A Swampland Tour
from global symmetries to axion physics

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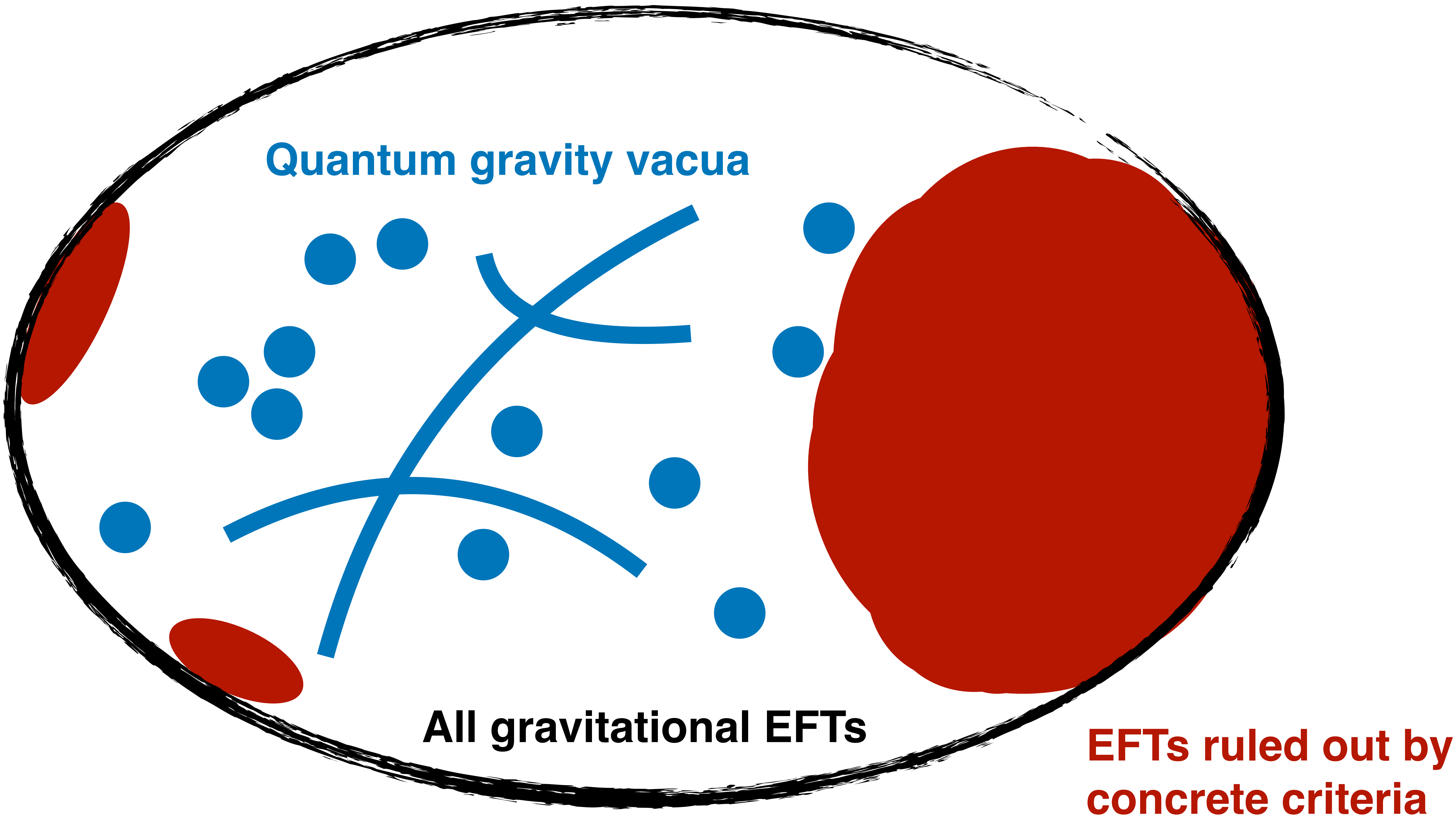
w/ Ben Heidenreich, Jake McNamara, Miguel Montero,
Tom Rudelius, and Irene Valenzuela
[arXiv:2012.00009 and other recent papers]

1. **The Swampland Program and the Weak Gravity Conjecture**

2. Generalized Global Symmetries and the Weak Gravity Conjecture

3. Chern-Weil Global Symmetries and the Necessity of Axions

The Landscape vs. The Swampland



The Swampland is the complement of the Landscape.
Our goal is to characterize it. Many suggestions.

Hopes for phenomenology

String theorists can build quantum gravity theories in several ways: heterotic string constructions, Type II models, F-theory models, M-theory on G_2 manifolds…

These share common features that are relevant for phenomenology:

- **Axions exist** with couplings to $\mathrm{tr}(F \wedge F)$, obtaining mass only from instanton effects
- **No very light Stückelberg photon masses** (only with low cutoff)
- Chiral matter comes in **small reps** (generally 2-index) of gauge groups
- Scalar potentials are not flat over ranges >> the Planck scale
- Scalar moduli exist with couplings to $\mathrm{tr}(F_{\mu\nu}F^{\mu\nu})$
- …

Are there deep principles behind these, or are the common features just because we only know simple examples of QG theories?

This Talk

I will focus on **“no global symmetries”** and its cousin, the Weak Gravity Conjecture. Small subset of ongoing Swampland work.

I think that a sufficiently general version of “no global symmetries” is behind the observation that string constructions always have axions coupling to $\mathrm{tr}(F \wedge F)$ terms.

