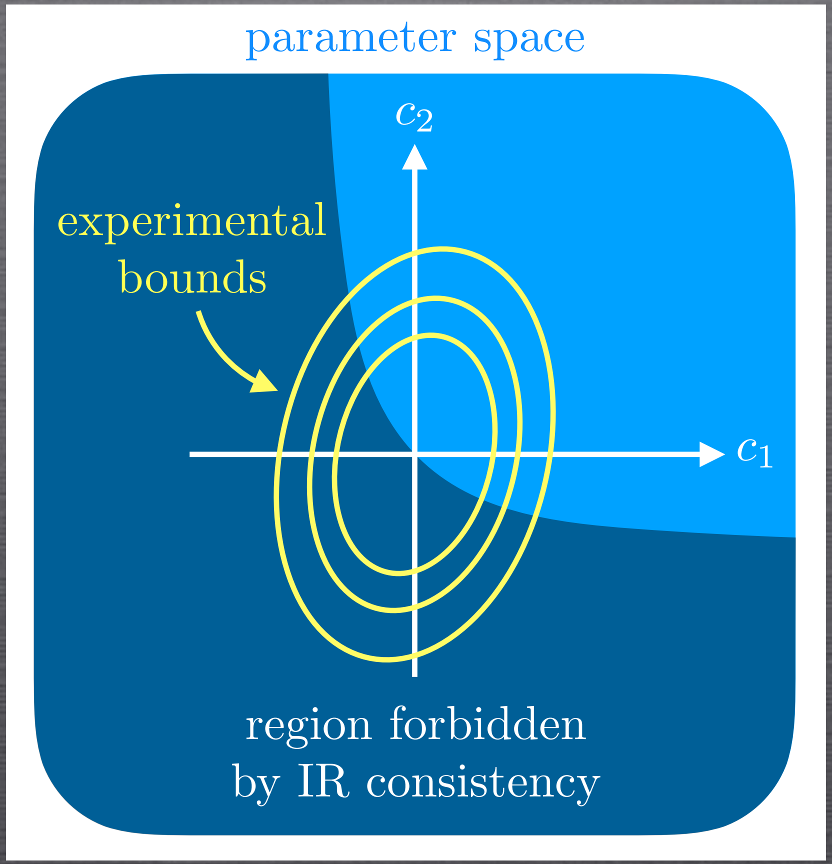
The Consistent SMEFT



Nick Rodd

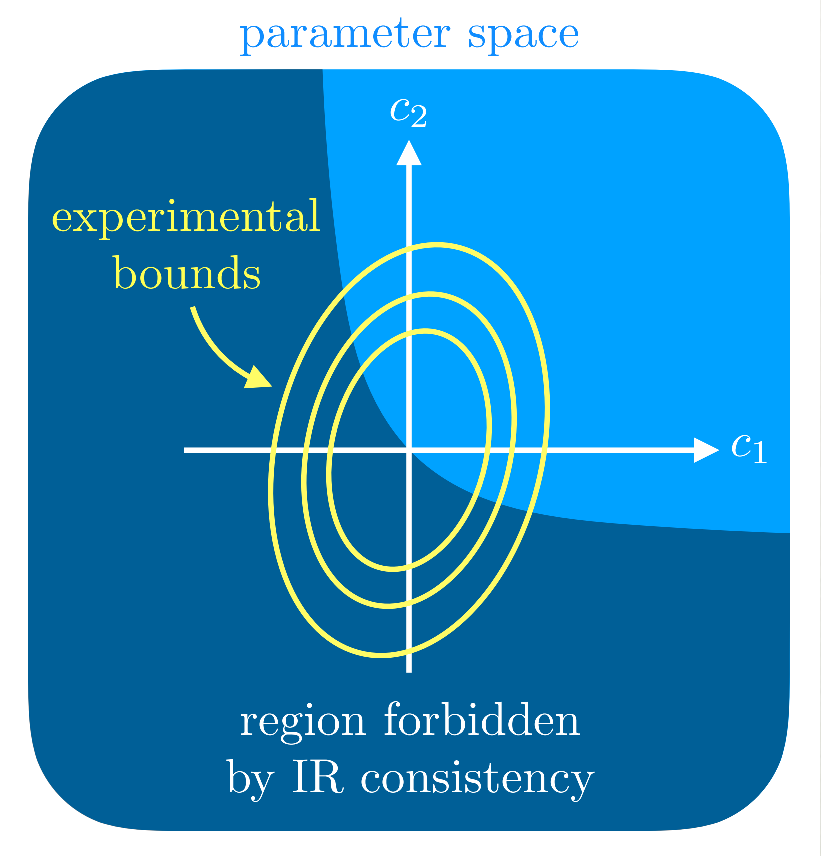
Bosons: JHEP 12 (2019) 032; Fermions: 2004.02885

w/ Grant Remmen

UC Davis, 27 April 2020

Motivation

Bedrock field theory principles constrain the SMEFT



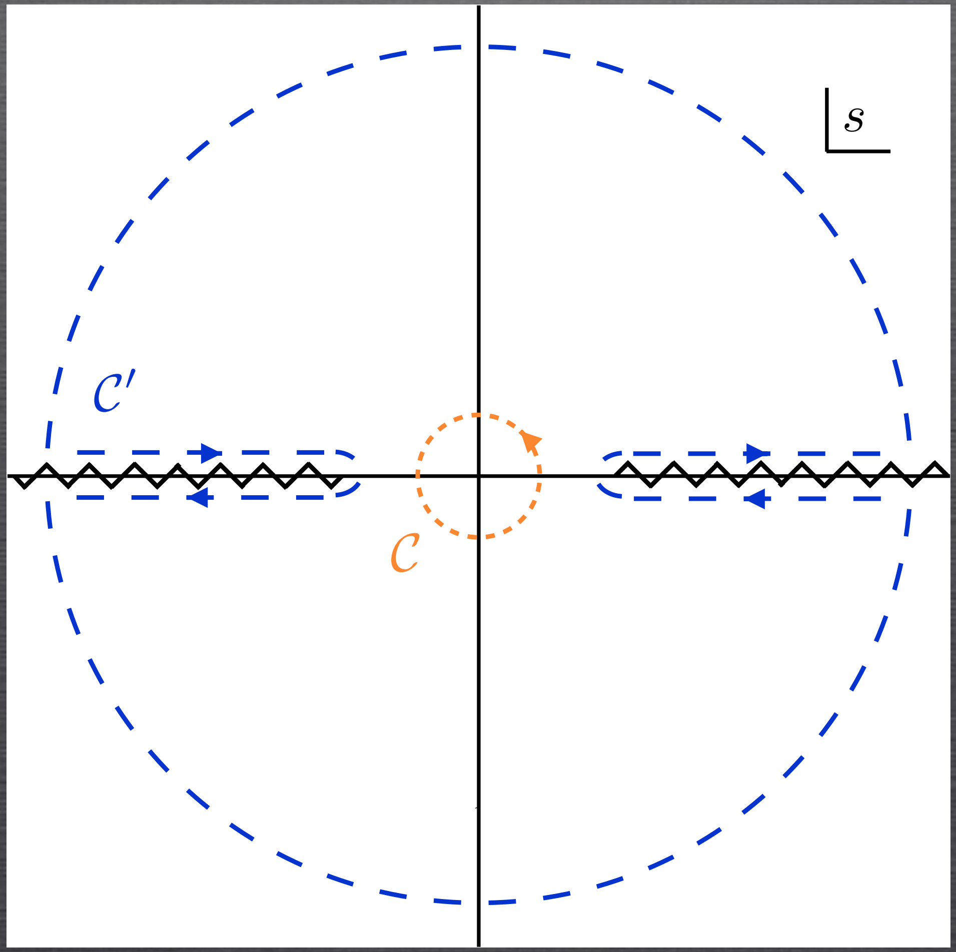
Unitarity, analyticity, and causality constraints on EFTs have found wide application in formal contexts - what do they reveal about the SMEFT?

Outline

Bedrock field theory principles constrain the SMEFT

1. Review IR consistency arguments
2. Bounds on the bosonic SMEFT
3. Bounds on the fermionic SMEFT
4. UV completions and our bounds
5. Phenomenology

1. IR Consistency



Unitarity and Analyticity

Consider a single massless scalar, invariant under $\phi \to \phi$ + const.

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{c}{M^4}(\partial\phi)^4$$

Example from [Adams+ hep-th/0602178], see also [Pham, Truong 1985]

Here follow [Remmen, **NLR** 1908.09845]

Unitarity and Analyticity

Consider a single massless scalar, invariant under $\phi \to \phi + \text{const.}$

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{c}{M^4}(\partial\phi)^4$$

What values of c are allowed?

Strategy:

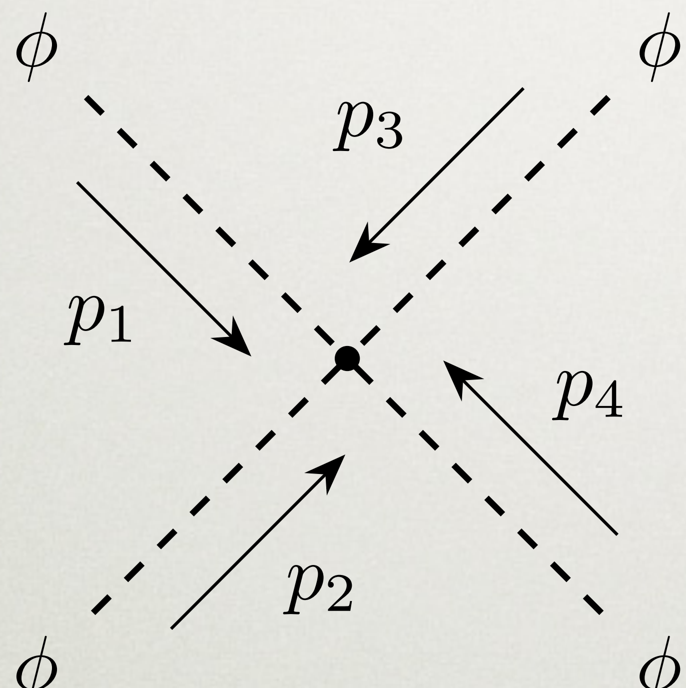
1. Use **analyticity** to connect c and $\text{Im}\mathcal{A}(s)$
2. Connect $\text{Im}\mathcal{A}(s)$ to σ via the optical theorem (**unitarity**)

Example from [Adams+ hep-th/0602178], see also [Pham, Truong 1985]

Here follow [Remmen, **NLR** 1908.09845]

Unitarity and Analyticity

2-2 scattering with $\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{c}{M^4}(\partial\phi)^4$



Recall

$$0 = p_1 + p_2 + p_3 + p_4$$

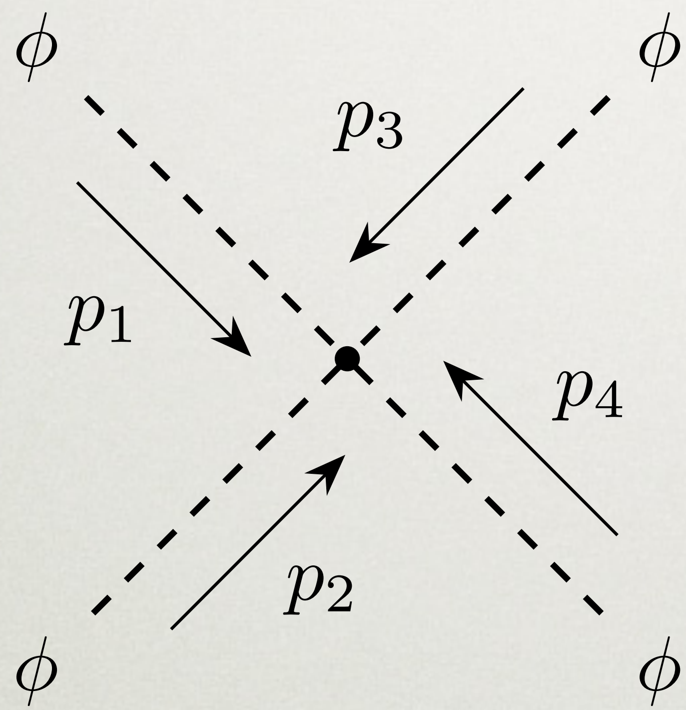
$$s = -(p_1 + p_2)^2$$

$$t = -(p_1 + p_3)^2$$

$$u = -s - t$$

Unitarity and Analyticity

2-2 scattering with $\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{c}{M^4}(\partial\phi)^4$



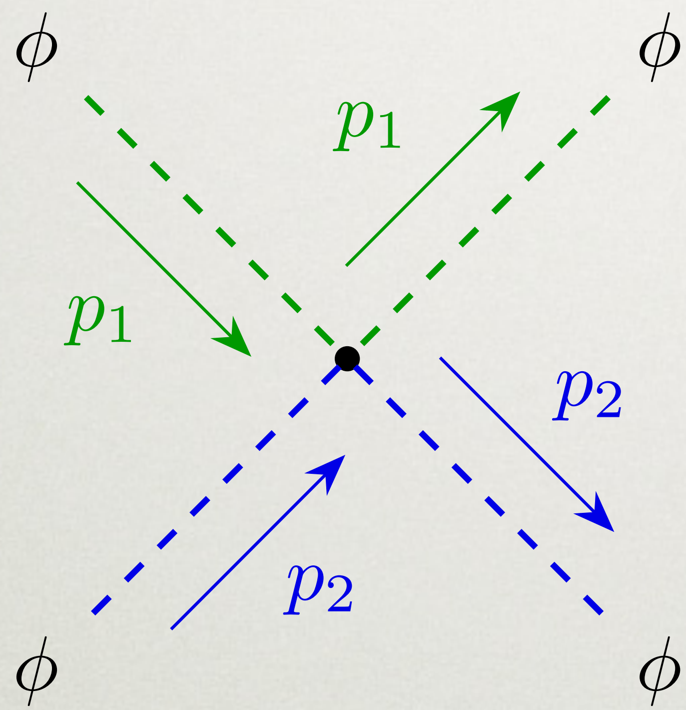
Recall

$$0 = p_1 + p_2 + p_3 + p_4$$
$$s = -(p_1 + p_2)^2$$
$$t = -(p_1 + p_3)^2$$
$$u = -s - t$$

$$\mathcal{M}(s,t) = \frac{2c}{M^4}(s^2 + t^2 + u^2)$$

Unitarity and Analyticity

2-2 scattering with $\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{c}{M^4}(\partial\phi)^4$ in the forward limit



Recall

$$0 = p_1 + p_2 + p_3 + p_4$$
$$s = -(p_1 + p_2)^2 \to s$$
$$t = -(p_1 + p_3)^2 \to 0$$
$$u = -s - t \to -s$$

$$\mathcal{A}(s) = \frac{4cs^2}{M^4}$$

Unitarity and Analyticity

2-2 scattering with $\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{c}{M^4}(\partial\phi)^4$ in the forward limit

Forward limit even in s

Cross 1 and 3 $s \to u = -s - t;\ t \to t$

$$\Rightarrow \mathcal{M}(s,t) = \mathcal{M}(-s-t,t)$$
$$\Rightarrow \mathcal{A}(s) = \mathcal{A}(-s)$$

Kills dim-6 operators*

Recall

$$0 = p_1 + p_2 + p_3 + p_4$$
$$s = -(p_1+p_2)^2 \to s$$
$$t = -(p_1+p_3)^2 \to 0$$
$$u = -s - t \to -s$$

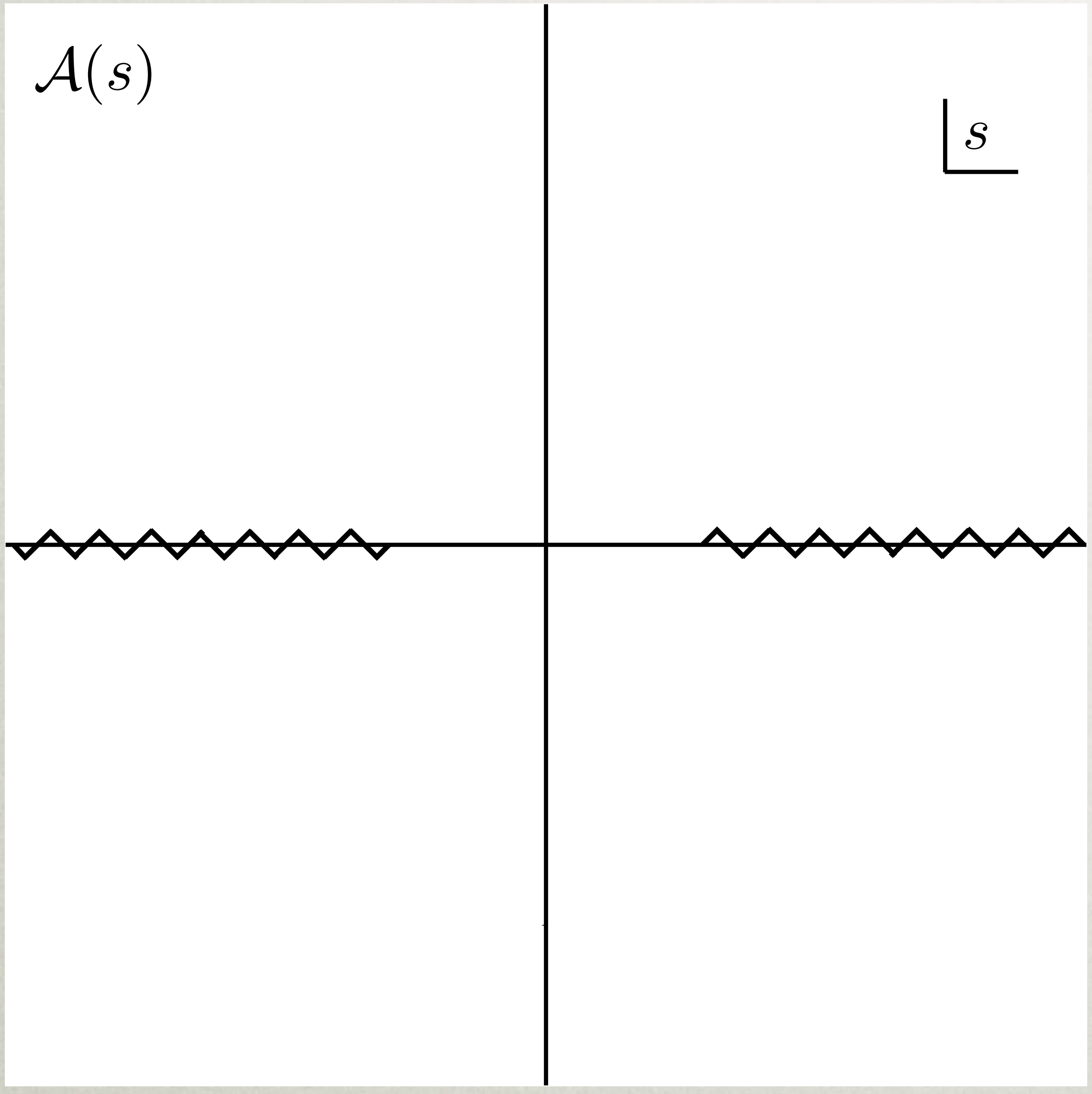
$$\mathcal{A}(s) = \frac{4cs^2}{M^4}$$

*Caveat: for particles with additional quantum numbers, crossing & forward limit aren't just kinematic and can mismatch

See e.g. [Low+ 0907.5413]

