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# Which EFT

Davis, 27 August 2018

**Based on unfinished work with Riccardo Rattazzi**

# Setting the stage

- Currently, there are no robust indications of the existence of new particles beyond the Standard Model within the reach of current or future colliders

- For the sake of this talk I will assume that there indeed aren't any new light particles

- On the other hand, we know for sure that new physics beyond the Standard Model does exist (because neutrino masses, dark matter, baryogenesis, inflation, gravity)

- These assumptions + experimental facts imply that the microscopic theory at E~1 TeV is a (relativistic) effective theory with the Standard Model particle content

# EFT ladder

| Energy Scale | Theory | Particles | Expansion Parameter |
|---|---|---|---|
| 1 TeV | Which EFT? |  | what expansion? |
| 100 GeV<br>10 GeV | WEFT<br>(aka Fermi theory, LEFT, WET) | γ,g,ν,e,μ,τ + $N_f$ q | $\frac{E}{m_W}$ |
| 1 GeV<br>100 MeV | $\chi$PT | γ,ν,e,μ,π,K | $\frac{E}{4\pi f_\pi}$ |
| 10 MeV | eQED | γ,ν,e | $\frac{E}{m_\mu}$ |
| 1 MeV | Euler-Heisenberg | γ,ν | $\frac{E}{m_e}$ |

# Open question

- The main practical questions is whether the particle interactions in the EFT are sufficiently different from those in the Standard Model so as to be observable in any of the current or near-future experiments

- However, there remains one  outstanding theoretical question

- Namely, whether electroweak symmetry is realized
  linearly or non-linearly in the EFT Lagrangian

# Linear vs non-linear

**Two mathematical formulations for effective theories with SM spectrum**

| **Linear** | **Non-linear** |
|---|---|
| $SU(3)_C \times SU(2)_L \times U(1)_Y$ | $SU(3)_C \times U(1)_{em}$ |

$$H \to LH, \qquad L \in SU(2)_L$$

$$H = \begin{pmatrix} iG_+ \\ \frac{v+h-iG_z}{\sqrt{2}} \end{pmatrix}$$

$$U \to g_L U g_Y^\dagger, \qquad h \to h$$

$$U = \exp\left(\frac{iG^a\sigma^a}{\mathrm{v}}\right)$$

125 GeV Higgs boson

Goldstone bosons eaten by W and Z

**In general, the two formulation lead to two physically distinct theories**

| **SMEFT** | **HEFT** |
|---|---|

# SMEFT

$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{\mathrm{SM}} + \frac{1}{\Lambda_L}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda_L^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \ldots$$

**Known SM Lagrangian**

**Higher-dimensional $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant interactions added to the SM**

**Dimensionful expansion parameter interpreted as mass scale of new physics**

$$1\ \mathrm{TeV} \lesssim \Lambda \lesssim ?$$

**Dimensionful expansion parameter for B-L violating interactions**

$$\Lambda_L \sim 10^{15}\ \mathrm{GeV}$$

# SMEFT

Buchmuller,Wyler (1986)