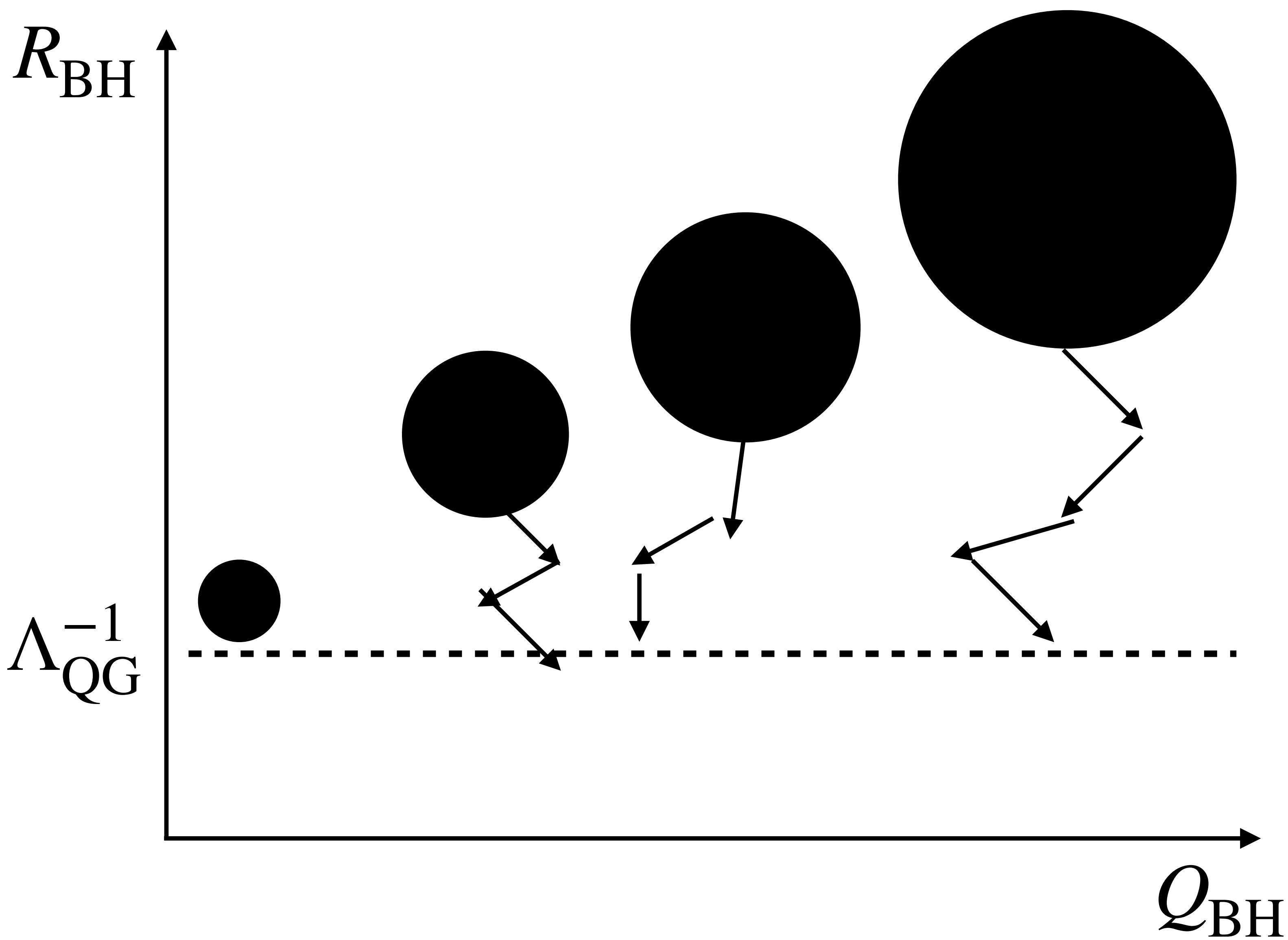
I will relate this to what we call “Chern-Weil global symmetries.”

Along the way I will have to spend some time introducing **generalized global symmetries** (Gaiotto, Kapustin, Seiberg, Willett).

It’s rather formal, but I think ultimately these ideas will play useful roles in particle physics. (Already, they’re often used in condensed matter theory.)

No global symmetries: continuous case

String worldsheet argument (Banks, Dixon '88):
Conserved current $J(z) \Longrightarrow$ vertex operator $J(z)\bar{\partial}X^{\mu}(z,\bar{z})\exp(ik^{\mu}X_{\mu}(z,\bar{z}))$
with $k^2 = 0$ creating a massless gauge boson.



Black hole Hawking evaporation would lead to infinite entropy in finite mass range.

Banks, Seiberg '10

Earlier work includes Georgi, Hall, Wise '81; Kamionkowski, March-Russell '92; Holman, Hsu, Kephart, Kolb, Watkins, Widrow '92; Kallosh, Linde, Linde, Susskind '95; …

No global symmetries: general case

It is believed that quantum gravity allows **no global symmetries**, including **discrete** and **p-form** global symmetries.

In the asymptotically AdS context, this has been argued by Harlow and Ooguri (1810.05337/8).

They define a global symmetry carefully to involve a “**splittability**” condition that avoids various pathological counterexamples.

Then, the non-existence of global symmetries in the AdS bulk follows from an argument using entanglement wedge reconstruction.