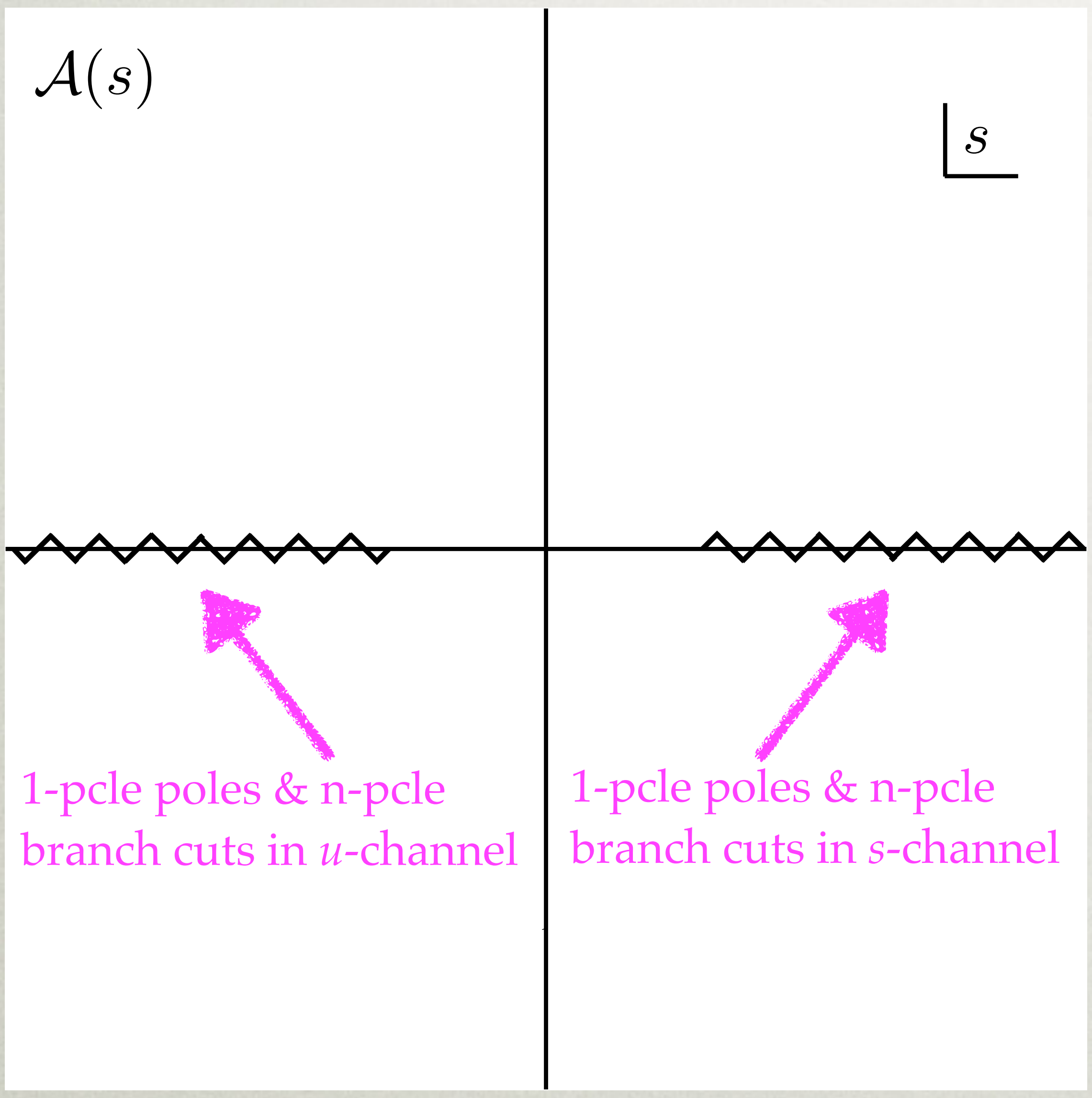
Unitarity and Analyticity

Study forward amplitude $\mathcal{A}(s) = 4cs^2/M^4$ in the complex plane



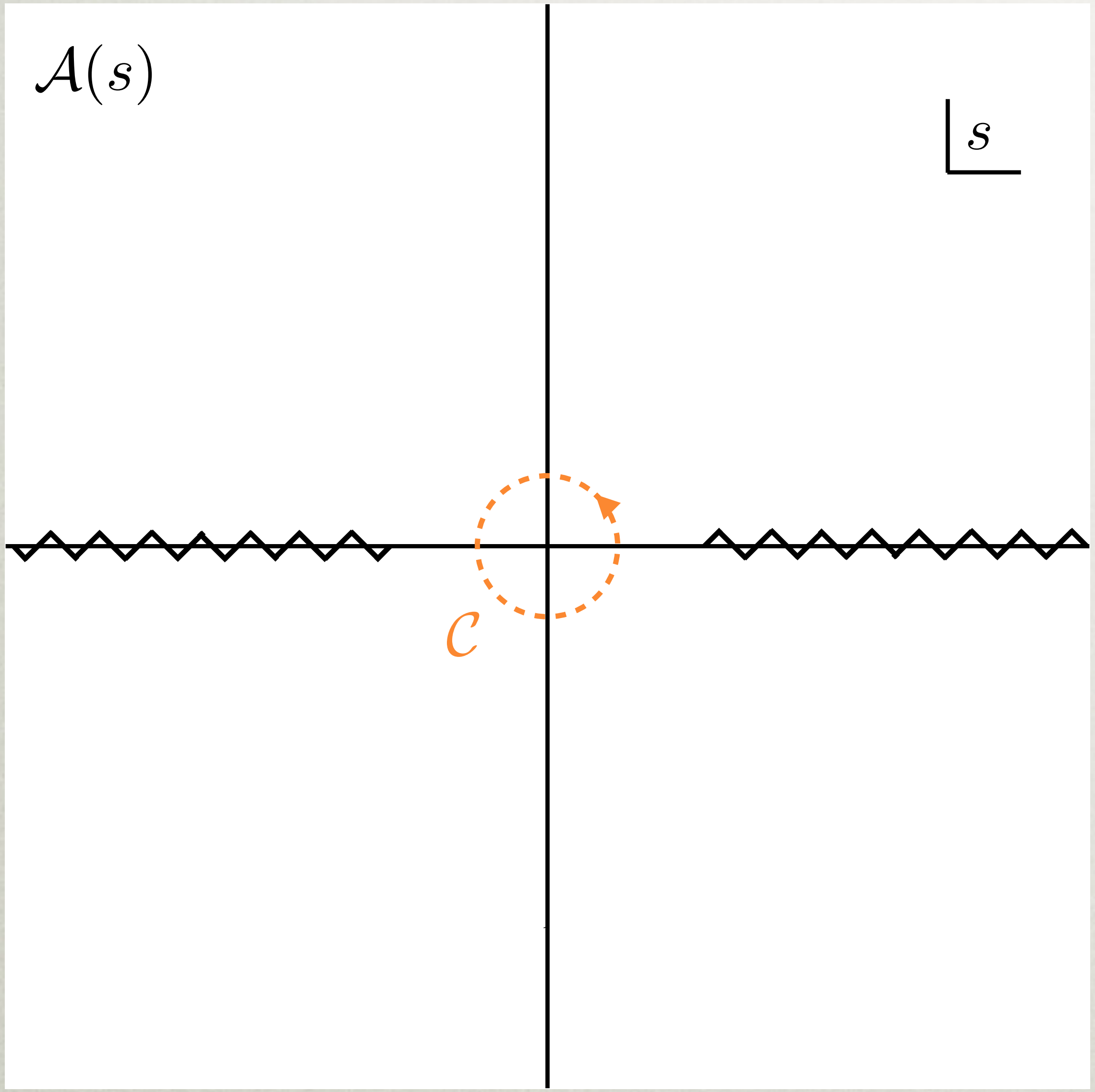
Unitarity and Analyticity

Study forward amplitude $\mathcal{A}(s) = 4cs^2/M^4$ in the complex plane



Unitarity and Analyticity

Study forward amplitude $\mathcal{A}(s) = 4cs^2/M^4$ in the complex plane

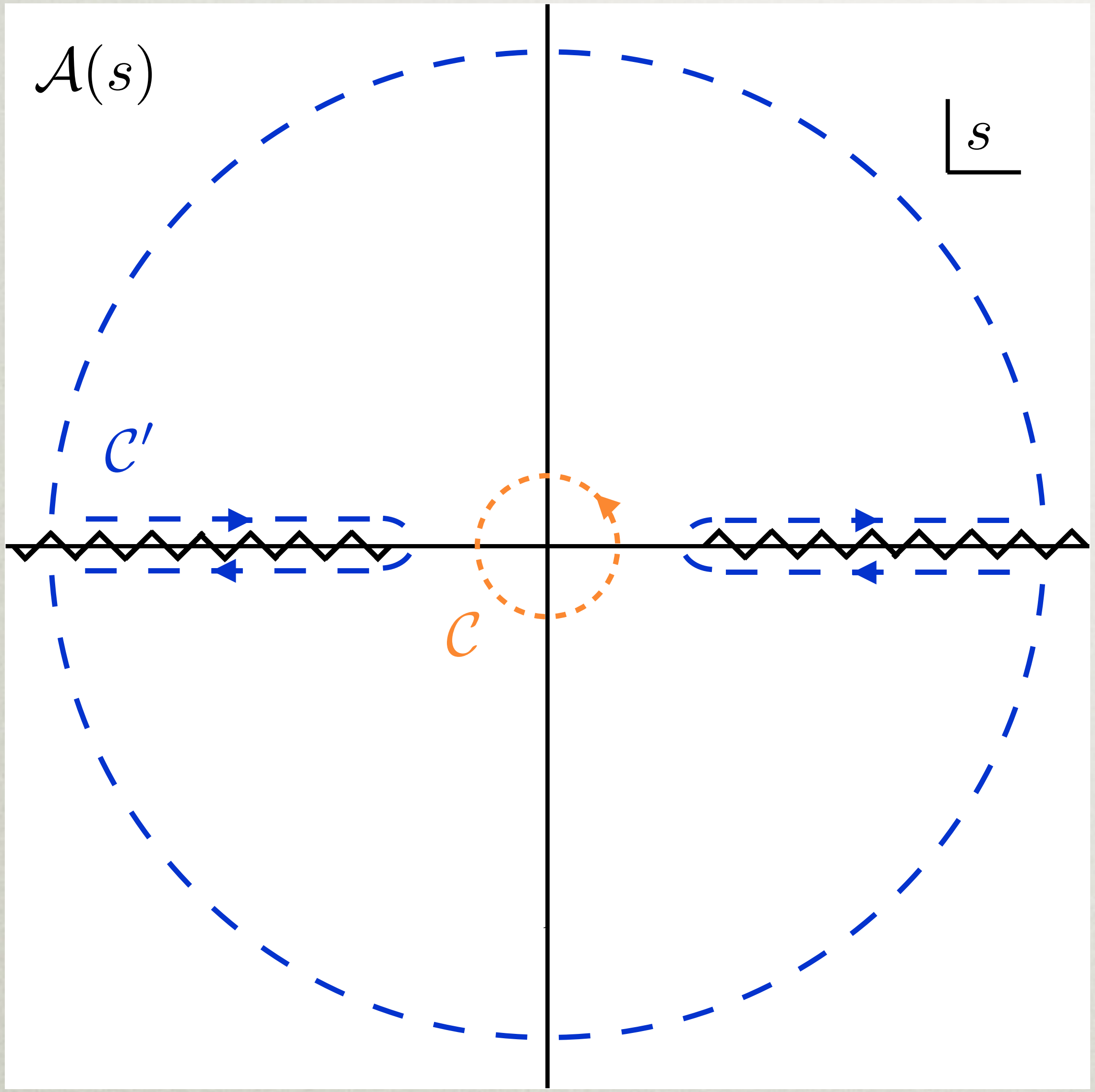


Isolate coeff. via residue theorem

$$\frac{4c}{M^4} = \frac{1}{2\pi i}\oint_{\mathcal{C}} \frac{ds}{s^3}\mathcal{A}(s)$$

Unitarity and Analyticity

Study forward amplitude $\mathcal{A}(s) = 4cs^2/M^4$ in the complex plane

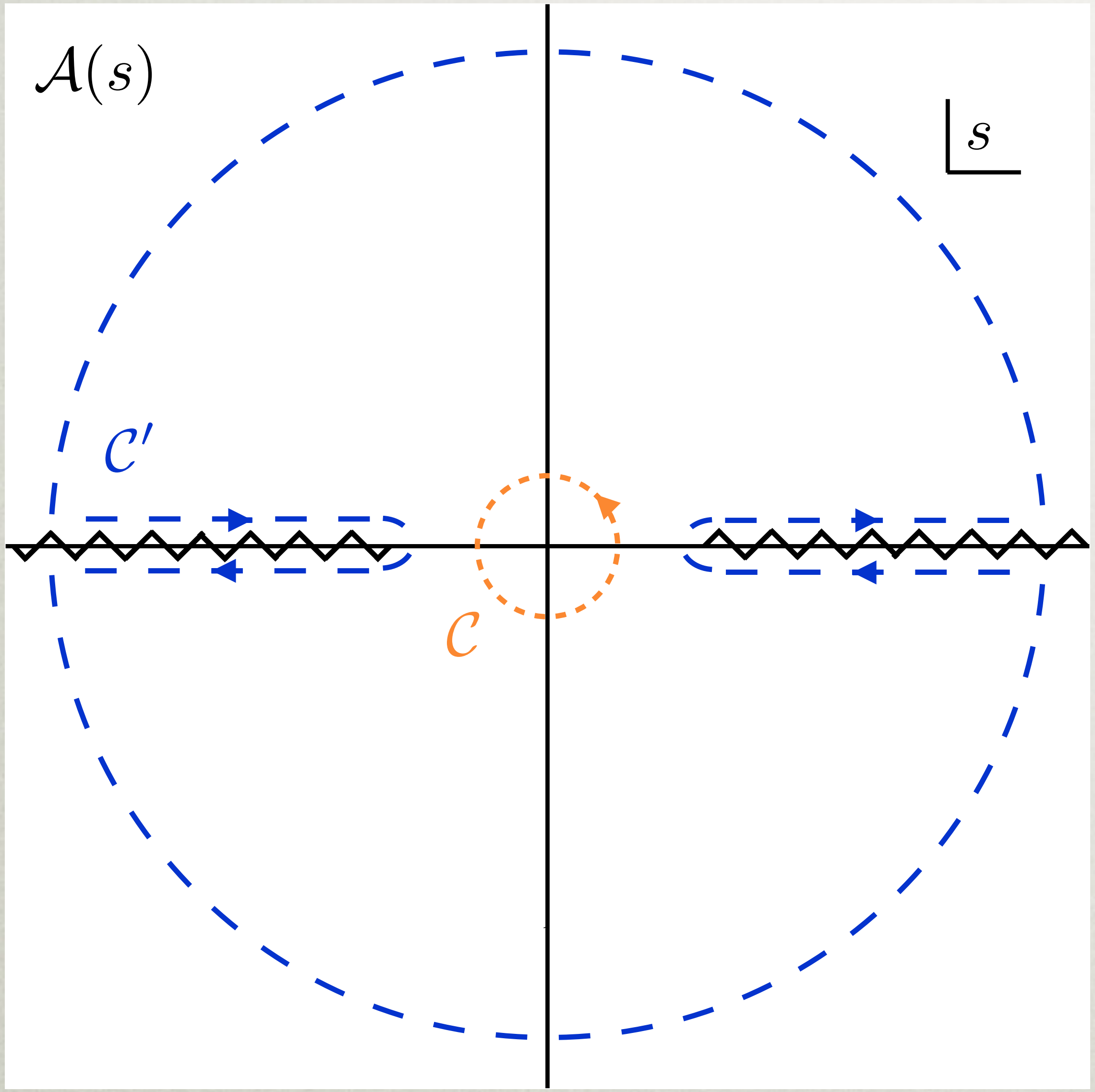


Exploit **analyticity** of the amplitude

$$\frac{4c}{M^4} = \frac{1}{2\pi i}\oint_{\mathcal{C}} \frac{ds}{s^3}\mathcal{A}(s)$$
$$= \frac{1}{2\pi i}\oint_{\mathcal{C}'} \frac{ds}{s^3}\mathcal{A}(s)$$

Unitarity and Analyticity

Study forward amplitude $\mathcal{A}(s) = 4cs^2/M^4$ in the complex plane



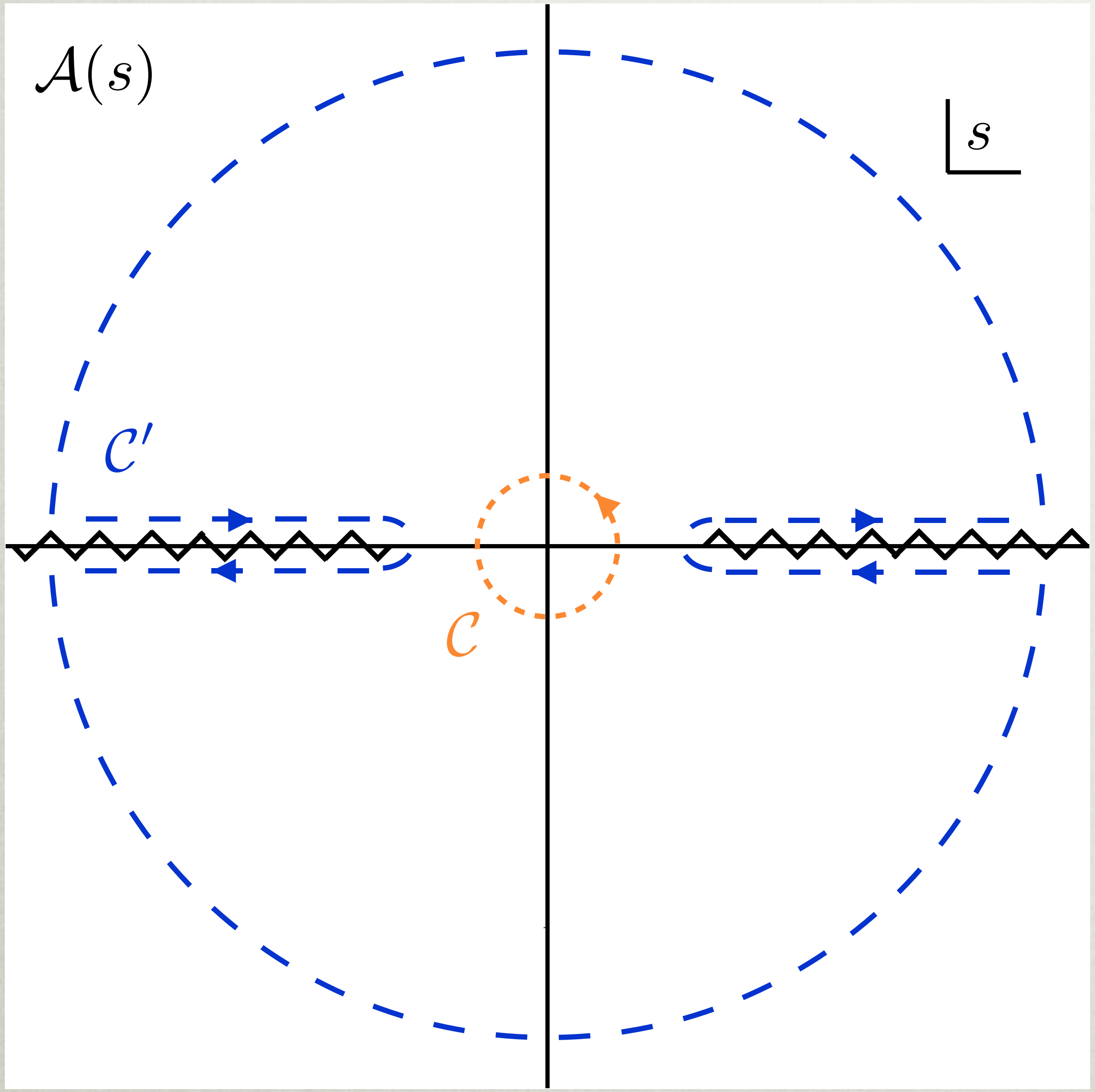
Remove boundary term via Froissart bound

$$\begin{aligned}\frac{4c}{M^4} &= \frac{1}{2\pi i}\oint_{\mathcal{C}} \frac{ds}{s^3}\mathcal{A}(s) \\ &= \frac{1}{2\pi i}\oint_{\mathcal{C}'} \frac{ds}{s^3}\mathcal{A}(s) \\ &= \frac{1}{2\pi i}\left(\int_{-\infty}^{-s_d} + \int_{s_d}^{\infty}\right)\frac{ds}{s^3}\text{Disc}\mathcal{A}(s)\end{aligned}$$

$$\text{Disc}\mathcal{A}(s) = \lim_{\epsilon\to 0}[\mathcal{A}(s+i\epsilon) - \mathcal{A}(s-i\epsilon)]$$

Unitarity and Analyticity

Study forward amplitude $\mathcal{A}(s) = 4cs^2/M^4$ in the complex plane



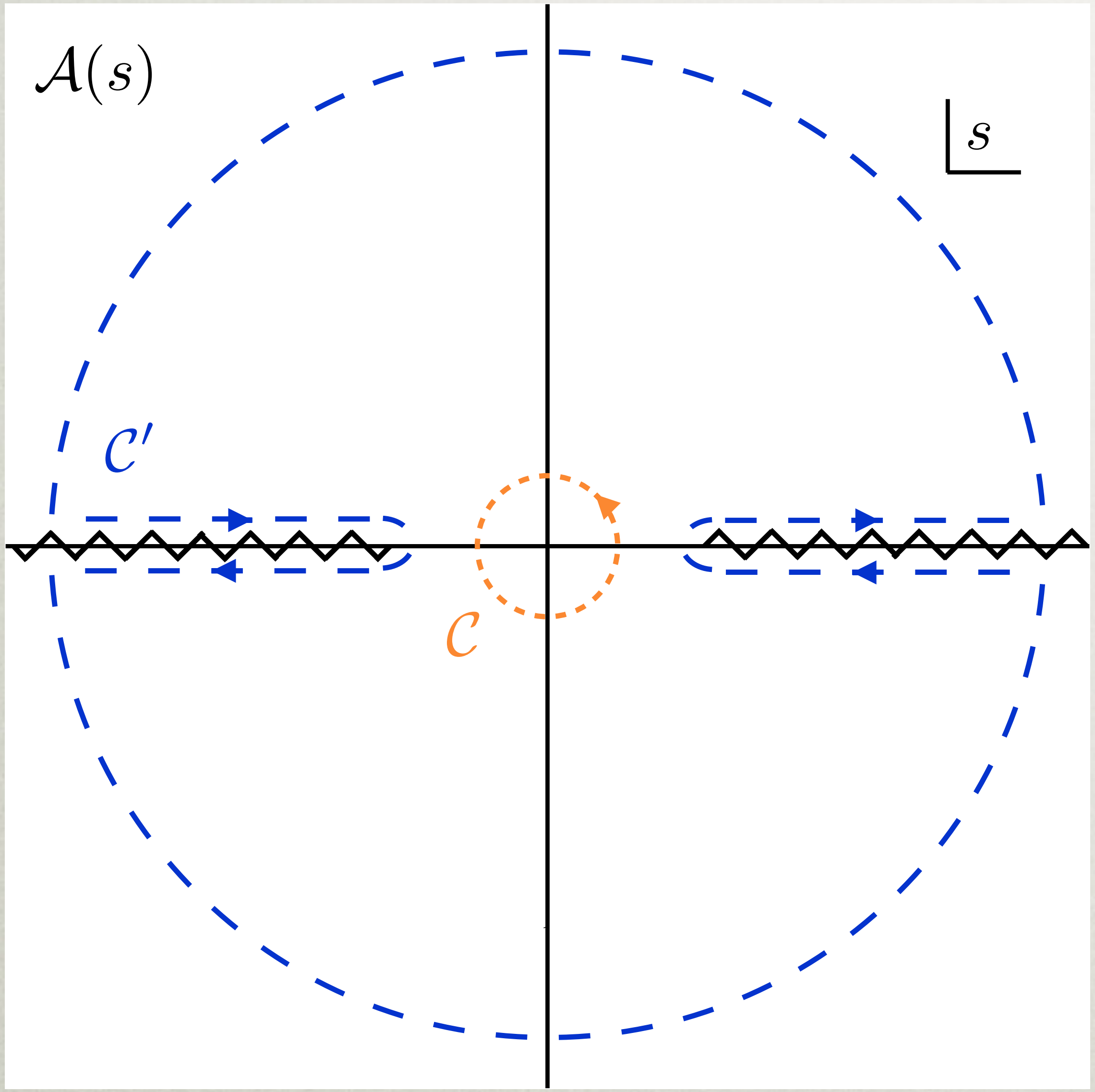
Invoke crossing symmetry

$$\begin{aligned}\frac{4c}{M^4} &= \frac{1}{2\pi i}\oint_{\mathcal{C}} \frac{ds}{s^3}\mathcal{A}(s)\\ &= \frac{1}{2\pi i}\oint_{\mathcal{C}'} \frac{ds}{s^3}\mathcal{A}(s)\\ &= \frac{1}{2\pi i}\left(\int_{-\infty}^{-s_d} + \int_{s_d}^{\infty}\right)\frac{ds}{s^3}\text{Disc}\mathcal{A}(s)\\ &= \frac{1}{i\pi}\int_{s_d}^{\infty}\frac{ds}{s^3}\text{Disc}\mathcal{A}(s)\end{aligned}$$

$$\text{Disc}\mathcal{A}(s) = \lim_{\epsilon\to 0}[\mathcal{A}(s+i\epsilon) - \mathcal{A}(s-i\epsilon)]$$

Unitarity and Analyticity

Study forward amplitude $\mathcal{A}(s) = 4cs^2/M^4$ in the complex plane



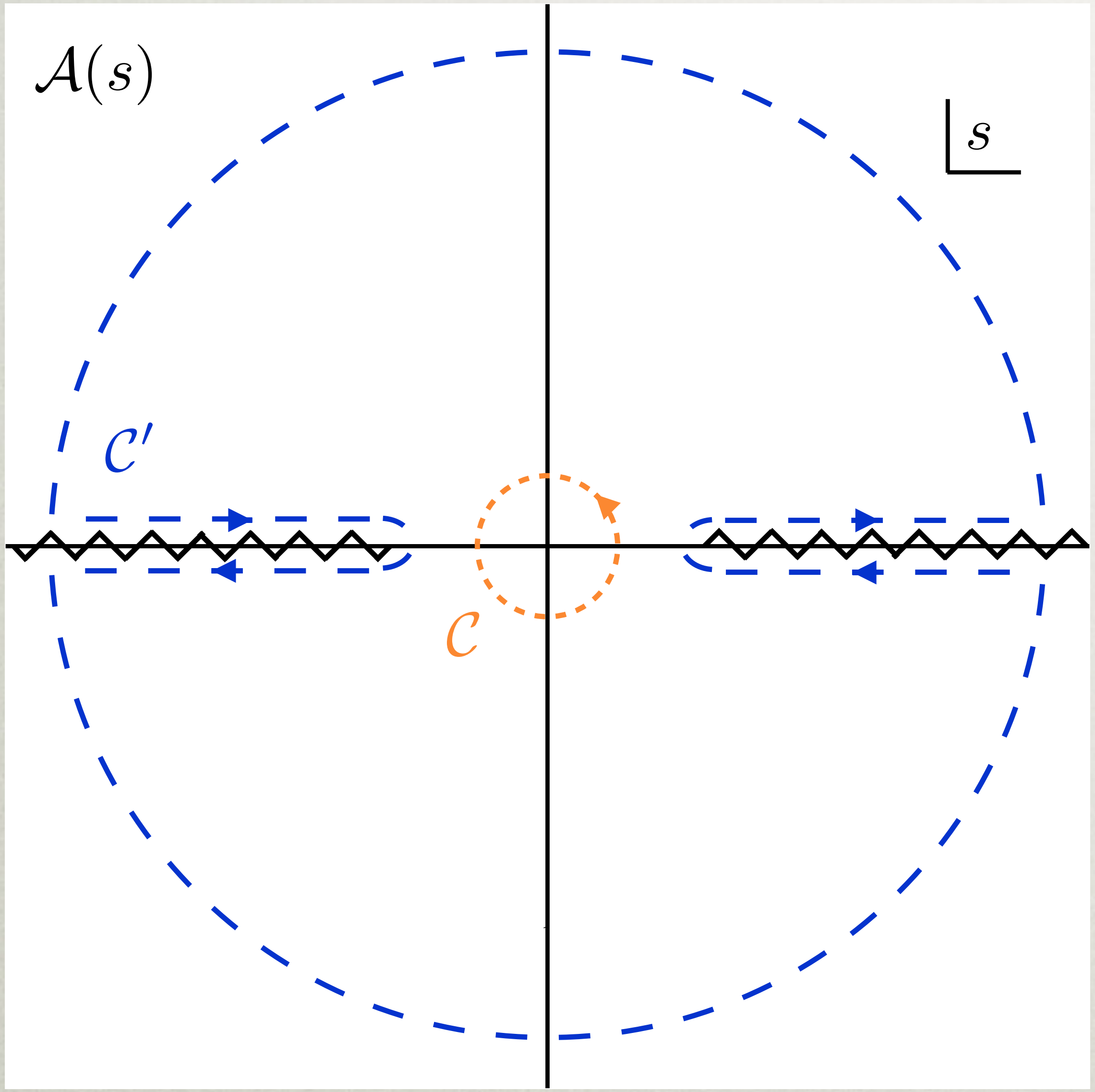
Relate Disc and Im

$$
\begin{aligned}
\frac{4c}{M^4} &= \frac{1}{2\pi i}\oint_{\mathcal{C}} \frac{ds}{s^3}\mathcal{A}(s) \\
&= \frac{1}{2\pi i}\oint_{\mathcal{C}'} \frac{ds}{s^3}\mathcal{A}(s) \\
&= \frac{1}{2\pi i}\left(\int_{-\infty}^{-s_d} + \int_{s_d}^{\infty}\right)\frac{ds}{s^3}\text{Disc}\mathcal{A}(s) \\
&= \frac{1}{i\pi}\int_{s_d}^{\infty}\frac{ds}{s^3}\text{Disc}\mathcal{A}(s) \\
&= \frac{2}{\pi}\int_{s_d}^{\infty}\frac{ds}{s^3}\text{Im}\mathcal{A}(s)
\end{aligned}
$$

$$\text{Disc}\mathcal{A}(s) = \lim_{\epsilon\to 0}[\mathcal{A}(s+i\epsilon) - \mathcal{A}(s-i\epsilon)]$$