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \ldots$$

**Known SM Lagrangian**

**Higher-dimensional $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant interactions added to the SM**

**Dimensionful expansion parameter interpreted as mass scale of new physics**

$$1\ \text{TeV} \lesssim \Lambda \lesssim ?$$

**In the following for simplicity we set** $\Lambda_L \to \infty$

**For $\Lambda >> v$ expansion can be truncated at dimension-6 level for most practical applications**

# Dimension-6 operators

Grządkowski et al.
1008.4884

| Bosonic CP-even | | Bosonic CP-odd | |
|---|---|---|---|
| $O_H$ | $(H^\dagger H)^3$ | | |
| $O_{H\Box}$ | $(H^\dagger H)\Box(H^\dagger H)$ | | |
| $O_{HD}$ | $\left\|H^\dagger D_\mu H\right\|^2$ | | |
| $O_{HG}$ | $H^\dagger H\, G^a_{\mu\nu} G^a_{\mu\nu}$ | $O_{H\widetilde{G}}$ | $H^\dagger H\, \widetilde{G}^a_{\mu\nu} G^a_{\mu\nu}$ |
| $O_{HW}$ | $H^\dagger H\, W^i_{\mu\nu} W^i_{\mu\nu}$ | $O_{H\widetilde{W}}$ | $H^\dagger H\, \widetilde{W}^i_{\mu\nu} W^i_{\mu\nu}$ |
| $O_{HB}$ | $H^\dagger H\, B_{\mu\nu} B_{\mu\nu}$ | $O_{H\widetilde{B}}$ | $H^\dagger H\, \widetilde{B}_{\mu\nu} B_{\mu\nu}$ |
| $O_{HWB}$ | $H^\dagger \sigma^i H\, W^i_{\mu\nu} B_{\mu\nu}$ | $O_{H\widetilde{W}B}$ | $H^\dagger \sigma^i H\, \widetilde{W}^i_{\mu\nu} B_{\mu\nu}$ |
| $O_W$ | $\epsilon^{ijk} W^i_{\mu\nu} W^j_{\nu\rho} W^k_{\rho\mu}$ | $O_{\widetilde{W}}$ | $\epsilon^{ijk} \widetilde{W}^i_{\mu\nu} W^j_{\nu\rho} W^k_{\rho\mu}$ |
| $O_G$ | $f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$ | $O_{\widetilde{G}}$ | $f^{abc} \widetilde{G}^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$ |

Table 2.2: Bosonic $D$=6 operators in the Warsaw basis.

**+ 2-fermion and 4-fermion operators**

$$H = \frac{1}{\sqrt{2}}\left( \begin{array}{c} \cdots \\ v + h(x) + \ldots \end{array} \right)$$

for review see e.g. LHCHXSWG 1610.07922

# HEFT

**Introduce triplet of Goldstone field ϕ via unitary matrix U:**

$$U = \exp\left(\frac{iG^a\sigma^a}{\mathrm{v}}\right)$$

**Linear transformation of U under $SU(2)_L \times U(1)_Y$ implies electroweak symmetry acts non-linearly on ϕ:**

$$U \to g_L U g_Y^\dagger, \qquad h \to h$$

**Higgs boson is perfect singlet under electroweak symmetry!**

**Lagrangian organized in derivative expansion:**

$$\mathscr{L}_{\mathrm{HEFT}} = \mathscr{L}_2 + \mathscr{L}_4 + \mathscr{L}_6 + \ldots$$

**Arbitrary polynomial of h allowed to multiply each term in $\mathscr{L}_{\mathrm{HEFT}}$ !**

$$\mathcal{L}_2 = -\frac{1}{2}\langle G_{\mu\nu}G^{\mu\nu}\rangle - \frac{1}{2}\langle W_{\mu\nu}W^{\mu\nu}\rangle - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \sum_{\psi=q_L,l_L,u_R,d_R,e_R}\bar{\psi}i\not{D}\psi$$
$$+\frac{v^2}{4}\langle D_\mu U^\dagger D^\mu U\rangle\left(1+F_U(h)\right) + \frac{1}{2}\partial_\mu h\partial^\mu h - V(h)$$
$$-v\left[\bar{q}_L\left(Y_u + \sum_{n=1}^{\infty}Y_u^{(n)}\left(\frac{h}{v}\right)^n\right)UP_+q_R + \bar{q}_L\left(Y_d + \sum_{n=1}^{\infty}Y_d^{(n)}\left(\frac{h}{v}\right)^n\right)UP_-q_R\right.$$
$$\left.+\bar{l}_L\left(Y_e + \sum_{n=1}^{\infty}Y_e^{(n)}\left(\frac{h}{v}\right)^n\right)UP_-l_R + \mathrm{h.c.}\right]$$

$$\mathcal{L}_4 = a_1 g' g\langle UT_3B_{\mu\nu}U^\dagger W^{\mu\nu}\rangle + ia_2g'\langle UT_3B_{\mu\nu}U^\dagger[V^\mu,V^\nu]\rangle - ia_3g\langle W_{\mu\nu}[V^\mu,V^\nu]\rangle$$
$$+a_4\langle V_\mu V_\nu\rangle\langle V^\mu V^\nu\rangle + a_5\langle V_\mu V^\mu\rangle\langle V_\nu V^\nu\rangle + \frac{e^2}{16\pi^2}c_{\gamma\gamma}\frac{h}{v}F_{\mu\nu}F^{\mu\nu} + \frac{g^{hh}}{v^4}(\partial_\mu h\partial^\mu h)^2$$
$$+\frac{d^{hh}}{v^2}(\partial_\mu h\partial^\mu h)\langle D_\nu U^\dagger D^\nu U\rangle + \frac{e^{hh}}{v^2}(\partial_\mu h\partial^\nu h)\langle D^\mu U^\dagger D_\nu U\rangle + \ldots$$