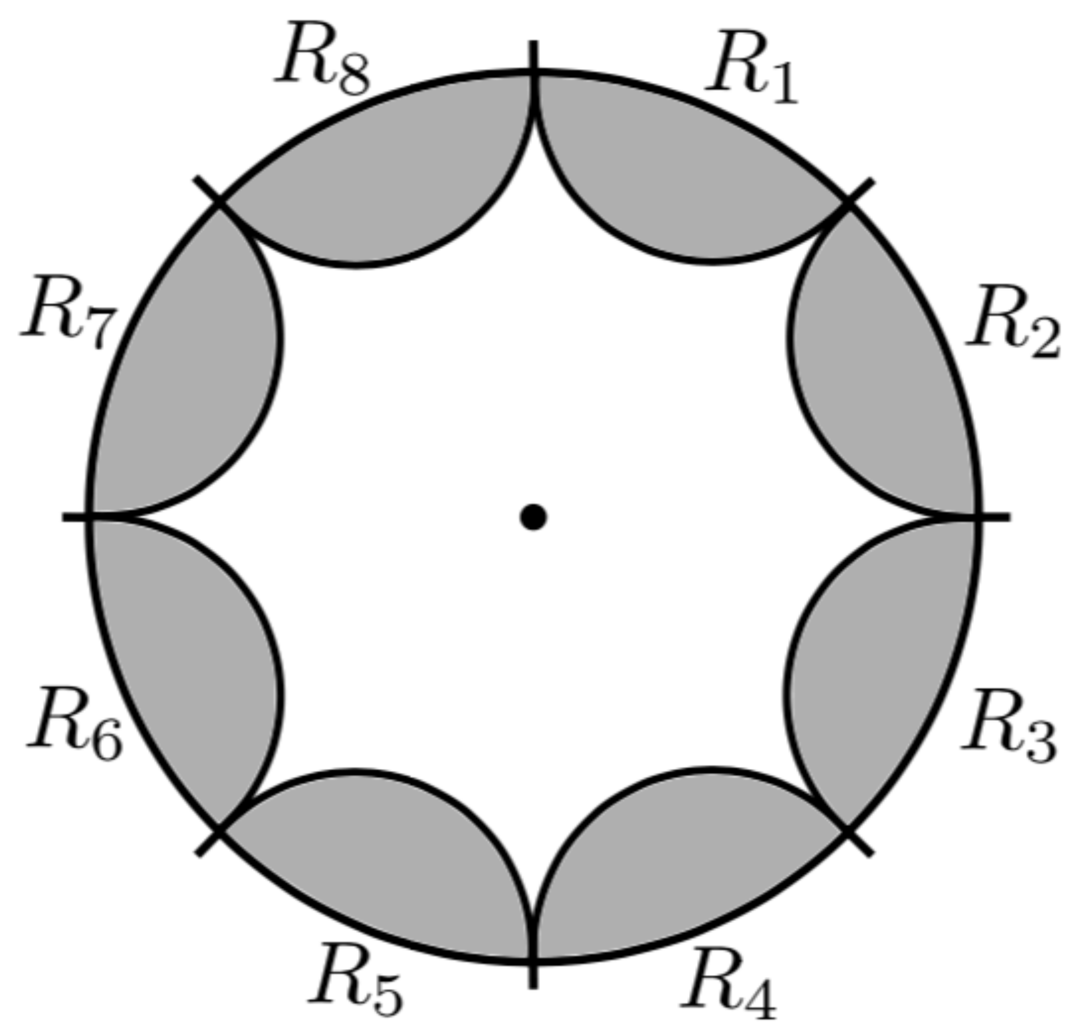


Fig. from 1810.05337 [Harlow/Ooguri]

What is the WGC? (Weak Gravity Conjecture)

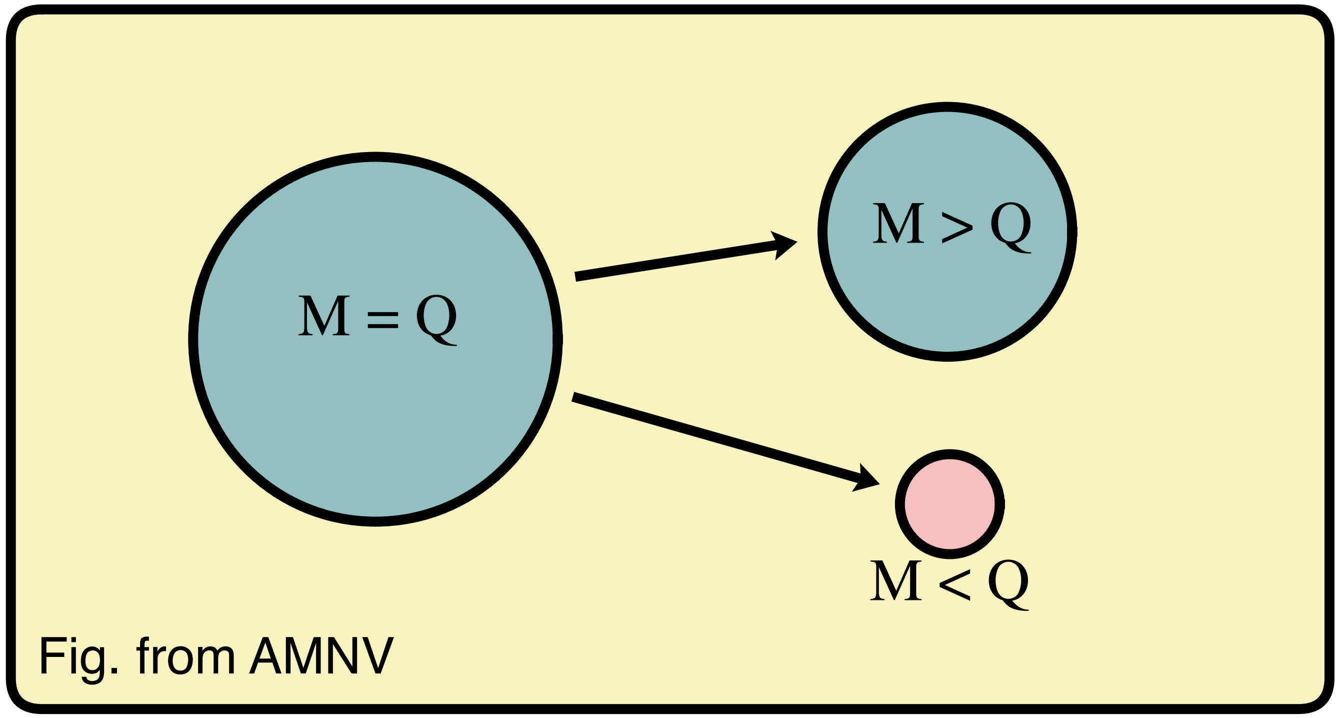


Fig. from AMNV

Arkani-Hamed, Motl, Nicolis, Vafa ("**AMNV**") hep-th/0601001

Particle exists with M<Q (superextremal).

Extremal BHs can shed charge.

Repulsive Force Conjecture:

A charged particle exists which is (long-range) **self-repulsive**. Gauge repulsion overcomes gravitational attraction.

Distinct conjectures when massless scalars exist.