Unitarity and Analyticity

Study forward amplitude $\mathcal{A}(s) = 4cs^2/M^4$ in the complex plane

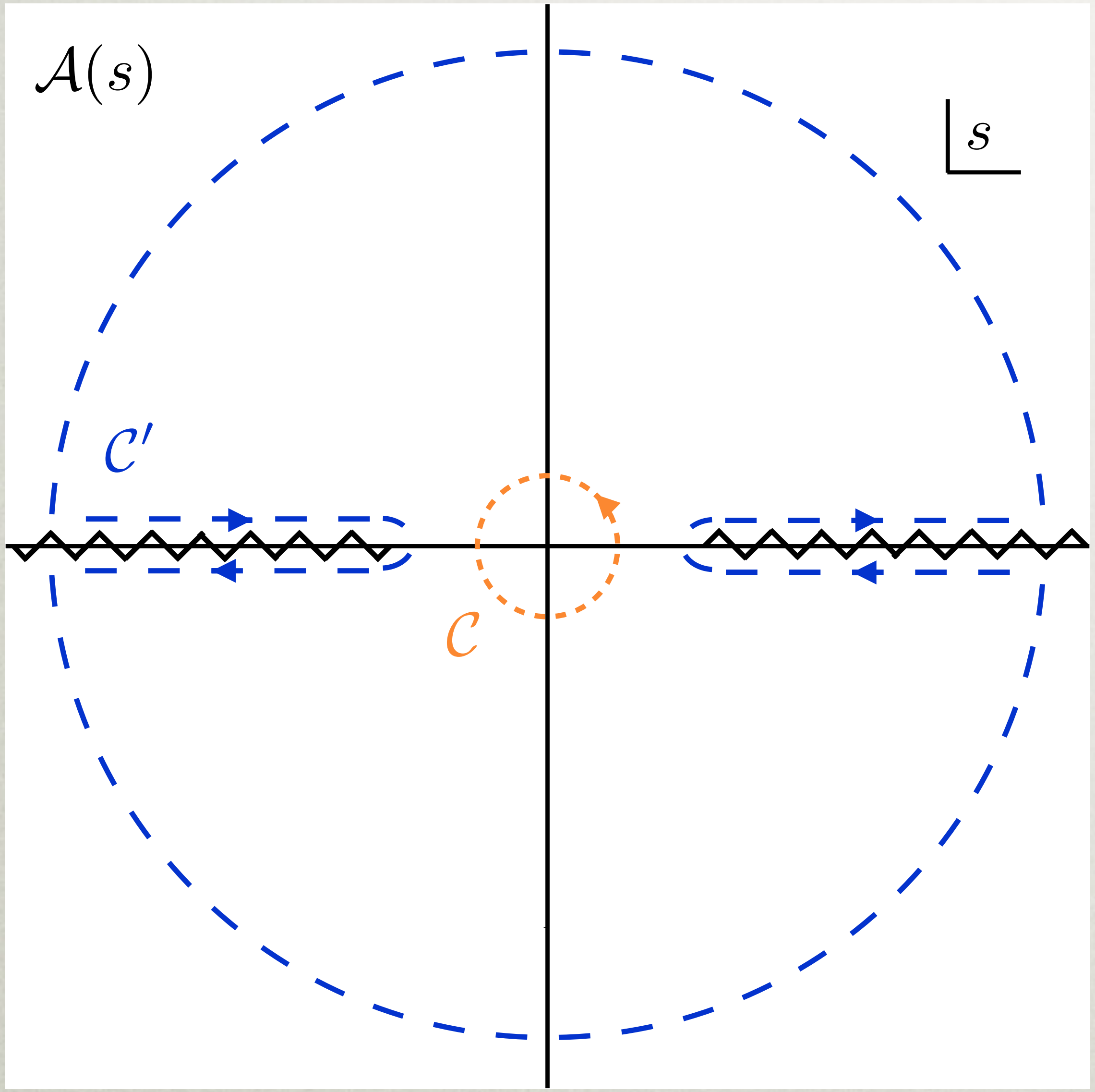


Exploit **unitarity** via the optical theorem

$$\begin{aligned}
\frac{4c}{M^4} &= \frac{1}{2\pi i}\oint_{\mathcal{C}} \frac{ds}{s^3}\mathcal{A}(s) \\
&= \frac{1}{2\pi i}\oint_{\mathcal{C}'} \frac{ds}{s^3}\mathcal{A}(s) \\
&= \frac{1}{2\pi i}\left(\int_{-\infty}^{-s_d} + \int_{s_d}^{\infty}\right)\frac{ds}{s^3}\mathrm{Disc}\mathcal{A}(s) \\
&= \frac{1}{i\pi}\int_{s_d}^{\infty}\frac{ds}{s^3}\mathrm{Disc}\mathcal{A}(s) \\
&= \frac{2}{\pi}\int_{s_d}^{\infty}\frac{ds}{s^3}\mathrm{Im}\mathcal{A}(s) \\
&= \frac{2}{\pi}\int_{s_d}^{\infty}\frac{ds}{s^2}\sigma(s)
\end{aligned}$$

Unitarity and Analyticity

Study forward amplitude $\mathcal{A}(s) = 4cs^2/M^4$ in the complex plane



Cross section is positive definite

$$
\begin{aligned}
\frac{4c}{M^4} &= \frac{1}{2\pi i}\oint_{\mathcal{C}} \frac{ds}{s^3}\mathcal{A}(s) \\
&= \frac{1}{2\pi i}\oint_{\mathcal{C}'} \frac{ds}{s^3}\mathcal{A}(s) \\
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&= \frac{1}{i\pi}\int_{s_d}^{\infty}\frac{ds}{s^3}\mathrm{Disc}\mathcal{A}(s) \\
&= \frac{2}{\pi}\int_{s_d}^{\infty}\frac{ds}{s^3}\mathrm{Im}\mathcal{A}(s) \\
&= \frac{2}{\pi}\int_{s_d}^{\infty}\frac{ds}{s^2}\sigma(s) > 0
\end{aligned}
$$

Unitarity and Analyticity

Study forward amplitude $\mathcal{A}(s) = 4cs^2/M^4$ in the complex plane

Final Result

$$c > 0$$

- More generally coefficients of dim-8 operators that support a non-vanishing forward limit are positive
- Holds also for fermions [Bellazzini 1605.06111]

Arrive at a positivity constraint

$$\begin{aligned}
\frac{4c}{M^4} &= \frac{1}{2\pi i}\oint_{\mathcal{C}} \frac{ds}{s^3}\mathcal{A}(s) \\
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\end{aligned}$$

Causality

Let's take a different approach to the same problem

Consider a single massless scalar, invariant under $\phi \to \phi + \text{const.}$

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{c}{M^4}(\partial\phi)^4$$

What values of c are allowed?

New Strategy:

1. Determine **classical EoM** for ϕ in a background field
2. Construct a **causal paradox** if $v > 1$

Example from [Adams+ hep-th/0602178]

Here follow [Remmen, **NLR** 1908.09845]

Causality

Classical EoM from $\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{c}{M^4}(\partial\phi)^4$

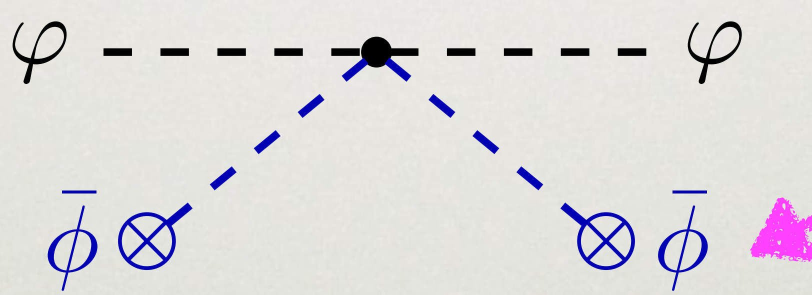
$$\Box\phi - \frac{4c}{M^4}\left[\Box\phi(\partial\phi)^2 + 2(\partial^\mu\phi)(\partial^\nu\phi)(\partial_\mu\partial_\nu\phi)\right] = 0$$

Causality

Classical EoM from $\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{c}{M^4}(\partial\phi)^4$

$$\Box\phi - \frac{4c}{M^4}\left[\Box\phi(\partial\phi)^2 + 2(\partial^\mu\phi)(\partial^\nu\phi)(\partial_\mu\partial_\nu\phi)\right] = 0$$

Consider propagation in a background field, $\phi = \varphi + \bar{\phi}$



Background condensate

$\overline{\partial_\mu\phi} = q_\mu$

Causality

Classical EoM from $\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{c}{M^4}(\partial\phi)^4$

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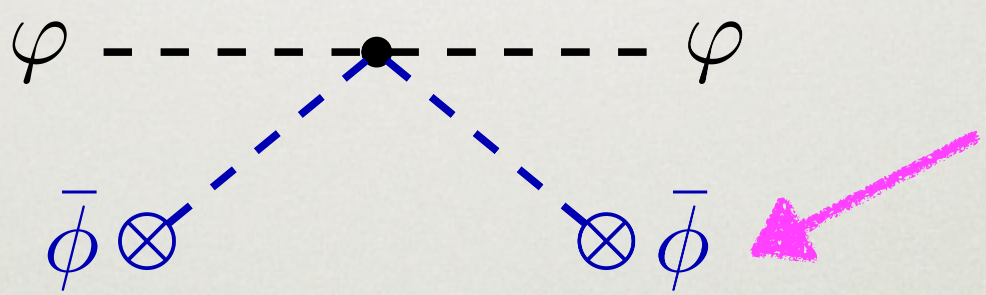

Background condensate

$\overline{\partial_\mu \phi} = q_\mu$

Obtain a dispersion relation for $\varphi \propto e^{ik\cdot x}$, from which

$$v \simeq 1 - \frac{4c(q\cdot k)^2}{M^4 k_0^2}$$

Causality

Classical EoM from $\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{c}{M^4}(\partial\phi)^4$

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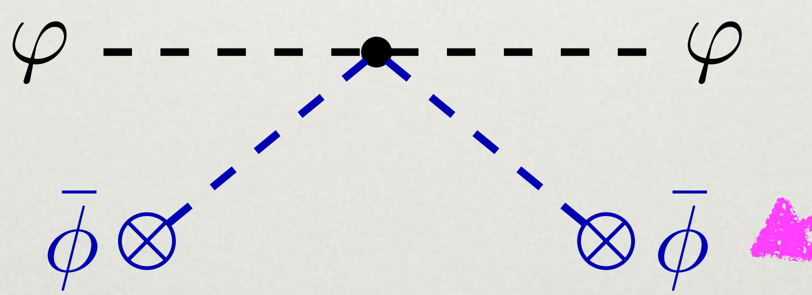


Background condensate

$\overline{\partial_\mu \phi} = q_\mu$

Obtain a dispersion relation for $\varphi \propto e^{ik\cdot x}$, from which

Final Result

$$c > 0$$

$$v \simeq 1 - \frac{4c(q\cdot k)^2}{M^4 k_0^2}$$

$c < 0 \Rightarrow v > 1$

Can construct a **causal paradox**

See [Adams+ hep-th/0602178]

2. Bosonic Bounds

B^4 operators

$\mathcal{O}_1^{B^4}$	$(BB)(BB)$
$\mathcal{O}_2^{B^4}$	$(B\widetilde{B})(B\widetilde{B})$
$\widetilde{\mathcal{O}}_1^{B^4}$	$(BB)(B\widetilde{B})$

W^4 operators

$\mathcal{O}_1^{W^4}$	$(W^IW^I)(W^JW^J)$
$\mathcal{O}_2^{W^4}$	$(W^I\widetilde{W}^I)(W^J\widetilde{W}^J)$
$\mathcal{O}_3^{W^4}$	$(W^IW^J)(W^IW^J)$
$\mathcal{O}_4^{W^4}$	$(W^I\widetilde{W}^J)(W^I\widetilde{W}^J)$
$\widetilde{\mathcal{O}}_1^{W^4}$	$(W^IW^I)(W^J\widetilde{W}^J)$
$\widetilde{\mathcal{O}}_2^{W^4}$	$(W^IW^J)(W^I\widetilde{W}^J)$

G^4 operators

$\mathcal{O}_1^{G^4}$	$(G^aG^a)(G^bG^b)$
$\mathcal{O}_2^{G^4}$	$(G^a\widetilde{G}^a)(G^b\widetilde{G}^b)$
$\mathcal{O}_3^{G^4}$	$(G^aG^b)(G^aG^b)$
$\mathcal{O}_4^{G^4}$	$(G^a\widetilde{G}^b)(G^a\widetilde{G}^b)$
$\mathcal{O}_5^{G^4}$	$d^{abe}d^{cde}(G^aG^b)(G^cG^d)$
$\mathcal{O}_6^{G^4}$	$d^{abe}d^{cde}(G^a\widetilde{G}^b)(G^c\widetilde{G}^d)$
$\widetilde{\mathcal{O}}_1^{G^4}$	$(G^aG^a)(G^b\widetilde{G}^b)$
$\widetilde{\mathcal{O}}_2^{G^4}$	$(G^aG^b)(G^a\widetilde{G}^b)$
$\widetilde{\mathcal{O}}_3^{G^4}$	$d^{abe}d^{cde}(G^aG^b)(G^c\widetilde{G}^d)$

$F_1^2F_2^2/F_1F_2^3$ cross-quartics

$\mathcal{O}_1^{B^2W^2}$	$(BB)(W^IW^I)$
$\mathcal{O}_2^{B^2W^2}$	$(B\widetilde{B})(W^I\widetilde{W}^I)$
$\mathcal{O}_3^{B^2W^2}$	$(BW^I)(BW^I)$
$\mathcal{O}_4^{B^2W^2}$	$(B\widetilde{W}^I)(B\widetilde{W}^I)$
$\widetilde{\mathcal{O}}_1^{B^2W^2}$	$(B\widetilde{B})(W^IW^I)$
$\widetilde{\mathcal{O}}_2^{B^2W^2}$	$(BB)(W^I\widetilde{W}^I)$
$\widetilde{\mathcal{O}}_3^{B^2W^2}$	$(BW^I)(B\widetilde{W}^I)$
$\mathcal{O}_1^{B^2G^2}$	$(BB)(G^aG^a)$
$\mathcal{O}_2^{B^2G^2}$	$(B\widetilde{B})(G^a\widetilde{G}^a)$
$\mathcal{O}_3^{B^2G^2}$	$(BG^a)(BG^a)$
$\mathcal{O}_4^{B^2G^2}$	$(B\widetilde{G}^a)(B\widetilde{G}^a)$
$\widetilde{\mathcal{O}}_1^{B^2G^2}$	$(B\widetilde{B})(G^aG^a)$
$\widetilde{\mathcal{O}}_2^{B^2G^2}$	$(BB)(G^a\widetilde{G}^a)$
$\widetilde{\mathcal{O}}_3^{B^2G^2}$	$(BG^a)(B\widetilde{G}^a)$
$\mathcal{O}_1^{W^2G^2}$	$(W^IW^I)(G^aG^a)$
$\mathcal{O}_2^{W^2G^2}$	$(W^I\widetilde{W}^I)(G^a\widetilde{G}^a)$
$\mathcal{O}_3^{W^2G^2}$	$(W^IG^a)(W^IG^a)$
$\mathcal{O}_4^{W^2G^2}$	$(W^I\widetilde{G}^a)(W^I\widetilde{G}^a)$
$\widetilde{\mathcal{O}}_1^{W^2G^2}$	$(W^I\widetilde{W}^I)(G^aG^a)$
$\widetilde{\mathcal{O}}_2^{W^2G^2}$	$(W^IW^I)(G^a\widetilde{G}^a)$
$\widetilde{\mathcal{O}}_3^{W^2G^2}$	$(W^IG^a)(W^I\widetilde{G}^a)$
$\mathcal{O}_1^{BG^3}$	$d^{abc}(BG^a)(G^bG^c)$
$\mathcal{O}_2^{BG^3}$	$d^{abc}(B\widetilde{G}^a)(G^b\widetilde{G}^c)$
$\widetilde{\mathcal{O}}_1^{BG^3}$	$d^{abc}(B\widetilde{G}^a)(G^bG^c)$
$\widetilde{\mathcal{O}}_2^{BG^3}$	$d^{abc}(BG^a)(G^b\widetilde{G}^c)$

$(DH)^4$ operators

$\mathcal{O}_1^{H^4}$	$(D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$
$\mathcal{O}_2^{H^4}$	$(D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$
$\mathcal{O}_3^{H^4}$	$(D^\mu H^\dagger D_\mu H)(D^\nu H^\dagger D_\nu H)$

$(DH)^2F^2$ cross-quartics

$\mathcal{O}_1^{H^2B^2}$	$(D^\mu H^\dagger D^\nu H)B_{\mu\rho}B_\nu{}^\rho$
$\mathcal{O}_2^{H^2B^2}$	$(D^\mu H^\dagger D_\mu H)B_{\rho\sigma}B^{\rho\sigma}$
$\widetilde{\mathcal{O}}_1^{H^2B^2}$	$(D^\mu H^\dagger D_\mu H)B_{\rho\sigma}\widetilde{B}^{\rho\sigma}$
$\mathcal{O}_1^{H^2W^2}$	$(D^\mu H^\dagger D^\nu H)W^I_{\mu\rho}W^{I\rho}_\nu$
$\mathcal{O}_2^{H^2W^2}$	$(D^\mu H^\dagger D_\mu H)W^I_{\rho\sigma}W^{I\rho\sigma}$
$\mathcal{O}_3^{H^2W^2}$	$i\,\epsilon^{IJK}(D^\mu H^\dagger\tau^I D^\nu H)W^J_{\mu\rho}W^{K\rho}_\nu$
$\widetilde{\mathcal{O}}_1^{H^2W^2}$	$(D^\mu H^\dagger D_\mu H)W^I_{\rho\sigma}\widetilde{W}^{I\rho\sigma}$
$\widetilde{\mathcal{O}}_2^{H^2W^2}$	$\epsilon^{IJK}(D^\mu H^\dagger\tau^I D^\nu H)(W^J_{\mu\rho}\widetilde{W}^{K\rho}_\nu - \widetilde{W}^J_{\mu\rho}W^{K\rho}_\nu)$
$\widetilde{\mathcal{O}}_3^{H^2W^2}$	$i\,\epsilon^{IJK}(D^\mu H^\dagger\tau^I D^\nu H)(W^J_{\mu\rho}\widetilde{W}^{K\rho}_\nu + \widetilde{W}^J_{\mu\rho}W^{K\rho}_\nu)$
$\mathcal{O}_1^{H^2G^2}$	$(D^\mu H^\dagger D^\nu H)G^a_{\mu\rho}G^{a\rho}_\nu$
$\mathcal{O}_2^{H^2G^2}$	$(D^\mu H^\dagger D_\mu H)G^a_{\rho\sigma}G^{a\rho\sigma}$
$\widetilde{\mathcal{O}}_1^{H^2G^2}$	$(D^\mu H^\dagger D_\mu H)G^a_{\rho\sigma}\widetilde{G}^{a\rho\sigma}$

$(DH)^2F_1F_2$ cross-quartics

$\mathcal{O}_1^{H^2BW}$	$(D^\mu H^\dagger\tau^I D_\mu H)B_{\rho\sigma}W^{I\rho\sigma}$
$\mathcal{O}_2^{H^2BW}$	$i(D^\mu H^\dagger\tau^I D^\nu H)(B_{\mu\rho}W^{I\rho}_\nu - B_{\nu\rho}W^{I\rho}_\mu)$
$\mathcal{O}_3^{H^2BW}$	$(D^\mu H^\dagger\tau^I D^\nu H)(B_{\mu\rho}W^{I\rho}_\nu + B_{\nu\rho}W^{I\rho}_\mu)$
$\widetilde{\mathcal{O}}_1^{H^2BW}$	$(D^\mu H^\dagger\tau^I D_\mu H)B_{\rho\sigma}\widetilde{W}^{I\rho\sigma}$
$\widetilde{\mathcal{O}}_2^{H^2BW}$	$i(D^\mu H^\dagger\tau^I D^\nu H)(B_{\rho[\mu}\widetilde{W}^{I\rho}_{\nu]} - \widetilde{B}_{\rho[\mu}W^{I\rho}_{\nu]})$
$\widetilde{\mathcal{O}}_3^{H^2BW}$	$(D^\mu H^\dagger\tau^I D^\nu H)(B_{\rho(\mu}\widetilde{W}^{I\rho}_{\nu)} + \widetilde{B}_{\rho(\mu}W^{I\rho}_{\nu)})$

2. Bosonic Bounds

B^4 operators

$\mathcal{O}_1^{B^4}$	$(BB)(BB)$
$\mathcal{O}_2^{B^4}$	$(B\widetilde{B})(B\widetilde{B})$
$\widetilde{\mathcal{O}}_1^{B^4}$	$(BB)(B\widetilde{B})$

W^4 operators

$\mathcal{O}_1^{W^4}$	$(W^IW^I)(W^JW^J)$
$\mathcal{O}_2^{W^4}$	$(W^I\widetilde{W}^I)(W^J\widetilde{W}^J)$
$\mathcal{O}_3^{W^4}$	$(W^IW^J)(W^IW^J)$
$\mathcal{O}_4^{W^4}$	$(W^I\widetilde{W}^J)(W^I\widetilde{W}^J)$
$\widetilde{\mathcal{O}}_1^{W^4}$	$(W^IW^I)(W^J\widetilde{W}^J)$
$\widetilde{\mathcal{O}}_2^{W^4}$	$(W^IW^J)(W^I\widetilde{W}^J)$

G^4 operators

$\mathcal{O}_1^{G^4}$	$(G^aG^a)(G^bG^b)$
$\mathcal{O}_2^{G^4}$	$(G^a\widetilde{G}^a)(G^b\widetilde{G}^b)$
$\mathcal{O}_3^{G^4}$	$(G^aG^b)(G^aG^b)$
$\mathcal{O}_4^{G^4}$	$(G^a\widetilde{G}^b)(G^a\widetilde{G}^b)$
$\mathcal{O}_5^{G^4}$	$d^{abe}d^{cde}(G^aG^b)(G^cG^d)$
$\mathcal{O}_6^{G^4}$	$d^{abe}d^{cde}(G^a\widetilde{G}^b)(G^c\widetilde{G}^d)$
$\widetilde{\mathcal{O}}_1^{G^4}$	$(G^aG^a)(G^b\widetilde{G}^b)$
$\widetilde{\mathcal{O}}_2^{G^4}$	$(G^aG^b)(G^a\widetilde{G}^b)$
$\widetilde{\mathcal{O}}_3^{G^4}$	$d^{abe}d^{cde}(G^aG^b)(G^c\widetilde{G}^d)$