$$D_\mu U = \partial_\mu U + igW_\mu U - ig'B_\mu UT_3,$$

$$F_U(h) = \sum_{n=1}^{\infty}f_{U,n}\left(\frac{h}{v}\right)^n, \qquad V(h) = v^4\sum_{n=2}^{\infty}f_{V,n}\left(\frac{h}{v}\right)^n$$

# SMEFT vs HEFT - Higgs self-couplings

**In SM self-coupling completely fixed...**

$$\mathscr{L}_{\rm SM} \supset m^2|H|^2 - \lambda|H|^4$$

$$\to -\frac{1}{2}m_h^2 h^2 - \frac{m_h^2}{2{\rm v}}h^3 - \frac{m_h^2}{8{\rm v}^2}h^4$$

**...but they can be deformed by BSM effects**

## SMEFT: D=6

$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{\rm SM} - \frac{c_6}{\Lambda^2}(H^\dagger H)^3$$

$$\mathscr{L}_{\rm SMEFT} \supset -\frac{m_h^2}{2{\rm v}}(1+\delta\lambda_3)h^3 - \frac{m_h^2}{8{\rm v}^2}(1+\delta\lambda_4)h^4 - \frac{\lambda_5}{{\rm v}}h^5 - \frac{\lambda_6}{{\rm v}^2}h^6$$

$$\delta\lambda_3 = \frac{2c_6{\rm v}^4}{m_h^2\Lambda^2},\ \delta\lambda_4 = \frac{12c_6{\rm v}^4}{m_h^2\Lambda^2},\ \lambda_5 = \frac{3c_6{\rm v}^2}{4\Lambda^2},\ \lambda_6 = \frac{c_6{\rm v}^2}{8\Lambda^2}$$

**(Truncated) SMEFT: Predicts correlations**

## HEFT

$$\mathscr{L}_{\rm HEFT} \supset -c_3\frac{m_h^2}{2{\rm v}}h^3 - c_4\frac{m_h^2}{8{\rm v}^2} - \frac{c_5}{{\rm v}}h^5 - \frac{c_6}{{\rm v}^2}h^6 + \ldots$$

**HEFT: in general no correlations**

# SMEFT vs HEFT - Higgs couplings to matter

$$\mathcal{L}_{\rm EFT} \supset \sqrt{g_L^2+g_Y^2}\left[\left(1+\delta g_L^{Ze}\right) Z_\mu \bar{e}_L \gamma_\mu e_L + \left(1+\delta g_R^{Ze}\right) Z_\mu \bar{e}_R \gamma_\mu e_R + \ldots\right]$$
$$+\frac{1}{v}\left[(d_{Ae} A_{\mu\nu} \bar{e}_L \sigma_{\mu\nu} e_R + {\rm h.c.}) + (d_{Ze} Z_{\mu\nu} \bar{e}_L \sigma_{\mu\nu} e_R + {\rm h.c.}) + \ldots\right]$$

$$\mathcal{L}_{h,\rm EFT} \supset \frac{h}{v}\sqrt{g_L^2+g_Y^2}\left[\delta \tilde{g}_L^{Ze} Z_\mu \bar{e}_L \gamma_\mu e_L + \delta \tilde{g}_R^{Ze} Z_\mu \bar{e}_R \gamma_\mu e_R + \ldots\right]$$
$$+\frac{h}{v^2}\left[(\tilde{d}_{Ae} A_{\mu\nu} \bar{e}_L \sigma_{\mu\nu} e_R + {\rm h.c.}) + (\tilde{d}_{Ze} Z_{\mu\nu} \bar{e}_L \sigma_{\mu\nu} e_R + {\rm h.c.}) + \ldots\right]$$