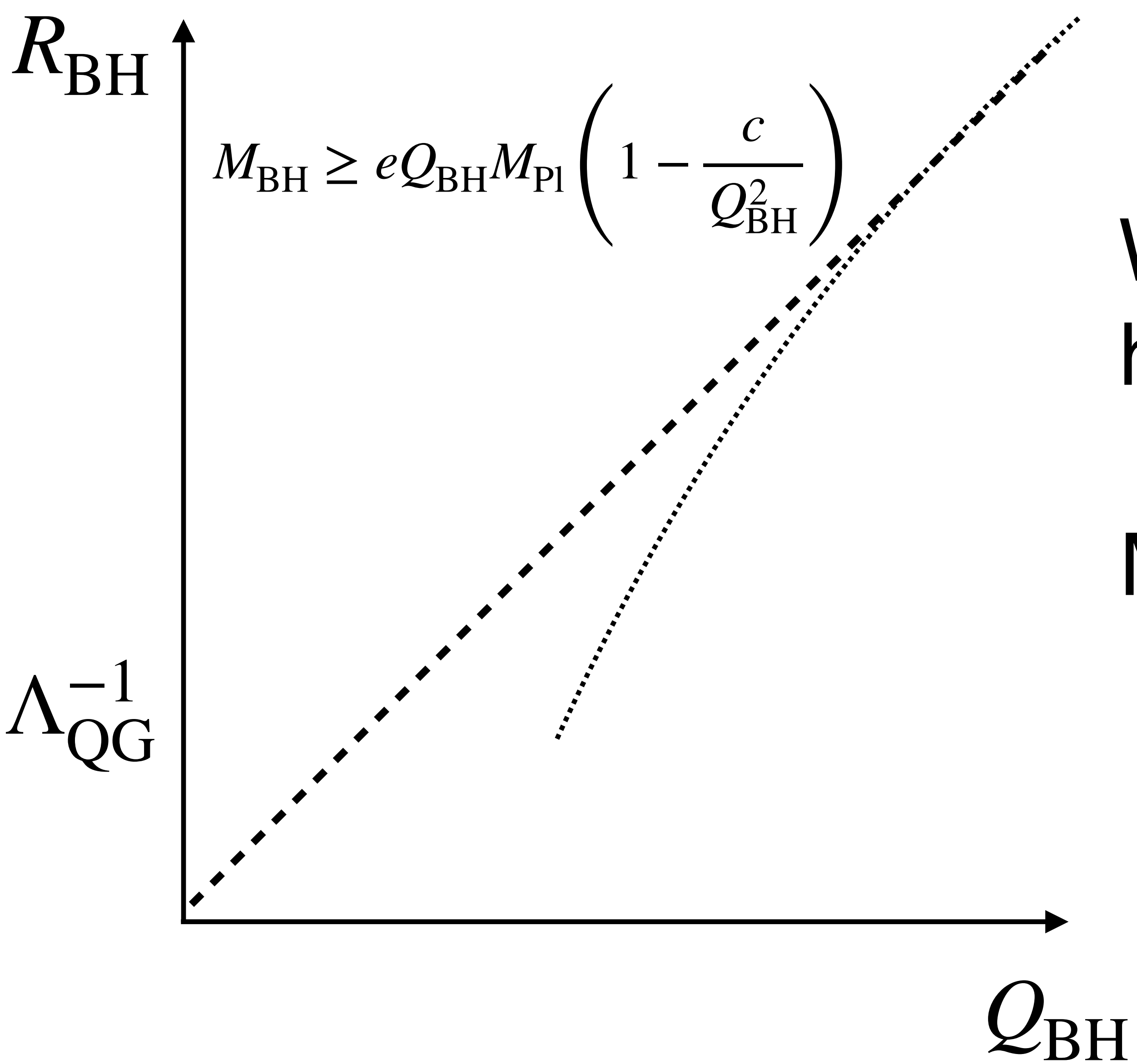
Palti '17; Lee, Lerche, Weigand '18; Heidenreich, MR, Rudelius '19

Is the minimal WGC obeyed by black holes?

Go beyond the 2-derivative action:

$$c_1(F_{\mu\nu}^2)^2 + c_2 F_{\mu\nu}F^{\nu\rho}F_{\rho\sigma}F^{\sigma\mu} + c_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + c_4 R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$$

AMNV; Kats, Motl, Padi '06



WGC obeyed by big black holes with small corrections!

Minimal WGC is **very weak.**

(Cheung, Remmen '18; Hamada, Noumi, Shiu '18; Bellazzini, Lewandowski, Serra '19; Mirbabayi '19; Charles, '19; Arkani-Hamed, Huang, Liu, to appear)

How to make the WGC less weak?

For many applications we would like a stronger statement to be true: the WGC is obeyed by a particle with mass below the Planck scale.

However, some simple "**Strong WGC**" statements are **known to be false**: the WGC need not be satisfied by the particle of smallest charge or by the lightest charged particle (AMNV '06; Heidenreich, MR, Rudelius '16 [w/ suggestions from Vafa]).

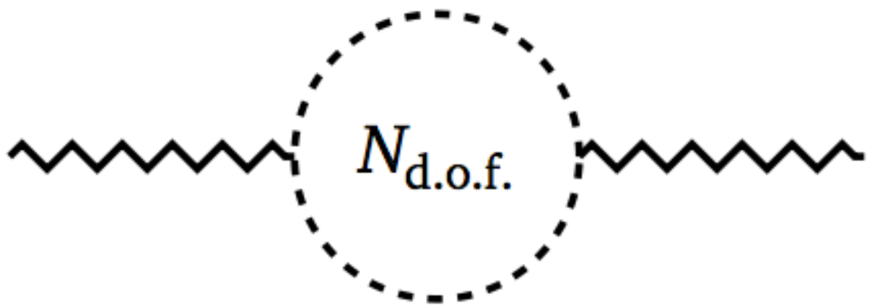
AMNV gave an argument that **the WGC scale serves as a UV cutoff**, by combining "**Magnetic WGC**" with the statement that **the classical radius of a magnetic monopole is a UV cutoff:**

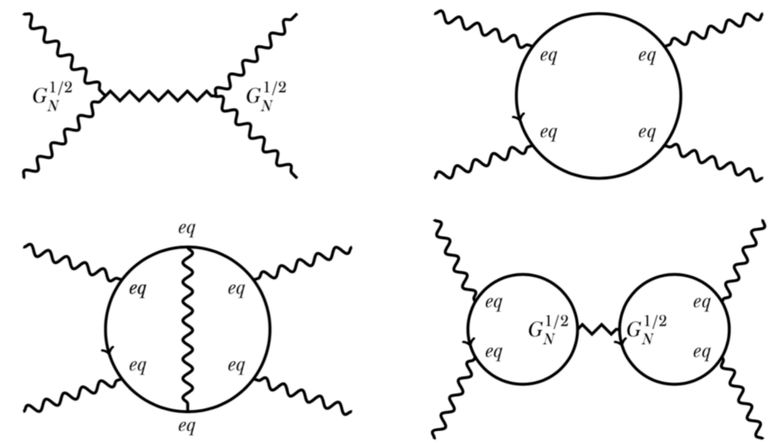
$$\Lambda \lesssim e M_{\rm Pl}$$

Sending $e \to 0$ to restore a global symmetry is then pathological.