$F_1^2F_2^2/F_1F_2^3$ cross-quartics

$\mathcal{O}_1^{B^2W^2}$	$(BB)(W^IW^I)$
$\mathcal{O}_2^{B^2W^2}$	$(B\widetilde{B})(W^I\widetilde{W}^I)$
$\mathcal{O}_3^{B^2W^2}$	$(BW^I)(BW^I)$
$\mathcal{O}_4^{B^2W^2}$	$(B\widetilde{W}^I)(B\widetilde{W}^I)$
$\widetilde{\mathcal{O}}_1^{B^2W^2}$	$(B\widetilde{B})(W^IW^I)$
$\widetilde{\mathcal{O}}_2^{B^2W^2}$	$(BB)(W^I\widetilde{W}^I)$
$\widetilde{\mathcal{O}}_3^{B^2W^2}$	$(BW^I)(B\widetilde{W}^I)$
$\mathcal{O}_1^{B^2G^2}$	$(BB)(G^aG^a)$
$\mathcal{O}_2^{B^2G^2}$	$(B\widetilde{B})(G^a\widetilde{G}^a)$
$\mathcal{O}_3^{B^2G^2}$	$(BG^a)(BG^a)$
$\mathcal{O}_4^{B^2G^2}$	$(B\widetilde{G}^a)(B\widetilde{G}^a)$
$\widetilde{\mathcal{O}}_1^{B^2G^2}$	$(B\widetilde{B})(G^aG^a)$
$\widetilde{\mathcal{O}}_2^{B^2G^2}$	$(BB)(G^a\widetilde{G}^a)$
$\widetilde{\mathcal{O}}_3^{B^2G^2}$	$(BG^a)(B\widetilde{G}^a)$
$\mathcal{O}_1^{W^2G^2}$	$(W^IW^I)(G^aG^a)$
$\mathcal{O}_2^{W^2G^2}$	$(W^I\widetilde{W}^I)(G^a\widetilde{G}^a)$
$\mathcal{O}_3^{W^2G^2}$	$(W^IG^a)(W^IG^a)$
$\mathcal{O}_4^{W^2G^2}$	$(W^I\widetilde{G}^a)(W^I\widetilde{G}^a)$
$\widetilde{\mathcal{O}}_1^{W^2G^2}$	$(W^I\widetilde{W}^I)(G^aG^a)$
$\widetilde{\mathcal{O}}_2^{W^2G^2}$	$(W^IW^I)(G^a\widetilde{G}^a)$
$\widetilde{\mathcal{O}}_3^{W^2G^2}$	$(W^IG^a)(W^I\widetilde{G}^a)$
$\mathcal{O}_1^{BG^3}$	$d^{abc}(BG^a)(G^bG^c)$
$\mathcal{O}_2^{BG^3}$	$d^{abc}(B\widetilde{G}^a)(G^b\widetilde{G}^c)$
$\widetilde{\mathcal{O}}_1^{BG^3}$	$d^{abc}(B\widetilde{G}^a)(G^bG^c)$
$\widetilde{\mathcal{O}}_2^{BG^3}$	$d^{abc}(BG^a)(G^b\widetilde{G}^c)$

$(DH)^4$ operators

$\mathcal{O}_1^{H^4}$	$(D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$
$\mathcal{O}_2^{H^4}$	$(D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$
$\mathcal{O}_3^{H^4}$	$(D^\mu H^\dagger D_\mu H)(D^\nu H^\dagger D_\nu H)$

$(DH)^2F^2$ cross-quartics

$\mathcal{O}_1^{H^2B^2}$	$(D^\mu H^\dagger D^\nu H)B_{\mu\rho}B_\nu{}^\rho$
$\mathcal{O}_2^{H^2B^2}$	$(D^\mu H^\dagger D_\mu H)B_{\rho\sigma}B^{\rho\sigma}$
$\widetilde{\mathcal{O}}_1^{H^2B^2}$	$(D^\mu H^\dagger D_\mu H)B_{\rho\sigma}\widetilde{B}^{\rho\sigma}$
$\mathcal{O}_1^{H^2W^2}$	$(D^\mu H^\dagger D^\nu H)W^I_{\mu\rho}W^{I\rho}_\nu$
$\mathcal{O}_2^{H^2W^2}$	$(D^\mu H^\dagger D_\mu H)W^I_{\rho\sigma}W^{I\rho\sigma}$
$\mathcal{O}_3^{H^2W^2}$	$i\,\epsilon^{IJK}(D^\mu H^\dagger\tau^I D^\nu H)W^J_{\mu\rho}W^{K\rho}_\nu$
$\widetilde{\mathcal{O}}_1^{H^2W^2}$	$(D^\mu H^\dagger D_\mu H)W^I_{\rho\sigma}\widetilde{W}^{I\rho\sigma}$
$\widetilde{\mathcal{O}}_2^{H^2W^2}$	$\epsilon^{IJK}(D^\mu H^\dagger\tau^I D^\nu H)(W^J_{\mu\rho}\widetilde{W}^{K\rho}_\nu - \widetilde{W}^J_{\mu\rho}W^{K\rho}_\nu)$
$\widetilde{\mathcal{O}}_3^{H^2W^2}$	$i\,\epsilon^{IJK}(D^\mu H^\dagger\tau^I D^\nu H)(W^J_{\mu\rho}\widetilde{W}^{K\rho}_\nu + \widetilde{W}^J_{\mu\rho}W^{K\rho}_\nu)$
$\mathcal{O}_1^{H^2G^2}$	$(D^\mu H^\dagger D^\nu H)G^a_{\mu\rho}G^{a\rho}_\nu$
$\mathcal{O}_2^{H^2G^2}$	$(D^\mu H^\dagger D_\mu H)G^a_{\rho\sigma}G^{a\rho\sigma}$
$\widetilde{\mathcal{O}}_1^{H^2G^2}$	$(D^\mu H^\dagger D_\mu H)G^a_{\rho\sigma}\widetilde{G}^{a\rho\sigma}$

$(DH)^2F_1F_2$ cross-quartics

$\mathcal{O}_1^{H^2BW}$	$(D^\mu H^\dagger\tau^I D_\mu H)B_{\rho\sigma}W^{I\rho\sigma}$
$\mathcal{O}_2^{H^2BW}$	$i(D^\mu H^\dagger\tau^I D^\nu H)(B_{\mu\rho}W^{I\rho}_\nu - B_{\nu\rho}W^{I\rho}_\mu)$
$\mathcal{O}_3^{H^2BW}$	$(D^\mu H^\dagger\tau^I D^\nu H)(B_{\mu\rho}W^{I\rho}_\nu + B_{\nu\rho}W^{I\rho}_\mu)$
$\widetilde{\mathcal{O}}_1^{H^2BW}$	$(D^\mu H^\dagger\tau^I D_\mu H)B_{\rho\sigma}\widetilde{W}^{I\rho\sigma}$
$\widetilde{\mathcal{O}}_2^{H^2BW}$	$i(D^\mu H^\dagger\tau^I D^\nu H)(B_{\rho[\mu}\widetilde{W}^{I\rho}_{\nu]} - \widetilde{B}_{\rho[\mu}W^{I\rho}_{\nu]})$
$\widetilde{\mathcal{O}}_3^{H^2BW}$	$(D^\mu H^\dagger\tau^I D^\nu H)(B_{\rho(\mu}\widetilde{W}^{I\rho}_{\nu)} + \widetilde{B}_{\rho(\mu}W^{I\rho}_{\nu)})$

Bosonic Bounds: Warmup

Consider higher order corrections to QED*

$$\mathcal{L} = -\frac{1}{4}(FF) + c_1(FF)^2 + c_2(F\tilde{F})^2 + \tilde{c}(FF)(F\tilde{F})$$

$*\,(AB) = A_{\mu\nu}B^{\mu\nu}$

Bosonic Bounds: Warmup

Consider higher order corrections to QED

$$\mathcal{L} = -\frac{1}{4}(FF) + c_1(FF)^2 + c_2(F\tilde{F})^2 + \tilde{c}(FF)(F\tilde{F})$$

$\gamma_L\gamma_L \to \gamma_L\gamma_L$ $c_1 > 0$	$\gamma_L\gamma_R \to \gamma_L\gamma_R$ $c_2 > 0$	$\gamma_L\gamma_L \to \gamma_L\gamma_R$ $\tilde{c} = ?$

Bosonic Bounds: Warmup

Consider higher order corrections to QED

$$\mathcal{L} = -\frac{1}{4}(FF) + c_1(FF)^2 + c_2(F\tilde{F})^2 + \tilde{c}(FF)(F\tilde{F})$$

$\gamma_L\gamma_L \to \gamma_L\gamma_L$	$\gamma_L\gamma_R \to \gamma_L\gamma_R$	$\gamma_L\gamma_L \to \gamma_L\gamma_R$
$c_1 > 0$	$c_2 > 0$	$\tilde{c} = ?$

- These are well known results [Adams+ hep-th/0602178]
- Satisfied by Euler-Heisenberg

$$c_1 = \frac{\alpha^2}{90 m_e^4} \qquad c_2 = \frac{7\alpha^2}{360 m_e^4}$$

- How can we hope to bound $\tilde{c}$?

Bosonic Bounds: Warmup

Consider higher order corrections to QED

$$\mathcal{L} = -\frac{1}{4}(FF) + c_1(FF)^2 + c_2(F\tilde{F})^2 + \tilde{c}(FF)(F\tilde{F})$$

Let's scatter more general polarisations (~a superposition)

$$p_1 = \sqrt{s}/2(1,0,0,1) \qquad \epsilon_1 = (0,1,0,0) \qquad \epsilon_2 = (0,\cos\theta,\sin\theta,0)$$

$$\Rightarrow \mathcal{A}(s) = 16s^2(c_1\cos^2\theta + c_2\sin^2\theta + \tilde{c}\sin\theta\cos\theta)$$

Bosonic Bounds: Warmup

Consider higher order corrections to QED

$$\mathcal{L} = -\frac{1}{4}(FF) + c_1(FF)^2 + c_2(F\tilde{F})^2 + \tilde{c}(FF)(F\tilde{F})$$

Let's scatter more general polarisations (~a superposition)

$$p_1 = \sqrt{s}/2(1,0,0,1) \qquad \epsilon_1 = (0,1,0,0) \qquad \epsilon_2 = (0,\cos\theta,\sin\theta,0)$$

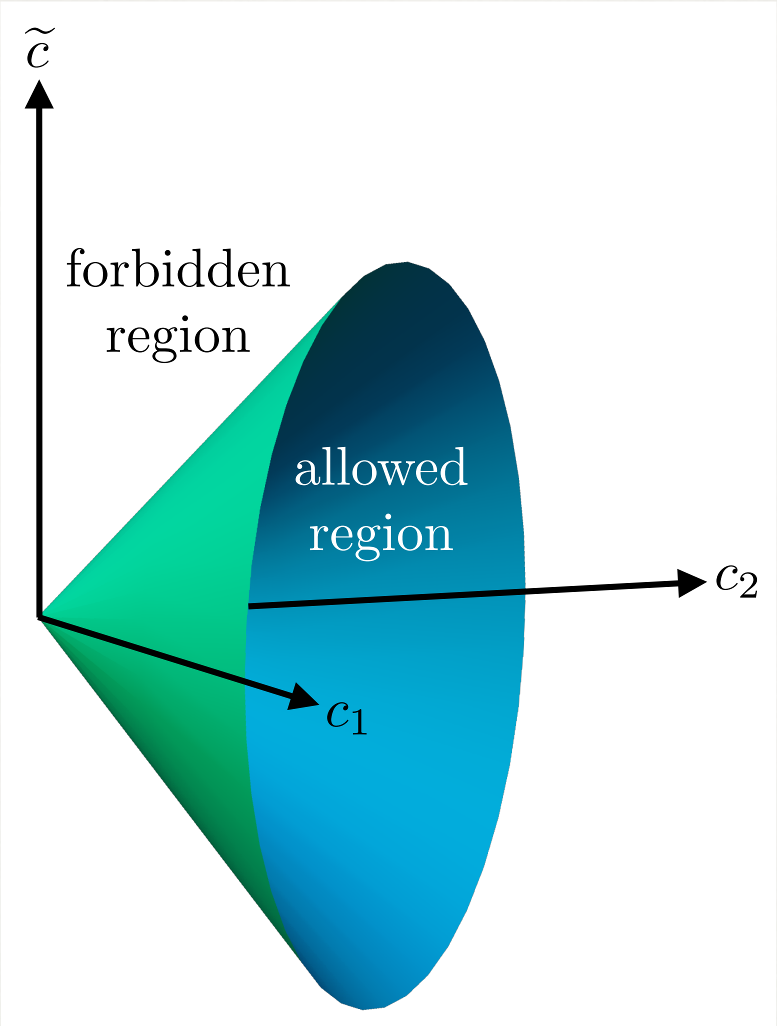
$$\Rightarrow \mathcal{A}(s) = 16s^2(c_1\cos^2\theta + c_2\sin^2\theta + \tilde{c}\sin\theta\cos\theta)$$

$$\theta = 0 \quad c_1 > 0$$

$$\theta = \pi/2 \quad c_2 > 0$$

$$\theta = \pm\arctan(\sqrt{c_1/c_2}) \quad 4c_1c_2 > \tilde{c}^2$$

Bosonic Bounds: Warmup



$$\theta = 0$$
$$c_1 > 0$$

$$\theta = \pi/2$$
$$c_2 > 0$$

$$\theta = \pm \arctan(\sqrt{c_1/c_2})$$
$$4c_1 c_2 > \tilde{c}^2$$

Interplay between CP even and odd will reappear

Bosonic Bounds: In Full

Key ingredient: complete operator basis

- Self-quartics [Morozov 1984]
- Cross-quartics [Remmen, **NLR** 2019]
- Higgs [Hays+ 1808.00442]

Result: 64 operators; 39 (25) CP even (odd)

$(DH)^4$ operators

$\mathcal{O}_1^{H^4}$	$(D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$
$\mathcal{O}_2^{H^4}$	$(D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$
$\mathcal{O}_3^{H^4}$	$(D^\mu H^\dagger D_\mu H)(D^\nu H^\dagger D_\nu H)$

$(DH)^2F^2$ cross-quartics

$\mathcal{O}_1^{H^2B^2}$	$(D^\mu H^\dagger D^\nu H)B_{\mu\rho}B_\nu{}^\rho$
$\mathcal{O}_2^{H^2B^2}$	$(D^\mu H^\dagger D_\mu H)B_{\rho\sigma}B^{\rho\sigma}$
$\widetilde{\mathcal{O}}_1^{H^2B^2}$	$(D^\mu H^\dagger D_\mu H)B_{\rho\sigma}\widetilde{B}^{\rho\sigma}$
$\mathcal{O}_1^{H^2W^2}$	$(D^\mu H^\dagger D^\nu H)W^I_{\mu\rho}W^{I\rho}_\nu$
$\mathcal{O}_2^{H^2W^2}$	$(D^\mu H^\dagger D_\mu H)W^I_{\rho\sigma}W^{I\rho\sigma}$
$\mathcal{O}_3^{H^2W^2}$	$i\,\epsilon^{IJK}(D^\mu H^\dagger \tau^I D^\nu H)W^J_{\mu\rho}W^{K\rho}_\nu$
$\widetilde{\mathcal{O}}_1^{H^2W^2}$	$(D^\mu H^\dagger D_\mu H)W^I_{\rho\sigma}\widetilde{W}^{I\rho\sigma}$
$\widetilde{\mathcal{O}}_2^{H^2W^2}$	$\epsilon^{IJK}(D^\mu H^\dagger \tau^I D^\nu H)(W^J_{\mu\rho}\widetilde{W}^{K\rho}_\nu - \widetilde{W}^J_{\mu\rho}W^{K\rho}_\nu)$
$\widetilde{\mathcal{O}}_3^{H^2W^2}$	$i\,\epsilon^{IJK}(D^\mu H^\dagger \tau^I D^\nu H)(W^J_{\mu\rho}\widetilde{W}^{K\rho}_\nu + \widetilde{W}^J_{\mu\rho}W^{K\rho}_\nu)$

$\mathcal{O}_1^{H^2G^2}$	$(D^\mu H^\dagger D^\nu H)G^a_{\mu\rho}G^{a\rho}_\nu$
$\mathcal{O}_2^{H^2G^2}$	$(D^\mu H^\dagger D_\mu H)G^a_{\rho\sigma}G^{a\rho\sigma}$
$\widetilde{\mathcal{O}}_1^{H^2G^2}$	$(D^\mu H^\dagger D_\mu H)G^a_{\rho\sigma}\widetilde{G}^{a\rho\sigma}$

$(DH)^2F_1F_2$ cross-quartics

$\mathcal{O}_1^{H^2BW}$	$(D^\mu H^\dagger \tau^I D_\mu H)B_{\rho\sigma}W^{I\rho\sigma}$
$\mathcal{O}_2^{H^2BW}$	$i(D^\mu H^\dagger \tau^I D^\nu H)(B_{\mu\rho}W^{I\rho}_\nu - B_{\nu\rho}W^{I\rho}_\mu)$
$\mathcal{O}_3^{H^2BW}$	$(D^\mu H^\dagger \tau^I D^\nu H)(B_{\mu\rho}W^{I\rho}_\nu + B_{\nu\rho}W^{I\rho}_\mu)$
$\widetilde{\mathcal{O}}_1^{H^2BW}$	$(D^\mu H^\dagger \tau^I D_\mu H)B_{\rho\sigma}\widetilde{W}^{I\rho\sigma}$
$\widetilde{\mathcal{O}}_2^{H^2BW}$	$i(D^\mu H^\dagger \tau^I D^\nu H)(B_{\rho[\mu}\widetilde{W}^{I\rho}_{\nu]} - \widetilde{B}_{\rho[\mu}W^{I\rho}_{\nu]})$
$\widetilde{\mathcal{O}}_3^{H^2BW}$	$(D^\mu H^\dagger \tau^I D^\nu H)(B_{\rho(\mu}\widetilde{W}^{I\rho}_{\nu)} + \widetilde{B}_{\rho(\mu}W^{I\rho}_{\nu)})$

B^4 operators

$\mathcal{O}_1^{B^4}$	$(BB)(BB)$
$\mathcal{O}_2^{B^4}$	$(B\widetilde{B})(B\widetilde{B})$
$\widetilde{\mathcal{O}}_1^{B^4}$	$(BB)(B\widetilde{B})$

W^4 operators

$\mathcal{O}_1^{W^4}$	$(W^IW^I)(W^JW^J)$
$\mathcal{O}_2^{W^4}$	$(W^I\widetilde{W}^I)(W^J\widetilde{W}^J)$
$\mathcal{O}_3^{W^4}$	$(W^IW^J)(W^IW^J)$
$\mathcal{O}_4^{W^4}$	$(W^I\widetilde{W}^J)(W^I\widetilde{W}^J)$
$\widetilde{\mathcal{O}}_1^{W^4}$	$(W^IW^I)(W^J\widetilde{W}^J)$
$\widetilde{\mathcal{O}}_2^{W^4}$	$(W^IW^J)(W^I\widetilde{W}^J)$

G^4 operators

$\mathcal{O}_1^{G^4}$	$(G^aG^a)(G^bG^b)$
$\mathcal{O}_2^{G^4}$	$(G^a\widetilde{G}^a)(G^b\widetilde{G}^b)$
$\mathcal{O}_3^{G^4}$	$(G^aG^b)(G^aG^b)$
$\mathcal{O}_4^{G^4}$	$(G^a\widetilde{G}^b)(G^a\widetilde{G}^b)$
$\mathcal{O}_5^{G^4}$	$d^{abe}d^{cde}(G^aG^b)(G^cG^d)$
$\mathcal{O}_6^{G^4}$	$d^{abe}d^{cde}(G^a\widetilde{G}^b)(G^c\widetilde{G}^d)$
$\widetilde{\mathcal{O}}_1^{G^4}$	$(G^aG^a)(G^b\widetilde{G}^b)$
$\widetilde{\mathcal{O}}_2^{G^4}$	$(G^aG^b)(G^a\widetilde{G}^b)$
$\widetilde{\mathcal{O}}_3^{G^4}$	$d^{abe}d^{cde}(G^aG^b)(G^c\widetilde{G}^d)$

$F_1^2F_2^2/F_1F_2^3$ cross-quartics

$\mathcal{O}_1^{B^2W^2}$	$(BB)(W^IW^I)$
$\mathcal{O}_2^{B^2W^2}$	$(B\widetilde{B})(W^I\widetilde{W}^I)$
$\mathcal{O}_3^{B^2W^2}$	$(BW^I)(BW^I)$
$\mathcal{O}_4^{B^2W^2}$	$(B\widetilde{W}^I)(B\widetilde{W}^I)$
$\widetilde{\mathcal{O}}_1^{B^2W^2}$	$(B\widetilde{B})(W^IW^I)$
$\widetilde{\mathcal{O}}_2^{B^2W^2}$	$(BB)(W^I\widetilde{W}^I)$
$\widetilde{\mathcal{O}}_3^{B^2W^2}$	$(BW^I)(B\widetilde{W}^I)$
$\mathcal{O}_1^{B^2G^2}$	$(BB)(G^aG^a)$
$\mathcal{O}_2^{B^2G^2}$	$(B\widetilde{B})(G^a\widetilde{G}^a)$
$\mathcal{O}_3^{B^2G^2}$	$(BG^a)(BG^a)$
$\mathcal{O}_4^{B^2G^2}$	$(B\widetilde{G}^a)(B\widetilde{G}^a)$
$\widetilde{\mathcal{O}}_1^{B^2G^2}$	$(B\widetilde{B})(G^aG^a)$
$\widetilde{\mathcal{O}}_2^{B^2G^2}$	$(BB)(G^a\widetilde{G}^a)$
$\widetilde{\mathcal{O}}_3^{B^2G^2}$	$(BG^a)(B\widetilde{G}^a)$
$\mathcal{O}_1^{W^2G^2}$	$(W^IW^I)(G^aG^a)$
$\mathcal{O}_2^{W^2G^2}$	$(W^I\widetilde{W}^I)(G^a\widetilde{G}^a)$
$\mathcal{O}_3^{W^2G^2}$	$(W^IG^a)(W^IG^a)$
$\mathcal{O}_4^{W^2G^2}$	$(W^I\widetilde{G}^a)(W^I\widetilde{G}^a)$
$\widetilde{\mathcal{O}}_1^{W^2G^2}$	$(W^I\widetilde{W}^I)(G^aG^a)$
$\widetilde{\mathcal{O}}_2^{W^2G^2}$	$(W^IW^I)(G^a\widetilde{G}^a)$
$\widetilde{\mathcal{O}}_3^{W^2G^2}$	$(W^IG^a)(W^I\widetilde{G}^a)$
$\mathcal{O}_1^{BG^3}$	$d^{abc}(BG^a)(G^bG^c)$
$\mathcal{O}_2^{BG^3}$	$d^{abc}(B\widetilde{G}^a)(G^b\widetilde{G}^c)$
$\widetilde{\mathcal{O}}_1^{BG^3}$	$d^{abc}(B\widetilde{G}^a)(G^bG^c)$
$\widetilde{\mathcal{O}}_2^{BG^3}$	$d^{abc}(BG^a)(G^b\widetilde{G}^c)$