**Vff and hVff couplings correlated in SMEFT but not in HEFT**

$$\mathcal{L}_{\rm hvv} = \frac{h}{v}[2(1+\delta c_w) m_W^2 W_\mu^+ W_\mu^- + (1+\delta c_z) m_Z^2 Z_\mu Z_\mu$$
$$+ c_{ww}\frac{g_L^2}{2} W_{\mu\nu}^+ W_{\mu\nu}^- + \tilde{c}_{ww}\frac{g_L^2}{2} W_{\mu\nu}^+ \tilde{W}_{\mu\nu}^- + c_{w\Box} g_L^2 \left(W_\mu^- \partial_\nu W_{\mu\nu}^+ + {\rm h.c.}\right)$$
$$+ c_{gg}\frac{g_s^2}{4} G_{\mu\nu}^a G_{\mu\nu}^a + c_{\gamma\gamma}\frac{e^2}{4} A_{\mu\nu} A_{\mu\nu} + c_{z\gamma}\frac{e g_L}{2 c_\theta} Z_{\mu\nu} A_{\mu\nu} + c_{zz}\frac{g_L^2}{4 c_\theta^2} Z_{\mu\nu} Z_{\mu\nu}$$
$$+ c_{z\Box} g_L^2 Z_\mu \partial_\nu Z_{\mu\nu} + c_{\gamma\Box} g_L g_Y Z_\mu \partial_\nu A_{\mu\nu}$$
$$+ \tilde{c}_{gg}\frac{g_s^2}{4} G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a + \tilde{c}_{\gamma\gamma}\frac{e^2}{4} A_{\mu\nu} \tilde{A}_{\mu\nu} + \tilde{c}_{z\gamma}\frac{e g_L}{2 c_\theta} Z_{\mu\nu} \tilde{A}_{\mu\nu} + \tilde{c}_{zz}\frac{g_L^2}{4 c_\theta^2} Z_{\mu\nu} \tilde{Z}_{\mu\nu}]$$

**Higgs couplings to WW and to ZZ/Zγ/γγ correlated in SMEFT but not HEFT (unless custodial symmetry imposed in latter)**

# Linear vs Non-linear

**SMEFT**


**HEFT**

**Expansion parameter**

$$\frac{E}{\Lambda}$$

**For Λ>>v: correlations between various interaction terms of the Higgs boson**

**Expansion parameter**

$$\frac{E}{v}$$

**Each Higgs boson interaction term is a-priori uncorrelated with other interactions**

$$\text{SMEFT} \subset \text{HEFT}$$

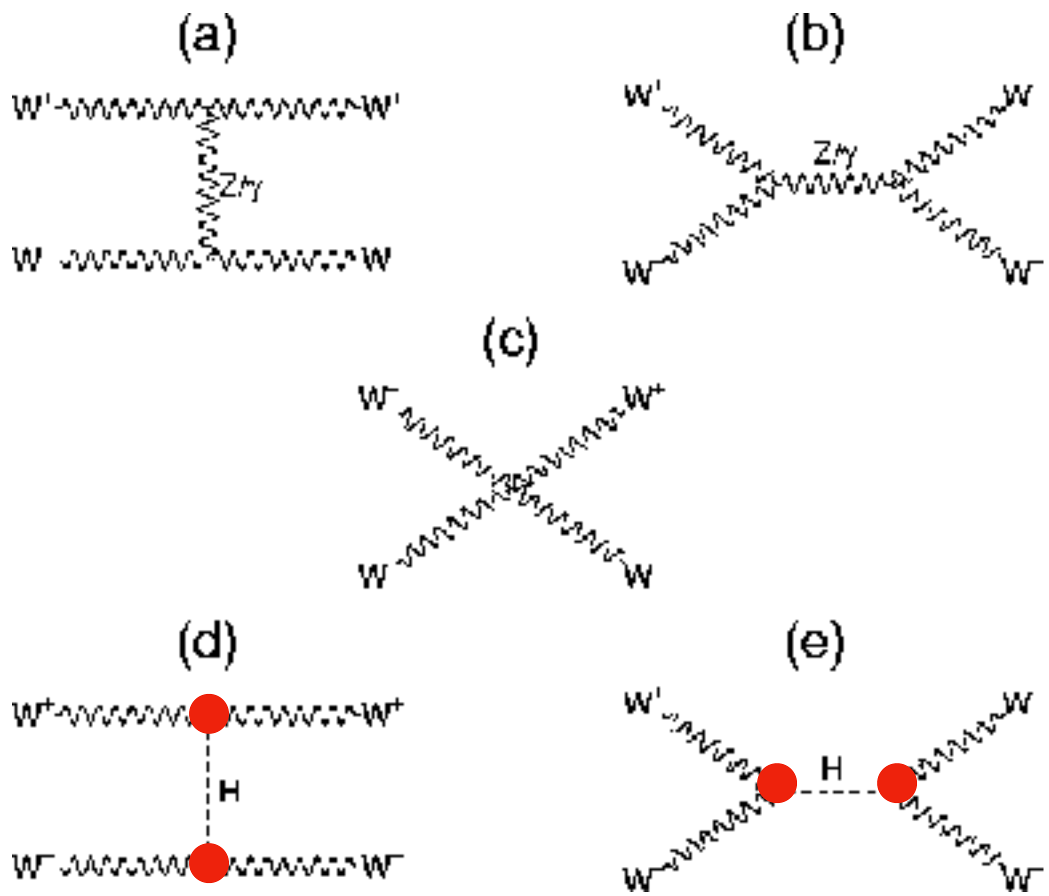
**HEFT is more general than (truncated) SMEFT, and it reduces to the latter in very special points of its parameter space**

