrap

$\mathcal{O}(p^4)$ soft blocks:

$$\begin{aligned}
\mathcal{S}_4^{(4)}(1,2,3,4) &= \frac{1}{f^2\Lambda^2}(c_1 s_{13}^2 + c_2 s_{12}s_{23}),\\
\mathcal{S}_4^{(4)}(1,2|3,4) &= \frac{1}{f^2\Lambda^2}(d_1 s_{12}^2 + d_2 s_{13}s_{23}),\\
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- The special case, starting with $\mathcal{S}_4^{(2)}(1,2|3,4)$:
  2 independent P-even operators for SO(N), while the WZW term exists only if $N=5$

Example: the WZW term vs. the pair basis

Consider the $SO(N)$ fundamental NLSM:

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What if we actually have only 5 flavors?

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$$\begin{aligned} M(1,2,3,4,\{5,6,7\}) &= M(1,2,3,4,5|6,7) + M(1,2,3,4,6|5,7) \\ &\quad + M(1,2,3,4,7|5,6). \end{aligned}$$

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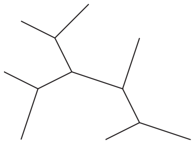
WZW term exists only if $N = 5$!

Outline

The color-kinematics duality

Gauge theory:

$$\mathcal{M}_n^{\text{YM}} = \sum_{g \in \{g_n\}} \frac{c_g \; n_g}{d_g}$$



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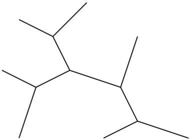
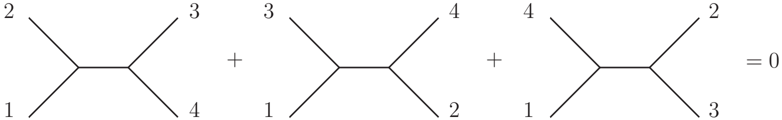


- c_g satisfies anti-symmetry and the Jacobi identity



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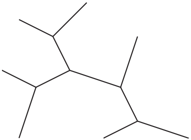

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Bern, Carrasco, Johansson, 0805.3993

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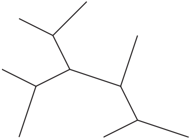

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- Replace c_g with n_g leads to gravity amplitudes!

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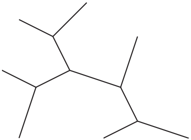

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- Double copy: $\text{GR} = \text{YM} \otimes \text{YM}$.

Review: Bern, Carrasco, Chiodaroli, Johansson, Roiban, 1909.01358

The flavor-kinematics duality

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Chen, Du, 1311.1133; Du, Fu, 1606.05846; Carrasco, Mafra, Schlotterer, 1608.02569

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$$\mathcal{M}_n^{\mathrm{sGal}} = \sum_{g\in\{g_n\}} \frac{n_g \, n_g}{d_g}.$$

- Double copy: sGal $=$ NLSM $\otimes$ NLSM, BI $=$ NLSM $\otimes$ YM...

Flavor-kinematics at $\mathcal{O}(p^4)$?

At 4-pt, $\mathcal{O}(p^2)$:

$$\mathcal{M}_4 = \frac{\mathrm{f}_s n_s}{s} + \frac{\mathrm{f}_t n_t}{t} + \frac{\mathrm{f}_u n_u}{u},$$

with $\mathrm{f}_s = T^i_{a_1 a_2} T^i_{a_3 a_4}$, $n_s = s(t-u)$.

- Lie algebra/"closure condition": $\mathrm{f}_s + \mathrm{f}_t + \mathrm{f}_u = 0$
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At $\mathcal{O}(p^4)$, correcting n_i fails

Elvang, Hadjiantonis, Jones, Paranjape, 1806.06079; González, Penco, Trodden, 1908.07531

New ideas: correcting the f_g!

Carrasco, Rodina, Yin, Zekioglu, 1910.12850

Flavor-kinematics at $\mathcal{O}(p^4)$!

4 different ways to correct f_i leads to 4 $\mathcal{O}(p^4)$ soft blocks

Low, **ZY**, 1911.08490

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- Single-trace:

$$\hat{f}_s^{(1)} = f_t(u-s) - f_u(s-t),$$
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$$\hat{\mathrm{f}}_s^{(3)} = \mathrm{f}'_t(u-s) - \mathrm{f}'_u(s-t),$$
$$\hat{\mathrm{f}}_s^{(4)} \propto \frac{1}{3!}\sum_{\sigma\in S_3} \delta^{a_{\sigma(1)} a_{\sigma(2)}} \delta^{a_{\sigma(3)} a_{\sigma(4)}}(t-u),$$

where $\mathrm{f}'_s = \delta^{a_1 a_3}\delta^{a_2 a_4} - \delta^{a_1 a_4}\delta^{a_2 a_3}$

Higher multiplicity

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Low, Rodina, **ZY**, 2009.00008

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  Low, Rodina, **ZY**, 2009.00008

- Replacing n_g with c_g:

$$\mathcal{M}_n^{\mathrm{YM}+\phi^3} \supset \sum_{g \in \{g_n\}} \frac{\hat{\mathrm{f}}_g \; \mathrm{c}_g}{d_g}$$

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Proposal:

$$\mathcal{M}_n^{(4)} = \sum_{g \in \{g_n\}} \frac{\hat{\mathrm{f}}_g \; n_g}{d_g},$$

- Works for $\mathcal{S}_4^{(4)}(1,2|3,4) \propto s_{13}s_{23}$ if we assume local $\hat{\mathrm{f}}$

  Low, Rodina, **ZY**, 2009.00008

- Replacing n_g with c_g:

$$\mathcal{M}_n^{\mathrm{YM}+\phi^3} \supset \sum_{g \in \{g_n\}} \frac{\hat{\mathrm{f}}_g \; \mathrm{c}_g}{d_g}$$

- Double copy: $\mathrm{NLSM}^{(4)} \subset \mathrm{NLSM}^{(2)} \otimes (\mathrm{YM} + \phi^3)$

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- Other theories: YMS, DBI, goldstini, the fundamental Higgs, SM...