Bosonic Bounds: In Full

Calculated 27 independent bounds $(2^{27} \sim 10^8)$

CP Even > 0

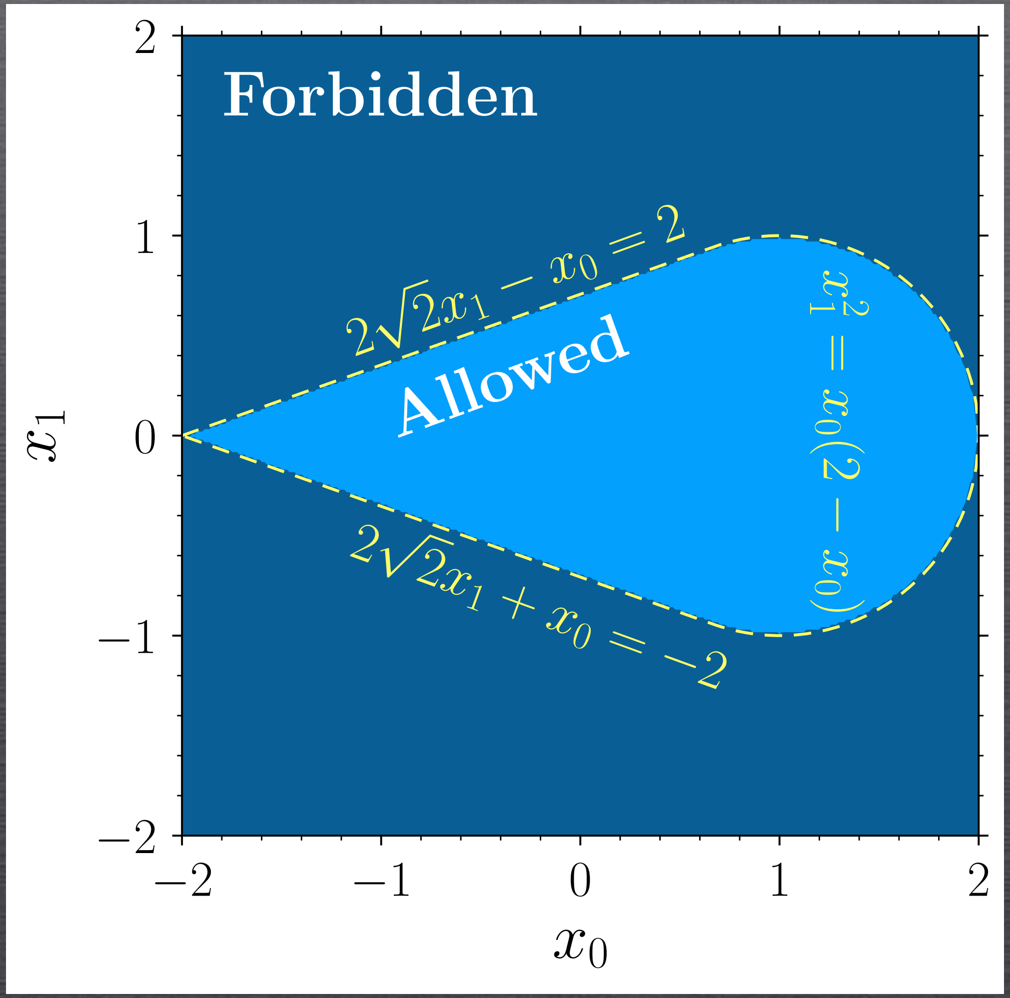
$$3c_1^{G^4} + 3c_3^{G^4} + c_5^{G^4} > 0 \qquad c_3^{B^2W^2} > 0$$
$$3c_3^{G^4} + 2c_5^{G^4} > 0 \qquad c_4^{B^2W^2} > 0$$
$$3c_2^{G^4} + 3c_4^{G^4} + c_6^{G^4} > 0 \qquad c_3^{B^2G^2} > 0$$
$$3c_4^{G^4} + 2c_6^{G^4} > 0 \qquad c_4^{B^2G^2} > 0$$
$$c_1^{W^4} + c_3^{W^4} > 0 \qquad c_3^{W^2G^2} > 0$$
$$c_2^{W^4} + c_4^{W^4} > 0 \qquad c_4^{W^2G^2} > 0$$
$$c_1^{B^4} > 0 \qquad c_1^{H^2B^2} > 0$$
$$c_2^{B^4} > 0 \qquad c_1^{H^2W^2} > 0$$
$$c_1^{H^4} + c_2^{H^4} + c_3^{H^4} > 0 \qquad c_1^{H^2G^2} > 0$$
$$c_1^{H^4} + c_2^{H^4} > 0$$
$$c_2^{H^4} > 0$$

CP Even > CP Odd

$$(3\tilde{c}_1^{G^4} + 3\tilde{c}_2^{G^4} + \tilde{c}_3^{G^4})^2 < 4(3c_1^{G^4} + 3c_3^{G^4} + c_5^{G^4})(3c_2^{G^4} + 3c_4^{G^4} + c_6^{G^4})$$
$$(3\tilde{c}_2^{G^4} + 2\tilde{c}_3^{G^4})^2 < 4(3c_3^{G^4} + 2c_5^{G^4})(3c_4^{G^4} + 2c_6^{G^4})$$
$$(\tilde{c}_1^{W^4} + \tilde{c}_2^{W^4})^2 < 4(c_1^{W^4} + c_3^{W^4})(c_2^{W^4} + c_4^{W^4})$$
$$(\tilde{c}_1^{B^4})^2 < 4c_1^{B^4}c_2^{B^4}$$
$$(\tilde{c}_3^{B^2W^2})^2 < 4c_3^{B^2W^2}c_4^{B^2W^2}$$
$$(\tilde{c}_3^{B^2G^2})^2 < 4c_3^{B^2G^2}c_4^{B^2G^2}$$
$$(\tilde{c}_3^{W^2G^2})^2 < 4c_3^{W^2G^2}c_4^{W^2G^2}$$

Can determine more by considering superpositions of H, B, W, G

3. Fermionic Bounds



Fermionic Bounds: Warmup

Consider four-fermion scattering mediated by

$e = e_R \sim (1, 1, -1)$

$$\mathcal{O} = c_{mnpq} \partial_\mu (\bar{e}_m \gamma_\nu e_n) \partial^\mu (\bar{e}_p \gamma^\nu e_q)$$

$m, n, p, q \in \{1, \ldots, N_f\}$

Fermionic Bounds: Warmup

Consider four-fermion scattering mediated by

$$\mathcal{O} = c_{mnpq}\partial_\mu(\bar{e}_m\gamma_\nu e_n)\partial^\mu(\bar{e}_p\gamma^\nu e_q)$$

Aside: How Many Operators?

Two conditions:

1. Hermiticity:

$$c_{mnpq} = c^*_{nmqp}$$

2. Symmetrization:

$$c_{mnpq} = c_{pqmn}$$

Result:

$$N_f^2(N_f^2+1)/2$$

Fermionic Bounds: Warmup

Consider four-fermion scattering mediated by

$$\mathcal{O} = c_{mnpq}\partial_\mu(\bar{e}_m\gamma_\nu e_n)\partial^\mu(\bar{e}_p\gamma^\nu e_q)$$

Aside: How Many Operators?

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Result:

$$N_f^2(N_f^2+1)/2$$

2, 84, 30, 993, 560, 15456, 11962, 261485,...: Higher dimension operators in the SM EFT

Brian Henning,[a] Xiaochuan Lu,[b] Tom Melia[c,d] and Hitoshi Murayama[c,d,e]

Class $\psi^4\mathcal{D}^2$:
Self conjugate:

$$(N_f^4+N_f^2)d^2d^{\dagger 2}\mathcal{D}^2,\ 2N_f^4dd^\dagger ee^\dagger\mathcal{D}^2,\ 4N_f^4dd^\dagger uu^\dagger\mathcal{D}^2,\ 2N_f^4dd^\dagger LL^\dagger\mathcal{D}^2,$$
$$4N_f^4dd^\dagger QQ^\dagger\mathcal{D}^2,\ \frac{1}{2}\left(N_f^4+N_f^2\right)e^2e^{\dagger 2}\mathcal{D}^2,\ 2N_f^4ee^\dagger uu^\dagger\mathcal{D}^2,\ 2N_f^4ee^\dagger LL^\dagger\mathcal{D}^2,$$
$$2N_f^4ee^\dagger QQ^\dagger\mathcal{D}^2,\ (N_f^4+N_f^2)u^2u^{\dagger 2}\mathcal{D}^2,\ 2N_f^4uu^\dagger LL^\dagger\mathcal{D}^2,\ 4N_f^4uu^\dagger QQ^\dagger\mathcal{D}^2,$$
$$(N_f^4+N_f^2)L^2L^{\dagger 2}\mathcal{D}^2,\ 4N_f^4QQ^\dagger LL^\dagger\mathcal{D}^2,\ 2\left(N_f^4+N_f^2\right)Q^2Q^{\dagger 2}\mathcal{D}^2 \qquad \text{(A.16)}$$

Fermionic Bounds: Warmup

Consider four-fermion scattering mediated by

$$\mathcal{O} = c_{mnpq}\partial_\mu(\bar{e}_m\gamma_\nu e_n)\partial^\mu(\bar{e}_p\gamma^\nu e_q)$$

Scatter flavour superpositions: $|\psi_1\rangle = \alpha_m|\bar{e}_m\rangle \quad |\psi_2\rangle = \beta_m|e_m\rangle$

$$\mathcal{A}(s) = 4c_{mnpq}\alpha_m\beta_n\beta_p^*\alpha_q^* s^2$$

$$\Rightarrow c_{\alpha\beta} = c_{mnpq}\rho^\alpha_{mq}\rho^\beta_{np} > 0$$

$\rho^\alpha_{mq} = \alpha_m\alpha_q^*$, pure density matrix

Fermionic Bounds: Warmup

A few simple examples

1. $\alpha_m = \delta_{1m} \qquad \beta_m = \delta_{2m}$

   $\Rightarrow c_{1221} > 0$

2. $\alpha_m = \delta_{1m} \qquad \beta_m = \delta_{3m}$

   $\Rightarrow c_{1331} > 0$

3. $\alpha_m = \delta_{1m} \qquad \beta_m = \delta_{2m} \cos\theta + \delta_{3m} \sin\theta e^{i\phi}$

   $\Rightarrow c_{1221} c_{1331} > |c_{1231}|^2$

Fermionic Bounds: Warmup

A few simple examples

1. $\alpha_m = \delta_{1m} \qquad \beta_m = \delta_{2m}$
   $\Rightarrow c_{1221} > 0$
   $\mu\,\bar{e} \to \mu\,\bar{e}$

2. $\alpha_m = \delta_{1m} \qquad \beta_m = \delta_{3m}$
   $\Rightarrow c_{1331} > 0$
   $\tau\,\bar{e} \to \tau\,\bar{e}$

3. $\alpha_m = \delta_{1m} \qquad \beta_m = \delta_{2m}\cos\theta + \delta_{3m}\sin\theta e^{i\phi}$
   $\Rightarrow c_{1221}c_{1331} > |c_{1231}|^2$
   $\mu\,\bar{e} \to \tau\,\bar{e}$

Flavour violating effects bounded by flavour conserving!

Reminiscent of our conical boson bounds

- For these operators CP and flavour violation linked, so again CP conserving > CP violating
- Orthogonal to other flavour structures such as MFV
- But there is more structure!

Fermionic Bounds: Warmup

Visualising the bounds in a simple case with $N_f = 2$, assuming

1. $c_{mnpq} \in \mathbb{R}$ (CP conservation)
2. $c = c_{1111} = c_{2222} = c_{1221}$
3. $c_0 = c_{1122}$
4. $c_1 = c_{1112} = c_{1222}$
5. $c_2 = c_{1212}$

Fermionic Bounds: Warmup

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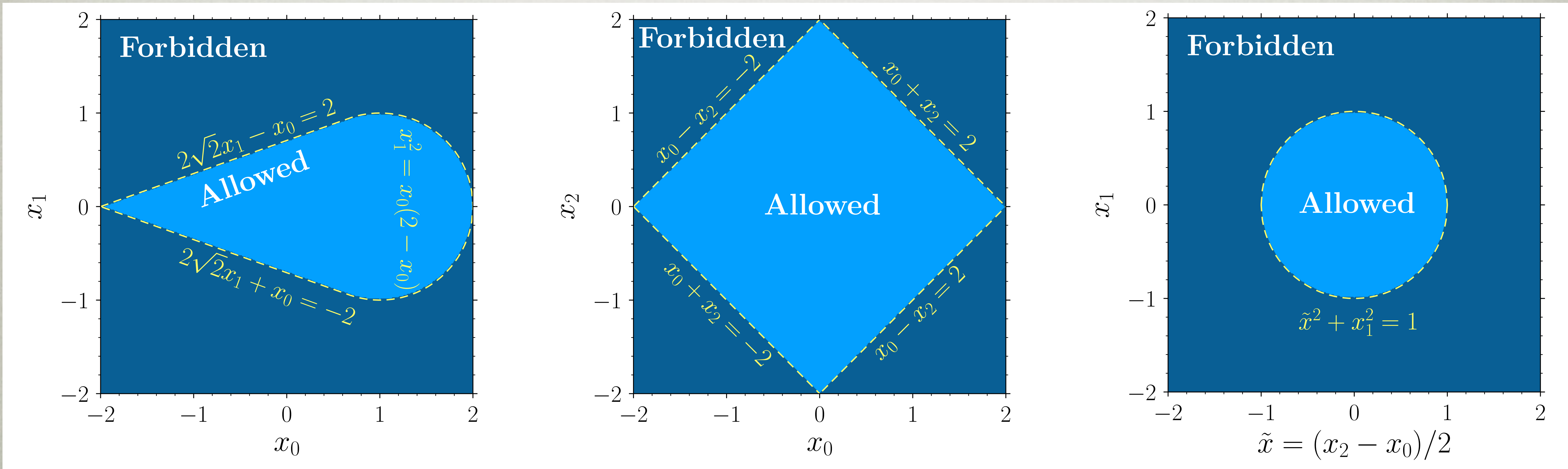
Bounds

$c > 0 \qquad x_i = c_i / c$



Fermionic Bounds: In Full

Operators relevant for scattering definite SM representations*

Self-Quartic

$$\mathcal{O}_1[\psi] = c^{\psi,1}_{mnpq}\partial_\mu J_\nu[\psi]_{mn}\partial^\mu J^\nu[\psi]_{pq} \quad \psi = \text{any}$$
$$\mathcal{O}_2[\psi] = c^{\psi,2}_{mnpq}\partial_\mu J_\nu[\psi]^I_{mn}\partial^\mu J^\nu[\psi]^I_{pq} \quad \psi = L,Q$$
$$\mathcal{O}_3[\psi] = c^{\psi,3}_{mnpq}\partial_\mu J_\nu[\psi]^a_{mn}\partial^\mu J^\nu[\psi]^a_{pq} \quad \psi = d,u,Q$$
$$\mathcal{O}_4[Q] = c^{Q,4}_{mnpq}\partial_\mu J_\nu[Q]^{Ia}_{mn}\partial^\mu J^\nu[Q]^{Ia}_{pq}$$

Cross-Quartic

$$\mathcal{O}_{J1}[\psi,\chi] = b^{\psi\chi,1}_{mnpq}\partial_\mu J_\nu[\psi]_{mq}\partial^\mu J^\nu[\chi]_{np} \quad \psi,\chi = \text{any}$$
$$\mathcal{O}_{J2}[Q,L] = b^{QL,2}_{mnpq}\partial_\mu J_\nu[Q]^I_{mq}\partial^\mu J^\nu[L]^I_{np}$$
$$\mathcal{O}_{J3}[\psi,\chi] = b^{\psi\chi,3}_{mnpq}\partial_\mu J_\nu[\psi]^a_{mq}\partial^\mu J^\nu[\chi]^a_{np} \quad \psi,\chi \in \{d,u,Q\}$$
$$\mathcal{O}_{K1}[\psi,\chi] = -a^{\psi\chi,1}_{mnpq}K_{\mu\nu}[\psi]_{mq}K^{\nu\mu}[\chi]_{np} \quad \psi,\chi = \text{any}$$
$$\mathcal{O}_{K2}[Q,L] = -a^{QL,2}_{mnpq}K_{\mu\nu}[Q]^I_{mq}K^{\nu\mu}[L]^I_{np}$$
$$\mathcal{O}_{K3}[\psi,\chi] = -a^{\psi\chi,3}_{mnpq}K_{\mu\nu}[\psi]^a_{mq}K^{\nu\mu}[\chi]^a_{np} \quad \psi,\chi \in \{d,u,Q\},$$

where we define

$$J^\mu[\psi]_{mn} = \bar{\psi}_m\gamma_\mu\psi_n \qquad J^\mu[\psi]^a_{mn} = \bar{\psi}_m T^a\gamma_\mu\psi_n \qquad K_{\mu\nu}[\psi]_{mn} = \bar{\psi}_m\gamma_\mu\partial_\nu\psi_n \qquad K_{\mu\nu}[\psi]^a_{mn} = \bar{\psi}_m T^a\gamma_\mu\partial_\nu\psi_n$$
$$J^\mu[\psi]^I_{mn} = \bar{\psi}_m\tau^I\gamma_\mu\psi_n \qquad J^\mu[\psi]^{Ia}_{mn} = \bar{\psi}_m\tau^I T^a\gamma_\mu\psi_n \qquad K_{\mu\nu}[\psi]^I_{mn} = \bar{\psi}_m\tau^I\gamma_\mu\partial_\nu\psi_n$$

*More operators enter if scatter superpositions of representations - relevant for e.g. baryon/lepton number violation

Fermionic Bounds: In Full

… and we derive that the following must be positive

Self-Quartic

$$c^{e,1}_{\alpha\beta}$$

$$c^{L,1}_{\alpha\beta} + \tfrac{1}{4}c^{L,2}_{\alpha\beta} \qquad c^{L,2}_{\alpha\beta}$$

$$c^{u,1}_{\alpha\beta} + \tfrac{1}{3}c^{u,3}_{\alpha\beta} \qquad c^{Q,1}_{\alpha\beta} + \tfrac{1}{4}c^{Q,2}_{\alpha\beta} + \tfrac{1}{3}c^{Q,3}_{\alpha\beta} + \tfrac{1}{12}c^{Q,4}_{\alpha\beta}$$

$$c^{u,3}_{\alpha\beta} \qquad c^{Q,2}_{\alpha\beta} + \tfrac{1}{3}c^{Q,4}_{\alpha\beta}$$

$$c^{d,1}_{\alpha\beta} + \tfrac{1}{3}c^{d,3}_{\alpha\beta} \qquad c^{Q,3}_{\alpha\beta} + \tfrac{1}{4}c^{Q,4}_{\alpha\beta}$$

$$c^{d,3}_{\alpha\beta} \qquad c^{Q,4}_{\alpha\beta}$$

Cross-Quartic

$$a^{de,1}_{\alpha\beta} \qquad a^{ue,1}_{\alpha\beta}$$

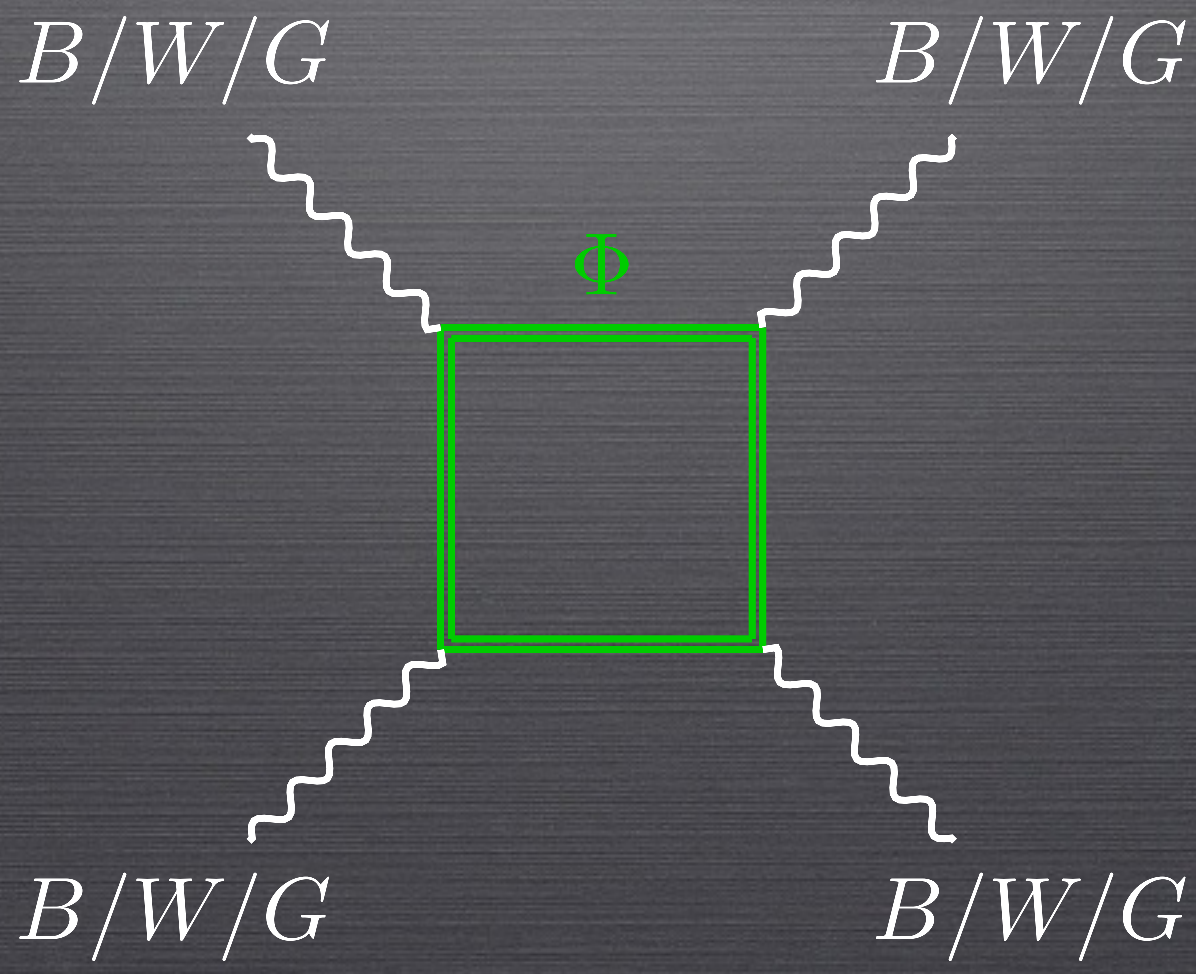
$$a^{eL,1}_{\alpha\beta} \qquad a^{dL,1}_{\alpha\beta}$$

$$a^{uL,1}_{\alpha\beta} \qquad a^{eQ,1}_{\alpha\beta}$$

$$a^{QL,1}_{\alpha\beta} \pm \tfrac{1}{4}a^{QL,2}_{\alpha\beta} \qquad a^{du,1}_{\alpha\beta} + \tfrac{1\pm 3}{12}a^{du,3}_{\alpha\beta}$$

$$a^{dQ,1}_{\alpha\beta} + \tfrac{1\pm 3}{12}a^{dQ,3}_{\alpha\beta} \qquad a^{uQ,1}_{\alpha\beta} + \tfrac{1\pm 3}{12}a^{uQ,3}_{\alpha\beta}$$

4. UV Completions



UV Completions

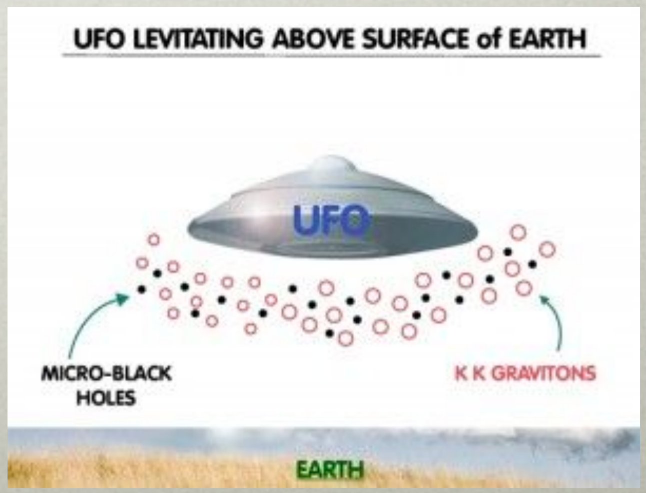
Strategy

Any BSM theory adding field content to the SM should satisfy our bounds - non-trivial cross check

UV Completions: Fermions

Consider a KK graviton coupled to the field strength of e_R

$$\mathcal{L} \supset \mathcal{L}_{\rm FP} + \kappa \phi^{\mu\nu} T_{\mu\nu}[e]$$



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Integrating out $\phi^{\mu\nu}$ generates $\mathcal{O}_1[e]$, with coefficient

$$c^{e,1}_{mnpq} = \frac{\kappa^2}{2m^2}(4\delta_{mq}\delta_{np} - \delta_{mn}\delta_{pq})$$