Tower and Sublattice WGCs

Substantial evidence that in string theory, weak coupling always emerges by integrating out loops of many degrees of freedom:





Stronger (Tower/Sublattice) version of the WGC: **infinitely many particles** in the weakly-coupled EFT below the Planck scale **each** obey the WGC.

(Tower WGC: Andriolo, Junghans, Noumi, Shiu ’18; Heidenreich, MR, Rudelius ’19; Sublattice WGC: Heidenreich, MR, Rudelius ’15/’16; Montero, Shiu, Soler ’16; String evidence: Grimm, Palti, Valenzuela ’18; Lee, Lerche, Weigand ’18/’19; Corvilain, Grimm, Valenzuela ’18; Grimm, Ruehle, van de Heisteeg ’19; Grimm, Li, Valenzuela ’19; Gendler, Valenzuela ’20)

One of the sharpest formulations (“String Emergence”):
weak coupling always arises as either a **decompactification limit** (many light KK modes) or a **tensionless string limit** (many light string modes).

(Lee, Lerche, Weigand ’19)

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Review: differential forms for currents

Conserved current: $\partial_\mu j^\mu = 0$

Rewrite in terms of $(d-1)$-form $J = \star j$:

$$J_{\mu_1 \cdots \mu_{d-1}} = \varepsilon_{\mu_1 \cdots \mu_d} j^{\mu_d} \quad \text{and} \quad \partial_\mu j^\mu = 0 \quad \Rightarrow \quad \mathrm{d}J = 0.$$

Conserved currents $\Longleftrightarrow$ **Closed forms** (related by $\star$)

Total charge:

$$Q = \int \mathrm{d}^{d-1}x\, j^0 \quad \Leftrightarrow \quad Q = \int_{M_{d-1}} J$$

Gauging a conserved current:

$$A_\mu j^\mu \quad \Leftrightarrow \quad A \wedge J_{d-1}$$

Equation of motion:

$$\partial^\mu F_{\mu\nu} = j_\nu \quad \Leftrightarrow \quad \mathrm{d}(\star F) = J$$

A current is gauged when it is *exact*, not just *closed*.
Gauging removes currents from the cohomology.

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Disclaimer:

I'm being sloppy by not writing the $\sqrt{|\det g|}$ factors, but they all work out so the equations on the right are exactly correct.

A current is gauged when it is *exact*, not just *closed*.

Gauging removes currents from the cohomology.

Ordinary Global Symmetries

For an ordinary U(1) global symmetry in Euclidean *d*-dimensional spacetime, we can associated a charge with ***any (d–1)-dimensional submanifold***,

$$Q = \int_{M_{d-1}} J \in \mathbb{Z}$$

In the quantum theory, this means that we have a family of ***operators***,

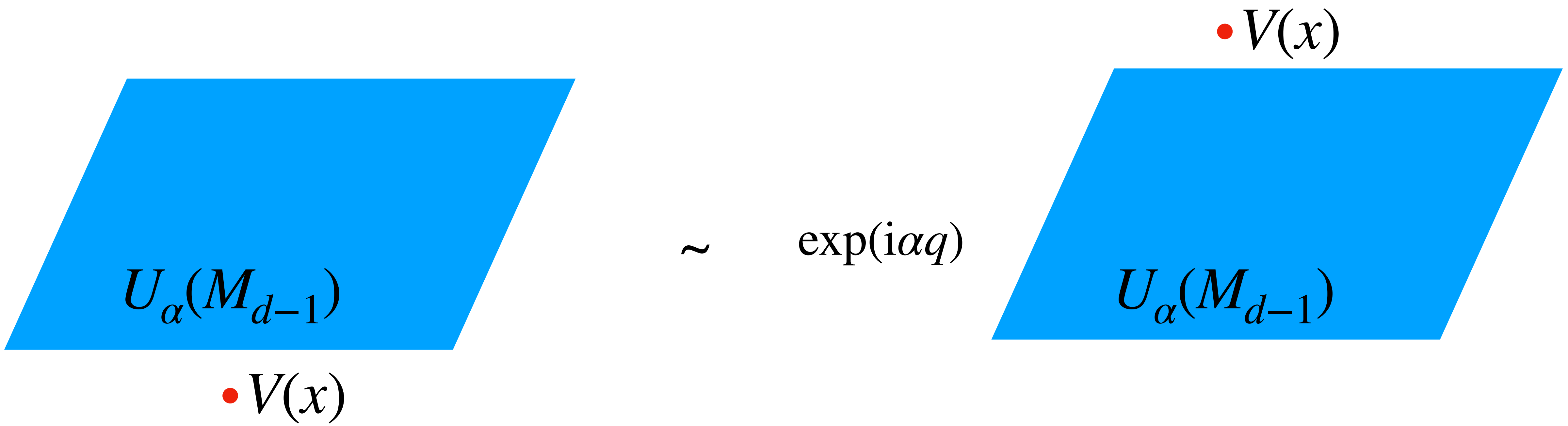
$$U_\alpha(M_{d-1}) = \exp\left(\mathrm{i}\alpha \int_{M_{d-1}} J \right).$$

associated to **codimension-1 surfaces**.

These operators are ***topological***: if a charged local operator is inserted in the theory, then the state picks up a phase when this operator **crosses through** the surface M_{d-1}.