# Linear vs Non-linear

| **SMEFT** | **HEFT** |
|---|---|
| **Higgs boson coupling to WW** | **Higgs boson coupling to WW** |
| $\mathcal{L}_{\rm EFT} \supset m_W^2 W_\mu^+ W_\mu^- + 2\frac{h}{v} m_W^2 W_\mu^+ W_\mu^- \left(1 + c\frac{g_*^2 v^2}{\Lambda^2}\right)$ | $\mathcal{L}_{\rm EFT} \supset m_W^2 W_\mu^+ W_\mu^- + c\frac{h}{v} m_W^2 W_\mu^+ W_\mu^-$ |
| ↑ free O(1) parameter | ↑ free O(1) parameter |
| **Parametric limit Λ→∞ where Higgs boson couplings become SM-like** | **No parametric where Higgs boson couplings become SM-like** |

LHC measurements of the Higgs boson couplings can be interpreted as a strong hint for linearly realized electroweak symmetry with $\Lambda >> v$

# Accidentally SM-like HEFT?




**Could be that HEFT couplings are SM-like as the consequence of HEFT UV completion being far above the electroweak scale?**

- As is well known, in the SM Higgs boson is crucial for unitarization of 2-to-2 WW scattering amplitudes
- For that to work, the Higgs boson coupling to WW is uniquely fixed, given the W boson mass and gauge coupling
- More generally, if the Higgs boson coupling to WW in the EFT is close to the SM one, the validity range of the EFT can extend far above the electroweak scale

# This talk

- There isn't really such thing as "small HEFT deformations", consistently with the intuition of the lack of physical scale other than electroweak scale v in the HEFT

- There is a dramatic difference between the SMEFT and HEFT, which shows in the high-energy behavior of multi-Higgs amplitudes

- As a result, the HEFT loses perturbative unitarity at the scale parametrically of order $4 \pi v$, even when deviations of the Higgs interactions from the SM is tiny

- The trick to see this is to realize that HEFT can be rewritten as an effective theory with linearly realized electroweak symmetry, but with non-analytic terms in the Lagrangian