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$$\mathcal{L} \supset \mathcal{L}_{\mathrm{FP}} + \kappa \phi^{\mu\nu} T_{\mu\nu}[e]$$

Integrating out $\phi^{\mu\nu}$ generates $\mathcal{O}_1[e]$, with coefficient

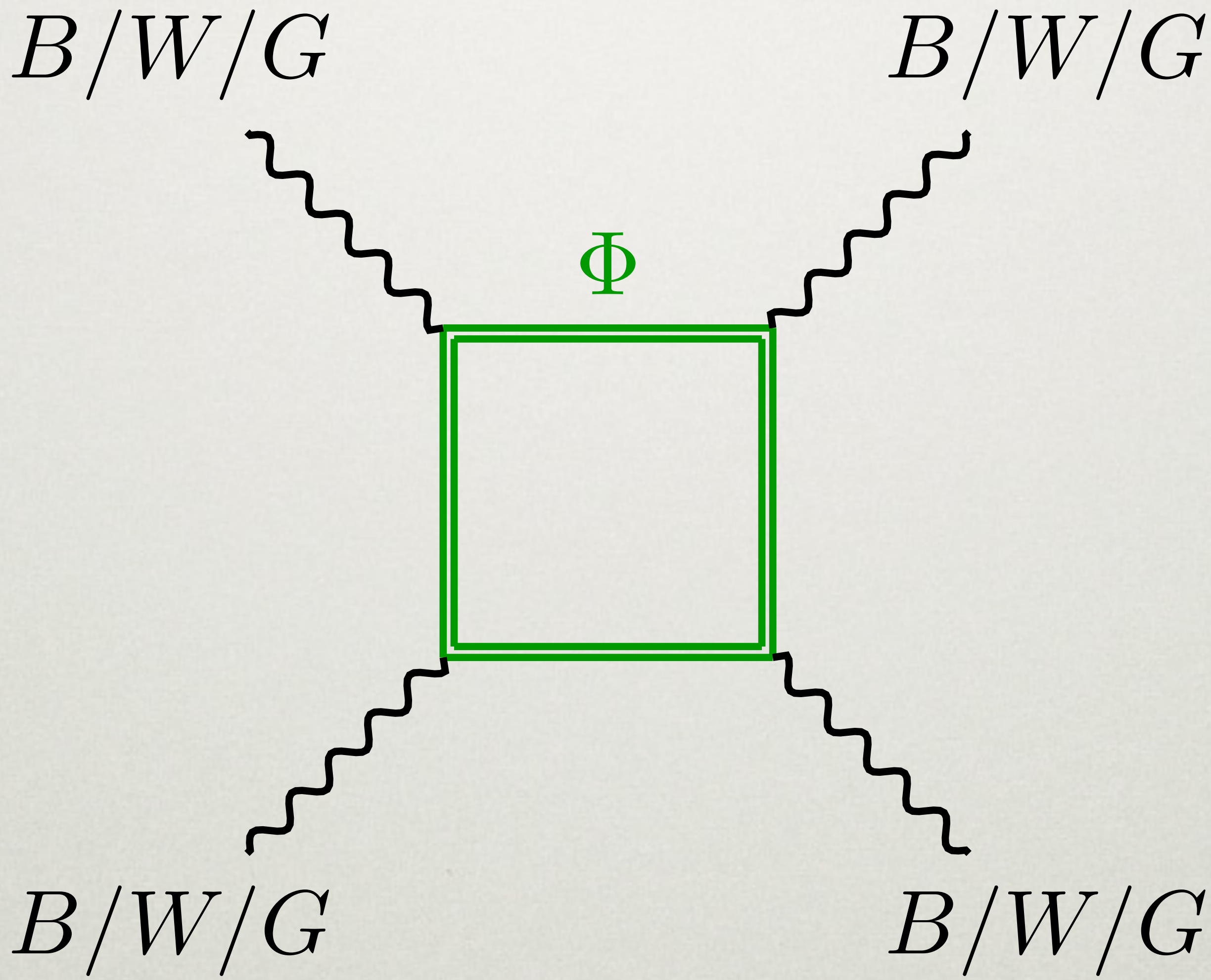
$$c^{e,1}_{mnpq} = \frac{\kappa^2}{2m^2}(4\delta_{mq}\delta_{np} - \delta_{mn}\delta_{pq})$$

Consistent with our bounds as

$$c^{e,1}_{\alpha\beta} = c^{e,1}_{mnpq}\rho^{\alpha}_{mq}\rho^{\beta}_{np} = \frac{\kappa^2}{2m^2}(4|\alpha|^2|\beta|^2 - |\alpha \cdot \beta|^2) > 0$$

UV Completions: Bosons

Imagine a scalar, fermion, or vector in an arbitrary rep of the SM



What if we get if we integrate it out?

UV Completions: Bosons

	scalar	fermion	vector
$c_1^{B^4}$	$\frac{7}{32}g_1^4Q^4$	$\frac{1}{2}g_1^4Q^4$	$\frac{261}{32}g_1^4Q^4$
$c_2^{B^4}$	$\frac{1}{32}g_1^4Q^4$	$\frac{7}{8}g_1^4Q^4$	$\frac{243}{32}g_1^4Q^4$
$c_1^{W^4}$	$g_2^4\left[\frac{7}{32}\Lambda(\mathbf{R}_2)+\frac{1}{48}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\frac{1}{2}\Lambda(\mathbf{R}_2)+\frac{1}{48}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\frac{261}{32}\Lambda(\mathbf{R}_2)-\frac{3}{16}I_2(\mathbf{R}_2)\right]$
$c_2^{W^4}$	$g_2^4\left[\frac{1}{32}\Lambda(\mathbf{R}_2)+\frac{1}{336}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\frac{7}{8}\Lambda(\mathbf{R}_2)+\frac{19}{336}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\frac{243}{32}\Lambda(\mathbf{R}_2)-\frac{27}{112}I_2(\mathbf{R}_2)\right]$
$c_3^{W^4}$	$g_2^4\left[\frac{7}{16}\Lambda(\mathbf{R}_2)-\frac{1}{48}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\Lambda(\mathbf{R}_2)-\frac{1}{48}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\frac{261}{16}\Lambda(\mathbf{R}_2)+\frac{3}{16}I_2(\mathbf{R}_2)\right]$
$c_4^{W^4}$	$g_2^4\left[\frac{1}{16}\Lambda(\mathbf{R}_2)-\frac{1}{336}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\frac{7}{4}\Lambda(\mathbf{R}_2)-\frac{19}{336}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\frac{243}{16}\Lambda(\mathbf{R}_2)+\frac{27}{112}I_2(\mathbf{R}_2)\right]$
$c_1^{G^4}$	$g_3^4\left[\frac{7}{32}\Lambda(\mathbf{R}_3)+\frac{1}{96}I_2(\mathbf{R}_3)\right]$	$g_3^4\left[\frac{1}{2}\Lambda(\mathbf{R}_3)+\frac{1}{96}I_2(\mathbf{R}_3)\right]$	$g_3^4\left[\frac{261}{32}\Lambda(\mathbf{R}_3)-\frac{3}{32}I_2(\mathbf{R}_3)\right]$
$c_2^{G^4}$	$g_3^4\left[\frac{1}{32}\Lambda(\mathbf{R}_3)+\frac{1}{672}I_2(\mathbf{R}_3)\right]$	$g_3^4\left[\frac{7}{8}\Lambda(\mathbf{R}_3)+\frac{19}{672}I_2(\mathbf{R}_3)\right]$	$g_3^4\left[\frac{243}{32}\Lambda(\mathbf{R}_3)-\frac{27}{224}I_2(\mathbf{R}_3)\right]$
$c_3^{G^4}$	$g_3^4\left[\frac{7}{16}\Lambda(\mathbf{R}_3)-\frac{1}{48}I_2(\mathbf{R}_3)\right]$	$g_3^4\left[\Lambda(\mathbf{R}_3)-\frac{1}{48}I_2(\mathbf{R}_3)\right]$	$g_3^4\left[\frac{261}{16}\Lambda(\mathbf{R}_3)+\frac{3}{16}I_2(\mathbf{R}_3)\right]$
$c_4^{G^4}$	$g_3^4\left[\frac{1}{16}\Lambda(\mathbf{R}_3)-\frac{1}{336}I_2(\mathbf{R}_3)\right]$	$g_3^4\left[\frac{7}{4}\Lambda(\mathbf{R}_3)-\frac{19}{336}I_2(\mathbf{R}_3)\right]$	$g_3^4\left[\frac{243}{16}\Lambda(\mathbf{R}_3)+\frac{27}{112}I_2(\mathbf{R}_3)\right]$
$c_5^{G^4}$	$\frac{1}{32}g_3^4I_2(\mathbf{R}_3)$	$\frac{1}{32}g_3^4I_2(\mathbf{R}_3)$	$-\frac{9}{32}g_3^4I_2(\mathbf{R}_3)$
$c_6^{G^4}$	$\frac{1}{224}g_3^4I_2(\mathbf{R}_3)$	$\frac{19}{224}g_3^4I_2(\mathbf{R}_3)$	$-\frac{81}{224}g_3^4I_2(\mathbf{R}_3)$
$c_1^{B^2W^2}$	$\frac{7}{16}g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$	$g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$	$\frac{261}{16}g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$
$c_2^{B^2W^2}$	$\frac{1}{16}g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$	$\frac{7}{4}g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$	$\frac{243}{16}g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$
$c_3^{B^2W^2}$	$\frac{7}{8}g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$	$2g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$	$\frac{261}{8}g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$
$c_4^{B^2W^2}$	$\frac{1}{8}g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$	$\frac{7}{2}g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$	$\frac{243}{8}g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$
$c_1^{B^2G^2}$	$\frac{7}{16}g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$	$g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$	$\frac{261}{16}g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$
$c_2^{B^2G^2}$	$\frac{1}{16}g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$	$\frac{7}{4}g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$	$\frac{243}{16}g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$
$c_3^{B^2G^2}$	$\frac{7}{8}g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$	$2g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$	$\frac{261}{8}g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$
$c_4^{B^2G^2}$	$\frac{1}{8}g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$	$\frac{7}{2}g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$	$\frac{243}{8}g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$
$c_1^{W^2G^2}$	$\frac{7}{16}g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$	$g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$	$\frac{261}{16}g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$
$c_2^{W^2G^2}$	$\frac{1}{16}g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$	$\frac{7}{4}g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$	$\frac{243}{16}g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$
$c_3^{W^2G^2}$	$\frac{7}{8}g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$	$2g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$	$\frac{261}{8}g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$
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$c_2^{BG^3}$	$\frac{1}{32}g_1g_3^3QI_3(\mathbf{R}_3)$	$\frac{7}{8}g_1g_3^3QI_3(\mathbf{R}_3)$	$\frac{243}{32}g_1g_3^3QI_3(\mathbf{R}_3)$

Coefficients from [Quevillon, Smith, Touati 1810.06994]

UV Completions: Bosons

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$c_2^{B^4}$	$\frac{1}{32}g_1^4Q^4$	$\frac{7}{8}g_1^4Q^4$	$\frac{243}{32}g_1^4Q^4$
$c_1^{W^4}$	$g_2^4\left[\frac{7}{32}\Lambda(\mathbf{R}_2)+\frac{1}{48}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\frac{1}{2}\Lambda(\mathbf{R}_2)+\frac{1}{48}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\frac{261}{32}\Lambda(\mathbf{R}_2)-\frac{3}{16}I_2(\mathbf{R}_2)\right]$
$c_2^{W^4}$	$g_2^4\left[\frac{1}{32}\Lambda(\mathbf{R}_2)+\frac{1}{336}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\frac{7}{8}\Lambda(\mathbf{R}_2)+\frac{19}{336}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\frac{243}{32}\Lambda(\mathbf{R}_2)-\frac{27}{112}I_2(\mathbf{R}_2)\right]$
$c_3^{W^4}$	$g_2^4\left[\frac{7}{16}\Lambda(\mathbf{R}_2)-\frac{1}{48}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\Lambda(\mathbf{R}_2)-\frac{1}{48}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\frac{261}{16}\Lambda(\mathbf{R}_2)+\frac{3}{16}I_2(\mathbf{R}_2)\right]$
$c_4^{W^4}$	$g_2^4\left[\frac{1}{16}\Lambda(\mathbf{R}_2)-\frac{1}{336}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\frac{7}{4}\Lambda(\mathbf{R}_2)-\frac{19}{336}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\frac{243}{16}\Lambda(\mathbf{R}_2)+\frac{27}{112}I_2(\mathbf{R}_2)\right]$
$c_1^{G^4}$	$g_3^4\left[\frac{7}{32}\Lambda(\mathbf{R}_3)+\frac{1}{96}I_2(\mathbf{R}_3)\right]$	$g_3^4\left[\frac{1}{2}\Lambda(\mathbf{R}_3)+\frac{1}{96}I_2(\mathbf{R}_3)\right]$	$g_3^4\left[\frac{261}{32}\Lambda(\mathbf{R}_3)-\frac{3}{32}I_2(\mathbf{R}_3)\right]$
$c_2^{G^4}$	$g_3^4\left[\frac{1}{32}\Lambda(\mathbf{R}_3)+\frac{1}{672}I_2(\mathbf{R}_3)\right]$	$g_3^4\left[\frac{7}{8}\Lambda(\mathbf{R}_3)+\frac{19}{672}I_2(\mathbf{R}_3)\right]$	$g_3^4\left[\frac{243}{32}\Lambda(\mathbf{R}_3)-\frac{27}{224}I_2(\mathbf{R}_3)\right]$
[illegible]	[illegible]	$\ldots(\mathbf{R}_3)-\frac{1}{48}I_2(\mathbf{R}_3)]$	$g_3^4\left[\frac{261}{16}\Lambda(\mathbf{R}_3)+\frac{3}{16}I_2(\mathbf{R}_3)\right]$
[illegible]	[illegible]	$\ldots(\mathbf{R}_3)-\frac{19}{336}I_2(\mathbf{R}_3)]$	$g_3^4\left[\frac{243}{16}\Lambda(\mathbf{R}_3)+\frac{27}{112}I_2(\mathbf{R}_3)\right]$
[illegible]	[illegible]	$\ldots\frac{1}{32}g_3^4I_2(\mathbf{R}_3)$	$-\frac{9}{32}g_3^4I_2(\mathbf{R}_3)$
[illegible]	[illegible]	$\ldots\frac{19}{224}g_3^4I_2(\mathbf{R}_3)$	$-\frac{81}{224}g_3^4I_2(\mathbf{R}_3)$
[illegible]	[illegible]	$\ldots g_2^2Q^2I_2(\mathbf{R}_2)$	$\frac{261}{16}g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$
[illegible]	[illegible]	$\ldots g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$	$\frac{243}{16}g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$
[illegible]	[illegible]	$\ldots g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$	$\frac{261}{8}g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$
[illegible]	[illegible]	$\ldots g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$	$\frac{243}{8}g_1^2g_2^2Q^2I_2(\mathbf{R}_2)$
[illegible]	[illegible]	$\ldots g_3^2Q^2I_2(\mathbf{R}_3)$	$\frac{261}{16}g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$
[illegible]	[illegible]	$\ldots g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$	$\frac{243}{16}g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$
[illegible]	[illegible]	$\ldots g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$	$\frac{261}{8}g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$
c_4	[illegible]	$\ldots\frac{}{2}g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$	$\frac{243}{8}g_1^2g_3^2Q^2I_2(\mathbf{R}_3)$
$c_1^{W^2G^2}$	$\frac{7}{16}g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$	$g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$	$\frac{261}{16}g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$
$c_2^{W^2G^2}$	$\frac{1}{16}g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$	$\frac{7}{4}g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$	$\frac{243}{16}g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$
$c_3^{W^2G^2}$	$\frac{7}{8}g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$	$2g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$	$\frac{261}{8}g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$
$c_4^{W^2G^2}$	$\frac{1}{8}g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$	$\frac{7}{2}g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$	$\frac{243}{8}g_2^2g_3^2I_2(\mathbf{R}_2)I_2(\mathbf{R}_3)$
$c_1^{BG^3}$	$\frac{7}{32}g_1g_3^3QI_3(\mathbf{R}_3)$	$\frac{1}{2}g_1g_3^3QI_3(\mathbf{R}_3)$	$\frac{261}{32}g_1g_3^3QI_3(\mathbf{R}_3)$
$c_2^{BG^3}$	$\frac{1}{32}g_1g_3^3QI_3(\mathbf{R}_3)$	$\frac{7}{8}g_1g_3^3QI_3(\mathbf{R}_3)$	$\frac{243}{32}g_1g_3^3QI_3(\mathbf{R}_3)$

U(1) charge

Group Theory Invariants

$$\mathrm{Tr}(T_{\mathbf{R}}^{(a}T_{\mathbf{R}}^{b)}) = I_2(\mathbf{R})\delta^{ab}$$

$$\mathrm{Tr}(T_{\mathbf{R}}^{(a}T_{\mathbf{R}}^{b}T_{\mathbf{R}}^{c)}) = \frac{1}{4}I_3(\mathbf{R})d^{abc}$$

$$\mathrm{Tr}(T_{\mathbf{R}}^{(a}T_{\mathbf{R}}^{b}T_{\mathbf{R}}^{c}T_{\mathbf{R}}^{d)}) = I_4(\mathbf{R})d^{abcd} + \Lambda(\mathbf{R})(\delta^{ab}\delta^{cd}+\delta^{ac}\delta^{bd}+\delta^{ad}\delta^{bc})$$

Coefficients from [Quevillon, Smith, Touati 1810.06994]

UV Completions: Bosons

	scalar	fermion	vector
$c_1^{B^4}$	$\frac{7}{32} g_1^4 Q^4$	$\frac{1}{2} g_1^4 Q^4$	$\frac{261}{32} g_1^4 Q^4$
$c_2^{B^4}$	$\frac{1}{32} g_1^4 Q^4$	$\frac{7}{8} g_1^4 Q^4$	$\frac{243}{32} g_1^4 Q^4$
$c_1^{W^4}$	$g_2^4 \left[\frac{7}{32}\Lambda(\mathbf{R}_2) + \frac{1}{48} I_2(\mathbf{R}_2)\right]$	$g_2^4 \left[\frac{1}{2}\Lambda(\mathbf{R}_2) + \frac{1}{48} I_2(\mathbf{R}_2)\right]$	$g_2^4 \left[\frac{261}{32}\Lambda(\mathbf{R}_2) - \frac{3}{16} I_2(\mathbf{R}_2)\right]$
$c_2^{W^4}$	$g_2^4 \left[\frac{1}{32}\Lambda(\mathbf{R}_2) + \frac{1}{336} I_2(\mathbf{R}_2)\right]$	$g_2^4 \left[\frac{7}{8}\Lambda(\mathbf{R}_2) + \frac{19}{336} I_2(\mathbf{R}_2)\right]$	$g_2^4 \left[\frac{243}{32}\Lambda(\mathbf{R}_2) - \frac{27}{112} I_2(\mathbf{R}_2)\right]$
$c_3^{W^4}$	$g_2^4 \left[\frac{7}{16}\Lambda(\mathbf{R}_2) - \frac{1}{48} I_2(\mathbf{R}_2)\right]$	$g_2^4 \left[\Lambda(\mathbf{R}_2) - \frac{1}{48} I_2(\mathbf{R}_2)\right]$	$g_2^4 \left[\frac{261}{16}\Lambda(\mathbf{R}_2) + \frac{3}{16} I_2(\mathbf{R}_2)\right]$
$c_4^{W^4}$	$g_2^4 \left[\frac{1}{16}\Lambda(\mathbf{R}_2) - \frac{1}{336} I_2(\mathbf{R}_2)\right]$	$g_2^4 \left[\frac{7}{4}\Lambda(\mathbf{R}_2) - \frac{19}{336} I_2(\mathbf{R}_2)\right]$	$g_2^4 \left[\frac{243}{16}\Lambda(\mathbf{R}_2) + \frac{27}{112} I_2(\mathbf{R}_2)\right]$
$c_1^{G^4}$	$g_3^4 \left[\frac{7}{32}\Lambda(\mathbf{R}_3) + \frac{1}{96} I_2(\mathbf{R}_3)\right]$	$g_3^4 \left[\frac{1}{2}\Lambda(\mathbf{R}_3) + \frac{1}{96} I_2(\mathbf{R}_3)\right]$	$g_3^4 \left[\frac{261}{32}\Lambda(\mathbf{R}_3) - \frac{3}{32} I_2(\mathbf{R}_3)\right]$
$c_2^{G^4}$	$g_3^4 \left[\frac{1}{32}\Lambda(\mathbf{R}_3) + \frac{1}{672} I_2(\mathbf{R}_3)\right]$	$g_3^4 \left[\frac{7}{8}\Lambda(\mathbf{R}_3) + \frac{19}{672} I_2(\mathbf{R}_3)\right]$	$g_3^4 \left[\frac{243}{32}\Lambda(\mathbf{R}_3) - \frac{27}{224} I_2(\mathbf{R}_3)\right]$
[illegible]	[illegible]	[illegible] $(\mathbf{R}_3) - \frac{1}{48} I_2(\mathbf{R}_3)]$	$g_3^4 \left[\frac{261}{16}\Lambda(\mathbf{R}_3) + \frac{3}{16} I_2(\mathbf{R}_3)\right]$
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible] $g_1^2 g_3^2 Q^2 I_2(\mathbf{R}_3)$	$\frac{261}{8} g_1^2 g_3^2 Q^2 I_2(\mathbf{R}_3)$
[illegible]	[illegible]	[illegible] $g_1^2 g_3^2 Q^2 I_2(\mathbf{R}_3)$	$\frac{243}{8} g_1^2 g_3^2 Q^2 I_2(\mathbf{R}_3)$
$c_1^{W^2G^2}$	$\frac{7}{16} g_2^2 g_3^2 I_2(\mathbf{R}_2) I_2(\mathbf{R}_3)$	$g_2^2 g_3^2 I_2(\mathbf{R}_2) I_2(\mathbf{R}_3)$	$\frac{261}{16} g_2^2 g_3^2 I_2(\mathbf{R}_2) I_2(\mathbf{R}_3)$
$c_2^{W^2G^2}$	$\frac{1}{16} g_2^2 g_3^2 I_2(\mathbf{R}_2) I_2(\mathbf{R}_3)$	$\frac{7}{4} g_2^2 g_3^2 I_2(\mathbf{R}_2) I_2(\mathbf{R}_3)$	$\frac{243}{16} g_2^2 g_3^2 I_2(\mathbf{R}_2) I_2(\mathbf{R}_3)$
$c_3^{W^2G^2}$	$\frac{7}{8} g_2^2 g_3^2 I_2(\mathbf{R}_2) I_2(\mathbf{R}_3)$	$2 g_2^2 g_3^2 I_2(\mathbf{R}_2) I_2(\mathbf{R}_3)$	$\frac{261}{8} g_2^2 g_3^2 I_2(\mathbf{R}_2) I_2(\mathbf{R}_3)$
$c_4^{W^2G^2}$	$\frac{1}{8} g_2^2 g_3^2 I_2(\mathbf{R}_2) I_2(\mathbf{R}_3)$	$\frac{7}{2} g_2^2 g_3^2 I_2(\mathbf{R}_2) I_2(\mathbf{R}_3)$	$\frac{243}{8} g_2^2 g_3^2 I_2(\mathbf{R}_2) I_2(\mathbf{R}_3)$
$c_1^{BG^3}$	$\frac{7}{32} g_1 g_3^3 Q I_3(\mathbf{R}_3)$	$\frac{1}{2} g_1 g_3^3 Q I_3(\mathbf{R}_3)$	$\frac{261}{32} g_1 g_3^3 Q I_3(\mathbf{R}_3)$
$c_2^{BG^3}$	$\frac{1}{32} g_1 g_3^3 Q I_3(\mathbf{R}_3)$	$\frac{7}{8} g_1 g_3^3 Q I_3(\mathbf{R}_3)$	$\frac{243}{32} g_1 g_3^3 Q I_3(\mathbf{R}_3)$

