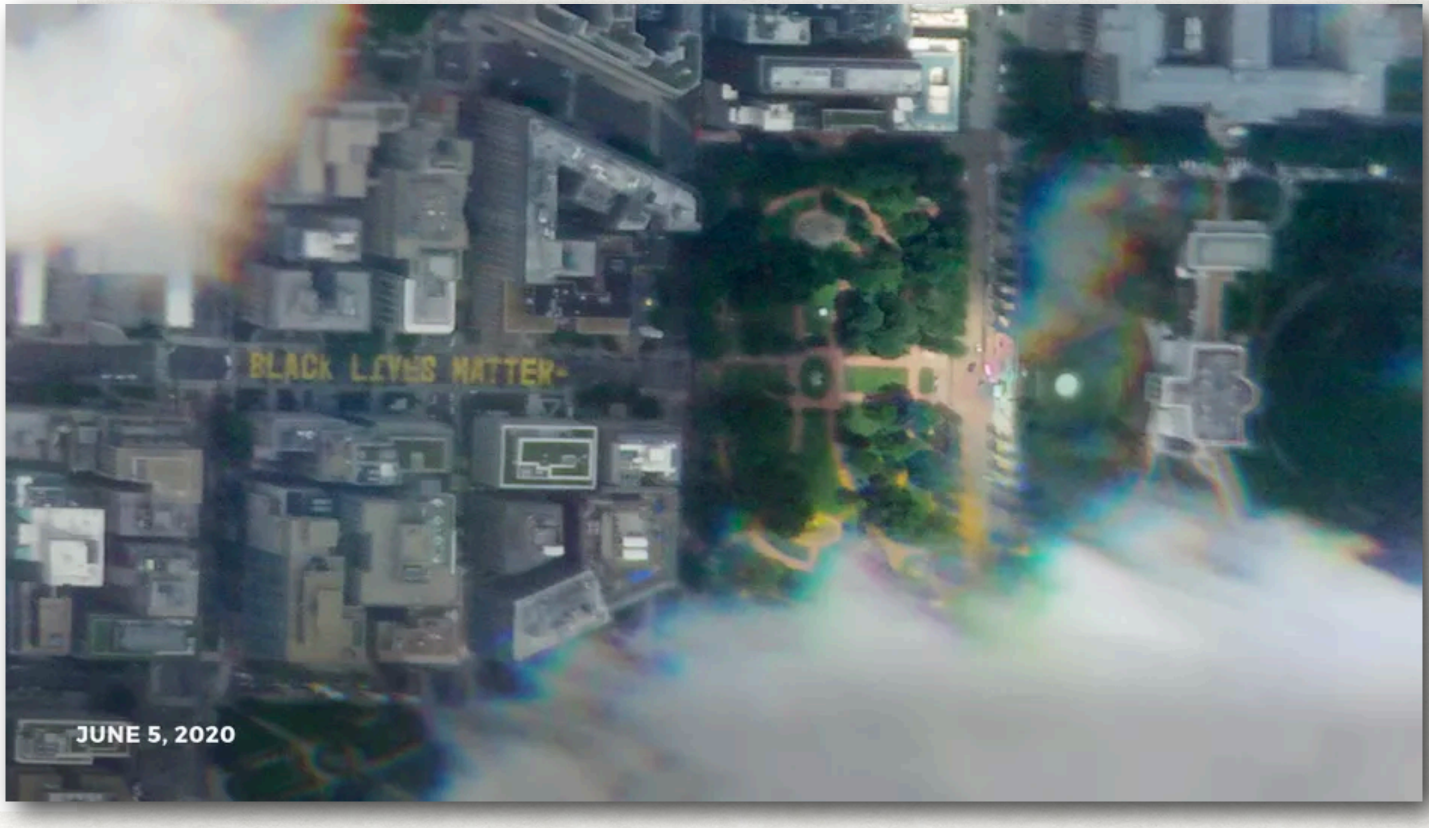
PULSAR TIMING PROBES OF SMALL SCALE STRUCTURE

HARIKRISHNAN RAMANI
BERKELEY TO STANFORD

ARXIV:1901.04490
WITH JEFF DROR, TANNER TRICKLE, KATHRYN ZUREK
ARXIV:2005.03030
WITH TANNER TRICKLE, KATHRYN ZUREK

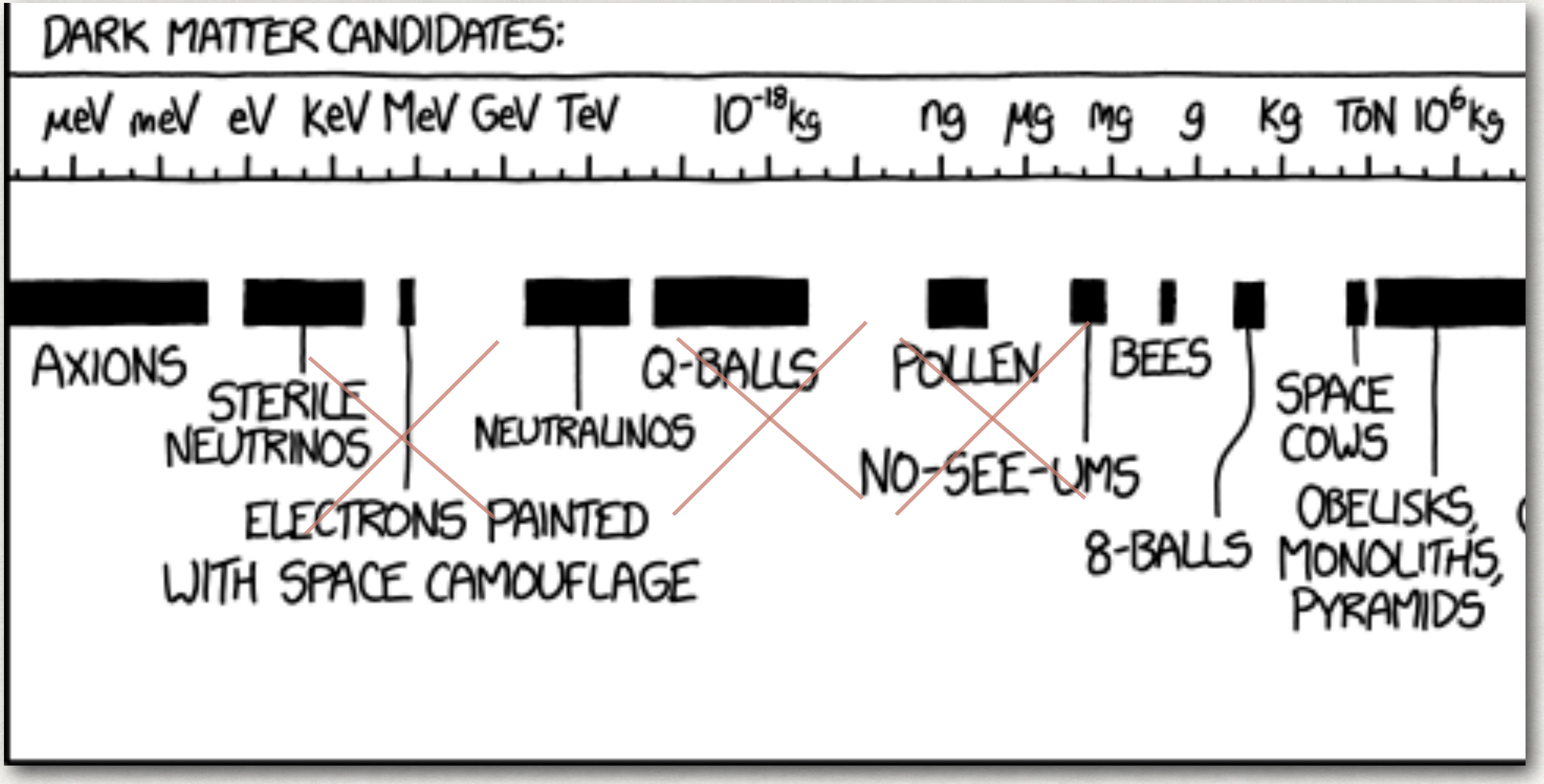
TO PEOPLE ON THE FRONTLINES



OUTLINE

- Dark matter substructure and particle physics
- Millisecond pulsars
- Deterministic and Stochastic Probes of PBH
- Probes of Diffuse Halos
- Results for extended Halo Mass functions
- Outlook

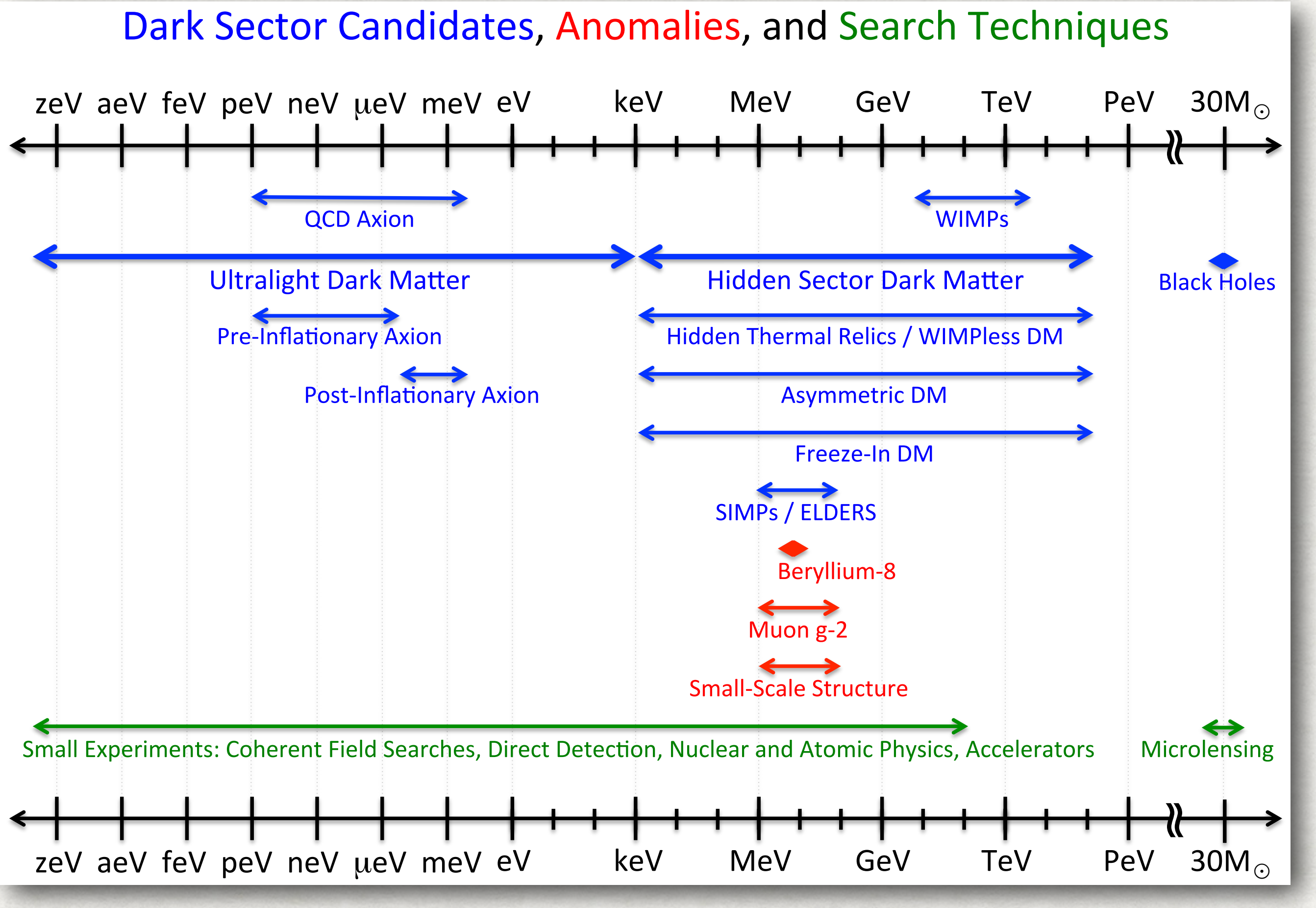
NIGHTMARE SCENARIOS FOR 2050



source: XKCD

WHAT DO WE KNOW ABOUT DARK MATTER?

- Ample Gravitational Evidence
- No confirmed positive signal in the lamp-post paradigm
- A bevy of promising experiments to probe interactions with SM and several more on the anvil



Cosmic visions

- What about gravitational probes?

COME BACK TO ME ONLY IF YOU LEARN ABOUT THE UNDERLYING PARTICLE PHYSICS

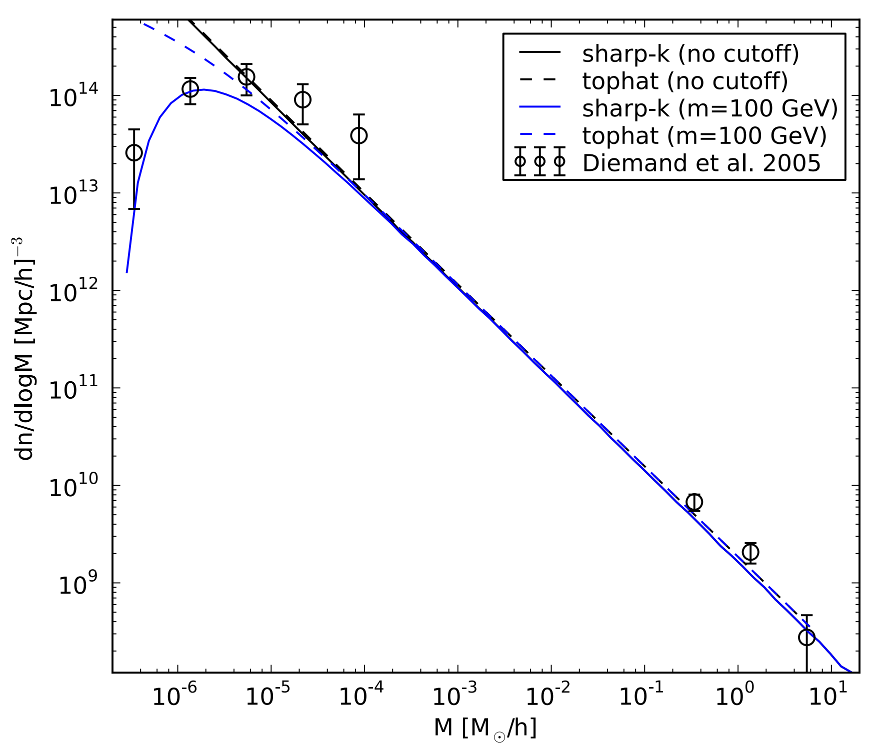
GRAVITATIONAL PROBES

Provide a wealth of information about particle nature

- Bullet Cluster - self interactions
- Dwarf Galaxies - lower limit on mass
- Super-radiance, other gravity probes of fuzzy DM
- Clues from “small scale” challenges viz. core vs cusp, missing satellites etc.
- Recent hints of subhalos from gaps in stellar streams
- How about substructure at even small scales (intra-galactic)?

CDM

- Vanilla CDM predicts diffuse structure, concentrated at larger masses
- WIMP paradigm predicts $M>10^{-6}\,M_{sun}$ corresponding to kinetic decoupling

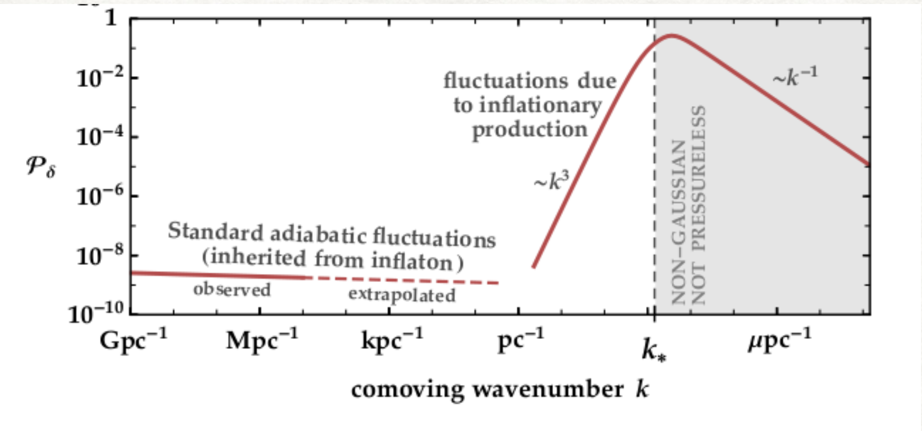


Source:1303.0839: Schneider et. al.

- Non-trivial models can predict drastically different halo mass functions and densities.

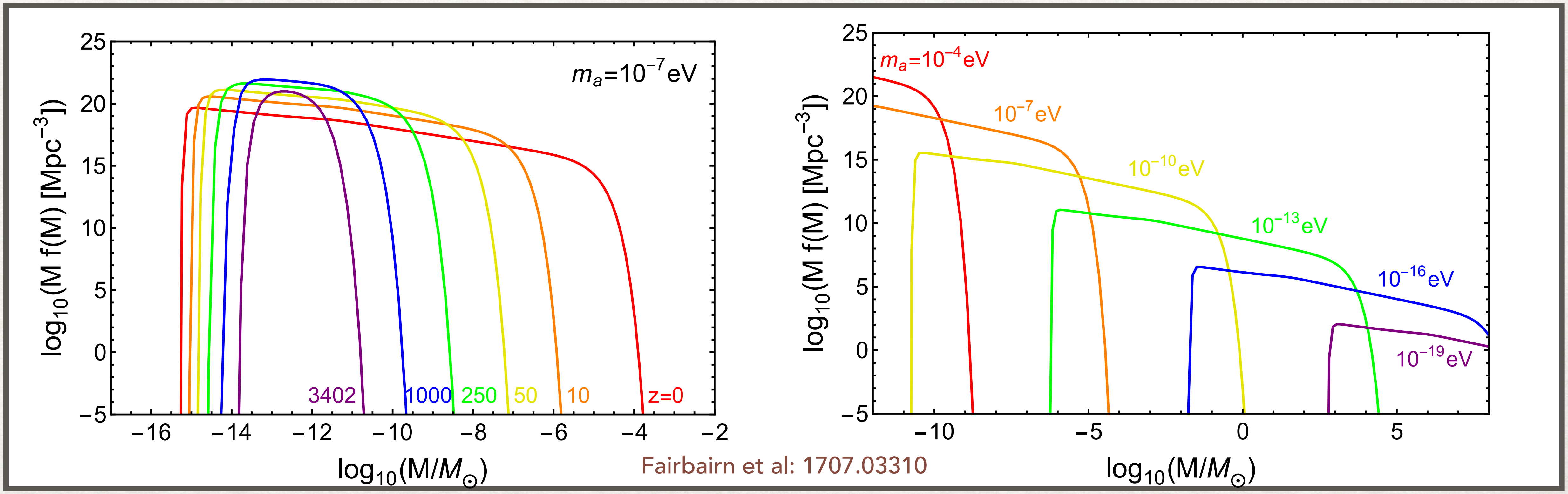
EXAMPLE MODELS

- Inflationary vector model



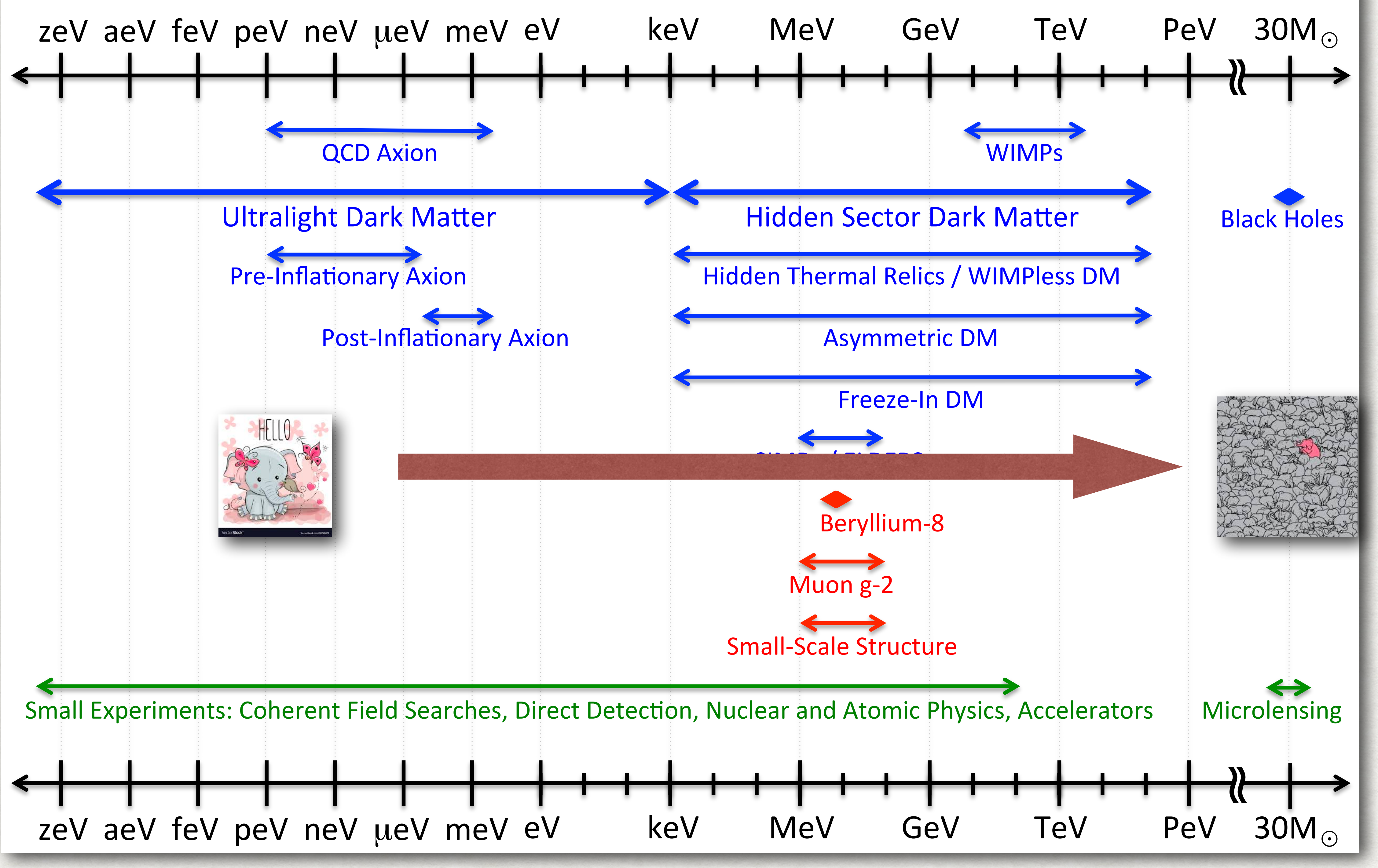
[Graham, Mardon, Rajendran - 1504.02102]