Galloway et al
1306.6534

# Example: tadpole model

**Integrating out 2nd Higgs doublet with large quartic yields effective theory with Higgs boson tadpole:**

$$V_h = \frac{m_H^2}{2} h^2 - \epsilon f h \left[ 1 + c_1 \left( \frac{\epsilon v_H}{m_{\rm aux}^2 f} \right) \frac{h}{v_H} + c_2 \left( \frac{\epsilon v_H}{m_{\rm aux}^2 f} \right)^2 \frac{h^2}{v_H^2} + \dots \right]$$

$$\frac{\epsilon v}{m_{\rm aux}^2 f} \ll 1$$

**The limit where only tadpole is kept is part of the HEFT but not SMEFT parameter space**

**Tadpole model be equivalently represented in SU(2)$_L$ x U(1)$_Y$ invariant form, with non-analytic term in Higgs potential:**

$$V = m_h^2 H^\dagger H - m_h^2 v \sqrt{2 H^\dagger H},$$

$$H = \begin{pmatrix} i G_+ \\ \frac{v + h - i G_z}{\sqrt{2}} \end{pmatrix}$$

**In unitary gauge, G=0, this reduces to the tadpole model above, up to field redefinition**

$$h - \frac{\epsilon f}{m_H^2} \to h$$

Galloway et al
1306.6534

# Example: tadpole model

**Tadpole model be equivalently represented in SU(2)$_L$ x U(1)$_Y$ invariant form, with non-analytic term in Higgs potential:**

$$V = m_h^2 H^\dagger H - m_h^2 v\sqrt{2H^\dagger H},$$

$$H = \begin{pmatrix} iG_+ \\ \frac{v+h-iG_z}{\sqrt{2}} \end{pmatrix}$$

**Away from unitary gauge, Lagrangian contains infinite series of interactions between Goldstone and Higgs bosons, suppressed only by electroweak scale v**

$$\begin{aligned} V &= \frac{m_h^2}{2}(v+h)^2 - m_h^2 v(v+h)\sqrt{1+\frac{G^2}{(v+h)^2}} \\ &= \frac{m_h^2}{2}h^2 - m_h^2 v\frac{G^2}{2(v+h)} + m_h^2 v\frac{G^4}{4(v+h)^3} + \dots \\ &= \frac{m_h^2}{2}h^2 - \frac{m_h^2}{2}G^2\sum_{n=0}^{\infty}\left(\frac{-h}{v}\right)^n + \frac{m_h^2}{8v^2}G^4\sum_{n=0}^{\infty}(n+2)(n+1)\left(\frac{-h}{v}\right)^n + \dots \end{aligned}$$

$$G^2 = 2G_+G_- + G_z^2.$$

**This can be used to calculate Goldstone scattering amplitudes, and translating them into scattering amplitudes of longitudinal W and Z bosons at large energies**

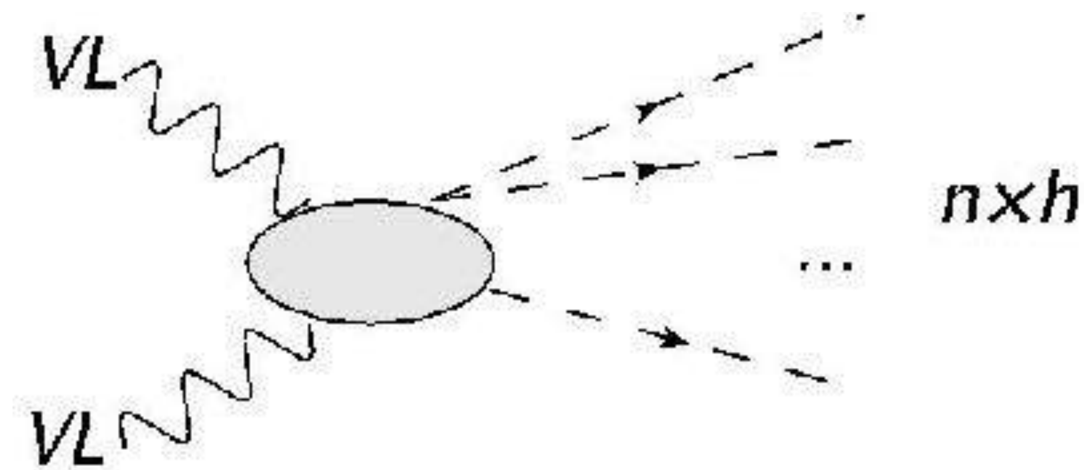
# Multi-Higgs production in tadpole model

**Consider VBF production of n ≥ 2 Higgs bosons:** $V_L V_L \to n \times h$

**Expanded V contains interactions**

$$V \supset \quad -\frac{m_h^2}{2} G^2 \sum_{n=0}^{\infty} \left(\frac{-h}{v}\right)^n .$$