Ordinary Global Symmetries

When a local operator $V(x)$ of charge q crosses the surface operator associated with the element $\exp(\mathrm{i}\alpha)$ of the global group U(1), it gains a phase $\exp(\mathrm{i}q\alpha)$

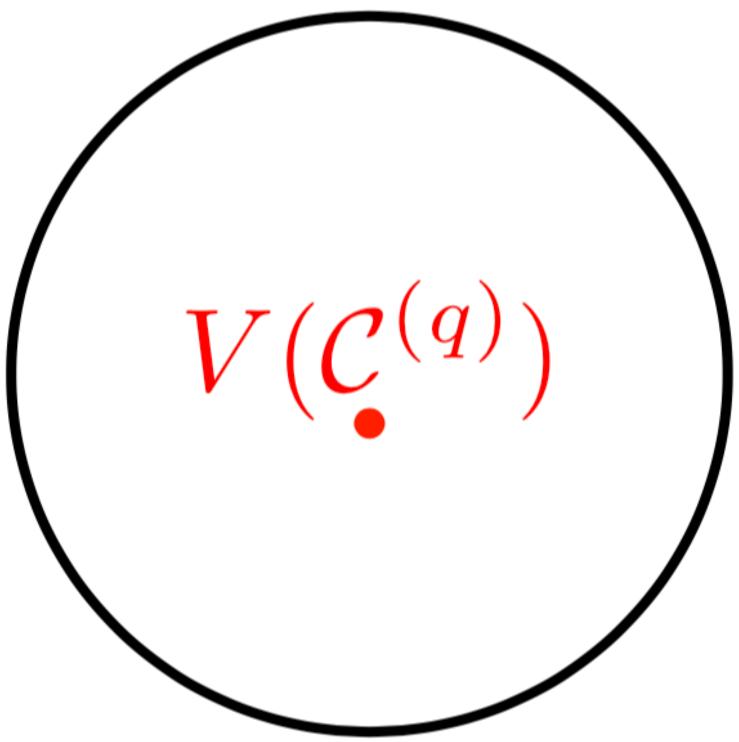


When the operator U is constructed out of the conserved current j^μ, this is familiar.

This formulation also works nicely for discrete symmetries, which have no local conserved current.

Generalized Global Symmetries

(Figure from a nice talk by Tom Rudelius at the 2019 Madrid workshop “Navigating the Swampland”)



$$U_g(S^{d-q-1}) \;=\; V(\mathcal{C}^{(q)}) \odot U_g(S^{d-q-1}) \;=\; \omega_g(V) \times V(\mathcal{C}^{(q)})$$

representation of *g*

Generalized Global Symmetries

arXiv:1412.5148 by Gaiotto, Kapustin, Seiberg, and Willett

A ***p*-form** G **global symmetry** has:

- Charge/symmetry operators $U_g(M_{(d-p-1)})$ which are **topological**
- *Charged* operators $V(M_p)$ associated with p-dimensional manifolds, which can be **"linked"** with the charge operators on $(d-p-1)$-manifolds.
- Dynamical charged *objects* with $(p+1)$-dimensional worldvolumes.
- Continuous G: local conserved $(d-p-1)$-form currents J
- Group law $U_g(M_{d-p-1})U_{g'}(M_{d-p-1}) = U_{gg'}(M_{d-p-1})$
- If $p > 0$, the only symmetries acting nontrivially are **abelian**

1-form Symmetries of U(1) Gauge Theory

In free Maxwell theory, we have no electric or magnetic sources, so

$$\mathrm{d}(\star F) = 0$$

Closed (d–2)-form current
$\Longrightarrow$ **Global (d–3)-form symmetry**

$$\mathrm{d}F = 0$$

Closed 2-form current
$\Longrightarrow$ **Global 1-form symmetry**

The quantization of fluxes means that these are both U(1) symmetries.
In 4d, they are both *1-form* global symmetries.

- **Electric symmetry, current $\star$ *F*, charged objects are *Wilson loops*.**
- **Magnetic symmetry, current *F*, charged objects are *'t Hooft loops*.**

The symmetries basically *count* Wilson or 't Hooft loops.

Existence of charged particles vs. presence of global symmetries

$$\mathrm{d}(\star F) = J$$

Charged particles break the *1-form symmetry*'s conservation law
(while gauging a *0-form symmetry* with current *J*)

The symmetry operators *exist*, but are no longer topological. Wilson operators can end on local operators that create charged particles.

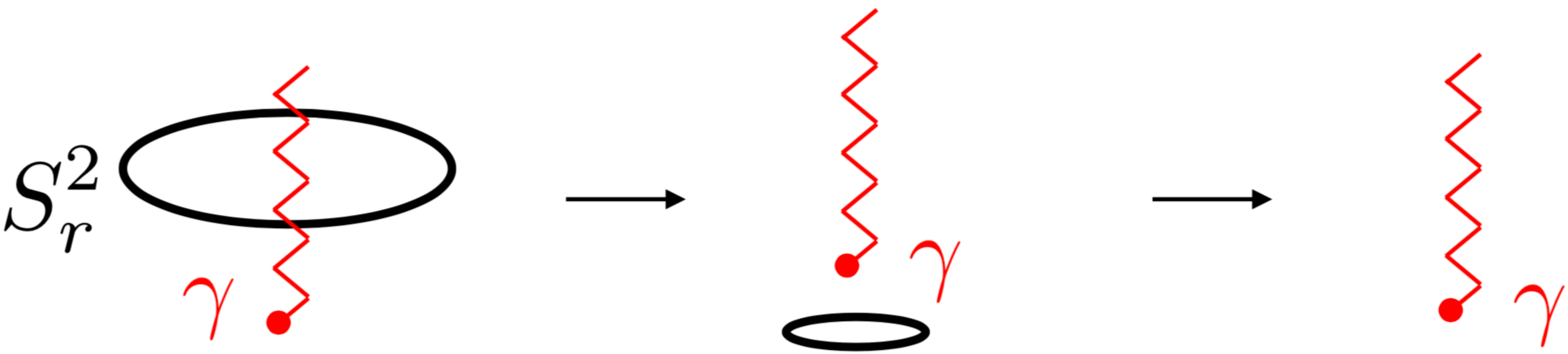


figure from Tom Rudelius

Wilson lines can *end* $\Longleftrightarrow$ 1-form electric symmetry is *explicitly broken*.

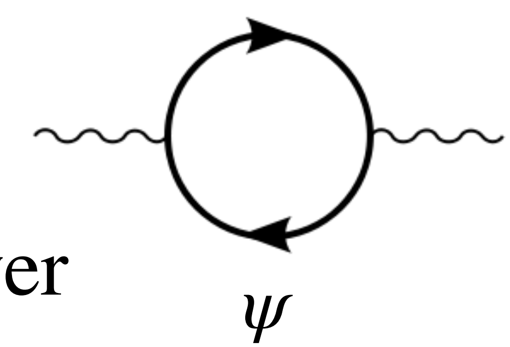
The WGC from no global symmetries?