- Blackholes from a plethora of models.
- Early Matter Domination - Dror et al. 1711.04773, Blinov et al. 1911.07853
- Axion/ Scalar miniclusters after a phase-transition - See Buschman et. al.1906.00967



Fairbairn et al: 1707.03310

Dark Sector Candidates, Anomalies, and Search Techniques



zeV aeV feV peV neV μeV meV eV keV MeV GeV TeV PeV 30M$_\odot$

QCD Axion

WIMPs

Ultralight Dark Matter

Hidden Sector Dark Matter

Black Holes

Pre-Inflationary Axion

Hidden Thermal Relics / WIMPless DM

Post-Inflationary Axion

Asymmetric DM

Freeze-In DM

[illegible]

Beryllium-8

Muon g-2

Small-Scale Structure

Small Experiments: Coherent Field Searches, Direct Detection, Nuclear and Atomic Physics, Accelerators

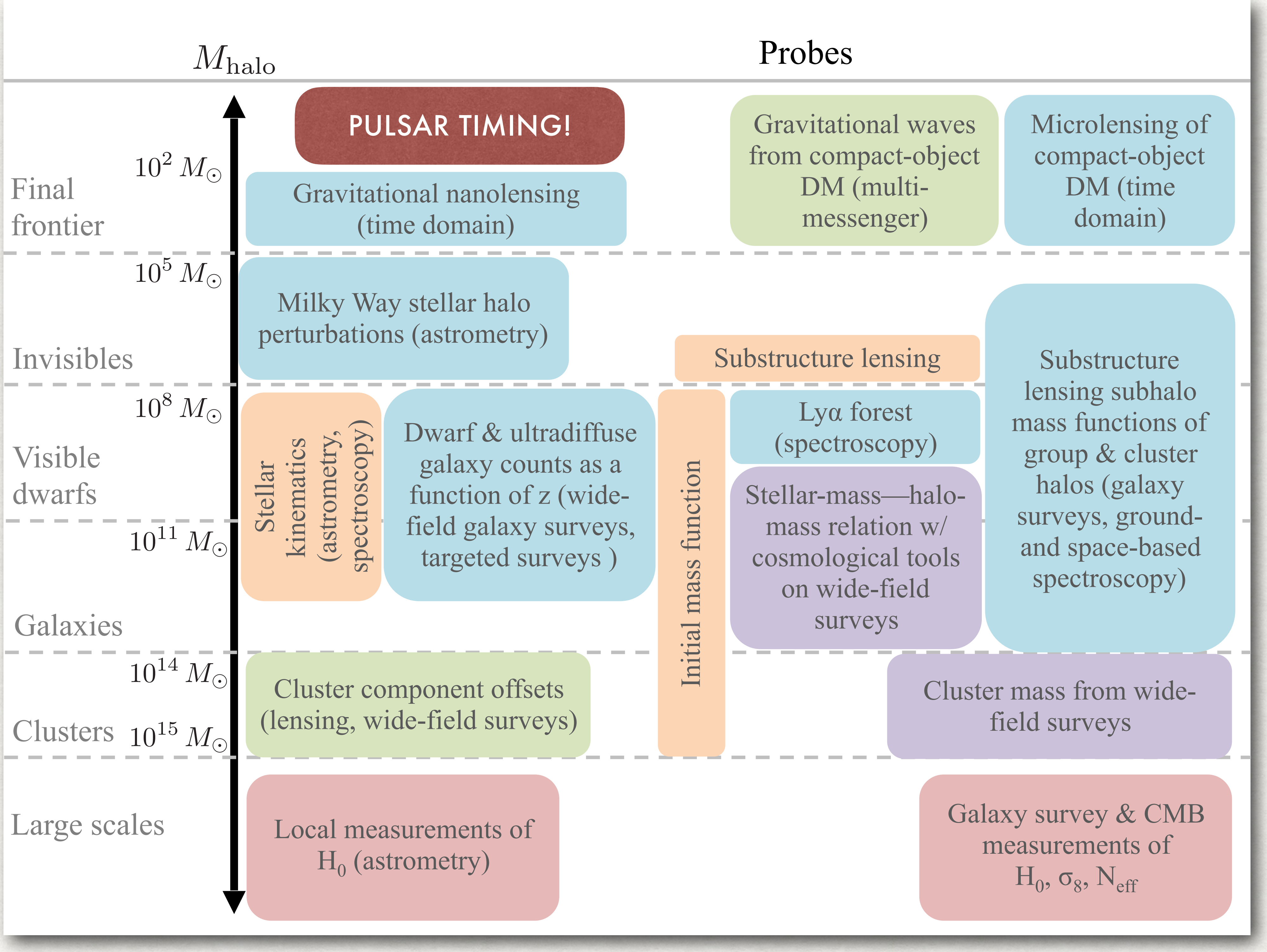
Microlensing

zeV aeV feV peV neV μeV meV eV keV MeV GeV TeV PeV 30M$_\odot$

SEVERAL UNKNOWNS

- Given an initial power spectrum, what is the substructure today?
- Well posed, hard to solve accurately
- Tidal stripping? Mergers?
- How much of the DM is still in these subhalos?
- Will take an agnostic view towards this issue and project constraints agnostically.
- Answers important for direct detection too.

PROBES OF DIFFERENT MASSES



M_{halo}

Probes

Final frontier

$10^2\ M_\odot$

PULSAR TIMING!

Gravitational nanolensing (time domain)

Gravitational waves from compact-object DM (multi-messenger)

Microlensing of compact-object DM (time domain)

$10^5\ M_\odot$

Invisibles

Milky Way stellar halo perturbations (astrometry)

Substructure lensing

Substructure lensing subhalo mass functions of group & cluster halos (galaxy surveys, ground- and space-based spectroscopy)

$10^8\ M_\odot$

Visible dwarfs

Stellar kinematics (astrometry, spectroscopy)

Dwarf & ultradiffuse galaxy counts as a function of z (wide-field galaxy surveys, targeted surveys)

Initial mass function

Lyα forest (spectroscopy)

Stellar-mass—halo-mass relation w/ cosmological tools on wide-field surveys

$10^{11}\ M_\odot$

Galaxies

$10^{14}\ M_\odot$

Clusters

$10^{15}\ M_\odot$

Cluster component offsets (lensing, wide-field surveys)

Cluster mass from wide-field surveys

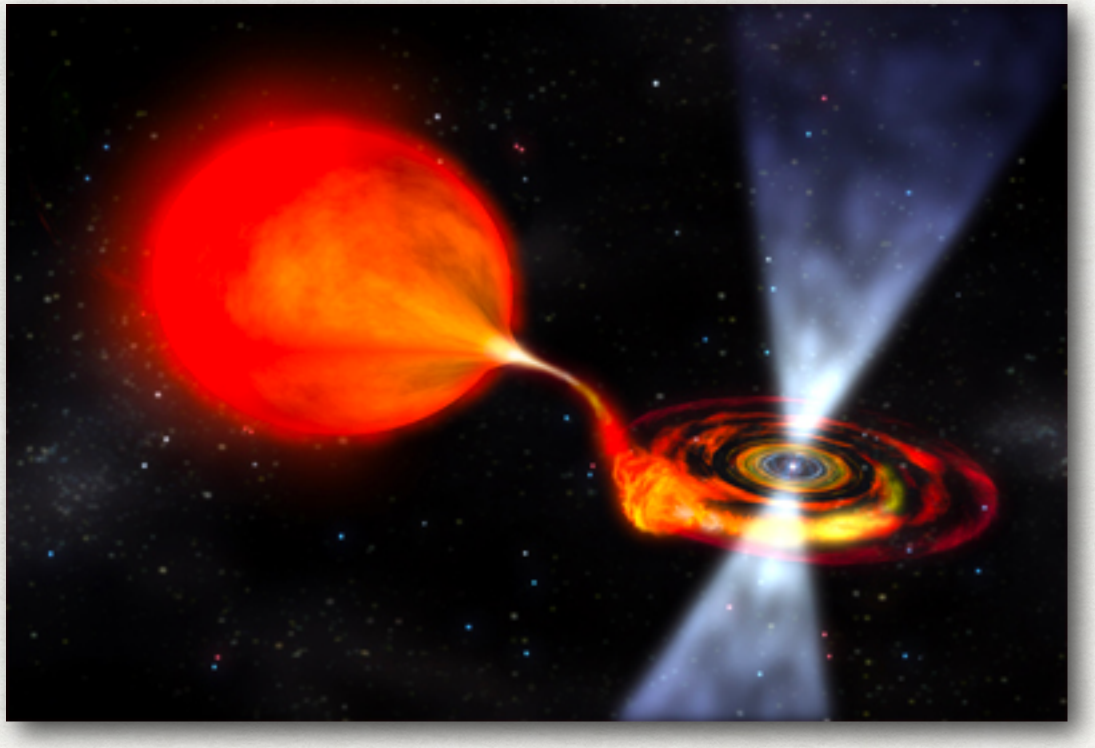
Large scales

Local measurements of H_0 (astrometry)

Galaxy survey & CMB measurements of H_0, σ_8, N_{eff}

Source: [Buckley, Peter:1712.06615]

MILLI-SECOND PULSARS



- Neutron stars sped up through accretion.
- Fastest rotating pulsars have frequencies of a few kHz.
- Stable over remarkable time-scales (T>20 years)
- Accurate timing models exist

PULSAR TIMING

- Phase: $\phi(t) = \phi_0 + \nu\, t + \frac{1}{2}\dot{\nu}\, t^2 + \frac{1}{6}\ddot{\nu}\, t^3 + \ldots$
- ν ~ kHz
- $\dot{\nu}/\nu$ ~ 10^{-23} to 10^{-20} Hz
- $\ddot{\nu}/\nu$ < 10^{-31} Hz^2, not included in fits
- After fitting away the period and derivative, residuals are remarkably small* (and stable).

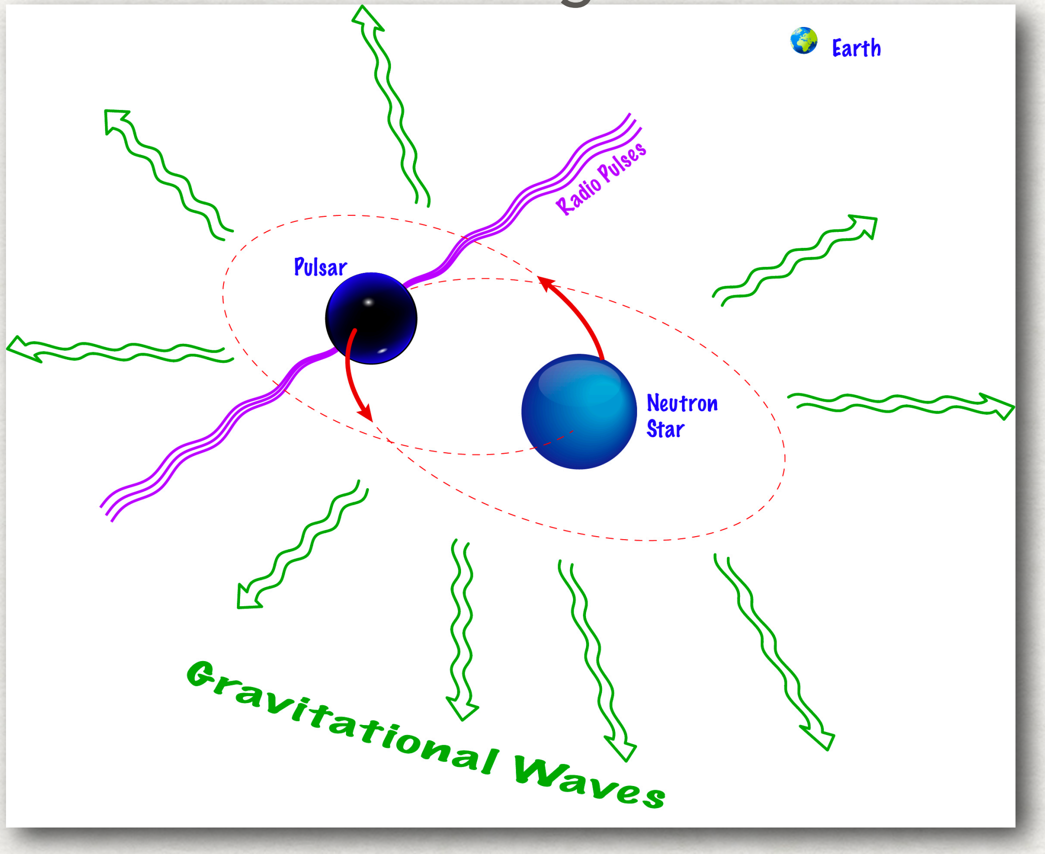
$$t_{\rm RMS} \equiv \sqrt{\frac{1}{N}\sum_n (t_n^{\rm data} - t_n^{\rm fit})^2}$$ ~ 50 ns

*in reality, some other delays, shall describe a relevant few later

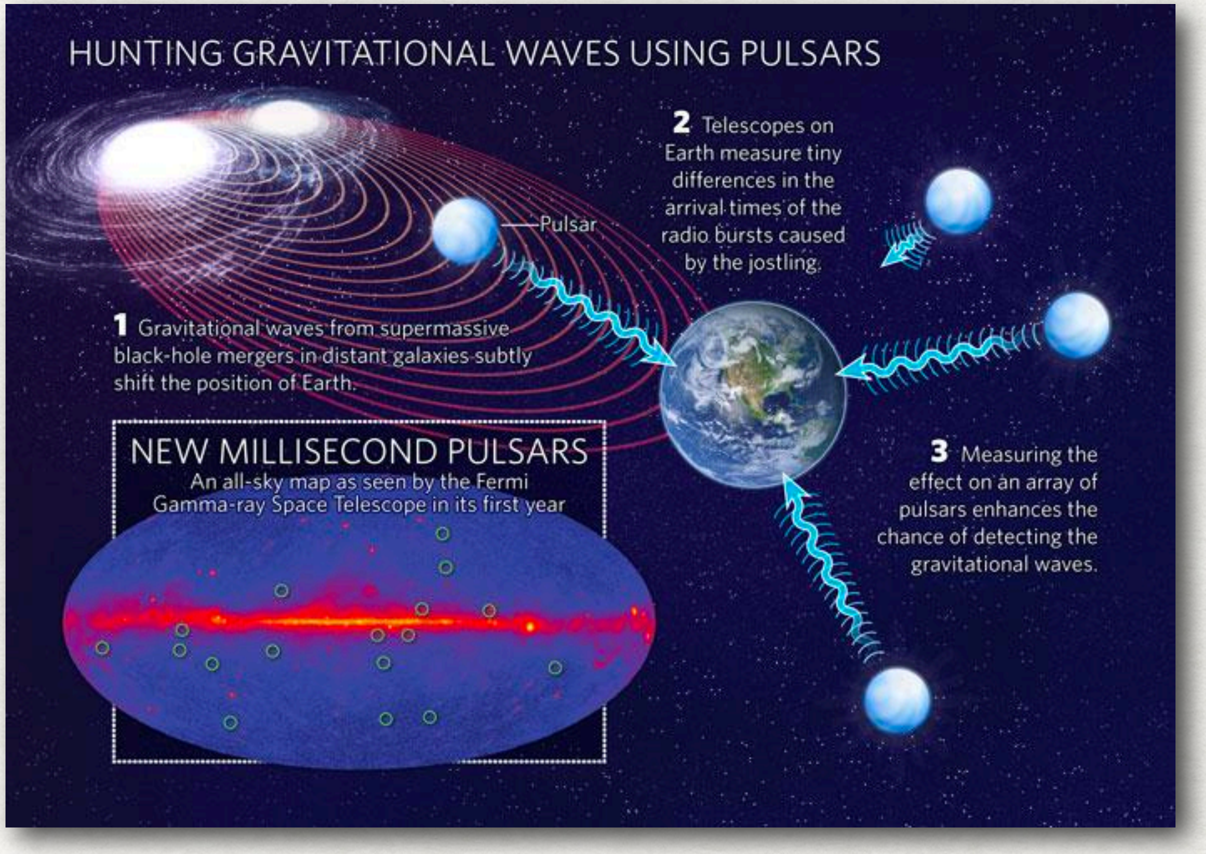
PHYSICS FROM PULSAR TIMING

- Any phenomenon that predicts time dependent $\delta\phi \equiv \int dt\, \delta\nu(t)$ can possibly be observed and constrained.

Hulse-Taylor binary used to "Detect" GW through its contracting orbit



Can be used as an extremely low frequency GW detector


HUNTING GRAVITATIONAL WAVES USING PULSARS

1 Gravitational waves from supermassive black-hole mergers in distant galaxies subtly shift the position of Earth.

2 Telescopes on Earth measure tiny differences in the arrival times of the radio bursts caused by the jostling.

3 Measuring the effect on an array of pulsars enhances the chance of detecting the gravitational waves.

NEW MILLISECOND PULSARS

An all-sky map as seen by the Fermi Gamma-ray Space Telescope in its first year

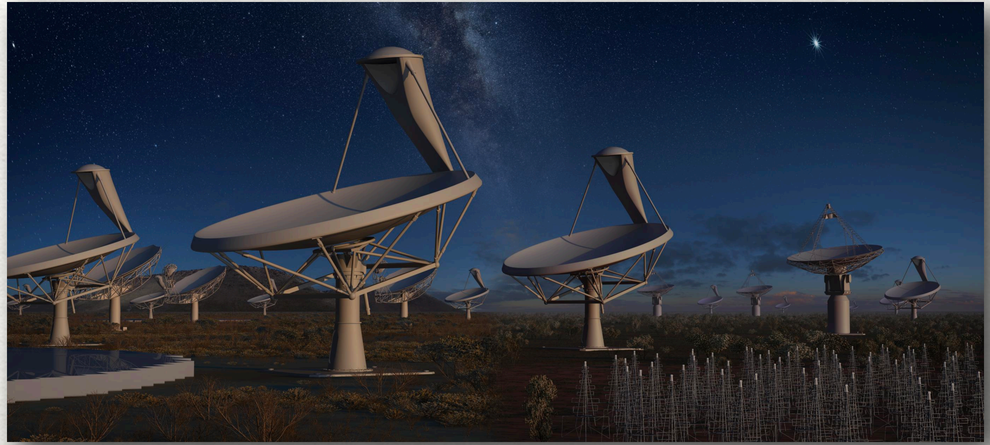
PTA COLLABORATIONS

Today

- Np ~ 73
- T =10 to 20 years
- 1 to 10 kpc away

Future

- Several precursors currently running
- Np ~ 200-1000
- Projected to start ~ 2030
- T =20+ years