**leading to interaction vertices with arbitrary number of Higgs bosons**




**s-wave isospin-0 amplitude for GG→hⁿ is momentum-independent constant proportional to the deformation**

$$|\mathcal{M}([GG]^{l=0}_{I=0} \to h^n)| \equiv |\mathcal{M}_n| = \frac{1}{4\sqrt{\pi}} \frac{\sqrt{3} n! m_h^2}{v^n} .$$

**Amplitudes for multi-Higgs production in W/Z boson fusion are only suppressed by scale v and not decay with growing energy, leading to unitarity loss at some scale above v**

# Unitarity primer

**S matrix unitarity** $S^\dagger S = 1$

symmetry factor
for n-body final state

**implies relation between forward scattering amplitude, and elastic and inelastic production cross sections**

$$2\mathrm{Im}\mathcal{M}(p_1,p_2\to p_1,p_2) = S_2\int d\Pi_2|\mathcal{M}_{\mathrm{el.}}(p_1,p_2\to k_1,k_2)|^2 + \sum S_n\int d\Pi_n|\mathcal{M}_{\mathrm{inel.}}(p_1,p_2\to k_1\dots k_n)|^2$$

**Equation is "diagonalized" after initial and final 2-body state are projected into partial waves**

$$a_l(s) = \frac{S_2}{16\pi}\sqrt{1-\frac{4m^2}{s}}\int_{-1}^{1} d\cos\theta P_l(\cos\theta)\mathcal{M}(s,\cos\theta),$$

$$2\mathrm{Im}\, a_l = |a_l|^2 + \sum_{n\in\mathrm{inel.}} S_n\int d\Pi_n|\mathcal{M}(E,0,l,m\to\{n\})|^2.$$

**This can be rewritten as the Argand circle equation**

$$(\mathrm{Re}\, a_l)^2 + (\mathrm{Im}\, a_l - 1)^2 = R_l^2, \qquad R_l = \sqrt{1-\sum_{n\in\mathrm{inel.}} S_n\int d\Pi_n|\mathcal{M}(E,0,l,m\to\{n\})|^2}.$$

$S^\dagger S = 1$

# Unitarity primer

**Argand circle equation**

$$(\mathrm{Re}\, a_l)^2 + (\mathrm{Im}\, a_l - 1)^2 = R_l^2, \qquad R_l = \sqrt{1 - \sum_{n \in \mathrm{inel.}} S_n \int d\Pi_n |\mathcal{M}(E, 0, l, m \to \{n\})|^2}.$$