For a U(1) gauge theory: absence of the 1-form generalized global symmetry requires electrically charged particles to exist.

Clay **Córdova**, Kantaro **Ohmori**, and Tom **Rudelius** (forthcoming work):

Asking that the 1-form symmetry be* badly *broken at the QG cutoff energy requires a tower of charged particles that parametrically obey the WGC.

$$\sum_{\psi \in \text{tower}}$$



$\Rightarrow$ $V(r)$ deviating strongly from $1/r$ for $r \sim \Lambda_{\text{QG}}^{-1}$

$\Rightarrow$ $U_\alpha(M) \sim \exp(\mathrm{i}\alpha \int_M \star F)$ is far from topological in the UV

(Effectively, reproduce the strong coupling argument for Tower WGC [Heidenreich, MR, Rudelius '17].)

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3. **Chern-Weil Global Symmetries and the Necessity of Axions**

arXiv:2012.00009 w/ Ben Heidenreich, Jake McNamara, Miguel Montero, Tom Rudelius, and Irene Valenzuela

Conservation of Chern-Weil currents

In an abelian gauge theory, if $\mathrm{d}F = 0$ (no magnetic monopoles), then

$$\mathrm{d}(F \wedge F) = \mathrm{d}F \wedge F + F \wedge \mathrm{d}F = 0,$$

so $F \wedge F$ is a conserved 4-form current, and generates a $(d-5)$-form symmetry. It is broken if magnetic monopoles exist (but the story is not so simple—stay tuned).

A generalization is true in nonabelian gauge theories:

$$\begin{aligned}\mathrm{d}\,\mathrm{tr}(F \wedge F) &= \mathrm{tr}(\mathrm{d}F \wedge F + F \wedge \mathrm{d}F) \\ &= \mathrm{tr}\big((\mathrm{d}F + [A,F]) \wedge F + F \wedge (\mathrm{d}F + [A,F])\big) \\ &= \mathrm{tr}(\mathrm{d}_A F \wedge F + F \wedge \mathrm{d}_A F) = 0\end{aligned}$$

This is a lemma in the construction of the Chern-Weil homomorphism, an important step in the theory of characteristic classes.

Conservation of Chern-Weil currents

More generally, we have a family of conservation laws,

$$\mathrm{d}\,\mathrm{tr}\left(\bigwedge^{k} F\right) = 0$$

Here $\bigwedge^k F$ denotes $F \wedge F \wedge \ldots \wedge F$, with k copies of F.

These conservation laws all follow from the nonabelian Bianchi identity,

$$\mathrm{d}_A F \equiv \mathrm{d}F + [A, F] = 0$$

Each ($2k$)-form conserved current means there is a generalized ($d - 2k - 1$)-form global symmetry, which we call a ***Chern-Weil global symmetry***.

Chern-Weil global symmetries vs. quantum gravity?

Chern-Weil global symmetries are ubiquitous in gauge theories.
They are not easy to break, as they follow from the Bianchi identity.

In 4 dimensions, the current $\mathrm{tr}(F \wedge F)$ is a 4-form, so it is trivially conserved. Nonetheless, there is a sense in which it generates a U(1) global "(–1)-form symmetry," because it has quantized (integer) integrals (periods). The charge is **instanton number**.

In 5 dimensions, this becomes an honest 0-form global symmetry and instantons are particles that carry a conserved charge.

Quantum gravity cannot have global symmetries. How does it remove these apparent Chern-Weil global symmetries?

Chern-Weil meets ’t Hooft-Polyakov

Consider *d*-dimensional SU(2) gauge theory higgsed to U(1) with an adjoint VEV. This theory contains the semiclassical, ’t Hooft-Polyakov magnetic monopole, whose worldvolume has codimension 3.
(We consider $d \geq 4$; the case $d = 4$ is somewhat degenerate, but I think it does make sense.)

UV: $\mathrm{d}\,\mathrm{tr}(F \wedge F) = 0$ Conserved 4-form current

IR: $\mathrm{d}\,(F \wedge F) = 2\,J_{\mathrm{mag}} \wedge F$

Broken 4-form current, due to monopoles

So, it appears that the Higgsing process has eliminated the symmetry from our IR theory.

Dyons and ’t Hooft-Polyakov

However, the story is more interesting. The classical ’t Hooft-Polyakov monopole solution has **collective coordinates** or **zero modes**.

The obvious zero modes are translations. However, there is a less obvious one, corresponding to a global U(1) rotation. This is realized as a ***compact scalar boson*** σ living on the monopole worldvolume.

In the 4d case, σ is described by the QM of a particle on a circle, which has a spectrum labeled by integers. Exciting this particle above its ground state **transforms the monopole into a dyon**, and the integer is the electric charge. σ shifts under U(1) gauge transformations.

For $d > 4$, σ is still a compact scalar, described by a *QFT* on the monopole worldvolume.

[Julia, Zee ’75; Jackiw, ’76; Tomboulis, Woo ’76; Christ, Guth, Weinberg ’76]

Chern-Weil, Dyons, and 't Hooft-Polyakov

We can *gauge* the SU(2) Chern-Weil current by adding a (d – 4)-form gauge field C with a (Chern-Simons) coupling,

$$\frac{1}{8\pi^2} C \wedge \mathrm{tr}(\mathrm{F} \wedge \mathrm{F})\,.$$

After Higgsing, this coupling is inherited not only by the U(1) gauge field but by the theory on the monopole worldline:

$$C \wedge F \wedge F - C \wedge \mathrm{d}_A \sigma \wedge J_{\mathrm{mag}}$$

(I am not being careful about normalization of the terms here and subsequently)

You can think of J_{mag} as the delta functions that localize the latter coupling on the worldline. Thus, the existence of the monopole breaks the conservation law of $F \wedge F$, but it *preserves* another closed 4-form current,

$$\mathrm{d}\left[F \wedge F - \mathrm{d}_A \sigma \wedge J_{\mathrm{mag}}\right] = 0.$$

This current had to exist, or our gauging with *C* would have been inconsistent!