SKA AND OPTIMISTIC

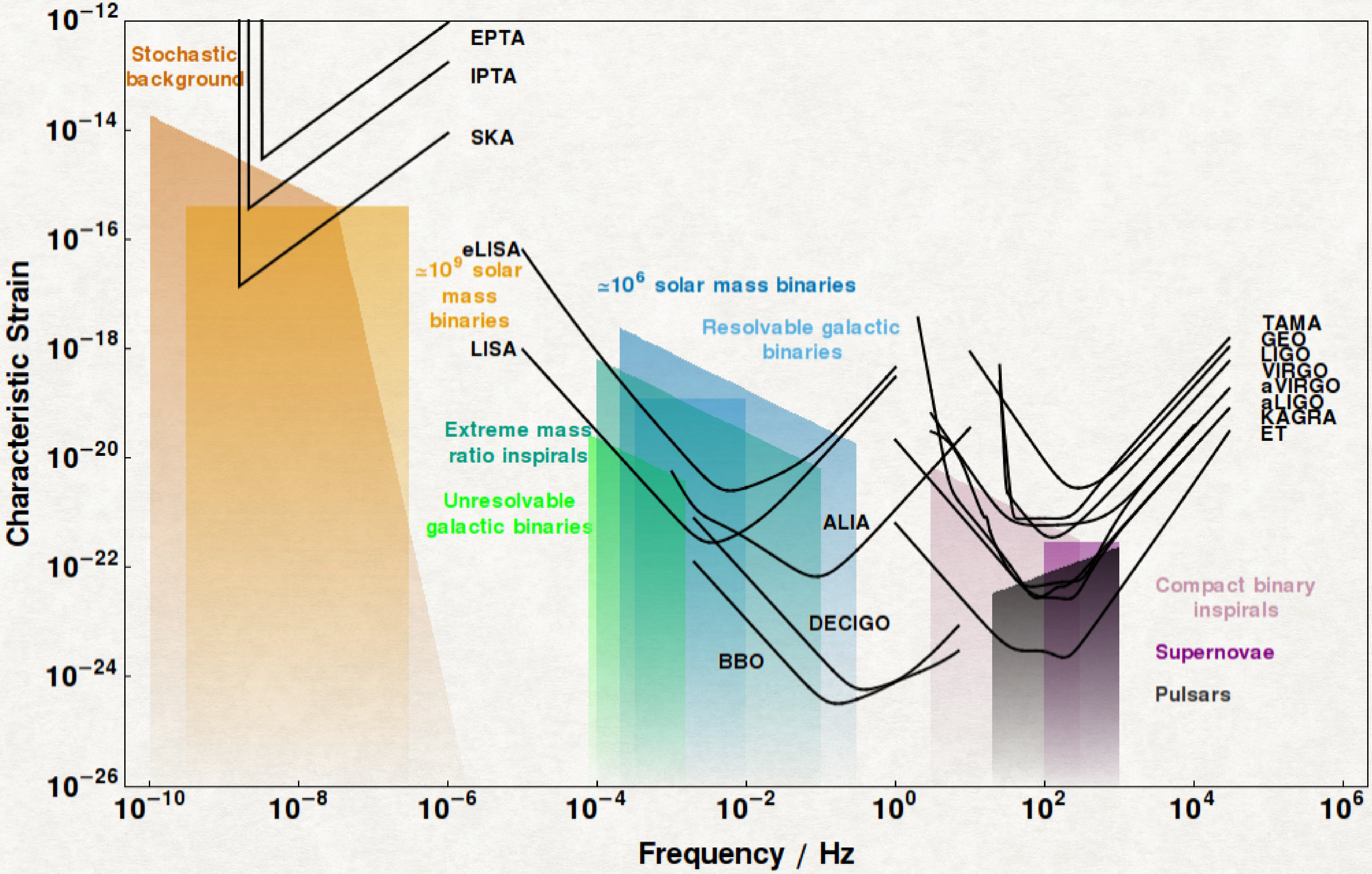
- SKA

$$N_P = 200,\ t_{\rm rms} = 50 \text{ ns},\ \Delta t = 2 \text{ week},\ T = 20 \text{ years},\ z_0 = 5 \text{ kpc}.$$

- Optimistic

$$N_P = 1000,\ T = 30 \text{ yr},\ t_{\rm rms} = 10 \text{ ns},\ \Delta t = 1 \text{ week},\ z_0 = 10 \text{ kpc}$$

PTA VS OTHER GRAVITY WAVE DETECTORS



Moore, Cole, Berry

SUBHALO PROBES

- Gravitational probes are broadly of two varieties
- Probe gravitational interaction between light and DM, e.g. Lensing
- Or probe gravitational interaction of DM with some test mass, i.e. Doppler effect e.g. Carney, Ghosh, Krnjaic, Taylor. arXiv:1903.00492
- PTAs have both kinds of signal (see also 1804.01991 van Tilburg, Taki, Weiner for larger masses with astrometry instead)

EXISTING LITERATURE

- Ultralight DM causing GW like delays - Not this work
- [Khmelnitsky, Rubakov - 1309.5888], [Graham, Kaplan, Mardon, Rajendran, Terrano - 1512.06165]
- PTAs are sensitive accelerometers: Doppler Delay - Discussed here
- [Seto,Corray - astro-ph/0702586] , [Baghram,Afshordi,Zurek - 1101.5487] [Kashiyama, Seto - 1208.4101],[Kazumi, Oguri, Masamune - 1801.07847]
- Gravitational potential wells along the light path: Shapiro Delay - Discussed here
- [Siegel, 0801.3458], [Siegel,Hertzberk,Fry - astro-ph/0702546], [Baghram,Afshordi,Zurek - 1101.5487], [Clark,Lewis,Scott - 1509.02938] , [Schutz, Liu - 1610.04234]

OUR WORK

- Explicit calculations of SNR
- Comprehensive analysis of all signal types
- Extension to diffuse halos

TYPES OF SIGNALS

- Type of effect: Doppler or Shapiro
- Length of signal: Dynamic or Static
- Number of signals accumulated:
  (single) Deterministic or (many) Stochastic
- Signal Affects Earth: shows up in all pulsars or on individual pulsars: Earth term (only for Doppler) vs Pulsar Term (for Doppler and Shapiro).
- There could be 8 (Doppler) + 4(Shapiro) distinct signal types!

START WITH MONOCHROMATIC PBH

DOPPLER DELAY

- Recognize the ratio $\frac{\delta\nu}{\nu}$ is v_{rel}/c
- Thus sensitive to tiny accelerations

$$\left(\frac{\delta\nu}{\nu}\right)_D = \hat{\mathbf{d}}\cdot\int \nabla\Phi\, dt,$$

- velocity shape for a point object transit looks like:

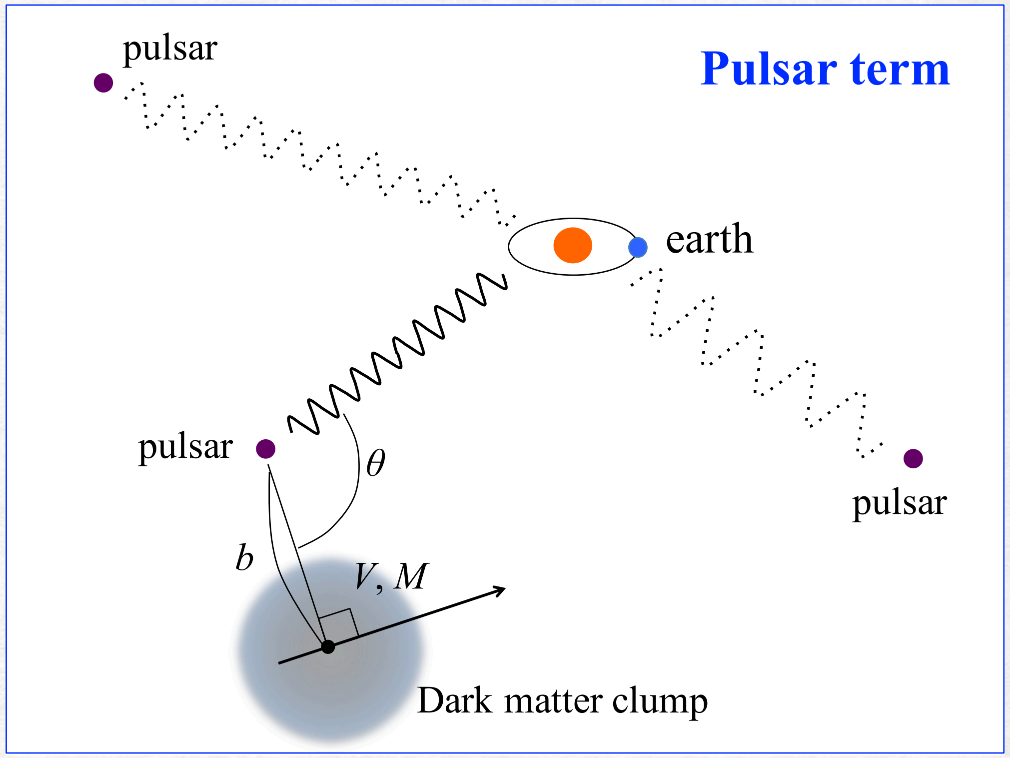
$$\left(\frac{\delta\nu}{\nu}\right)_D = \frac{GM}{v^2\tau_D}\frac{1}{\sqrt{1+x_D^2}}\left(x_D\hat{\mathbf{b}}-\hat{\mathbf{v}}\right)\cdot\hat{\mathbf{d}}.$$

Impact parameter

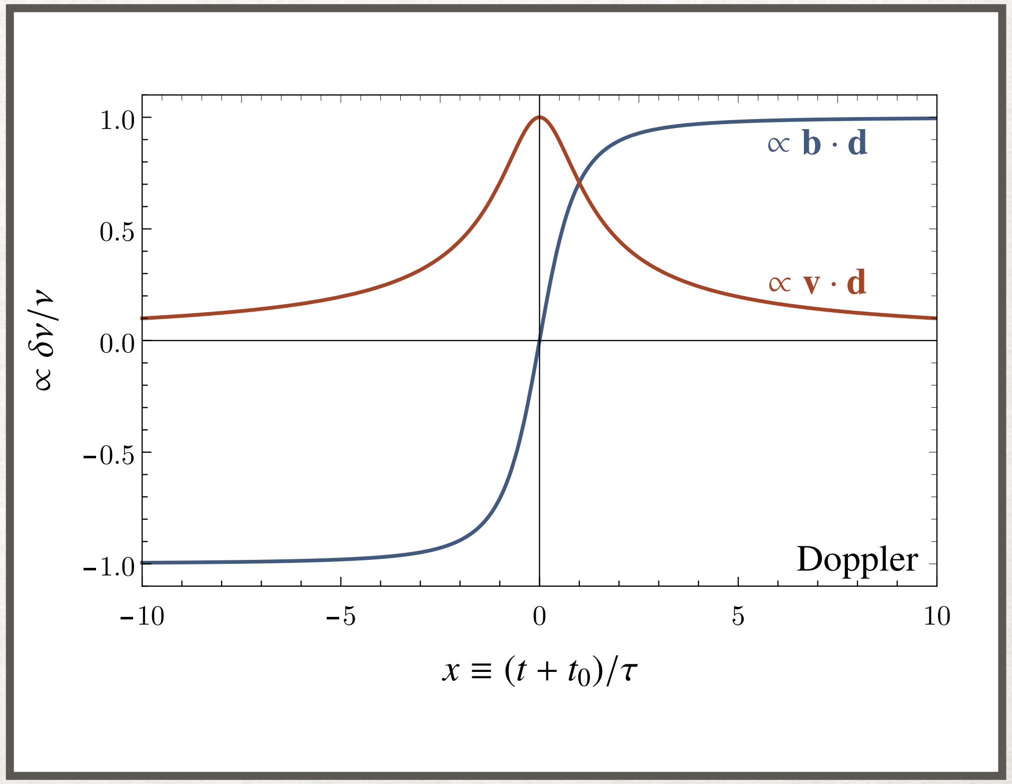
Signal period

Dimensionless time variable

$$\boxed{|\mathbf{b}| = \tau v}$$



Source: Kashiyama, Seto - 1208.4101



DOPPLER GEOMETRY

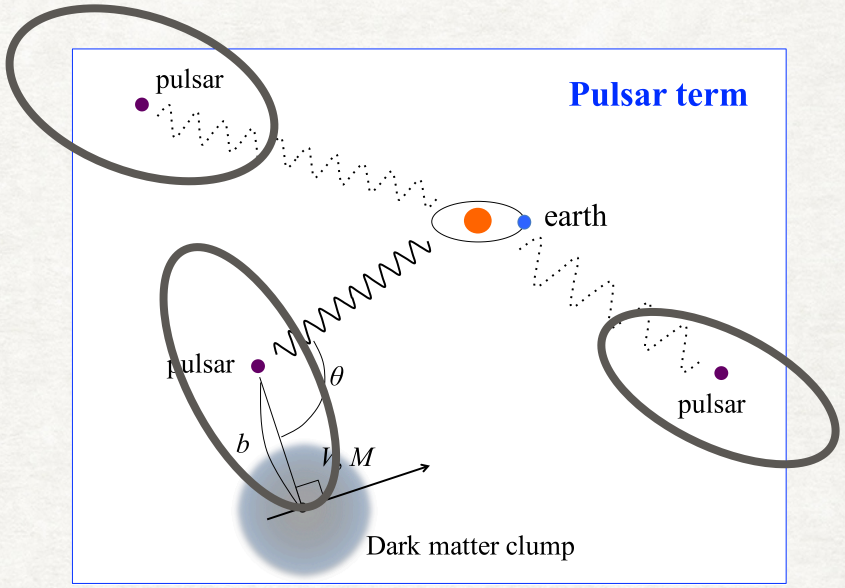
- To determine typical timescale, let us determine object of closest approach
- Cross-section for Doppler, is a circle.
- Remembering $|\mathbf{b}| = \tau v$

$$\tau_{\rm min} \simeq \frac{1}{v}\sqrt{\frac{M}{N_P f \rho_{\rm DM} v T}}$$

$$\sim \frac{20\ {\rm yr}}{\sqrt{N_P f}} \left(\frac{M}{10^{-9}\, M_\odot}\right)^{\frac{1}{2}} \left(\frac{20\ {\rm yr}}{T}\right)^{\frac{1}{2}}$$

Number of pulsars

Fraction of DM in M mass PBH



N_P pulsars

N_P x cross-section

DYNAMIC VS STATIC

Dynamic if

$$\underset{\text{Cadence}}{\Delta t} \ll \overset{\text{Characteristic signal period}}{\tau} < t_0 < \underset{\text{Total Time of observation}}{T} - \tau$$

Static otherwise

$$\tau \gtrsim T$$

DETECTING DYNAMIC SIGNALS

- Similar to a bump hunt / LIGO signal / Microlensing signal
- Doppler - leaves a permanent imprint
- Shapiro - Blip (As we will see)
- SNR is a solved problem in signal processing.

Fourier transform of Signal

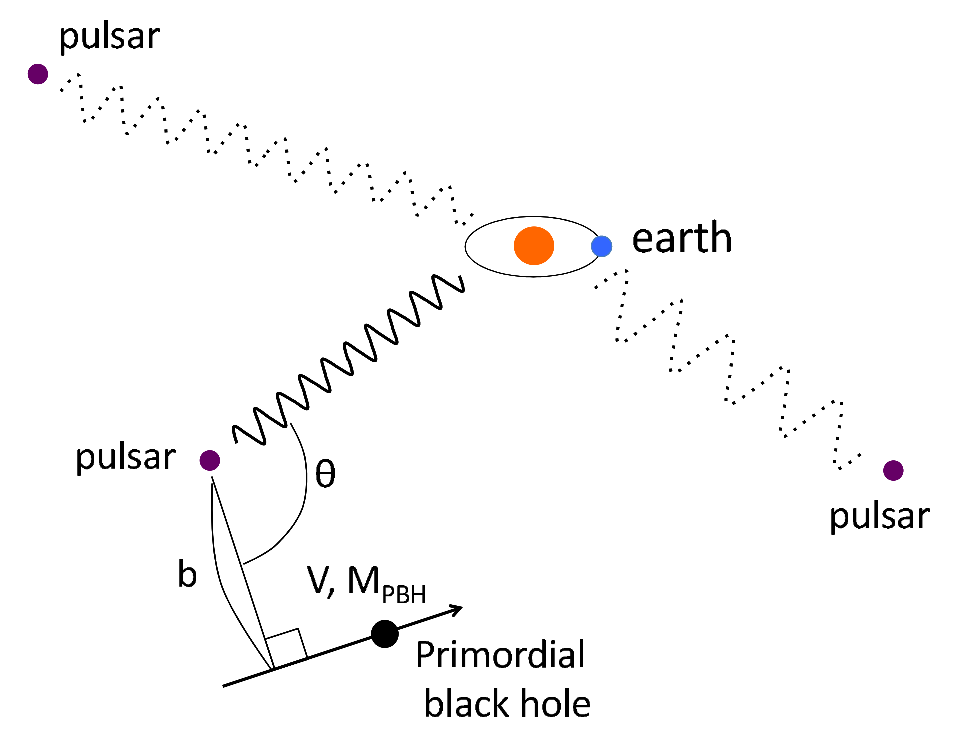
$$\mathrm{SNR}^2 = 4\int_0^\infty df\, \frac{|\widetilde{h}(f)|^2}{S_{\dot{\delta t}}(f)}$$

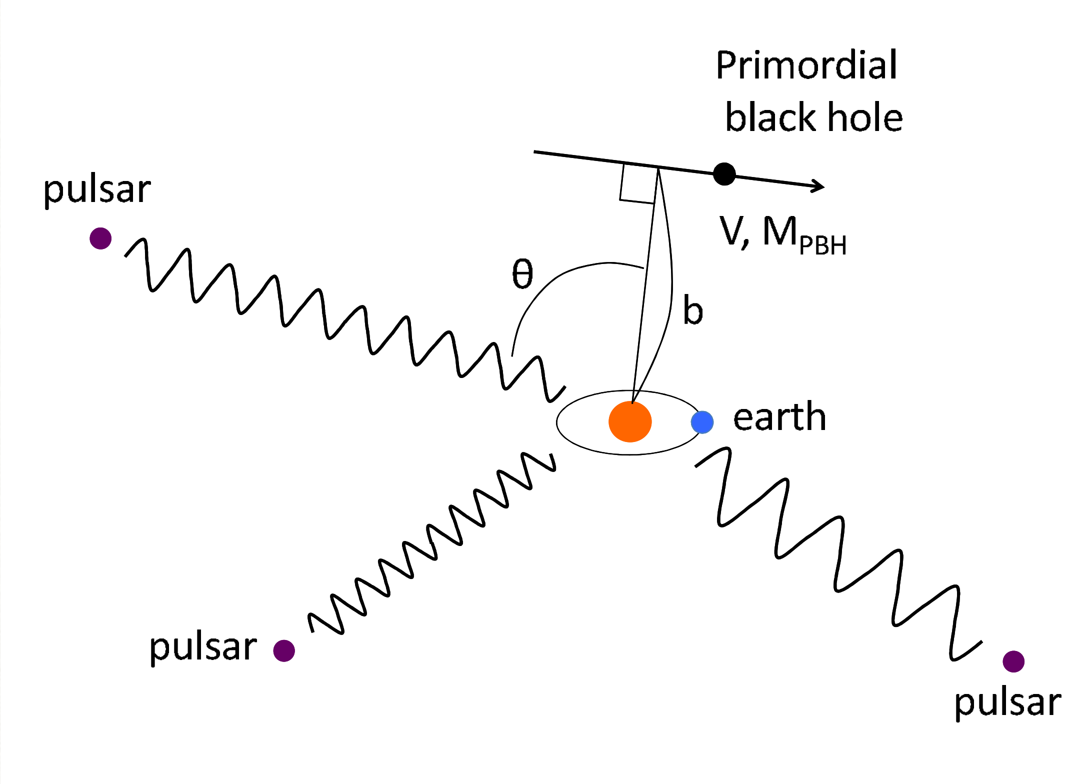
Cadence ~ 2 weeks

$$S_{\dot{\delta t}}(f) \equiv 8\pi^2 t_{\mathrm{RMS}}^2 \Delta t\, f^2$$

PULSAR TERM VS EARTH TERM FOR DOPPLER

- Many more pulsars → impact parameter far lower for one lucky pulsar.
- Angular correlations → sensitivity far higher for earth term





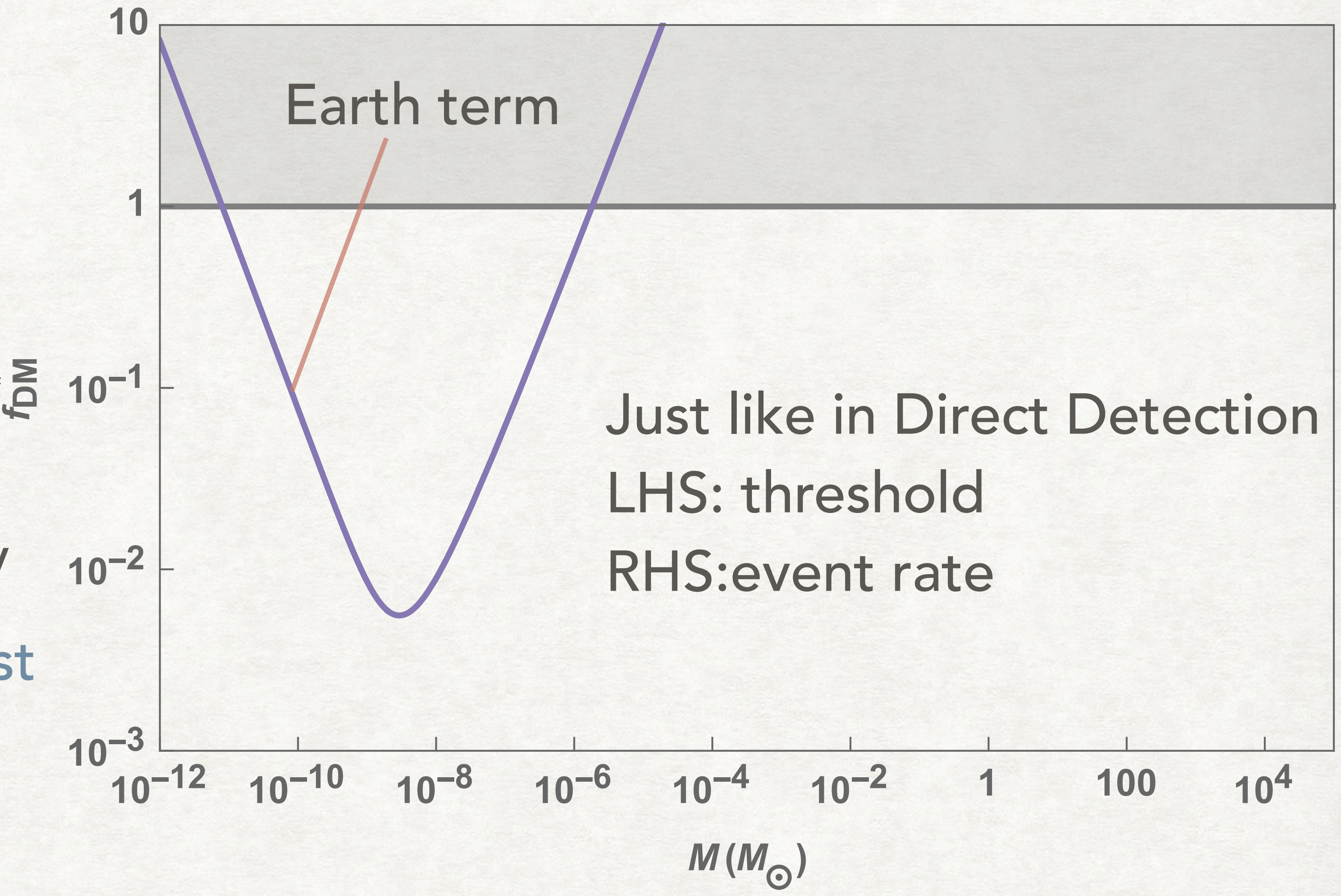
BOUNDS FROM DYNAMIC SIGNALS (DOPPLER)

- For Doppler Pulsar Term,
- LHS:

$$f^{L}_{\mathrm{D,\ dyn}} \lesssim 0.01 \left(\frac{10^{-9}\, M_{\odot}}{M}\right)\left(\frac{200}{N_P}\right)\left(\frac{20\ \mathrm{yr}}{T}\right)^{4}$$

Earth term scales the same way

- At some Mass M, even the nearest PBH starts failing dynamic constraint.