U(1) charge

Group Theor[illegible]

$$\mathrm{Tr}(T_{\mathbf{R}}^{(a} T_{\mathbf{R}}^{b)}) = I_2(\mathbf{R}$$ [illegible]

$$\mathrm{Tr}(T_{\mathbf{R}}^{(a} T_{\mathbf{R}}^{b} T_{\mathbf{R}}^{c)}) = \frac{1}{4} I_3($$ [illegible]

$$\mathrm{Tr}(T_{\mathbf{R}}^{(a} T_{\mathbf{R}}^{b} T_{\mathbf{R}}^{c} T_{\mathbf{R}}^{d)}) = I_4(\mathbf{R}$$ [illegible]

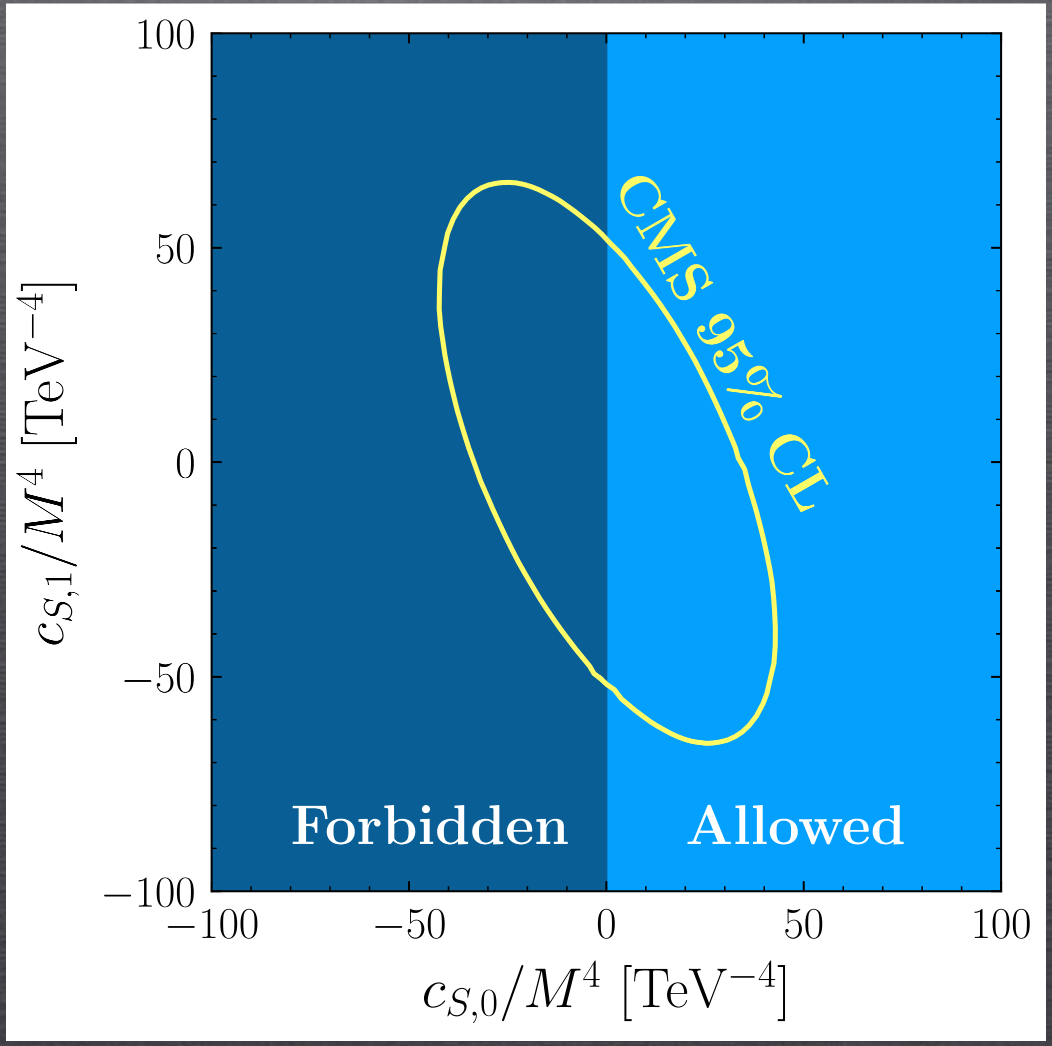
$$+ \Lambda(\mathbf{R})($$ [illegible]

Final Result

Bounds satisfied for any representation!

Coefficients from [Quevillon, Smith, Touati 1810.06994]

5. Phenomenology



Phenomenology: Strategy

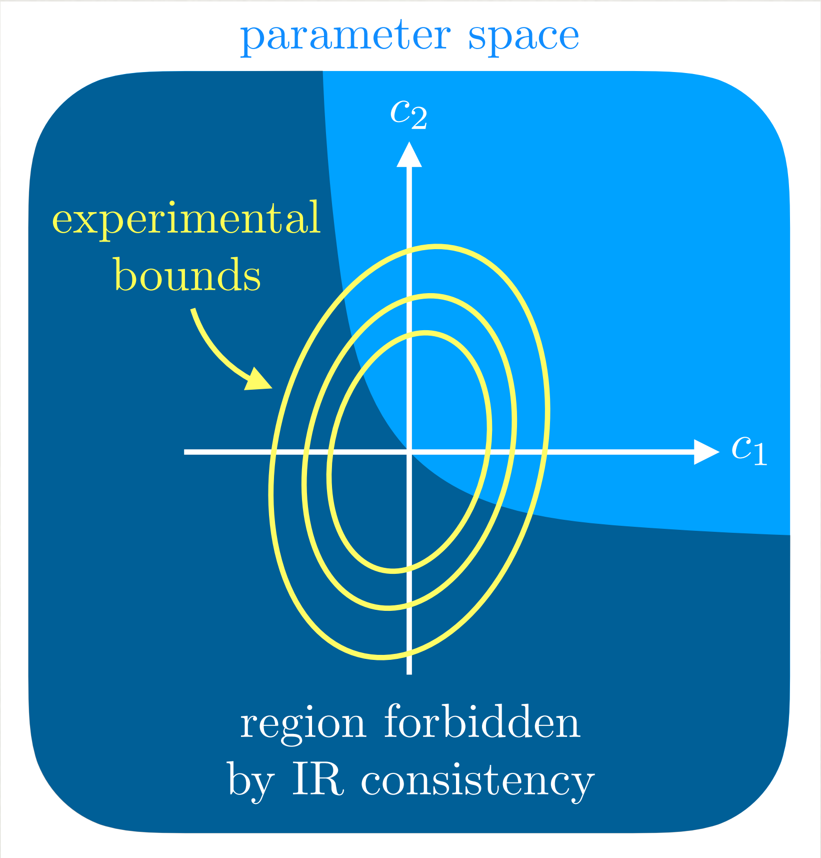
Two types of bounds*

*Caveat: all bounds are on dim-8 operators; generically harder to probe than dim-6

Phenomenology: Strategy

Two types of bounds

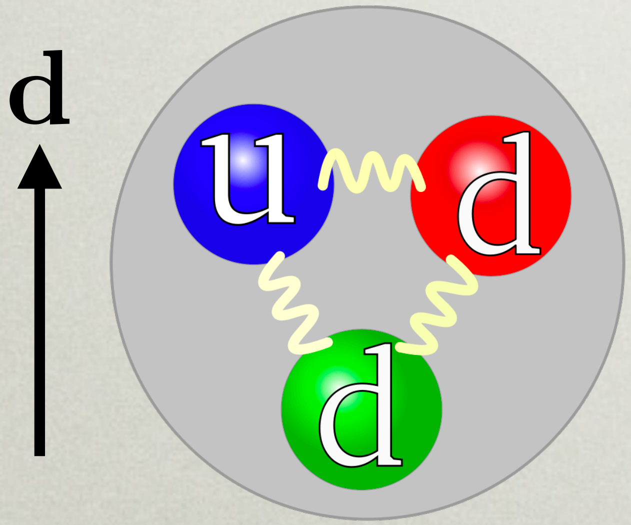
1. $c > 0$
   - Establish priors on the SMEFT
   - If sign measured: test unitarity, analyticity of UV theory



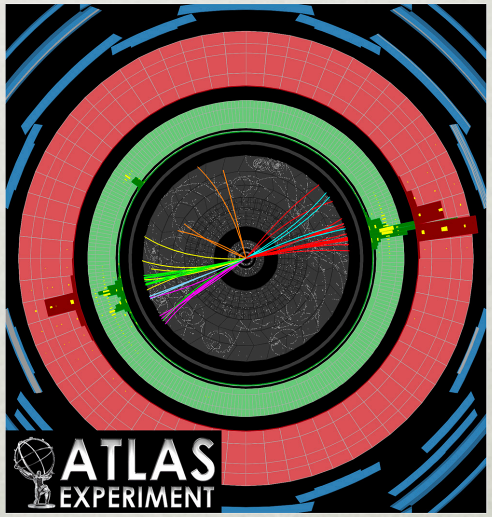
Phenomenology: Strategy

Two types of bounds

1. $c > 0$
    - Establish priors on the SMEFT
    - If sign measured: test unitarity, analyticity of UV theory
2. $c_1 c_2 > |\tilde{c}|^2$
    - Connects disparate experiments (CP & flavour)



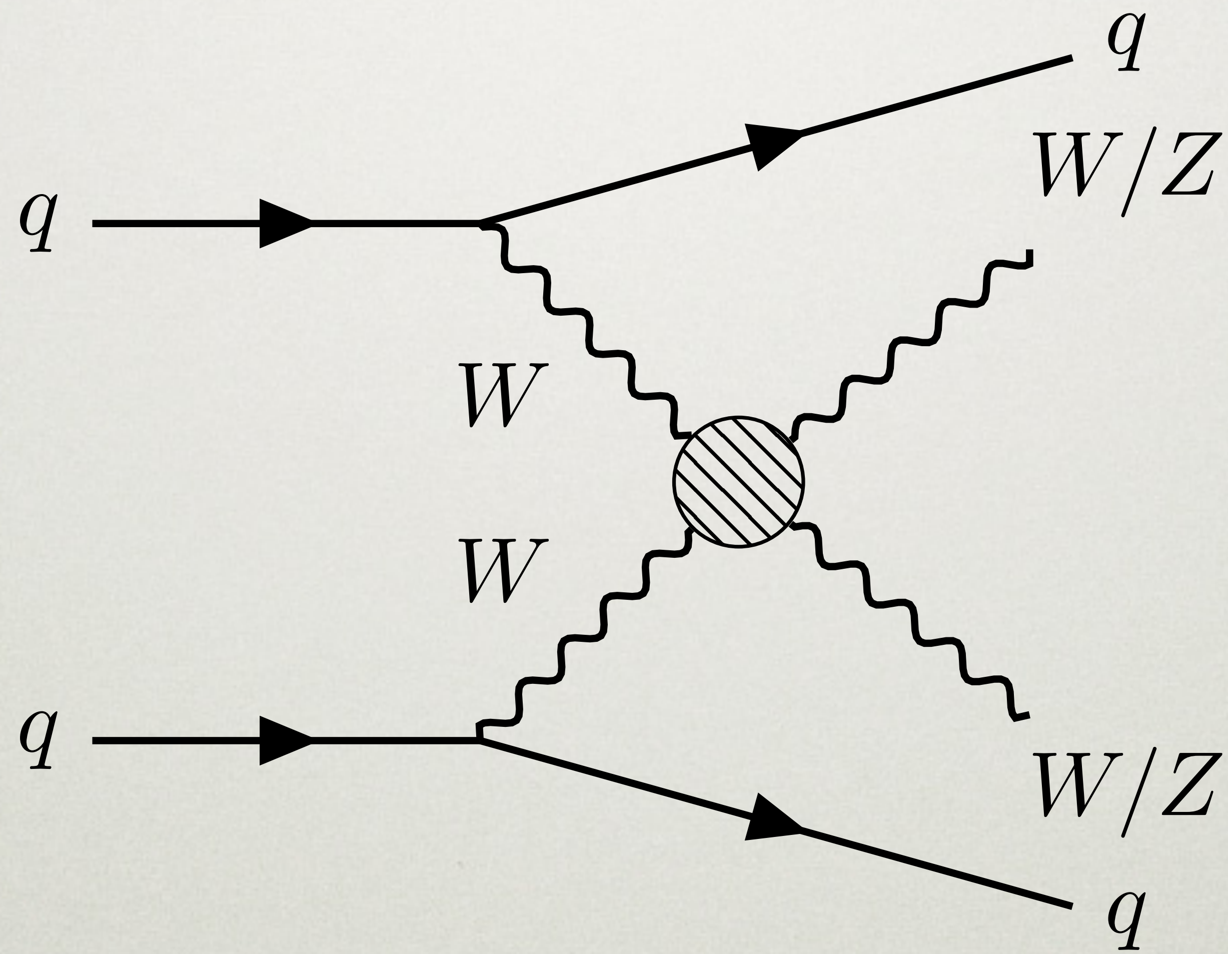
$\tilde{c}\, G^3 \tilde{G}$ $\Rightarrow$



$c\, G^4$

Phenomenology: aQGCs

Ongoing search for dim-8 ops is aQGCs (e.g. $WWWW$, $WWZZ$)



See for example [ATLAS 1906.03203], [CMS 1901.04060], [Green, Meade, Pleier 1610.07572]

Phenomenology: aQGCs

Start with basis of CP-even operators [Eboli+ hep-ph/0606118]

2 Scalar

$$\mathcal{O}_{S,0} = \left[(D_\mu\Phi)^\dagger D_\nu\Phi\right] \times \left[(D^\mu\Phi)^\dagger D^\nu\Phi\right] ,$$
$$\mathcal{O}_{S,1} = \left[(D_\mu\Phi)^\dagger D^\mu\Phi\right] \times \left[(D_\nu\Phi)^\dagger D^\nu\Phi\right] ,$$

8 Mixed

$$\mathcal{O}_{M,0} = \mathrm{Tr}\left[\widehat{W}_{\mu\nu}\widehat{W}^{\mu\nu}\right] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right] ,$$
$$\mathcal{O}_{M,1} = \mathrm{Tr}\left[\widehat{W}_{\mu\nu}\widehat{W}^{\nu\beta}\right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right] ,$$
$$\mathcal{O}_{M,2} = \left[\widehat{B}_{\mu\nu}\widehat{B}^{\mu\nu}\right] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right] ,$$
$$\mathcal{O}_{M,3} = \left[\widehat{B}_{\mu\nu}\widehat{B}^{\nu\beta}\right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right] ,$$
$$\mathcal{O}_{M,4} = \left[(D_\mu\Phi)^\dagger\widehat{W}_{\beta\nu}D^\mu\Phi\right] \times \widehat{B}^{\beta\nu} ,$$
$$\mathcal{O}_{M,5} = \left[(D_\mu\Phi)^\dagger\widehat{W}_{\beta\nu}D^\nu\Phi\right] \times \widehat{B}^{\beta\mu} ,$$
$$\mathcal{O}_{M,6} = \left[(D_\mu\Phi)^\dagger\widehat{W}_{\beta\nu}\widehat{W}^{\beta\nu}D^\mu\Phi\right] ,$$
$$\mathcal{O}_{M,7} = \left[(D_\mu\Phi)^\dagger\widehat{W}_{\beta\nu}\widehat{W}^{\beta\mu}D^\nu\Phi\right] .$$

10 Tensor

$$\mathcal{O}_{T,0} = \mathrm{Tr}\left[\widehat{W}_{\mu\nu}\widehat{W}^{\mu\nu}\right] \times \mathrm{Tr}\left[\widehat{W}_{\alpha\beta}\widehat{W}^{\alpha\beta}\right] ,$$
$$\mathcal{O}_{T,1} = \mathrm{Tr}\left[\widehat{W}_{\alpha\nu}\widehat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\widehat{W}_{\mu\beta}\widehat{W}^{\alpha\nu}\right] ,$$
$$\mathcal{O}_{T,2} = \mathrm{Tr}\left[\widehat{W}_{\alpha\mu}\widehat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\widehat{W}_{\beta\nu}\widehat{W}^{\nu\alpha}\right] ,$$
$$\mathcal{O}_{T,3} = \mathrm{Tr}\left[\widehat{W}_{\alpha\mu}\widehat{W}^{\mu\beta}\widehat{W}^{\nu\alpha}\right] \times \widehat{B}_{\beta\nu} ,$$
$$\mathcal{O}_{T,4} = \mathrm{Tr}\left[\widehat{W}_{\alpha\mu}\widehat{W}^{\alpha\mu}\widehat{W}^{\beta\nu}\right] \times \widehat{B}_{\beta\nu} ,$$
$$\mathcal{O}_{T,5} = \mathrm{Tr}\left[\widehat{W}_{\mu\nu}\widehat{W}^{\mu\nu}\right] \times \widehat{B}_{\alpha\beta}\widehat{B}^{\alpha\beta} ,$$
$$\mathcal{O}_{T,6} = \mathrm{Tr}\left[\widehat{W}_{\alpha\nu}\widehat{W}^{\mu\beta}\right] \times \widehat{B}_{\mu\beta}\widehat{B}^{\alpha\nu} ,$$
$$\mathcal{O}_{T,7} = \mathrm{Tr}\left[\widehat{W}_{\alpha\mu}\widehat{W}^{\mu\beta}\right] \times \widehat{B}_{\beta\nu}\widehat{B}^{\nu\alpha} ,$$
$$\mathcal{O}_{T,8} = \widehat{B}_{\mu\nu}\widehat{B}^{\mu\nu}\widehat{B}_{\alpha\beta}\widehat{B}^{\alpha\beta} ,$$
$$\mathcal{O}_{T,9} = \widehat{B}_{\alpha\mu}\widehat{B}^{\mu\beta}\widehat{B}_{\beta\nu}\widehat{B}^{\nu\alpha} ,$$

PHENOMENOLOGY: aQGCs

Corrected over time, see e.g. [Rauch 1610.08420]

3 Scalar

$$\mathcal{O}_{S,0} = \left[(D_\mu\Phi)^\dagger D_\nu\Phi\right] \times \left[(D^\mu\Phi)^\dagger D^\nu\Phi\right] ,$$
$$\mathcal{O}_{S,1} = \left[(D_\mu\Phi)^\dagger D^\mu\Phi\right] \times \left[(D_\nu\Phi)^\dagger D^\nu\Phi\right] ,$$
$$\mathcal{O}_{S,2} = \left[(D_\mu\Phi)^\dagger D_\nu\Phi\right] \times \left[(D^\nu\Phi)^\dagger D^\mu\Phi\right] ,$$

7 Mixed

$$\mathcal{O}_{M,0} = \mathrm{Tr}\left[\widehat{W}_{\mu\nu}\widehat{W}^{\mu\nu}\right] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right] ,$$
$$\mathcal{O}_{M,1} = \mathrm{Tr}\left[\widehat{W}_{\mu\nu}\widehat{W}^{\nu\beta}\right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right] ,$$
$$\mathcal{O}_{M,2} = \left[\widehat{B}_{\mu\nu}\widehat{B}^{\mu\nu}\right] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right] ,$$
$$\mathcal{O}_{M,3} = \left[\widehat{B}_{\mu\nu}\widehat{B}^{\nu\beta}\right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right] ,$$
$$\mathcal{O}_{M,4} = \left[(D_\mu\Phi)^\dagger\widehat{W}_{\beta\nu}D^\mu\Phi\right] \times \widehat{B}^{\beta\nu} ,$$
$$\mathcal{O}_{M,5} = \left[(D_\mu\Phi)^\dagger\widehat{W}_{\beta\nu}D^\nu\Phi\right] \times \widehat{B}^{\beta\mu} ,$$
$$\mathcal{O}_{M,7} = \left[(D_\mu\Phi)^\dagger\widehat{W}_{\beta\nu}\widehat{W}^{\beta\mu}D^\nu\Phi\right] .$$

8 Tensor

$$\mathcal{O}_{T,0} = \mathrm{Tr}\left[\widehat{W}_{\mu\nu}\widehat{W}^{\mu\nu}\right] \times \mathrm{Tr}\left[\widehat{W}_{\alpha\beta}\widehat{W}^{\alpha\beta}\right] ,$$
$$\mathcal{O}_{T,1} = \mathrm{Tr}\left[\widehat{W}_{\alpha\nu}\widehat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\widehat{W}_{\mu\beta}\widehat{W}^{\alpha\nu}\right] ,$$
$$\mathcal{O}_{T,2} = \mathrm{Tr}\left[\widehat{W}_{\alpha\mu}\widehat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\widehat{W}_{\beta\nu}\widehat{W}^{\nu\alpha}\right] ,$$
$$\mathcal{O}_{T,5} = \mathrm{Tr}\left[\widehat{W}_{\mu\nu}\widehat{W}^{\mu\nu}\right] \times \widehat{B}_{\alpha\beta}\widehat{B}^{\alpha\beta} ,$$
$$\mathcal{O}_{T,6} = \mathrm{Tr}\left[\widehat{W}_{\alpha\nu}\widehat{W}^{\mu\beta}\right] \times \widehat{B}_{\mu\beta}\widehat{B}^{\alpha\nu} ,$$
$$\mathcal{O}_{T,7} = \mathrm{Tr}\left[\widehat{W}_{\alpha\mu}\widehat{W}^{\mu\beta}\right] \times \widehat{B}_{\beta\nu}\widehat{B}^{\nu\alpha} ,$$
$$\mathcal{O}_{T,8} = \widehat{B}_{\mu\nu}\widehat{B}^{\mu\nu}\widehat{B}_{\alpha\beta}\widehat{B}^{\alpha\beta} ,$$
$$\mathcal{O}_{T,9} = \widehat{B}_{\alpha\mu}\widehat{B}^{\mu\beta}\widehat{B}_{\beta\nu}\widehat{B}^{\nu\alpha} ,$$

Phenomenology: aQGCs

Compared to our basis, *still missing operators!*

3 Scalar ✔

$$\mathcal{O}_{S,0} = \left[(D_\mu\Phi)^\dagger D_\nu\Phi\right] \times \left[(D^\mu\Phi)^\dagger D^\nu\Phi\right] ,$$
$$\mathcal{O}_{S,1} = \left[(D_\mu\Phi)^\dagger D^\mu\Phi\right] \times \left[(D_\nu\Phi)^\dagger D^\nu\Phi\right] ,$$
$$\mathcal{O}_{S,2} = \left[(D_\mu\Phi)^\dagger D_\nu\Phi\right] \times \left[(D^\nu\Phi)^\dagger D^\mu\Phi\right] ,$$

8 ~~7~~ Mixed

$$\mathcal{O}_{M,0} = \mathrm{Tr}\left[\widehat{W}_{\mu\nu}\widehat{W}^{\mu\nu}\right] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right] ,$$
$$\mathcal{O}_{M,1} = \mathrm{Tr}\left[\widehat{W}_{\mu\nu}\widehat{W}^{\nu\beta}\right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right] ,$$
$$\mathcal{O}_{M,2} = \left[\widehat{B}_{\mu\nu}\widehat{B}^{\mu\nu}\right] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right] ,$$
$$\mathcal{O}_{M,3} = \left[\widehat{B}_{\mu\nu}\widehat{B}^{\nu\beta}\right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right] ,$$
$$\mathcal{O}_{M,4} = \left[(D_\mu\Phi)^\dagger\widehat{W}_{\beta\nu}D^\mu\Phi\right] \times \widehat{B}^{\beta\nu} ,$$
$$\mathcal{O}_{M,5} = \left[(D_\mu\Phi)^\dagger\widehat{W}_{\beta\nu}D^\nu\Phi\right] \times \widehat{B}^{\beta\mu} ,$$
$$\mathcal{O}_{M,7} = \left[(D_\mu\Phi)^\dagger\widehat{W}_{\beta\nu}\widehat{W}^{\beta\mu}D^\nu\Phi\right] .$$