Chern-Weil and the Witten effect

In the 4d case, *C* is a **“0-form gauge field”**, which is to say, a periodic scalar boson—**an axion!**

$$\frac{1}{8\pi^2}\theta\,\mathrm{tr}(\mathrm{F}\wedge\mathrm{F})\,.$$

The localized coupling on the monopole worldline, that is, the familiar theta term of a particle on a circle in QM,

$$\theta\,\mathrm{d}_A\sigma$$

serves to implement the **Witten effect**: magnetic monopoles acquire an electric charge when a theta angle is turned on,

$$q_{\mathrm{el}} = q_{\mathrm{mag}}\frac{\theta}{2\pi}\,.$$

We see that this whole story fits together nicely: the Witten effect is essential in order to allow us to consistently gauge the Chern-Weil symmetry of the nonabelian theory.

Chern-Weil gauging on D-branes

In string theory, gauge fields can live on a stack of D*p*-branes, which have a (*p*+1)-dimensional worldvolume. In these cases, we always find that the Chern-Weil current $\mathrm{tr}(F \wedge F)$ is gauged by a closed string $(p-3)$-form field:

$$C_{p-3} \wedge \mathrm{tr}(F \wedge F)$$

So far, so good. But this field actually propagates into the bulk, where it couples to lower-dimensional membranes, so a more complete story is:

$$C_{p-3} \wedge \left[\mathrm{tr}(F \wedge F) \wedge J_{Dp} + J_{D(p-4)}\right]$$

Where J_{Dq} is a $(9-q)$-form (the number of delta functions needed to localize on the brane).

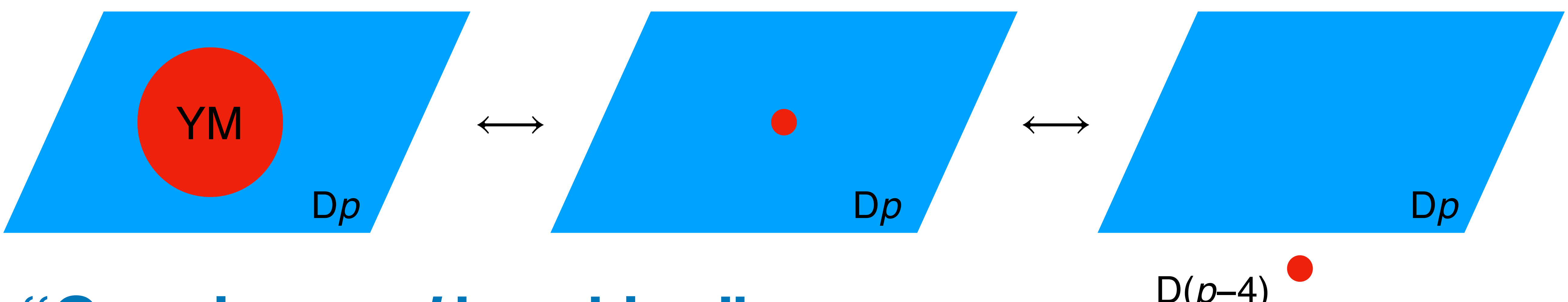
Chern-Weil gauging on D-branes

If the closed string gauge field C_{p-3} is gauging the current in brackets,

$$C_{p-3} \wedge \left[\mathrm{tr}(F \wedge F) \wedge J_{Dp} + J_{D(p-4)} \right]$$

then **what happens to the other linear combination of these two conserved currents?**

The answer is a well-known effect in string theory: **zero-size Yang-Mills instantons on the D*p*-brane are *the same thing* as D(*p* – 4)-branes.**
(Witten '95; Douglas '95; Green, Harvey, Moore '96).



"Gauging *and* breaking"

Chern-Weil and GUTs

Consider a nonabelian gauge group that is higgsed to a product group, as in the SM embedding in a GUT, for instance:

$$\mathrm{SU}(5) \to \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$$

The IR theory has more Chern-Weil currents than the UV theory. Some of these are “accidental”: selecting out SU(3) within SU(5) requires Higgs insertions, so the IR $\mathrm{tr}(F \wedge F)$ contains Higgses in the UV theory, and d(Higgs) is nonzero.

An IR theorist might overcount Chern-Weil symmetries and expect more gauge fields (or axions). However, there will always be at least one. This UV explicit breaking of IR Chern-Weil symmetries only happens for “unifiable” gauge groups.

Summary of examples

Once you start looking for Chern-Weil symmetries and mechanisms to remove them, you get a fresh perspective on many familiar phenomena.

Chern-Weil symmetries are ubiquitous in gauge theories. They are not easy to eliminate.

String theory removes many Chern-Weil symmetries by **gauging via Chern-Simons terms**. This might even be thought of as the reason why C-S terms are so generic in string theory.

Often, Chern-Weil symmetries are broken to the diagonal with another current through **intrinsically stringy UV effects**, e.g., turning YM instantons into branes.

[see also: “Chern-Simons pandemic”, Montero, Uranga, Valenzuela ’17]

Implications for axion physics

If SM gauge fields propagate in higher dimensions, the $\mathrm{tr}(F \wedge F)$ terms are symmetry currents. Expect at least one combination to be gauged. Reducing to 4d, this gives an axionic coupling,

$$\frac{1}{8\pi^2}\theta\,\mathrm{tr}(\mathrm{F} \wedge \mathrm{F})\,.$$

to a *fundamental* axion (compact scalar).

Even in 4d, the notion of a U(1) (–1)-form global symmetry may be well-defined and require such couplings, though this is subtle.

String theory *examples* with axions coupling to $\mathrm{tr}(F \wedge F)$ are common. Chern-Weil symmetry perspective sheds light on *why*—not just "looking under the lamp post."

Conclusions

Some messages to take away

The absence of charged particles often leads to generalized global symmetries (or related topological operators [Rudelius, Shao ’20]).

Towers of charged particles guarantee that **1-form symmetries are badly broken at the Planck scale**.

Chern-Weil global symmetries are ubiquitous in gauge theories. In gravitational theories, they must be gauged or broken.

Often they are gauged via Chern-Simons couplings. **Suggestive of why axions are necessary in QG.**

Future: what does it mean for those to be “badly broken”? What are implications for axion physics?

Where are we and where are we going?