Just like in Direct Detection

LHS: threshold

RHS:event rate

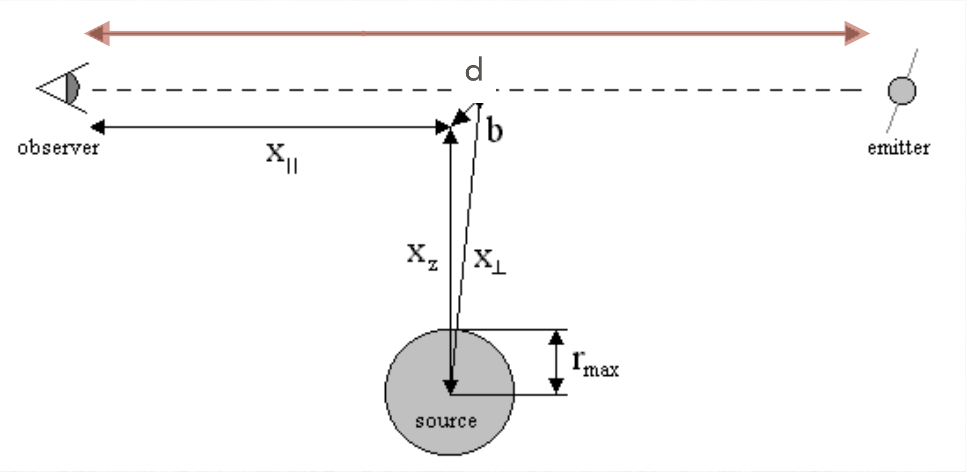
$$f^{R}_{\mathrm{D,\ dyn}} \lesssim 3 \left(\frac{M}{10^{-7}\, M_{\odot}}\right)\left(\frac{200}{N_P}\right)\left(\frac{20\ \mathrm{yr}}{T}\right)^{3}$$

Earth term has Np=1

SHAPIRO DELAY

- Similar to Sachs-Wolfe effect
- In frequency domain given by,

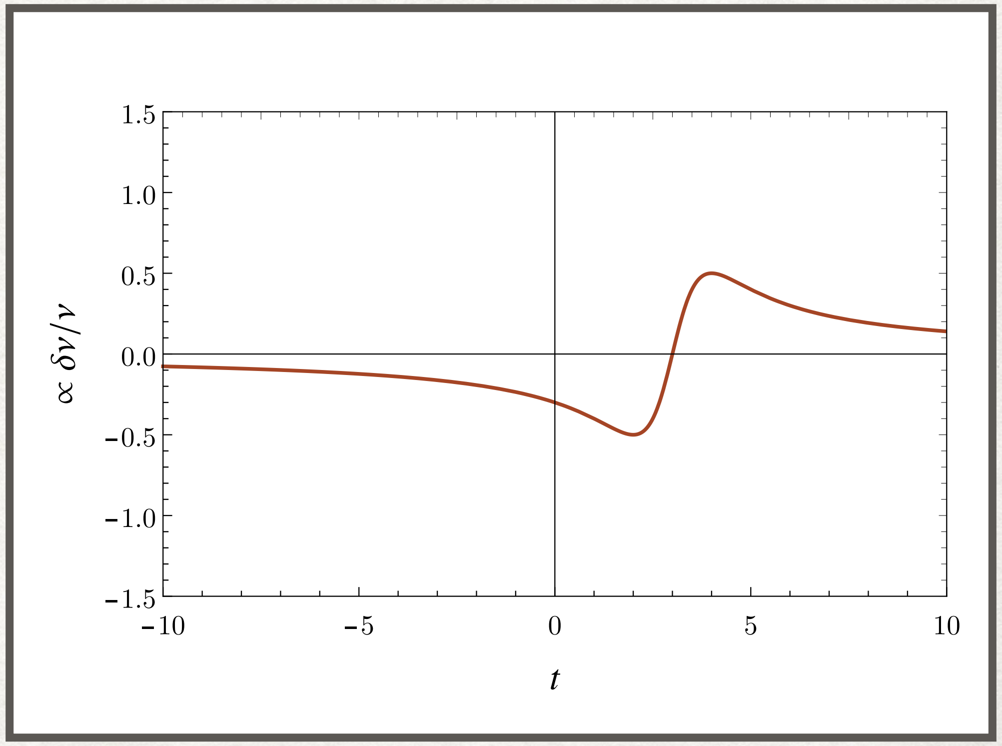
$$\left(\frac{\delta\nu}{\nu}\right)_S = -2\int \mathbf{v}\cdot\nabla\Phi\, dz$$



- For a point object,
- $$\left(\frac{\delta\nu}{\nu}\right)_S = \frac{4GM}{\tau_S}\frac{x_S}{1+x_S^2}$$

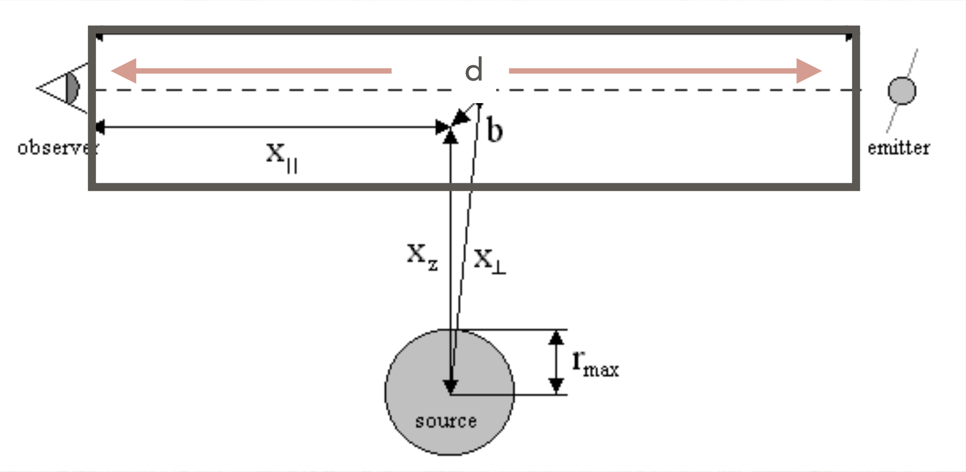
Duration of signal

Dimensionless time variable



SHAPIRO CROSS-SECTION

- Cross-section for Shapiro is a rectangle



$$\begin{aligned}\tau_{\rm min} &\simeq \frac{2}{v}\frac{M}{N_P f \rho_{\rm DM} v T d}\,,\\ &\sim \frac{20~{\rm yr}}{N_P f}\left(\frac{M}{10^{-4}\,M_\odot}\right)\left(\frac{20~{\rm yr}}{T}\right)\left(\frac{\rm kpc}{d}\right)\end{aligned}$$

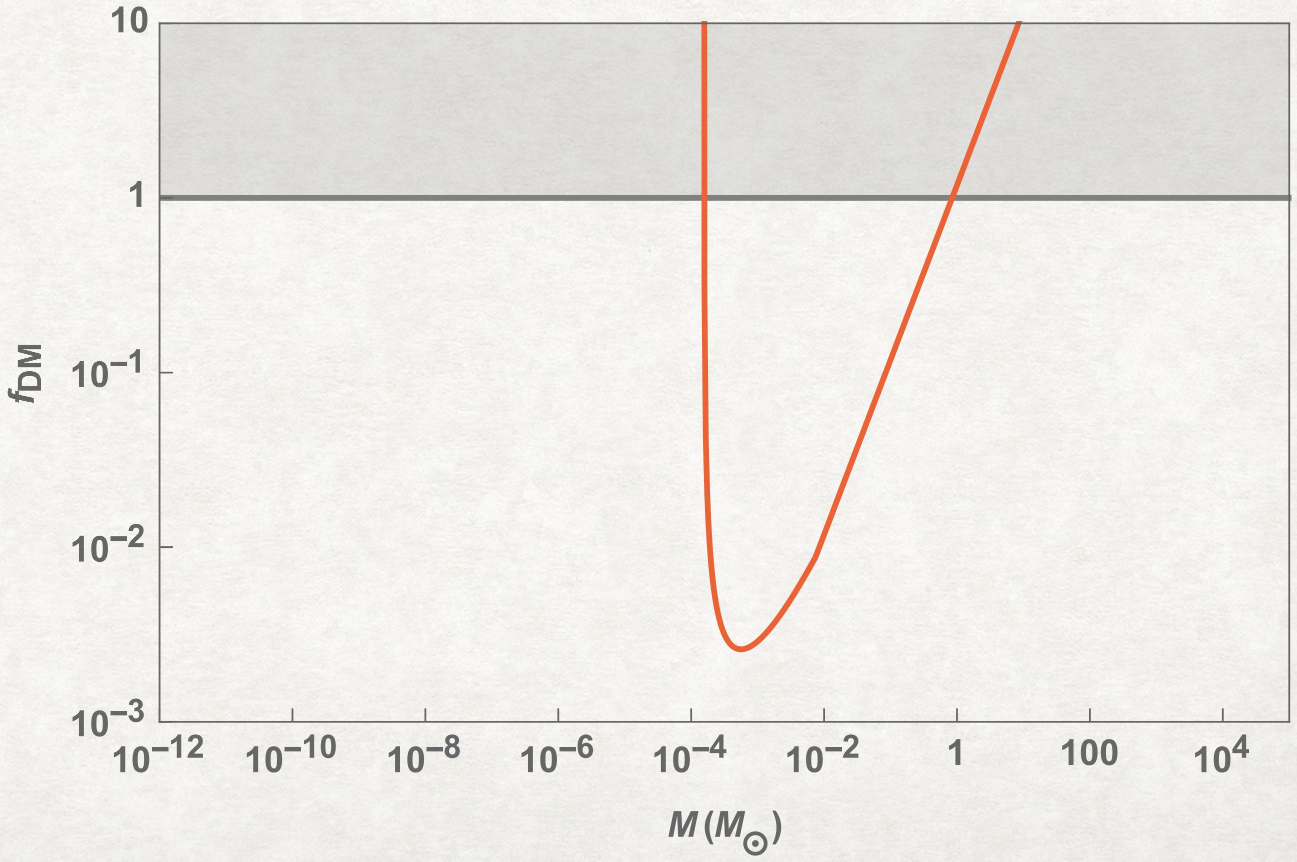
BOUNDS FROM DYNAMIC SIGNALS (SHAPIRO)

- For small enough $\tau_{\min}$,

$$\mathrm{SNR} = 4\frac{GM}{c^6 t_{\mathrm{rms}}}\sqrt{\frac{T}{\Delta t}}$$

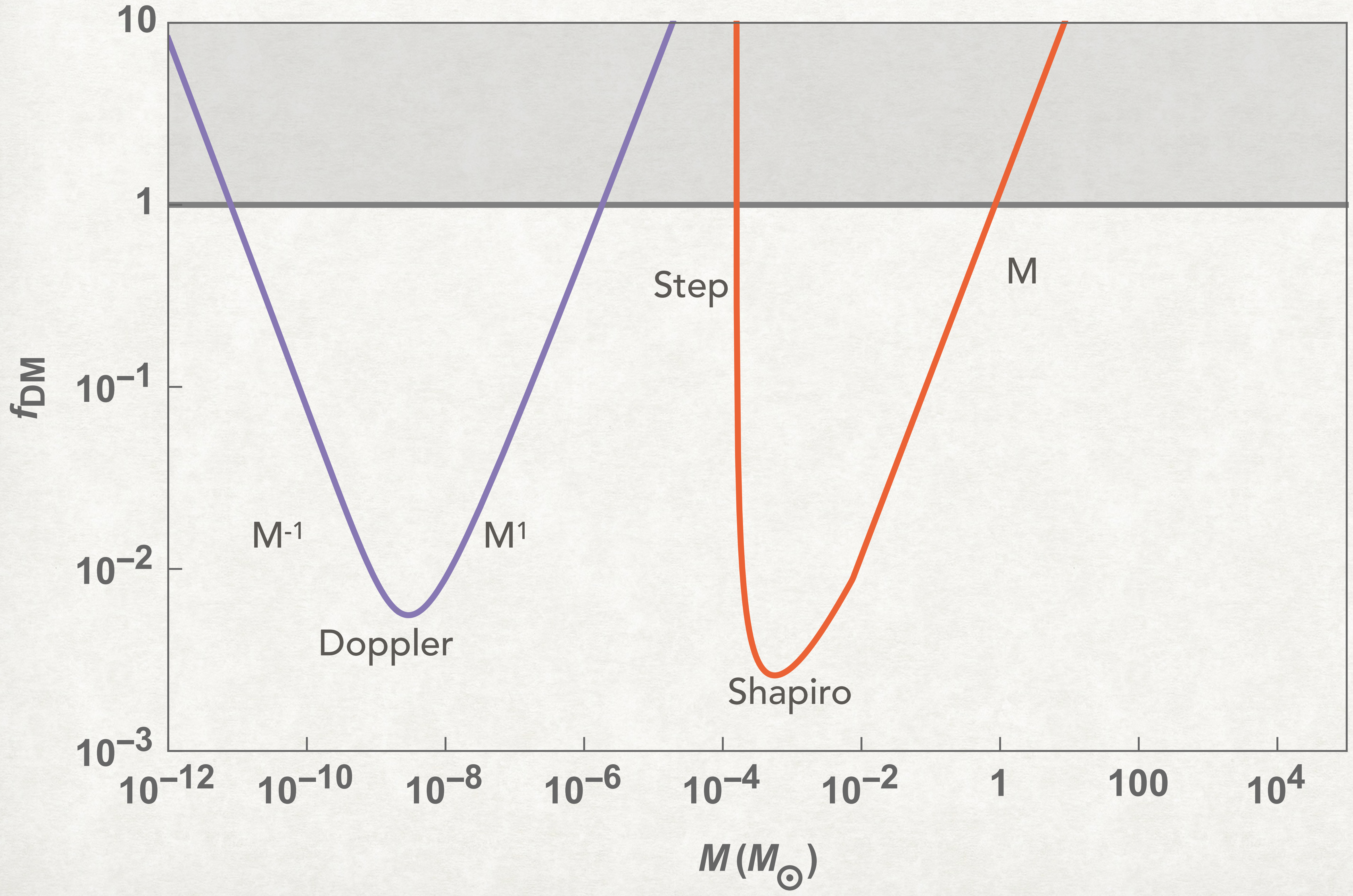
- Low enough masses are simply incapable of producing signal
- RHS just like before, f ~ M,

$$f^R_{\mathrm{S,\,dyn}} \lesssim 0.8\left(\frac{M}{10^{-2}\,M_\odot}\right)\left(\frac{200}{N_P}\right)\left(\frac{20\ \mathrm{yr}}{T}\right)^2$$



SHAPIRO SIGNAL CAN NEVER HAVE AN EARTH TERM: SAMPLING VOLUMES DO NOT OVERLAP

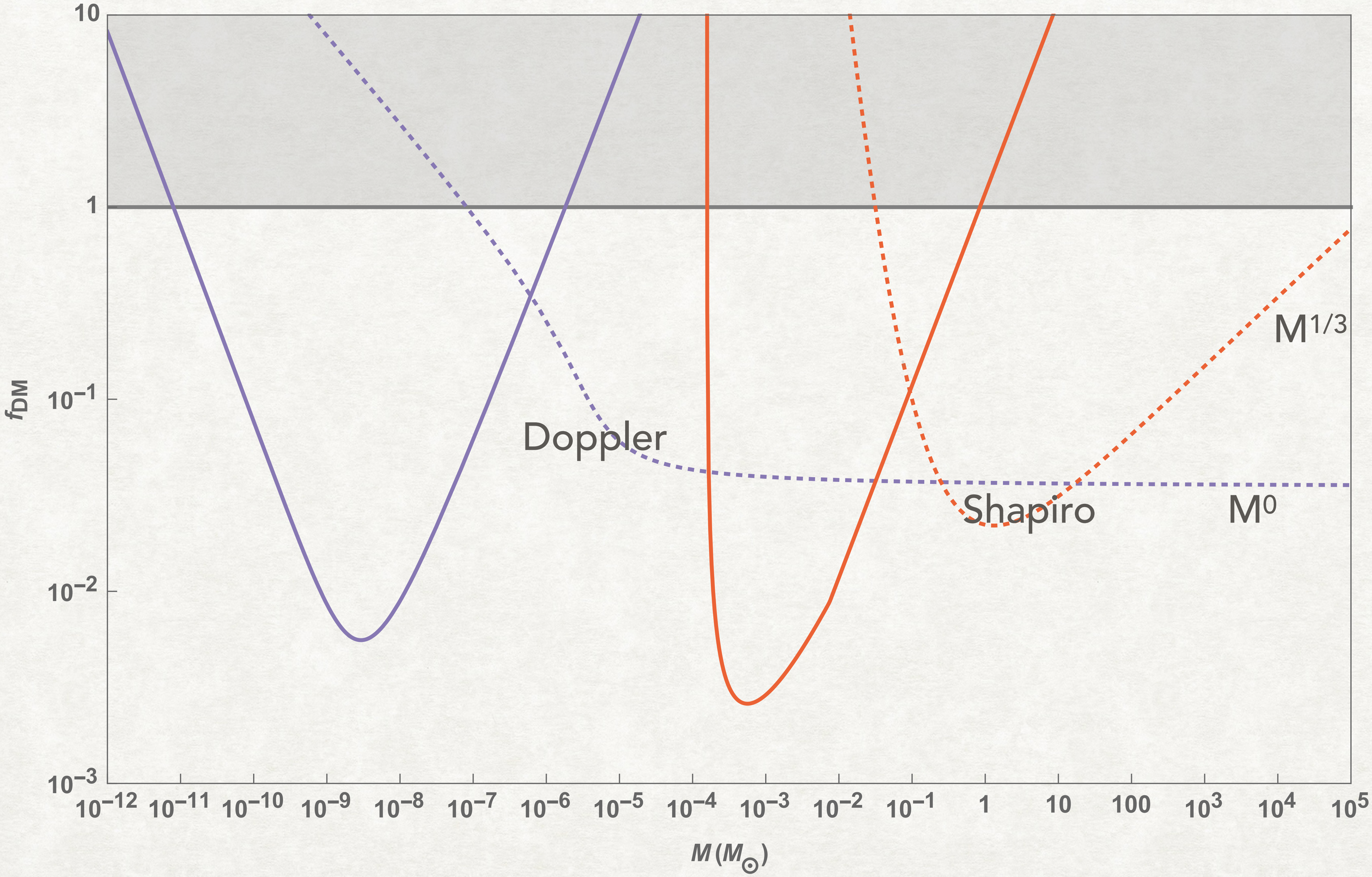
FRACTION VS M SCALING - DYNAMIC LIMIT



SIGNAL TO NOISE RATIO (STATIC SIGNALS)

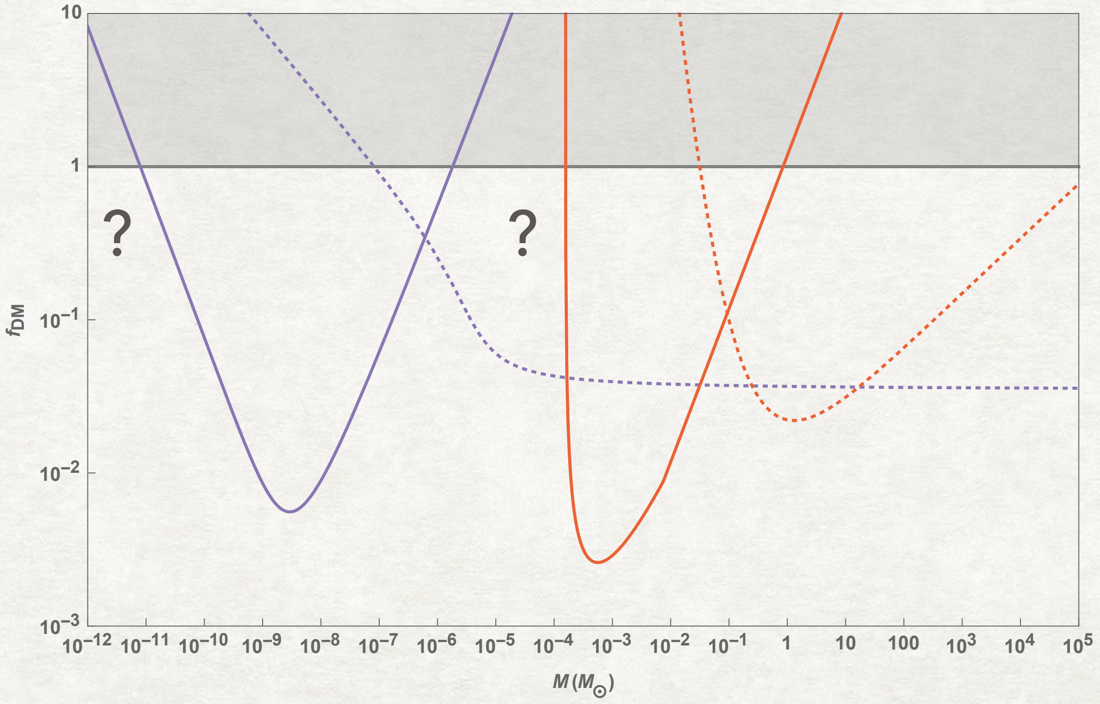
- In the limit that you don’t see the whole signal, Taylor expand.
- A constant first derivative i.e. spin-down or sometimes even spin-up is already observed (incalculable from first principles).
- Subtracted as part of the fitting procedure.
- Subtraction also relevant to dynamic signals (more on this later)
- Second derivative much less common.
- Non-observation of second derivative can be used to set constraints.