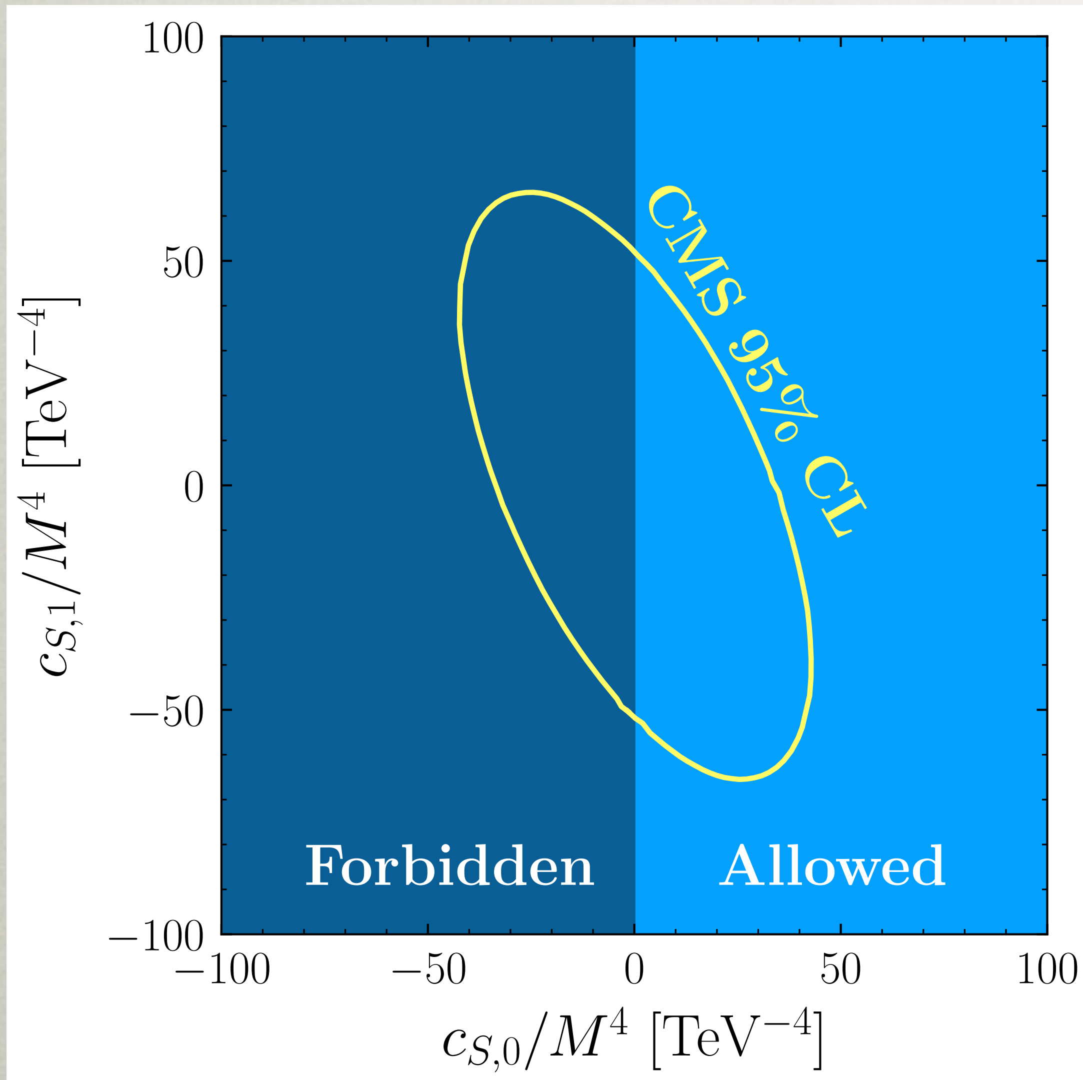
10 ~~8~~ Tensor

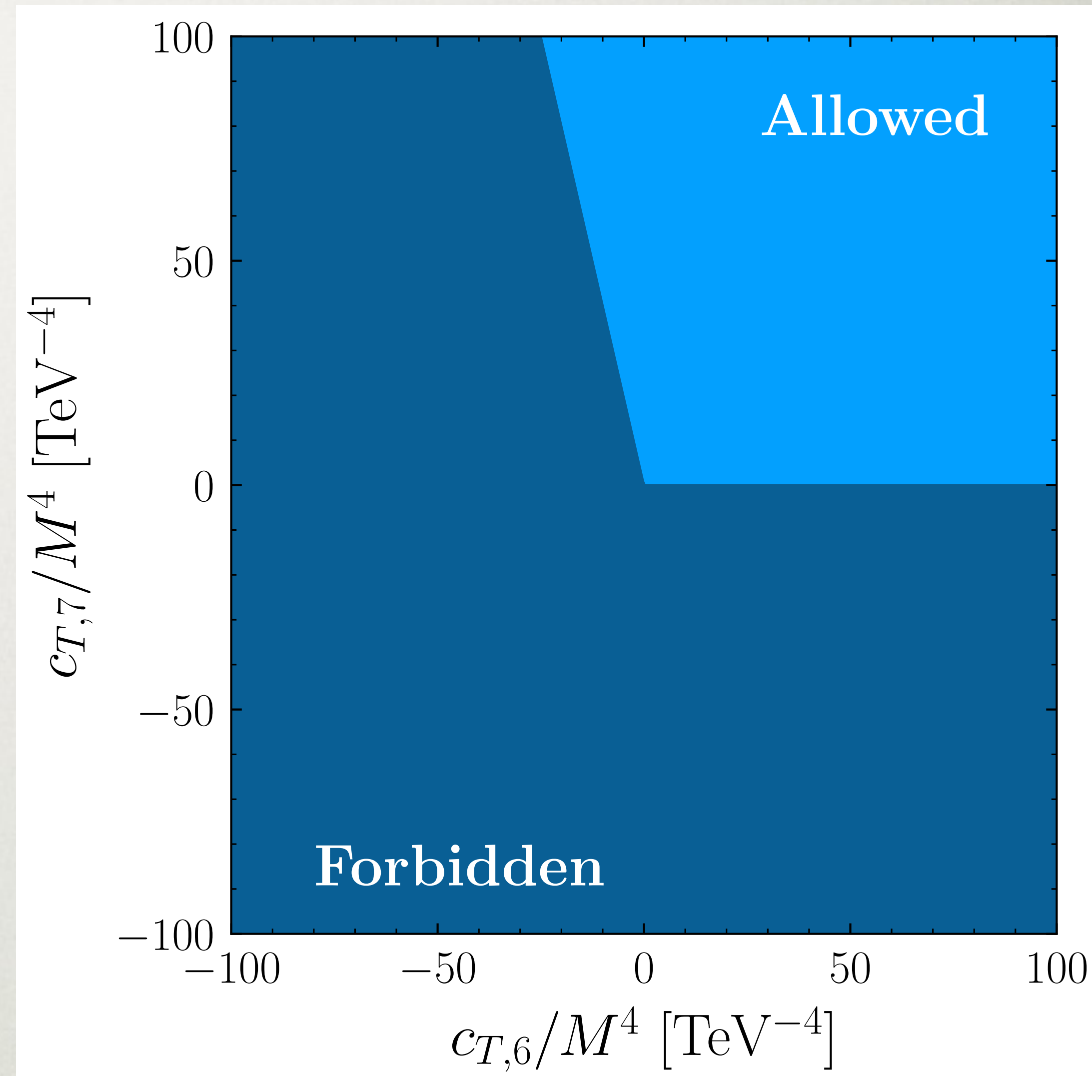
$$\mathcal{O}_{T,0} = \mathrm{Tr}\left[\widehat{W}_{\mu\nu}\widehat{W}^{\mu\nu}\right] \times \mathrm{Tr}\left[\widehat{W}_{\alpha\beta}\widehat{W}^{\alpha\beta}\right] ,$$
$$\mathcal{O}_{T,1} = \mathrm{Tr}\left[\widehat{W}_{\alpha\nu}\widehat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\widehat{W}_{\mu\beta}\widehat{W}^{\alpha\nu}\right] ,$$
$$\mathcal{O}_{T,2} = \mathrm{Tr}\left[\widehat{W}_{\alpha\mu}\widehat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\widehat{W}_{\beta\nu}\widehat{W}^{\nu\alpha}\right] ,$$
$$\mathcal{O}_{T,5} = \mathrm{Tr}\left[\widehat{W}_{\mu\nu}\widehat{W}^{\mu\nu}\right] \times \widehat{B}_{\alpha\beta}\widehat{B}^{\alpha\beta} ,$$
$$\mathcal{O}_{T,6} = \mathrm{Tr}\left[\widehat{W}_{\alpha\nu}\widehat{W}^{\mu\beta}\right] \times \widehat{B}_{\mu\beta}\widehat{B}^{\alpha\nu} ,$$
$$\mathcal{O}_{T,7} = \mathrm{Tr}\left[\widehat{W}_{\alpha\mu}\widehat{W}^{\mu\beta}\right] \times \widehat{B}_{\beta\nu}\widehat{B}^{\nu\alpha} ,$$
$$\mathcal{O}_{T,8} = \widehat{B}_{\mu\nu}\widehat{B}^{\mu\nu}\widehat{B}_{\alpha\beta}\widehat{B}^{\alpha\beta} ,$$
$$\mathcal{O}_{T,9} = \widehat{B}_{\alpha\mu}\widehat{B}^{\mu\beta}\widehat{B}_{\beta\nu}\widehat{B}^{\nu\alpha} ,$$

Phenomenology: aQGCs

Map our bounds onto the aQGC basis

Bounds from [CMS 1901.04060]





$$c_{S,0} > 0$$

$$\mathcal{O}_{S,0} = \left(D_\mu H^\dagger D_\nu H\right)\left(D^\mu H^\dagger D^\nu H\right)$$
$$\mathcal{O}_{S,1} = \left(D^\mu H^\dagger D_\mu H\right)\left(D^\nu H^\dagger D_\nu H\right)$$

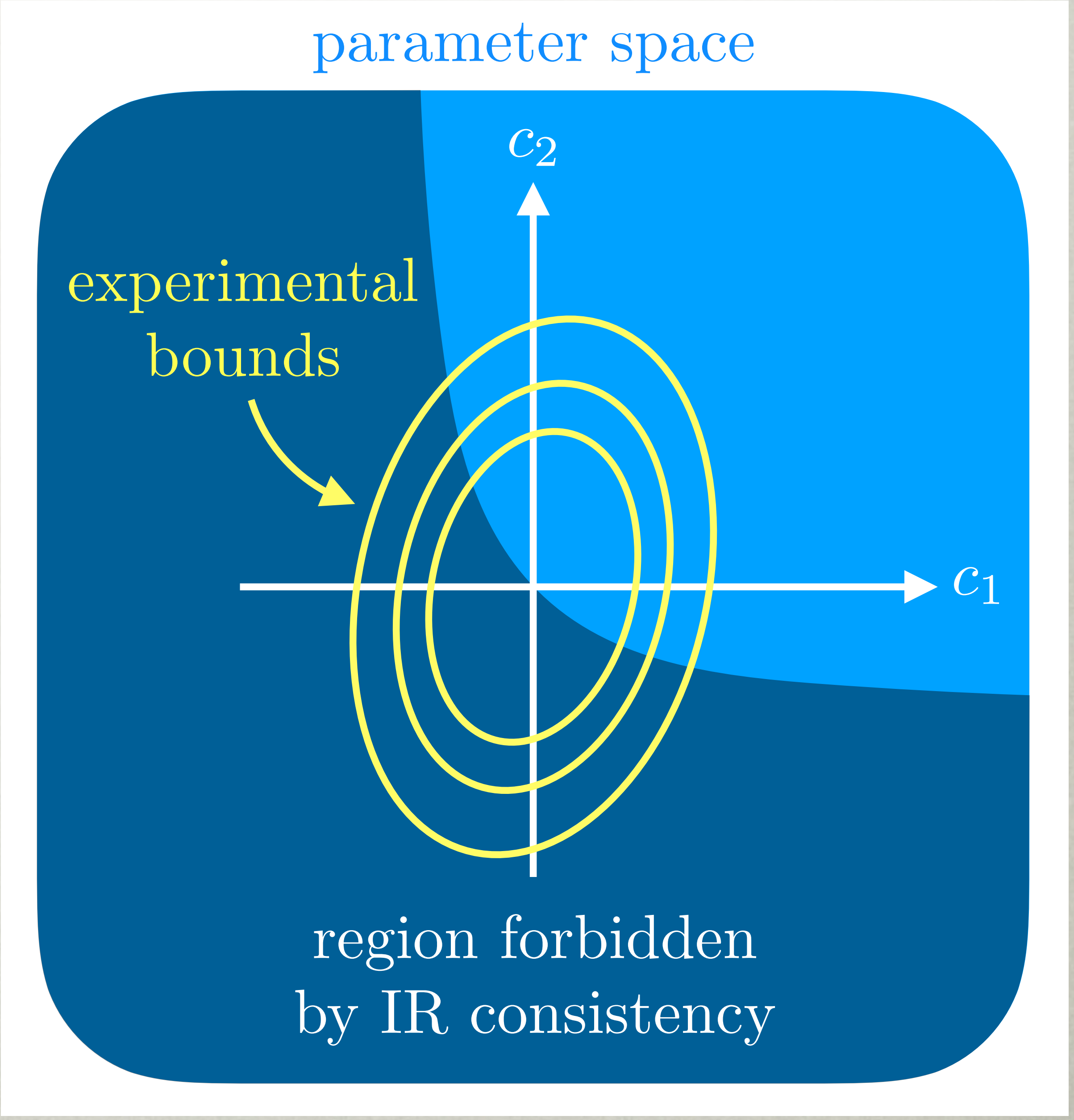
$$4c_{T,6} + c_{T,7} > 0$$
$$c_{T,7} > 0$$

$$\mathcal{O}_{T,6} = \frac{g_1^2 g_2^2}{8}\mathcal{O}_3^{B^2W^2}$$
$$\mathcal{O}_{T,7} = \frac{g_1^2 g_2^2}{32}\left(\mathcal{O}_1^{B^2W^2} + \mathcal{O}_3^{B^2W^2} + \mathcal{O}_4^{B^2W^2}\right)$$

Conclusion

Bedrock field theory principles constrain the SMEFT

- Many open directions
  - Superposition of representations (connection to B/L violation)
  - Detailed pheno studies
  - BSM extensions to the SMEFT
  - The story at dimension 6
  - Connection between causality and unitarity/analyticity
  - …



Backup Slides

Basis of SU(N) Operators

$\mathcal{O}_1^{F^4}$	$(F^a F^a)(F^b F^b)$
$\mathcal{O}_2^{F^4}$	$(F^a \widetilde{F}^a)(F^b \widetilde{F}^b)$
$\mathcal{O}_3^{F^4}$	$(F^a F^b)(F^a F^b)$
$\mathcal{O}_4^{F^4}$	$(F^a \widetilde{F}^b)(F^a \widetilde{F}^b)$
$\mathcal{O}_5^{F^4}$	$d^{abe} d^{cde} (F^a F^b)(F^c F^d)$
$\mathcal{O}_6^{F^4}$	$d^{abe} d^{cde} (F^a \widetilde{F}^b)(F^c \widetilde{F}^d)$
$\mathcal{O}_7^{F^4}$	$d^{ace} d^{bde} (F^a F^b)(F^c F^d)$
$\mathcal{O}_8^{F^4}$	$d^{ace} d^{bde} (F^a \widetilde{F}^b)(F^c \widetilde{F}^d)$
$\widetilde{\mathcal{O}}_1^{F^4}$	$(F^a F^a)(F^b \widetilde{F}^b)$
$\widetilde{\mathcal{O}}_2^{F^4}$	$(F^a F^b)(F^a \widetilde{F}^b)$
$\widetilde{\mathcal{O}}_3^{F^4}$	$d^{abe} d^{cde} (F^a F^b)(F^c \widetilde{F}^d)$
$\widetilde{\mathcal{O}}_4^{F^4}$	$d^{ace} d^{bde} (F^a F^b)(F^c \widetilde{F}^d)$

[Morozov 1984]

Dim-6 Operators

X^3		H^6 and H^4D^2		ψ^2H^3	
1) O_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	5) O_H	$(H^\dagger H)^3$	8) O_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$
2) $O_{\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	6) $O_{H\Box}$	$(H^\dagger H)\partial^2(H^\dagger H)$	9) O_{uH}	$(H^\dagger H)(\bar{Q}_p u_r \widetilde{H})$
3) O_W	$\varepsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	7) O_{HD}	$\left(H^\dagger D^\mu H\right)^\star\left(H^\dagger D_\mu H\right)$	10) O_{dH}	$(H^\dagger H)(\bar{Q}_p d_r H)$
4) $O_{\widetilde{W}}$	$\varepsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$				

X^2H^2		ψ^2XH		ψ^2H^2D	
11) O_{HG}	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$	19) O_{eW}	$(\bar{L}_p\sigma^{\mu\nu}e_r)\tau^I H W_{\mu\nu}^I$	27) $O_{Hl}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{L}_p\gamma^\mu L_r)$
12) $O_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	20) O_{eB}	$(\bar{L}_p\sigma^{\mu\nu}e_r)H B_{\mu\nu}$	28) $O_{Hl}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar{L}_p\tau^I\gamma^\mu L_r)$
13) O_{HW}	$H^\dagger H\, W_{\mu\nu}^I W^{I\mu\nu}$	21) O_{uG}	$(\bar{Q}_p\sigma^{\mu\nu}T^A u_r)\widetilde{H}\, G_{\mu\nu}^A$	29) O_{He}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}_p\gamma^\mu e_r)$
14) $O_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$	22) O_{uW}	$(\bar{Q}_p\sigma^{\mu\nu}u_r)\tau^I\widetilde{H}\, W_{\mu\nu}^I$	30) $O_{Hq}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{Q}_p\gamma^\mu Q_r)$
15) O_{HB}	$H^\dagger H\, B_{\mu\nu}B^{\mu\nu}$	23) O_{uB}	$(\bar{Q}_p\sigma^{\mu\nu}u_r)\widetilde{H}\, B_{\mu\nu}$	31) $O_{Hq}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar{Q}_p\tau^I\gamma^\mu Q_r)$
16) $O_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu}B^{\mu\nu}$	24) O_{dG}	$(\bar{Q}_p\sigma^{\mu\nu}T^A d_r)H\, G_{\mu\nu}^A$	32) O_{Hu}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}_p\gamma^\mu u_r)$
17) O_{HWB}	$H^\dagger\tau^I H\, W_{\mu\nu}^I B^{\mu\nu}$	25) O_{dW}	$(\bar{Q}_p\sigma^{\mu\nu}d_r)\tau^I H\, W_{\mu\nu}^I$	33) O_{Hd}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}_p\gamma^\mu d_r)$
18) $O_{H\widetilde{W}B}$	$H^\dagger\tau^I H\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$	26) O_{dB}	$(\bar{Q}_p\sigma^{\mu\nu}d_r)H\, B_{\mu\nu}$	34) O_{Hud}	$(\widetilde{H}^\dagger iD_\mu H)(\bar{u}_p\gamma^\mu d_r)$

[Grzadkowski, Iskrzynski, Misiak, Rosiek 1008.4884]

Dim-6 Operators

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
35) O_{ll}	$(\bar{L}_p\gamma_\mu L_r)(\bar{L}_s\gamma^\mu L_t)$	40) O_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	47) O_{le}	$(\bar{L}_p\gamma_\mu L_r)(\bar{e}_s\gamma^\mu e_t)$
36) $O_{qq}^{(1)}$	$(\bar{Q}_p\gamma_\mu Q_r)(\bar{Q}_s\gamma^\mu Q_t)$	41) O_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	48) O_{lu}	$(\bar{L}_p\gamma_\mu L_r)(\bar{u}_s\gamma^\mu u_t)$
37) $O_{qq}^{(3)}$	$(\bar{Q}_p\gamma_\mu\tau^I Q_r)(\bar{Q}_s\gamma^\mu\tau^I Q_t)$	42) O_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	49) O_{ld}	$(\bar{L}_p\gamma_\mu L_r)(\bar{d}_s\gamma^\mu d_t)$
38) $O_{lq}^{(1)}$	$(\bar{L}_p\gamma_\mu L_r)(\bar{Q}_s\gamma^\mu Q_t)$	43) O_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	50) O_{qe}	$(\bar{Q}_p\gamma_\mu Q_r)(\bar{e}_s\gamma^\mu e_t)$
39) $O_{lq}^{(3)}$	$(\bar{L}_p\gamma_\mu\tau^I L_r)(\bar{Q}_s\gamma^\mu\tau^I Q_t)$	44) O_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	51) $O_{qu}^{(1)}$	$(\bar{Q}_p\gamma_\mu Q_r)(\bar{u}_s\gamma^\mu u_t)$
		45) $O_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	52) $O_{qu}^{(8)}$	$(\bar{Q}_p\gamma_\mu T^A Q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		46) $O_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	53) $O_{qd}^{(1)}$	$(\bar{Q}_p\gamma_\mu Q_r)(\bar{d}_s\gamma^\mu d_t)$
				54) $O_{qd}^{(8)}$	$(\bar{Q}_p\gamma_\mu T^A Q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$	
55) O_{ledq}	$(\bar{L}_p^j e_r)(\bar{d}_s Q_t^j)$
56) $O_{quqd}^{(1)}$	$(\bar{Q}_p^j u_r)\varepsilon_{jk}(\bar{Q}_s^k d_t)$
57) $O_{quqd}^{(8)}$	$(\bar{Q}_p^j T^A u_r)\varepsilon_{jk}(\bar{Q}_s^k T^A d_t)$
58) $O_{lequ}^{(1)}$	$(\bar{L}_p^j e_r)\varepsilon_{jk}(\bar{Q}_s^k u_t)$
59) $O_{lequ}^{(3)}$	$(\bar{L}_p^j\sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{Q}_s^k\sigma^{\mu\nu} u_t)$

[Grzadkowski, Iskrzynski, Misiak, Rosiek 1008.4884]

UV Completions: Bosons

Revisit our simple U(1) theory

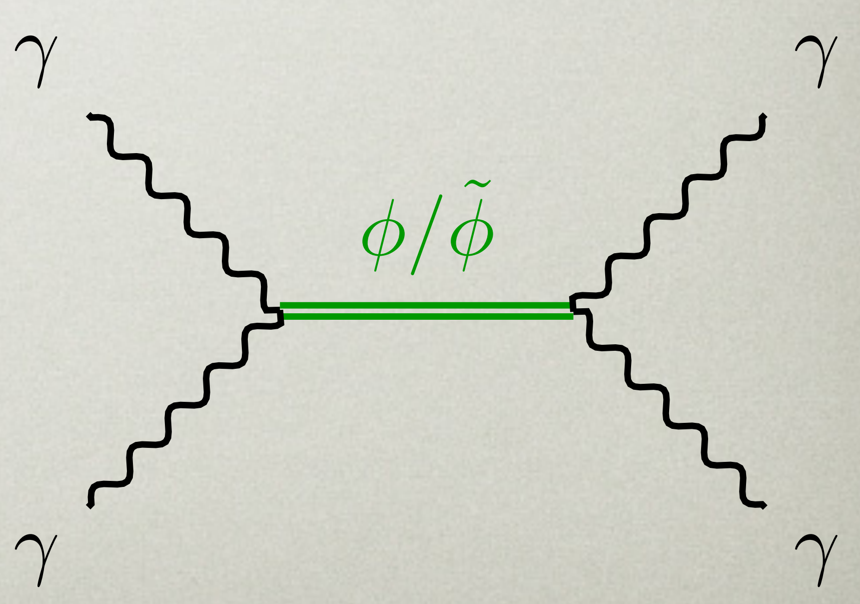
$$\Delta\mathcal{L} = \frac{1}{M^4}\left[c_1(FF)^2 + c_2(F\tilde{F})^2 + \tilde{c}(FF)(F\tilde{F})\right]$$

Bounds imply we can rewrite this as a sum of squares ($\alpha, \beta, \gamma \in \mathbb{R}$)

$$= \frac{\alpha^2}{2M^4}\left[[(FF) + \beta(F\tilde{F})]^2 + \gamma^2[(FF) - \beta(F\tilde{F})]^2\right]$$

Suggests we can complete with a mixed axion & dilaton

$$\Delta\mathcal{L} \to -\frac{M^2}{2}(\phi + a)^2 - \frac{M^2}{2\gamma^2}(\phi - a)^2$$
$$+ \frac{2\alpha}{M}\phi(FF) + \frac{2\alpha\beta}{M}a(F\tilde{F})$$



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Mixing controls CP Violation

$$\tilde{c} \propto (1 - \gamma^2)$$