FRACTION VS M SCALING -STATIC



STOCHASTIC SIGNAL

- In 1901.04490 we considered only deterministic single event.
- Left on the table: multiple events at lower masses which do not pass the threshold SNR individually
- Lose ability to fit for deterministic signal shape



STOCHASTIC SNR

$$\langle \delta\phi(t)\delta\phi(t')\rangle = \sum_{i=1}^{N}\langle\delta\phi_i(t)\delta\phi_i(t')\rangle + \sum_{i\neq j}^{N(N-1)}\langle\delta\phi_i(t)\delta\phi_j(t')\rangle \equiv R_1(t,t') + R_2(t,t')$$

1-halo 2-halo

$$\mathrm{SNR}_P^2 = \frac{N_P}{2\widetilde{N}^2}\int dtdt'\langle R_I^{\mathrm{sub}}(t,t')^2\rangle_{\mathcal{P}}$$

$$\mathrm{SNR}_E^2 = \frac{N_P(N_P-1)}{2\widetilde{N}^2}\int dtdt'\left\langle R_{IJ}^{\mathrm{sub}}(t,t')^2\right\rangle_{\mathcal{P}}$$

Sum over all events i,
Average over ensemble

Deterministic Signal: care about the single closest event. A random “Best pulsar” exists

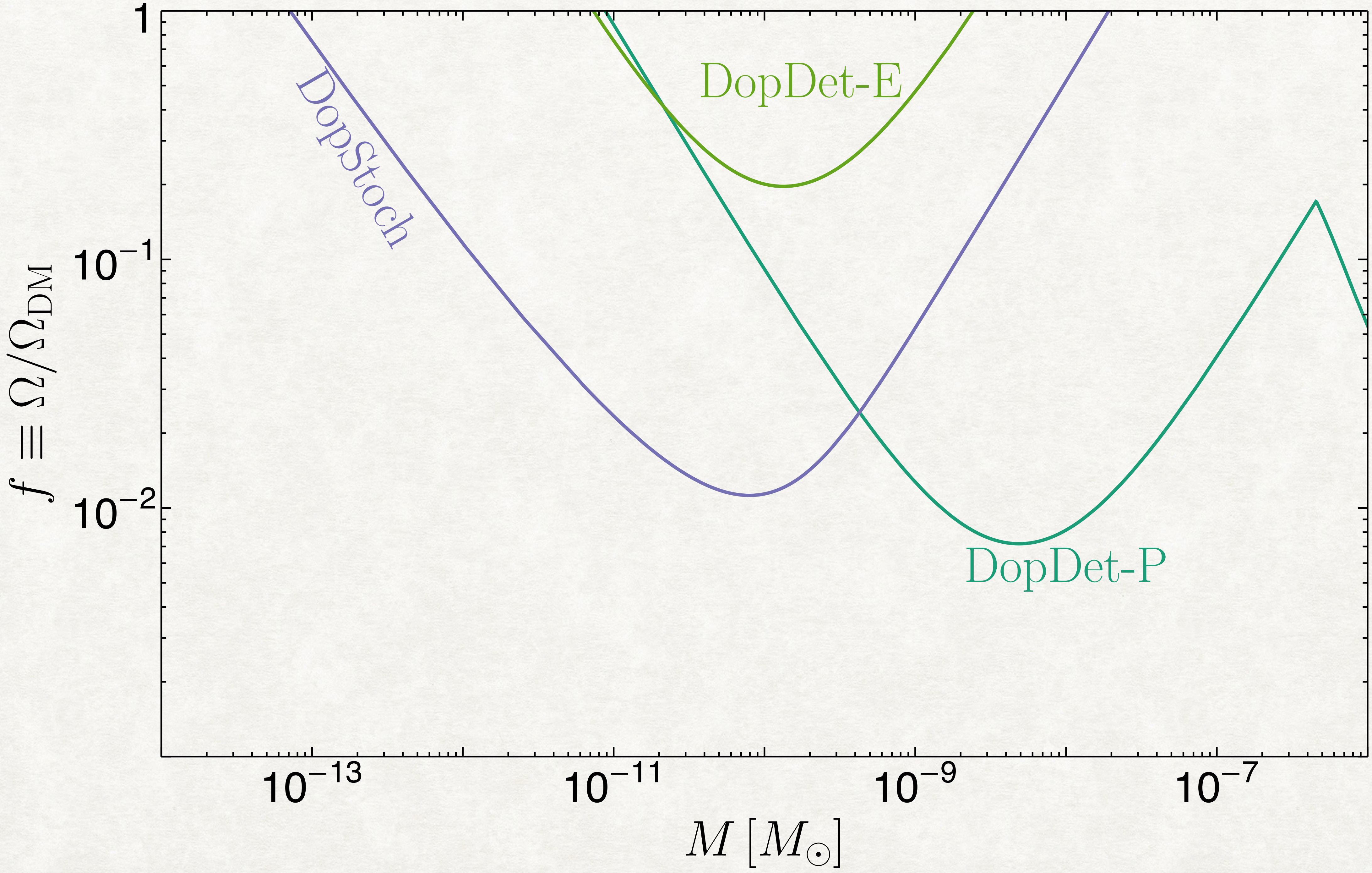
Stochastic signal:

Pulsar Term - N_P pulsars accumulating more statistics
Earth Term - can cross-correlate across pulsars with angular correlations.

For the highest single die roll, helps to roll die several times,
For sum of 100 die rolls, no point repeating the 100 roll.

DOPPLER SUMMARY

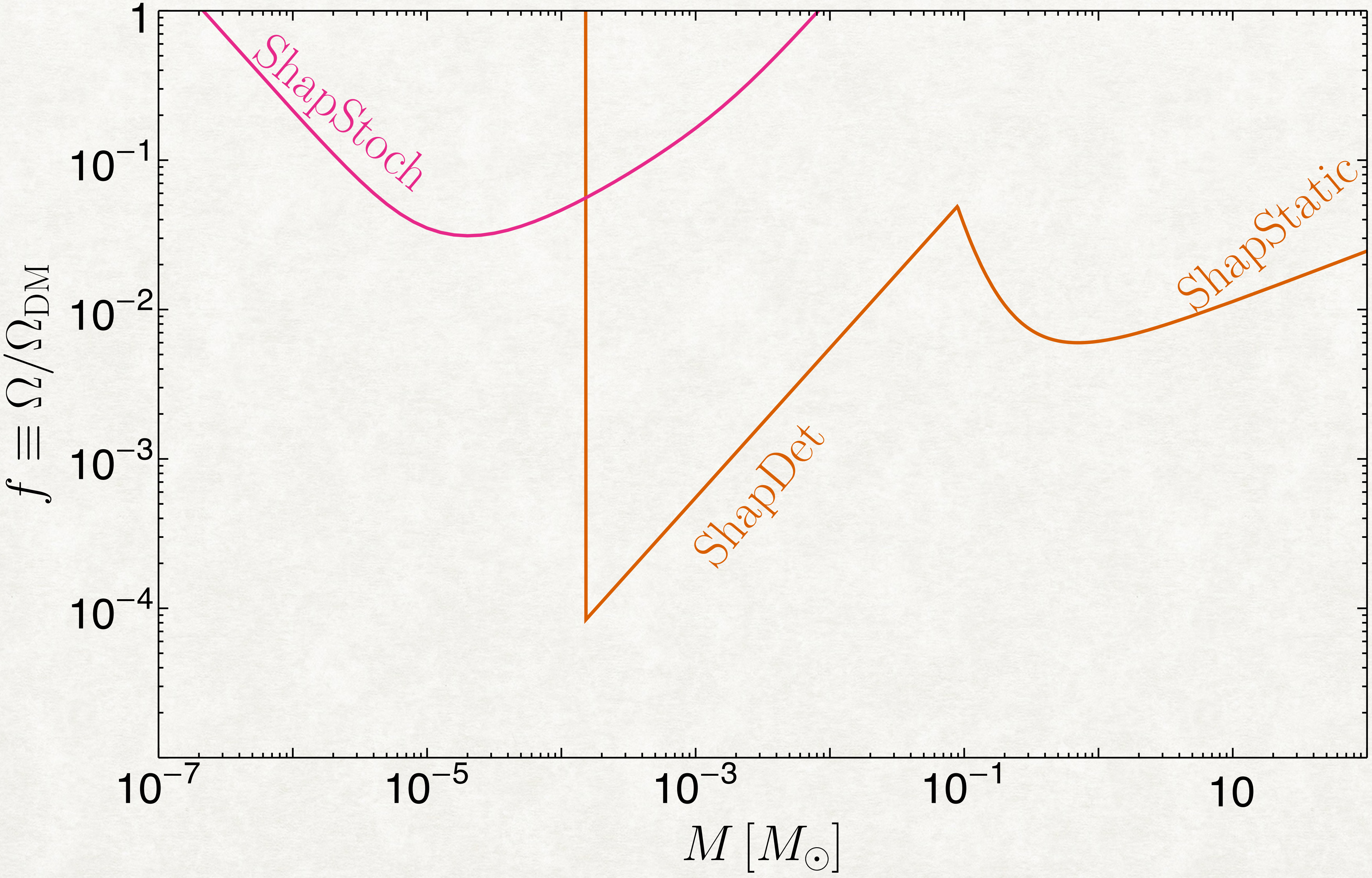
Stochastic Signal: Random walk in velocity



OPTIMISTIC

SHAPIRO SUMMARY

Stochastic Signal: Random addition of blips



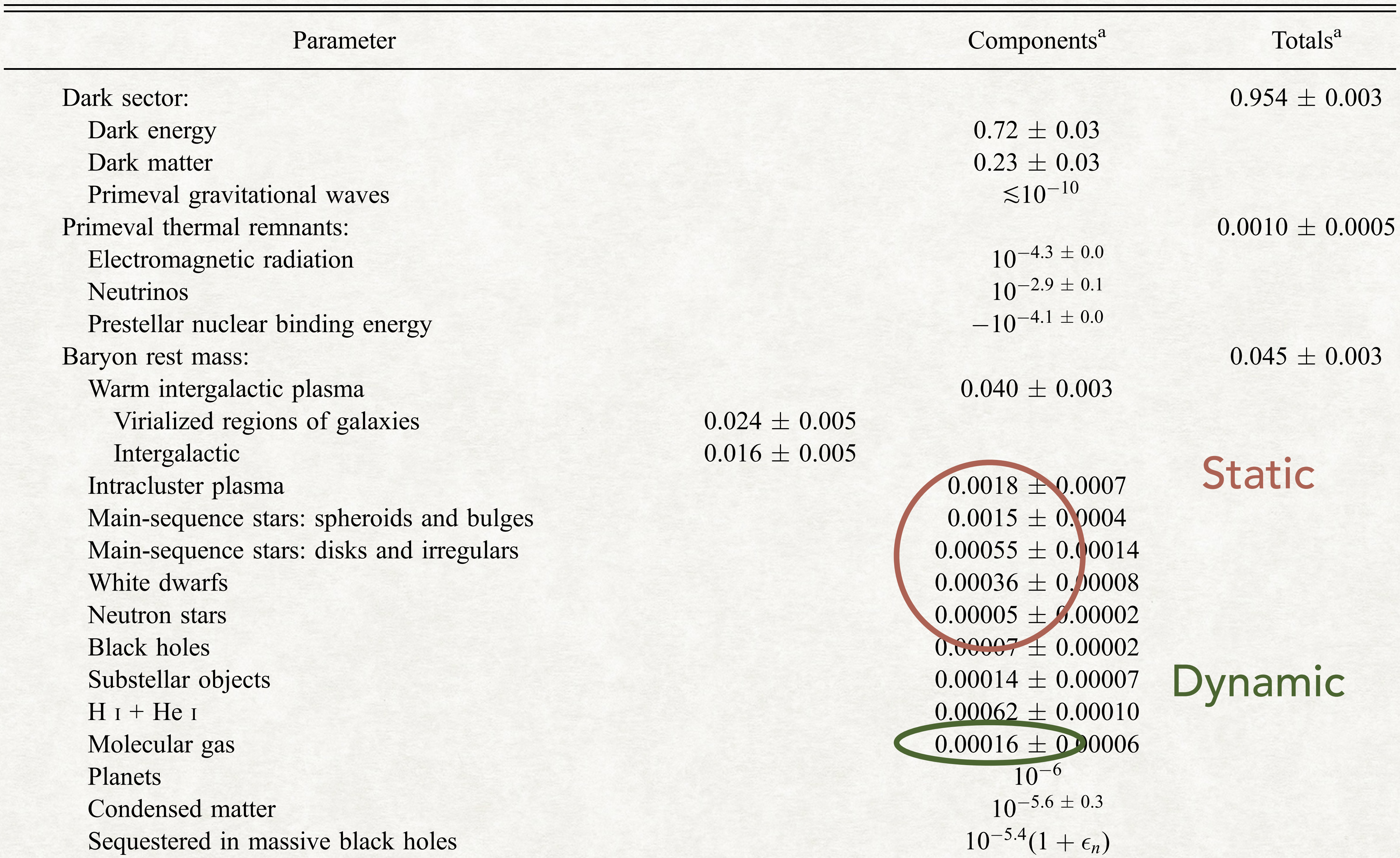
OPTIMISTIC

MAJOR BACKGROUNDS

BARYONS

THE COSMIC ENERGY INVENTORY

Masataka Fukugita, P. J. E. Peebles, 0406095



Parameter		Components[a]	Totals[a]
Dark sector:			0.954 ± 0.003
Dark energy		0.72 ± 0.03	
Dark matter		0.23 ± 0.03	
Primeval gravitational waves		$\lesssim 10^{-10}$	
Primeval thermal remnants:			0.0010 ± 0.0005
Electromagnetic radiation		$10^{-4.3 \pm 0.0}$	
Neutrinos		$10^{-2.9 \pm 0.1}$	
Prestellar nuclear binding energy		$-10^{-4.1 \pm 0.0}$	
Baryon rest mass:			0.045 ± 0.003
Warm intergalactic plasma		0.040 ± 0.003	
Virialized regions of galaxies	0.024 ± 0.005		
Intergalactic	0.016 ± 0.005		
Intracluster plasma		0.0018 ± 0.0007	
Main-sequence stars: spheroids and bulges		0.0015 ± 0.0004	
Main-sequence stars: disks and irregulars		0.00055 ± 0.00014	
White dwarfs		0.00036 ± 0.00008	
Neutron stars		0.00005 ± 0.00002	
Black holes		0.00007 ± 0.00002	
Substellar objects		0.00014 ± 0.00007	
H I + He I		0.00062 ± 0.00010	
Molecular gas		0.00016 ± 0.00006	
Planets		10^{-6}	
Condensed matter		$10^{-5.6 \pm 0.3}$	
Sequestered in massive black holes		$10^{-5.4}(1 + \epsilon_n)$	

Static

Dynamic

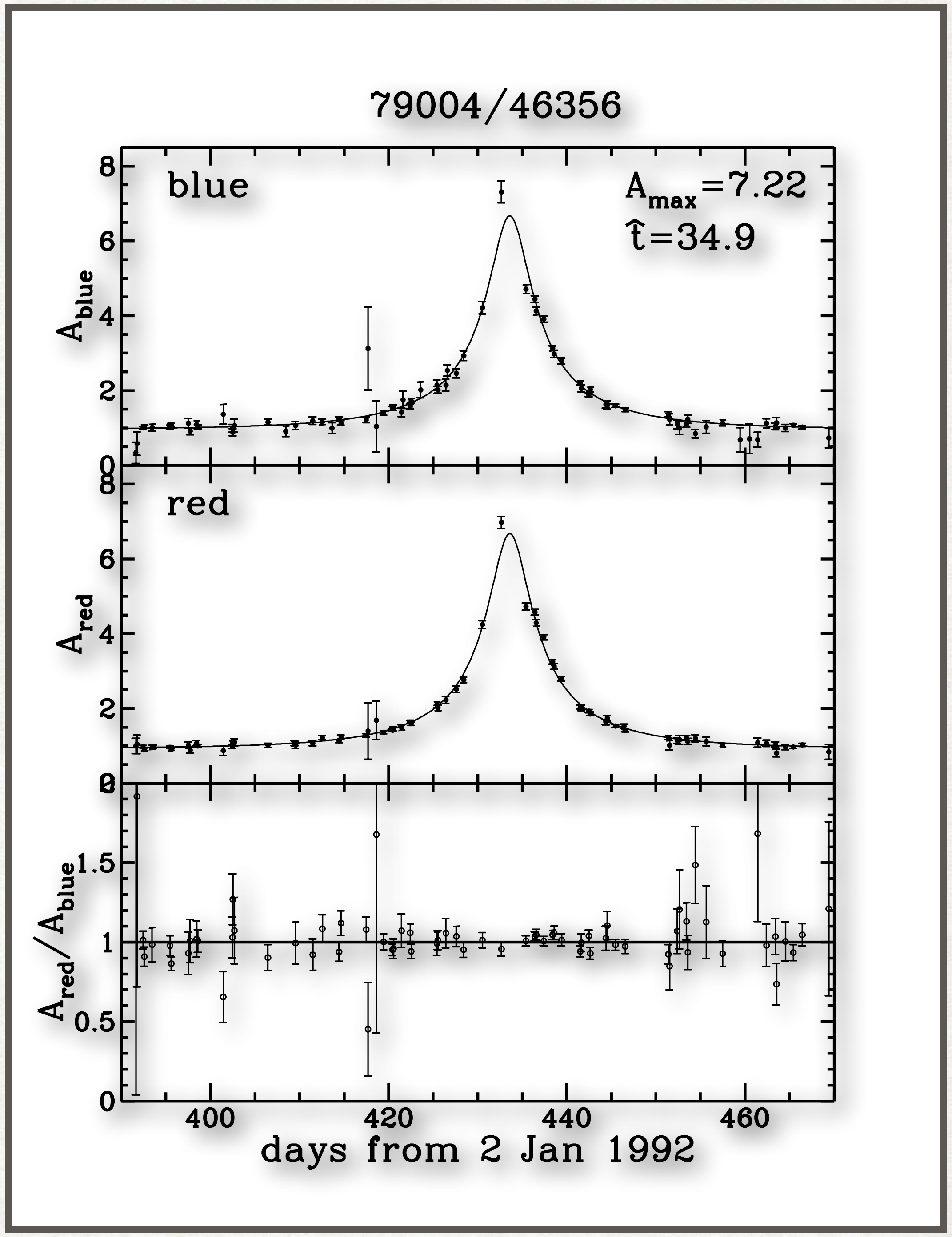
Most of the baryonic component will also be co-rotating with pulsar or earth

OTHER SOURCES OF BACKGROUND

- Glitches: Sudden increase in frequency, followed by a slow relaxation (days-year). Reduced significantly for Earth Term
- We considered a simplistic white noise
- In reality,
- Dispersion through interstellar medium - frequency dependent and red
- Some pulsars also suffer from intrinsic red noise
- Next step: use collaboration code to check signal survival

DYNAMIC BACKGROUNDS

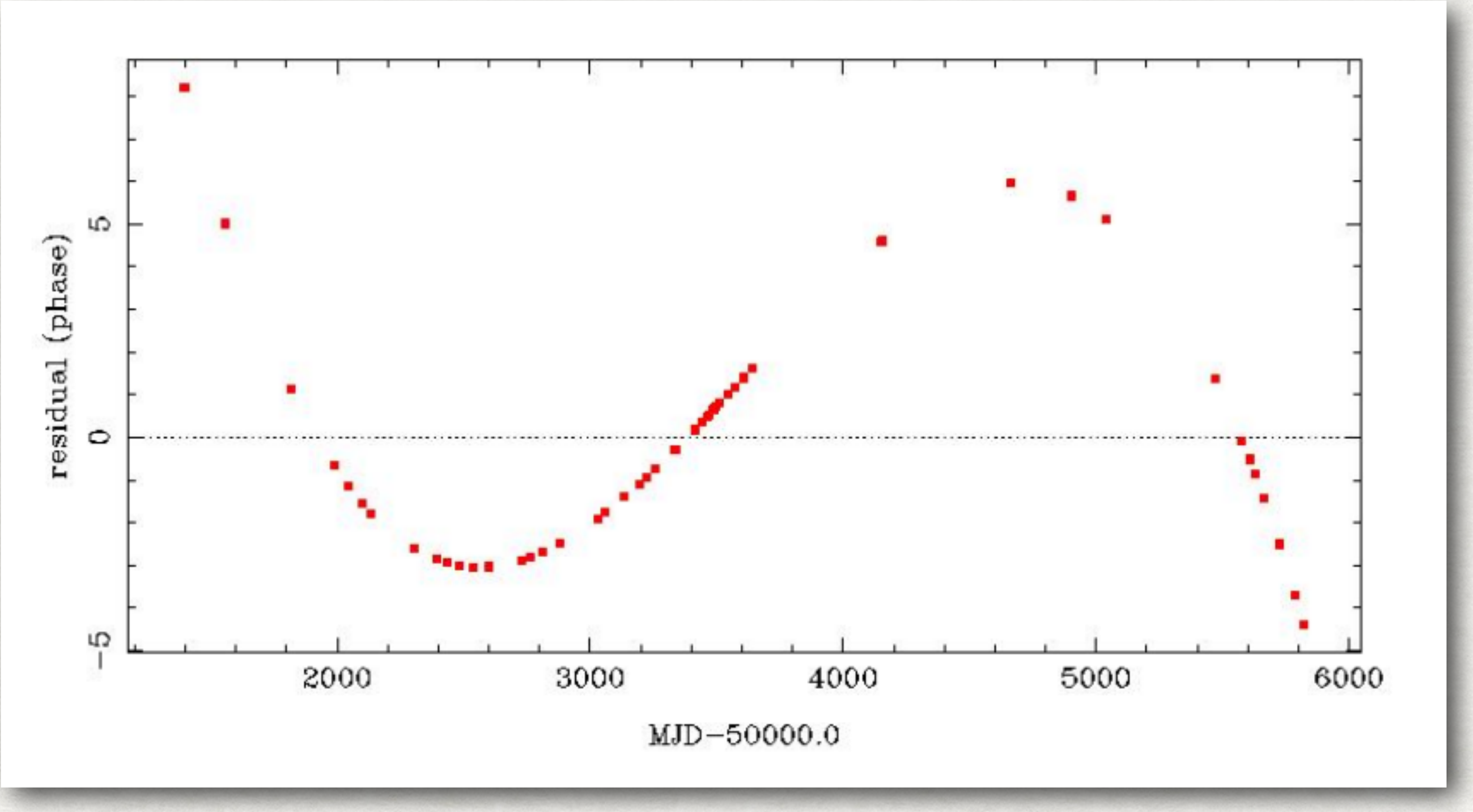
- Dynamic signal more spectacular than static signal.
- Shape differences could help differentiate from glitches etc.
- DM signals are non-dispersive
- Baryonic structure too few at these masses



Dispersion used in Microlensing to differentiate lensing blip from a dispersive blip

STATIC BACKGROUNDS

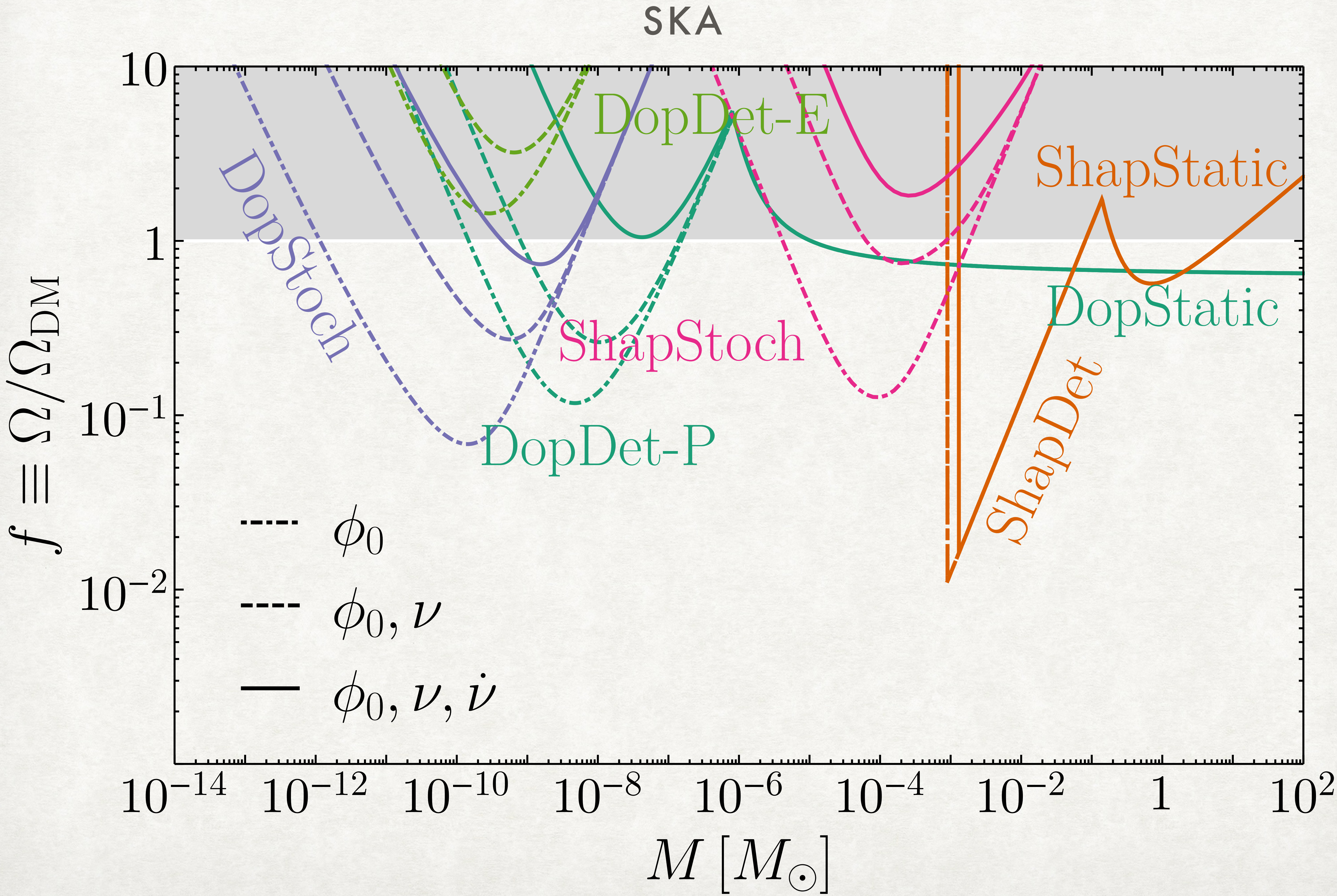
- A few pulsars already display non-zero second derivative.
- Will need to supplement with E&M observations to subtract known nearby objects.



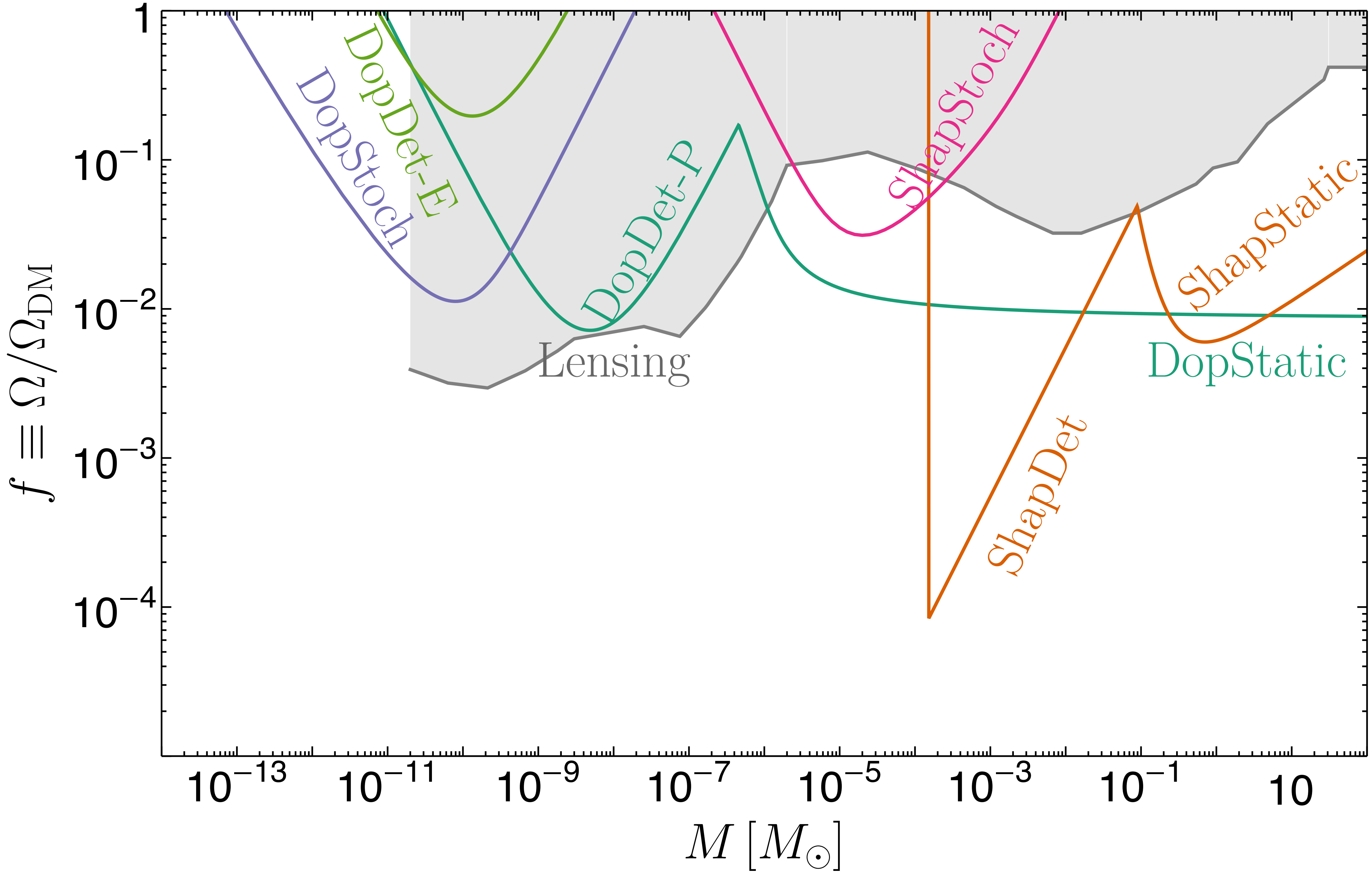
MONTECARLO SIMULATION

- Assume PBHs randomly distributed
- Isotropic Maxwell distribution with velocity truncated at v_{esc}.
- Simulate N_P randomly distributed pulsars at appropriate distances.
- Simulate order $O(10^5)$ universes and require more than 95% universes pass SNR cut.

SUBTRACTION OF INTRINSIC PULSAR PARAMETERS



RESULTS FOR PBH : OPTIMISTIC



Lensing constraint from Subaru, Machos, Eros, Ogle (MEO) and SN lensing

IS THIS A SILVER MEDAL?

Limits comparable but subdominant to lensing for the most part

MORE DIFFUSE OBJECTS

- We have seen point-like objects till now.
- If size of the object < impact parameter, Gauss’ law: treat object as point like
- Signal loss if object size > impact parameter.
- Can get conservative estimate with $M_{enc}(b)$.

EXTENDED OBJECTS

- Parametrize the profile as NFW.

$$\rho(r, M_{\rm vir}) = \frac{\rho_s}{(r/r_s)^{\alpha}\,(1 + r/r_s)^{\beta}}$$

$$\alpha = 1,\ \beta = 2$$

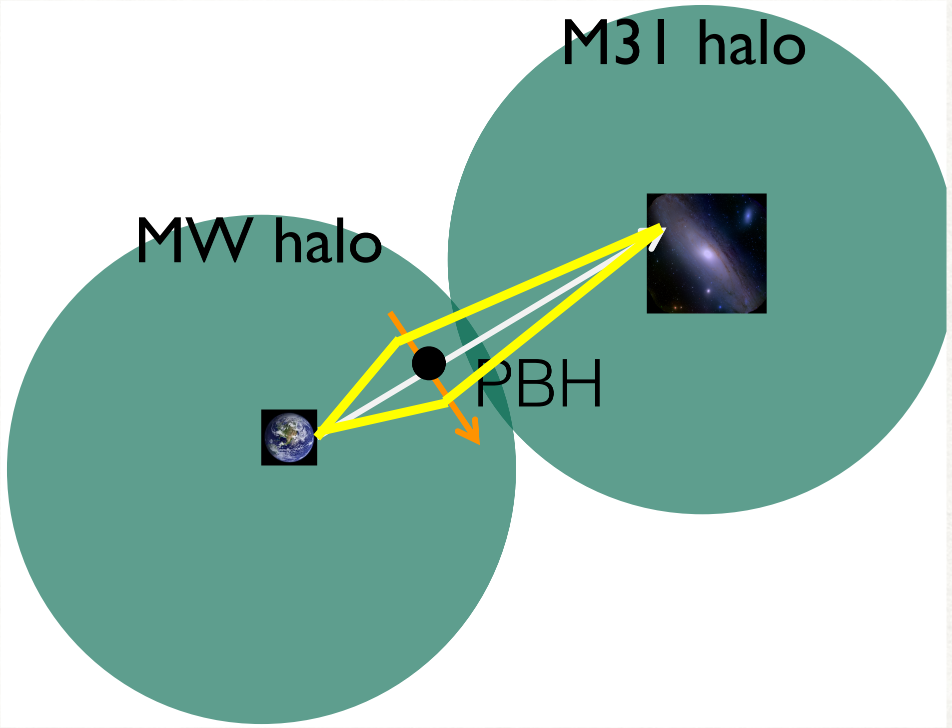
$$r_{\rm vir} \equiv (3M_{\rm vir}/800\pi\rho_c)^{1/3}$$

$$c \equiv r_{\rm vir}/r_s$$

Retrieve PBH in the large c limit

MICROLENSING

- Microlensing constraints from looking at M31/LMC



Source:Subaru

- Einstein radius

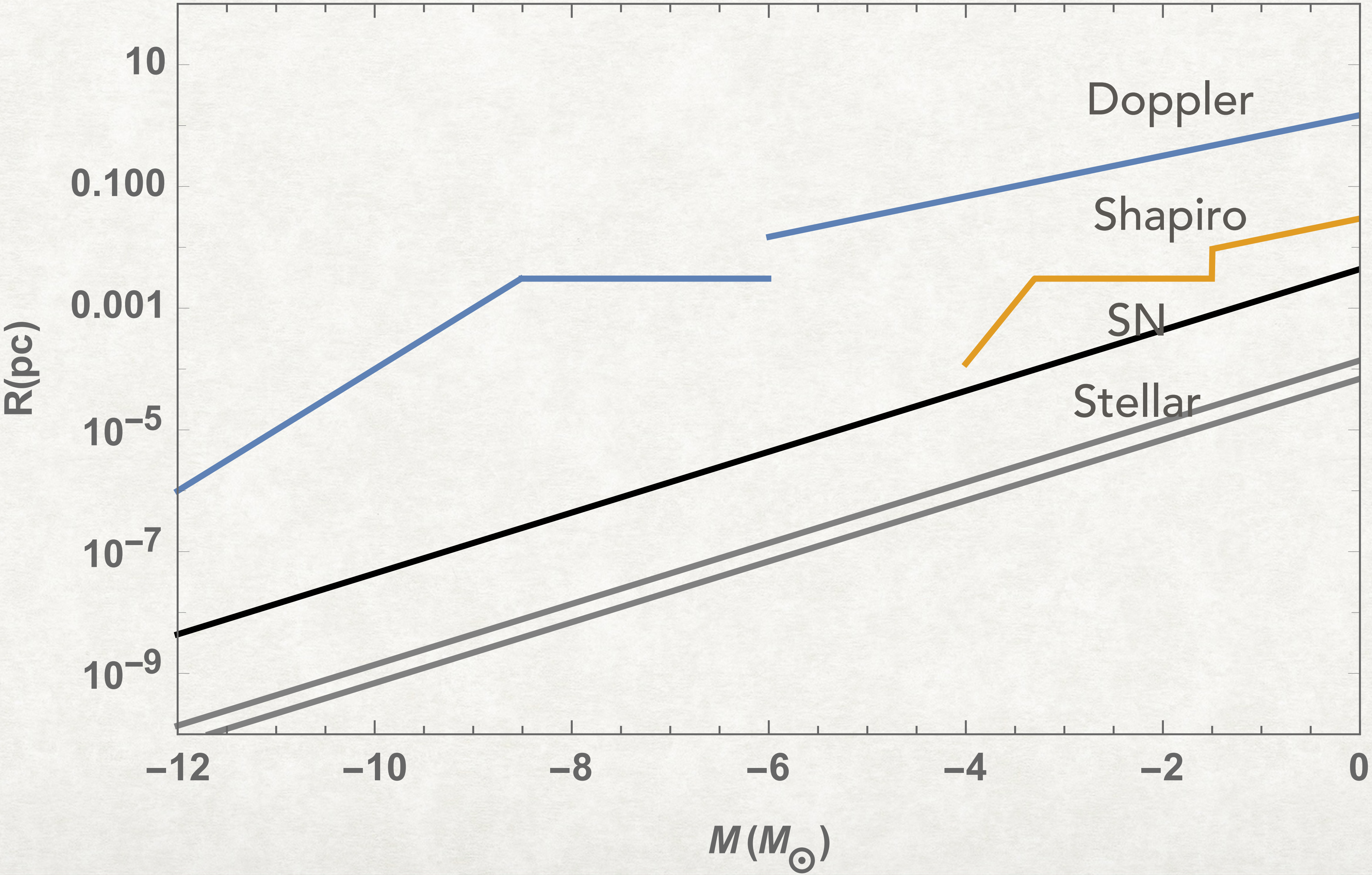
$$r_E \simeq \left(4GM\frac{(D_S - D_L)D_L}{D_S}\right)^{1/2}$$

D_S= Earth Star distance
D_L= Earth Lens distance

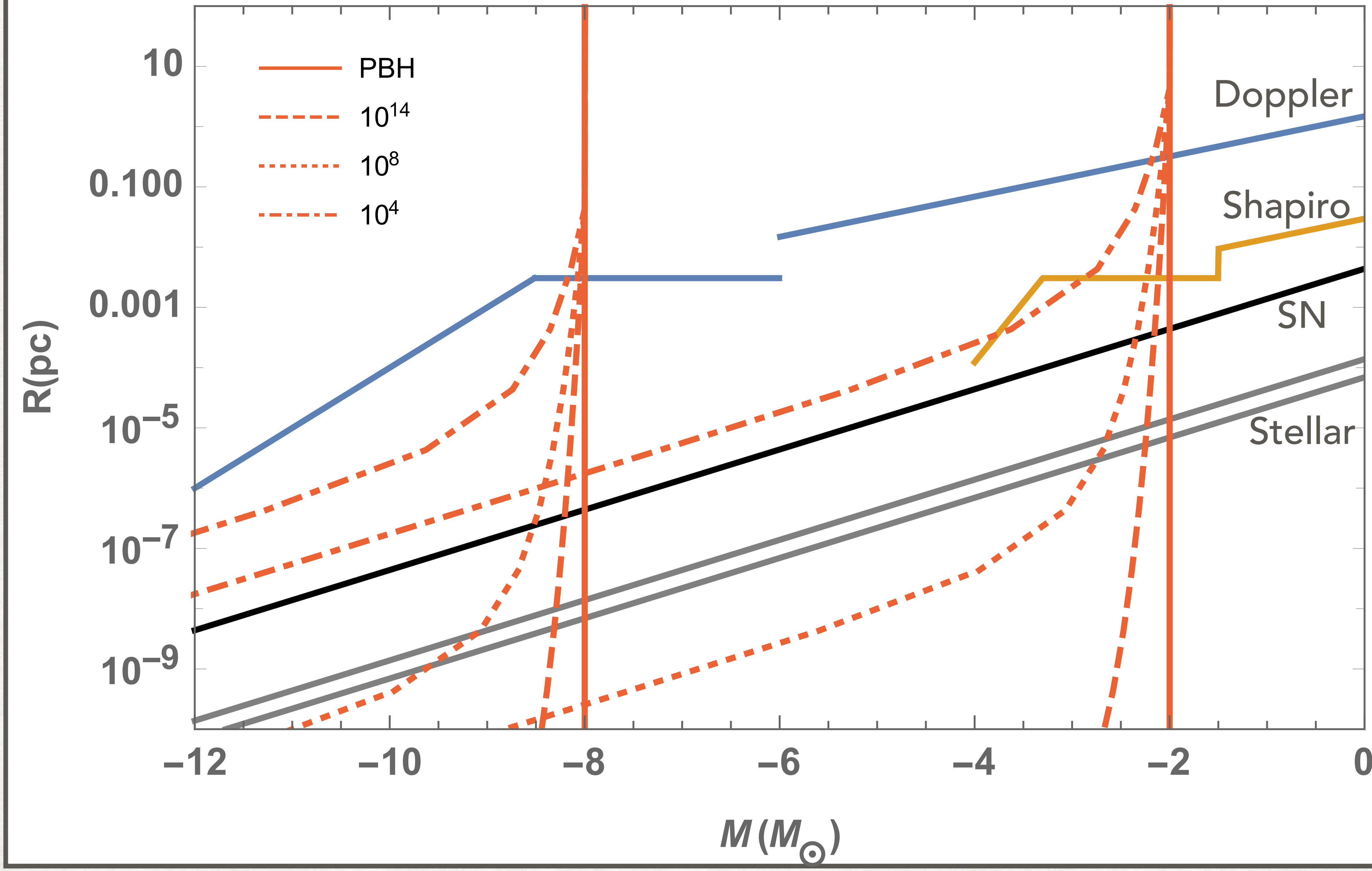
IMPACT PARAMETER: PTA VS LENSING

- Lensing: (Billion Stars x few hours) small impact parameter
- PTA: (100-1000 pulsars x few years) enormous impact parameter

$$r_{\mathrm{PTA}} \sim 10^{-3}\ \mathrm{pc} \times \begin{cases} \frac{M}{10^{-9}\, M_\odot} & \text{(Doppler Dynamic)} \\ \left(\frac{M}{10^{-3}\, M_\odot}\right)^2 & \text{(Shapiro Dynamic)} \end{cases} \qquad r_E \sim \begin{cases} 10^{-6}\ \mathrm{pc} \left(\frac{M}{10^{-4}\, M_\odot}\right)^{\frac{1}{2}} & \text{(Stellar Lensing)} \\ 10^{-2}\ \mathrm{pc} \left(\frac{M}{10\, M_\odot}\right)^{\frac{1}{2}} & \text{(Supernovae Lensing)} \end{cases}$$

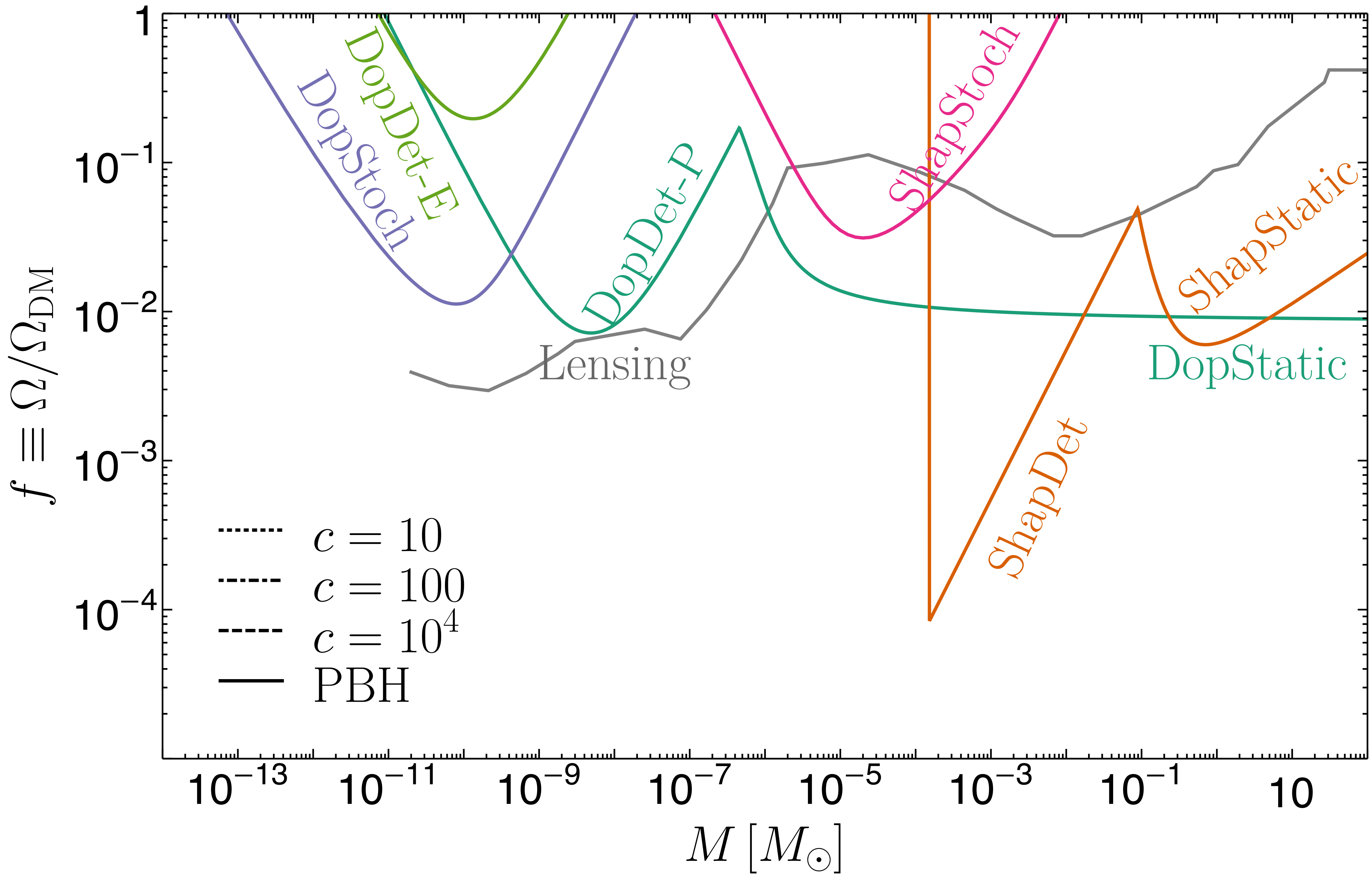


SENSITIVITY TO DIFFUSE HALOS

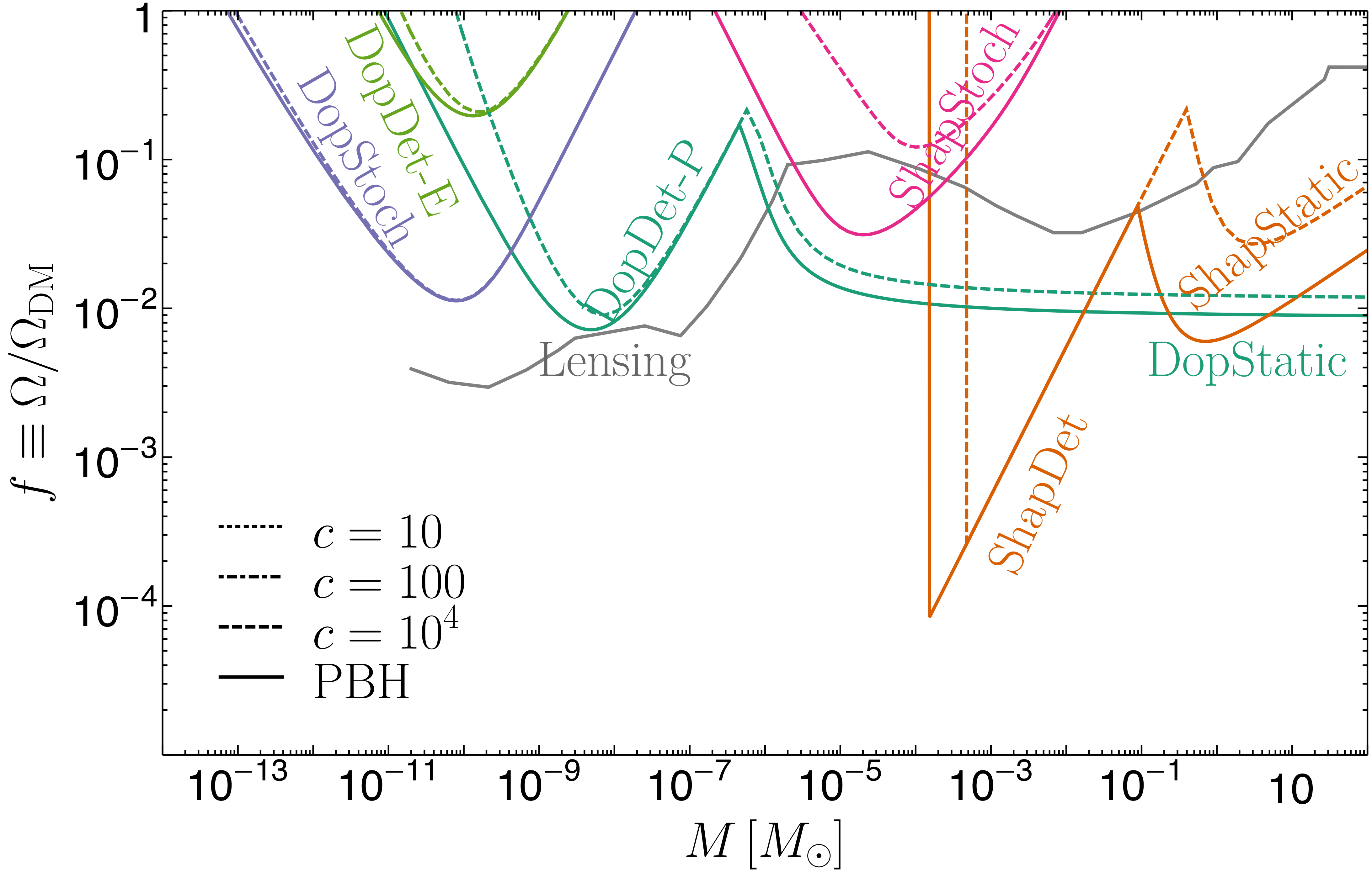


Limits iff red line intersects a probe radius

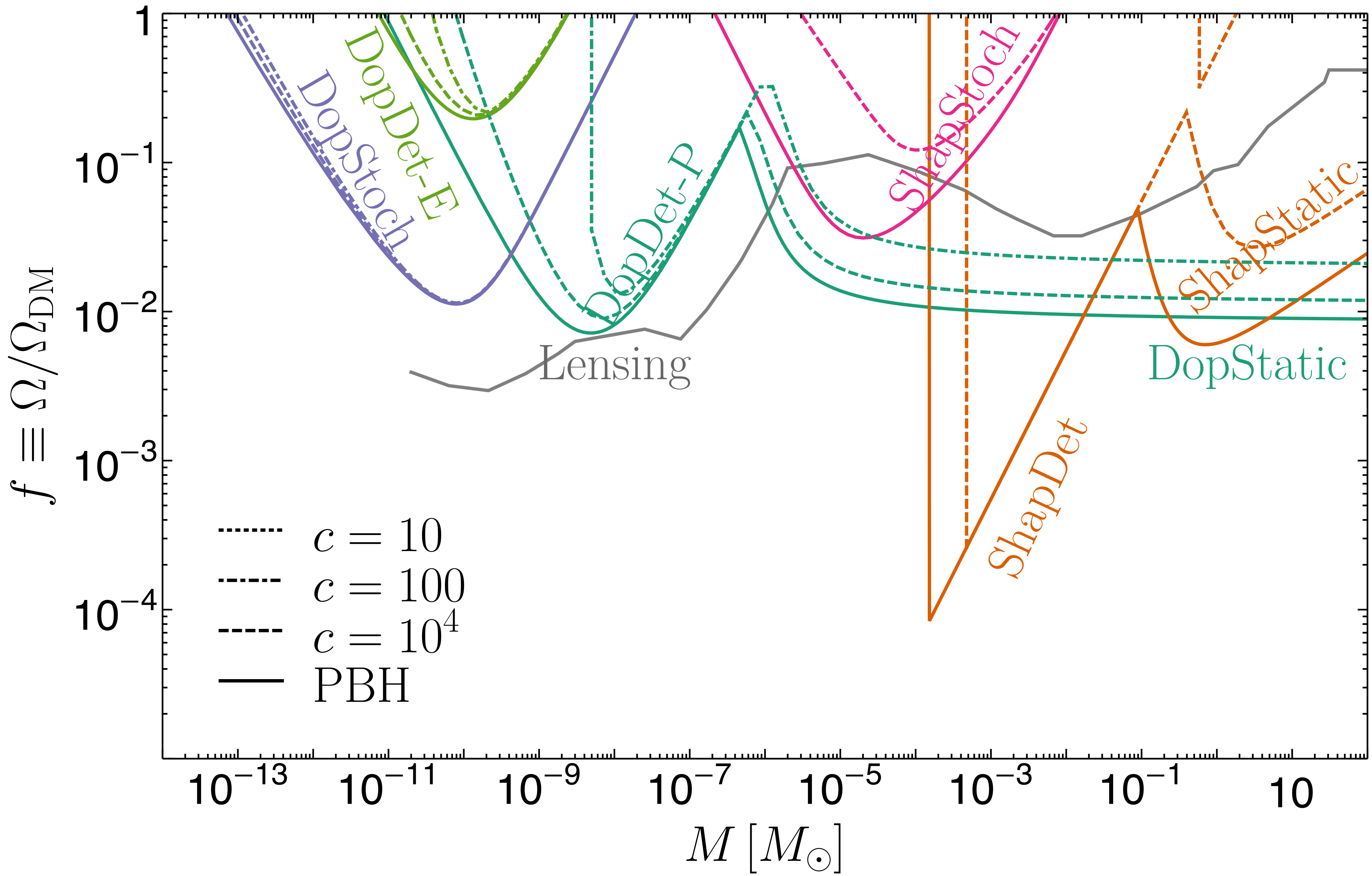
LIMITS FOR DIFFUSE OBJECTS



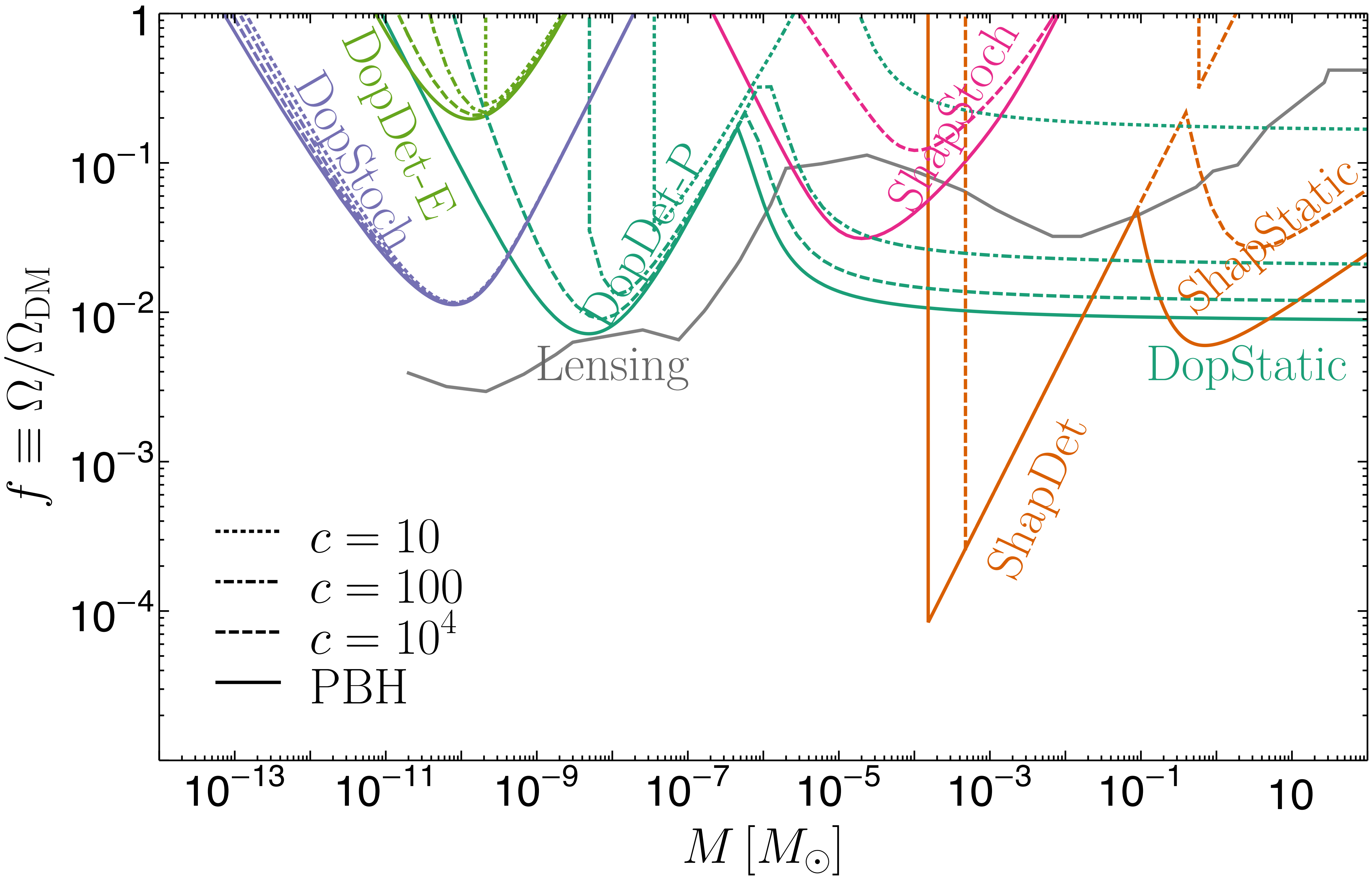
LIMITS FOR DIFFUSE OBJECTS



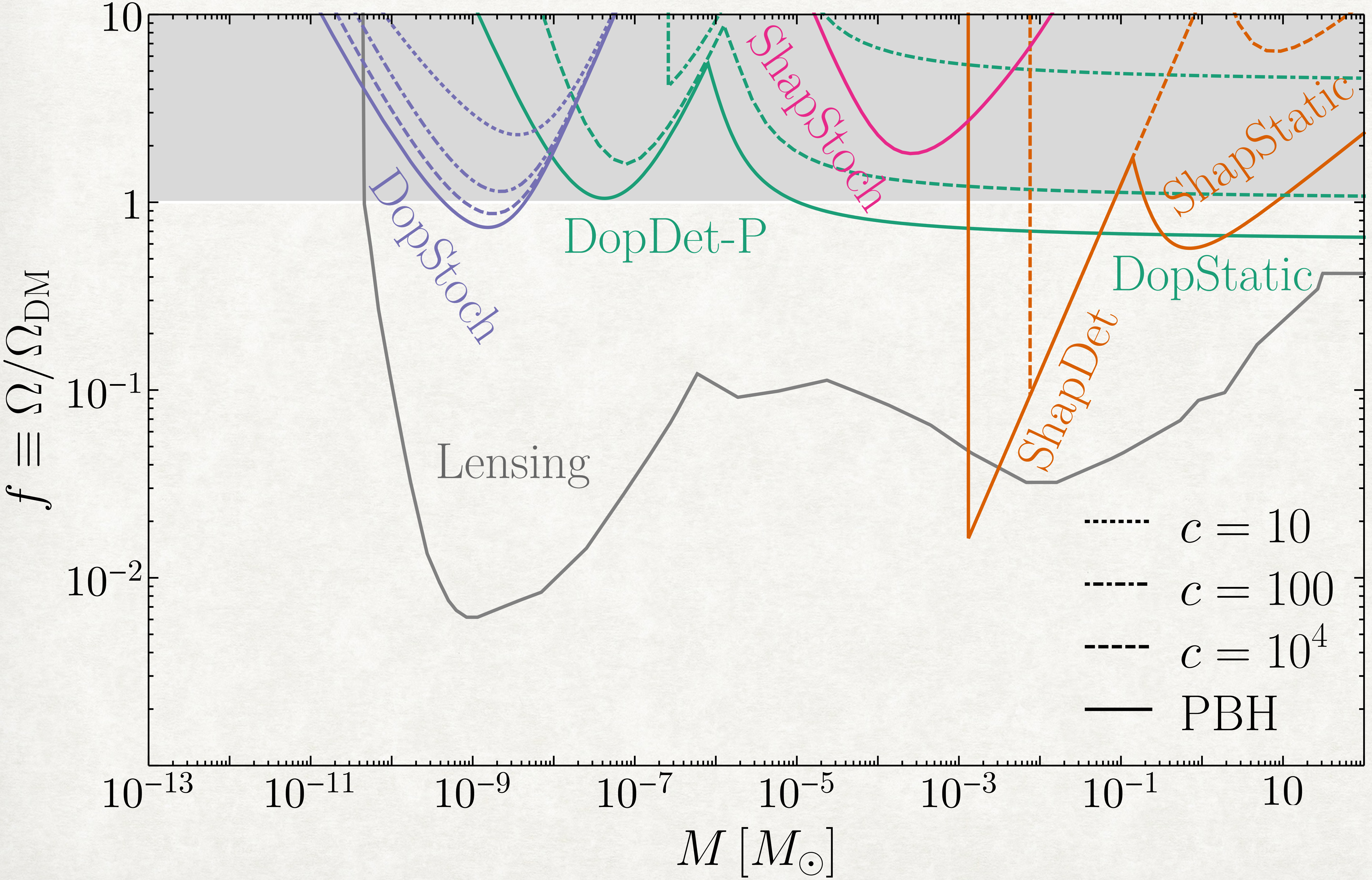
LIMITS FOR DIFFUSE OBJECTS



LIMITS FOR DIFFUSE OBJECTS

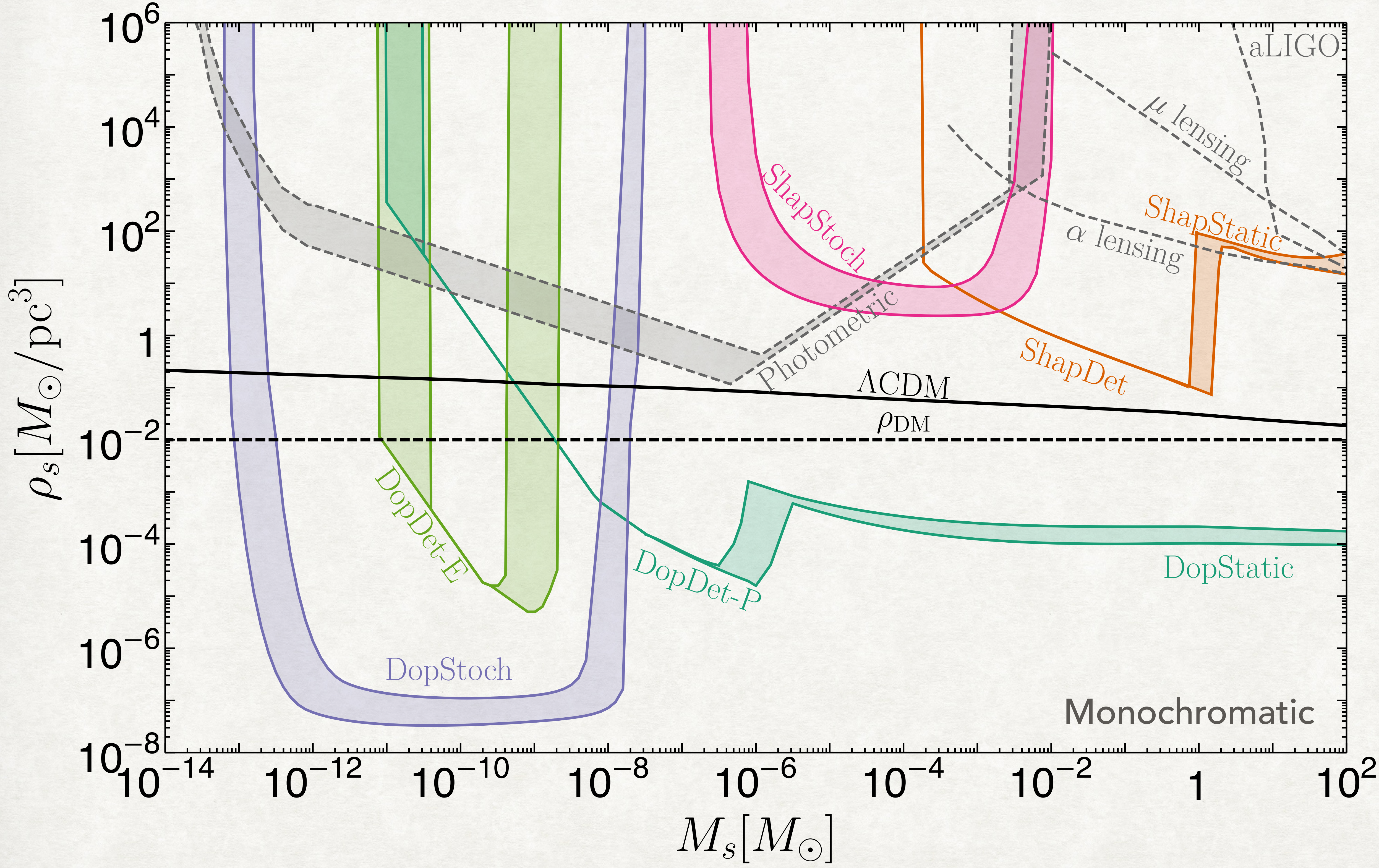


SKA



10
1
10^{-1}
10^{-2}
$f \equiv \Omega/\Omega_{\mathrm{DM}}$
10^{-13}
10^{-11}
10^{-9}
10^{-7}
10^{-5}
10^{-3}
10^{-1}
10
$M\ [M_\odot]$
DopStoch
DopDet-P
ShapStoch
ShapStatic
DopStatic
ShapDet
Lensing
$c = 10$
$c = 100$
$c = 10^4$
PBH

TIDALLY STRIPPED CORES - OPTIMISTIC



Error bands correspond to f=1 and f=0.3

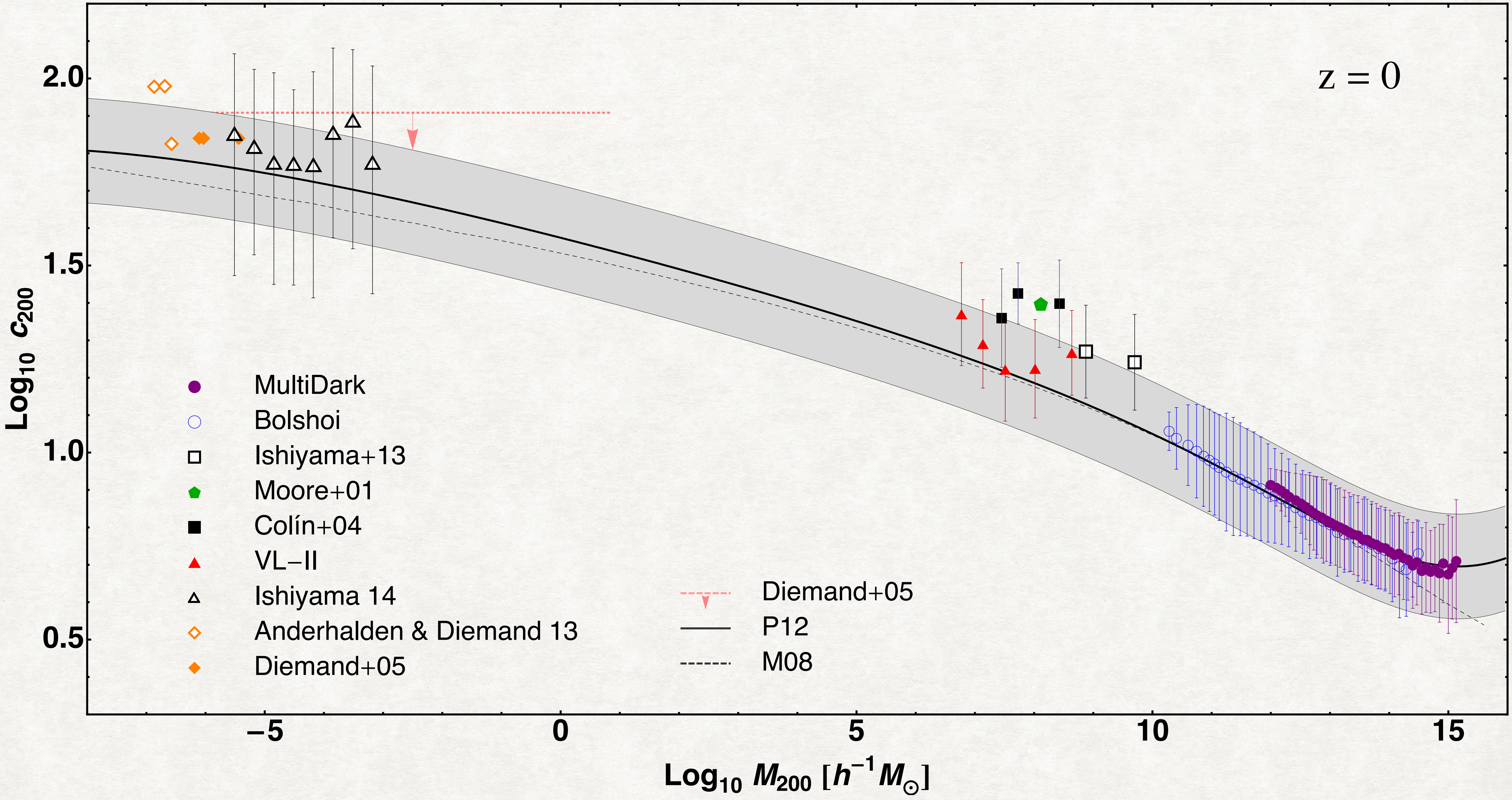
EXTENDED HALO MASS FUNCTION

- Assume typical scale-free Halo mass function from Press-Schechter.
- $dn/dM \sim M^{-2}$
- Abrupt cutoffs: M_{min}, and M_{max}
- Equal amount of DM in every decade of masses,
- Even large M_{max}/M_{min} can be probed using sensitivity solely in a small subset window.

LIMIT SETTING PARAMETERS

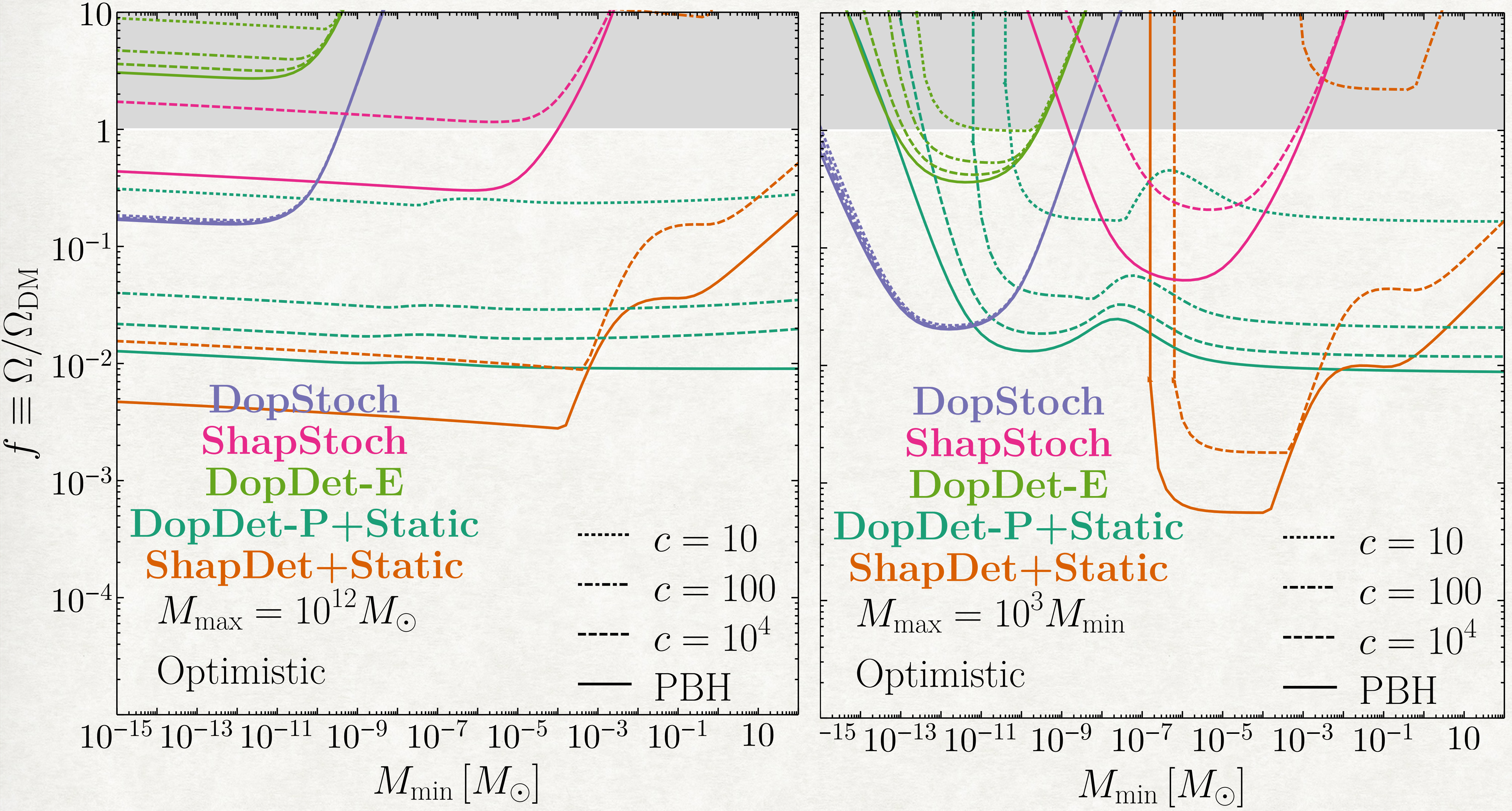
- Set Limits for
- c, the concentration parameter
- f the fraction of dark matter that has not disrupted
- Ignoring tidal disruption and sweeping it into c and f

GOAL: C=100



M usually cut off at 10^{-6} because WIMPs wash out small-scale structure…

EXTENDED HMF



OUTLOOK

- MSPs across the GC?
- in DM rich environments?
- Extra galactic MSPs?
- Non-gravitational long range forces?

- Better understanding of subhalos today given an initial Power Spectrum
- Limits on sub halos today into limits on primordial power spectrum?
- Understanding better the map between substructure or the lack thereof today and particle physics models.

CONCLUSIONS

- Pulsar timing can probe structure at a wide range of small scales.
- Doppler and Shapiro delays, especially in the dynamic regime, can provide a compelling discovery signal for DM subhalos.
- Probing CDM subhalos could be viable.

BACKUP

BOUNDS FROM DYNAMIC SIGNALS (DOPPLER)

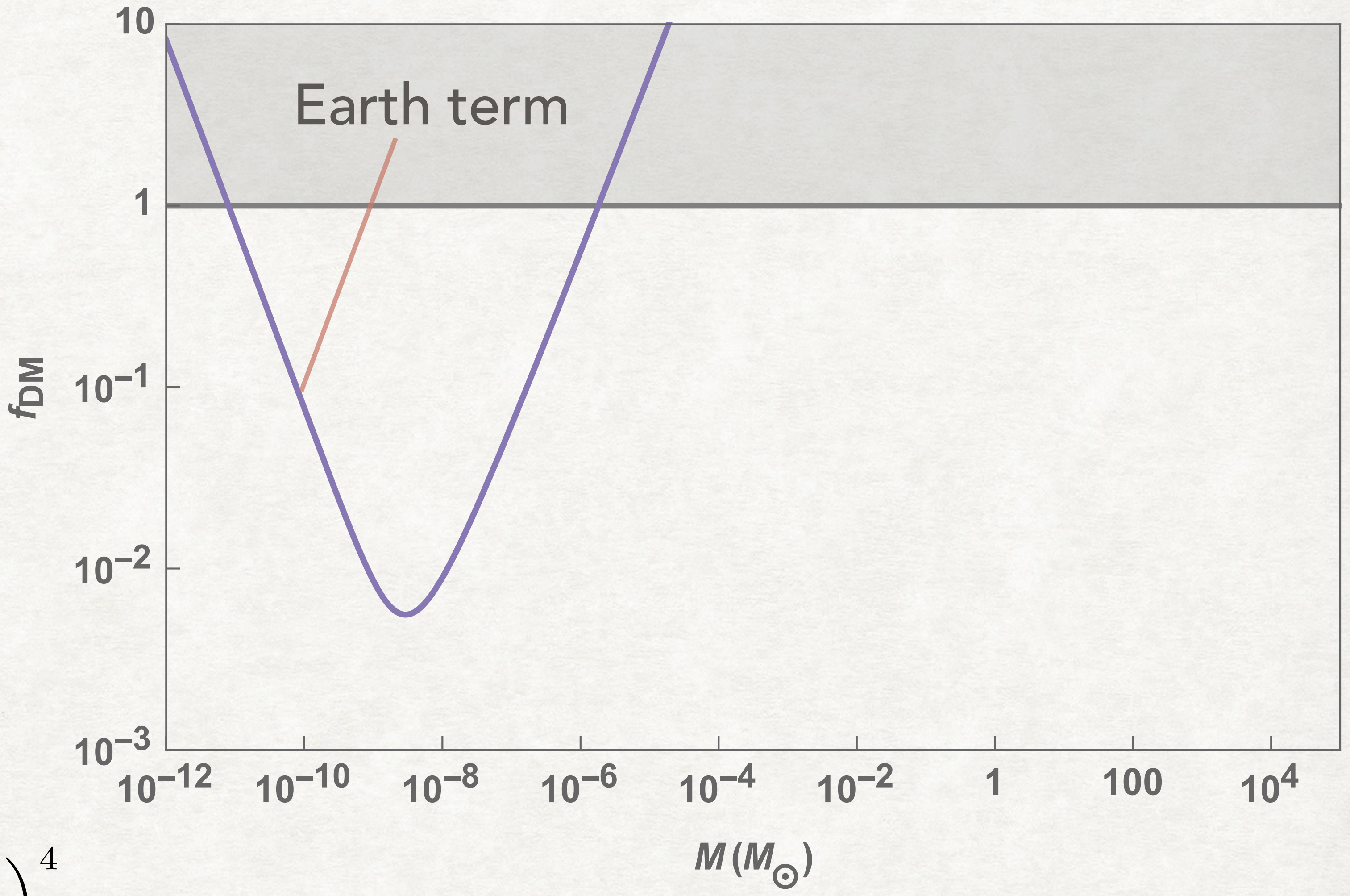
- For Doppler

$$\mathrm{SNR}_D = \frac{1}{2\sqrt{3}} \frac{GMT^{\frac{3}{2}}}{c\, t_{\mathrm{rms}} v^2 \sqrt{\Delta t}\, \tau}$$

$$\boxed{\tau_{\min} \propto \sqrt{\frac{M}{f}}}$$

- Requiring the closest approaching PBH to have SNR>4.
- f scales as 1/M

$$f^{L}_{\mathrm{D,\ dyn}} \lesssim 0.01 \left(\frac{10^{-9}\, M_\odot}{M}\right) \left(\frac{200}{N_P}\right) \left(\frac{20\ \mathrm{yr}}{T}\right)^4$$



Earth term scales the same way

- At some Mass M, even the nearest PBH starts failing dynamic constraint.
- This condition on f scales as M

$$f^{R}_{\mathrm{D,\ dyn}} \lesssim 3 \left(\frac{M}{10^{-7}\, M_\odot}\right) \left(\frac{200}{N_P}\right) \left(\frac{20\ \mathrm{yr}}{T}\right)^3$$

Earth term has Np=1

STATIC SIGNAL SENSITIVITY

Doppler

$$\frac{\ddot{\nu}}{\nu} \simeq \frac{2GMv}{r_{\min}^3} \sim 3 \times 10^{-32} \left(\frac{N_P f}{200}\right) \ \mathrm{Hz}^2$$

Shapiro

$$\frac{\ddot{\nu}}{\nu} \simeq \frac{16GMv^3}{r_{\times,\min}^3}$$

$$\sim 8 \times 10^{-33} \left(\frac{N_P f}{200}\right)^{\frac{3}{2}} \left(\frac{M_\odot}{M}\right)^{\frac{1}{2}} \left(\frac{d}{\mathrm{kpc}}\right)^{\frac{3}{2}} \ \mathrm{Hz}^2$$

Uncertainty in second derivative purely from rms fluctuations

$$\sigma_{\ddot{\nu}/\nu} = 6\sqrt{\frac{2800\Delta t}{T} \frac{t_{\mathrm{RMS}}}{T^3}}$$

$$f_{\mathrm{D,\ stat}} \lesssim 0.4 \left(\frac{200}{N_P}\right) \left(\frac{20\ \mathrm{yr}}{T}\right)^{\frac{7}{2}}$$

$$f_{\mathrm{S,\ stat}} \lesssim \left(\frac{200}{N_P}\right) \left(\frac{M}{M_\odot}\right)^{\frac{1}{3}} \left(\frac{20\ \mathrm{yr}}{T}\right)^{\frac{7}{3}} \left(\frac{\mathrm{kpc}}{d}\right)$$

Notice no M dependence here