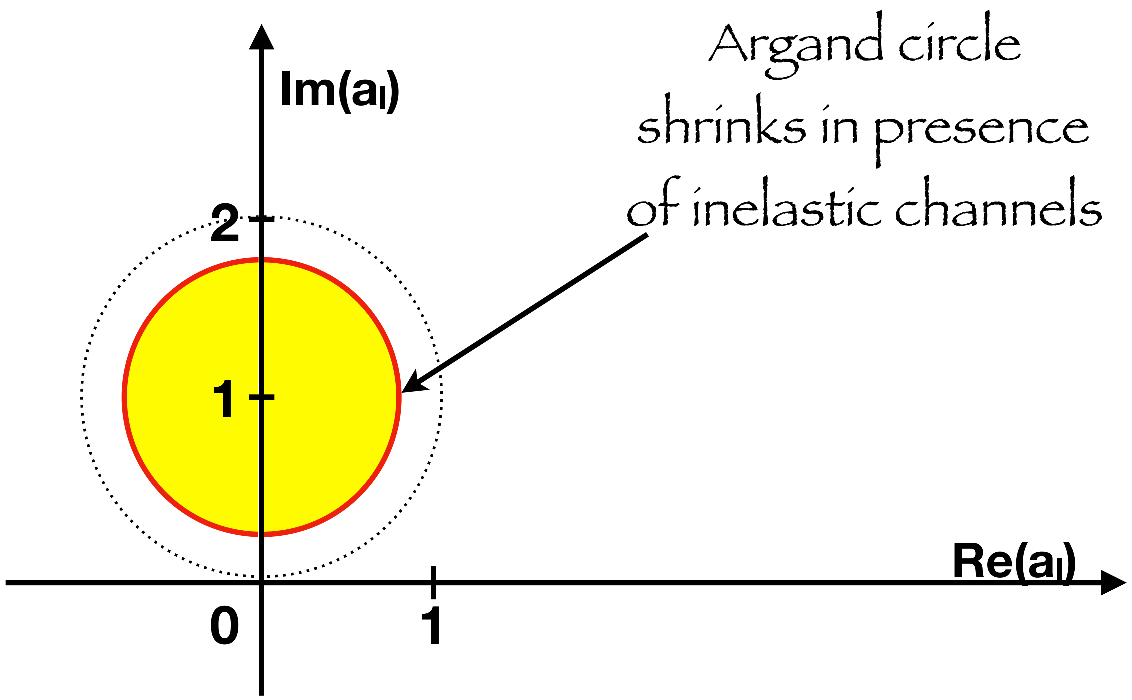
**leads to the constraint**

$$(\mathrm{Re}\, a_l)^2 + \sum_{n \in \mathrm{inel}} S_n \int d\Pi_n |\mathscr{M}(E,0,l,m \to \{n\})|^2 \leq 1$$

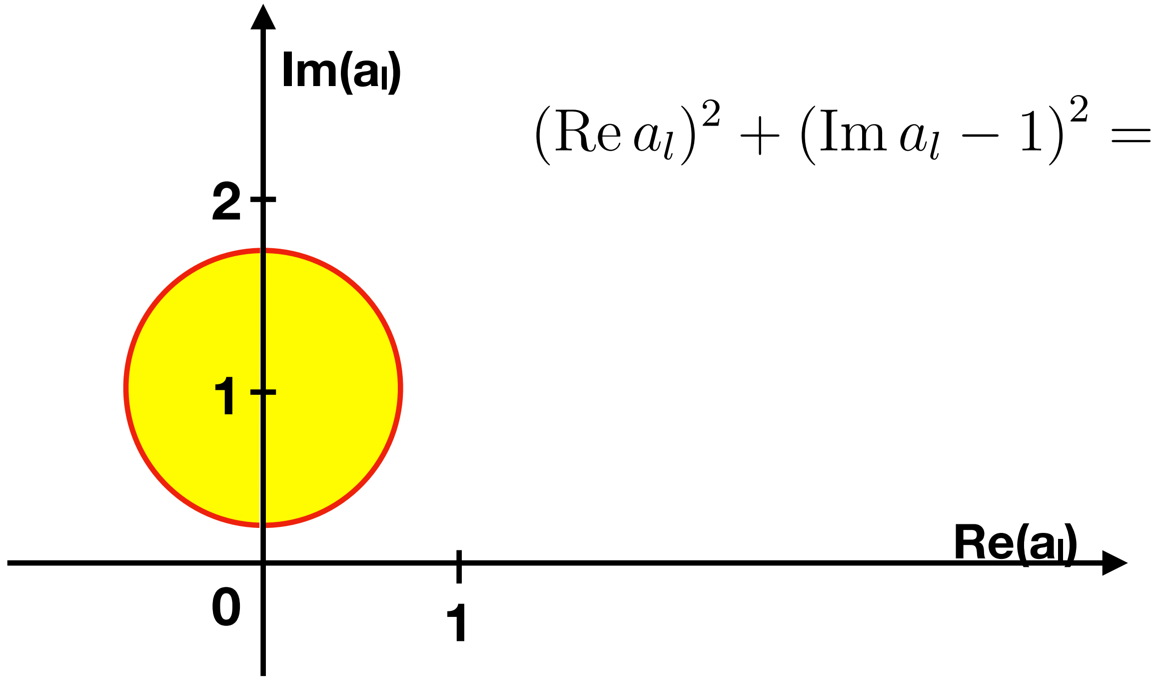
**which implies constraints on <u>both</u> elastic and inelastic amplitudes**

$$|\mathrm{Re}\, a_l| \leq 1$$

$$\sum_{n \in \mathrm{inel.}} S_n \int d\Pi_n |\mathcal{M}(E, 0, l, m \to \{n\})|^2 \leq 1.$$



# Unitarity primer




$$(\mathrm{Re}\, a_l)^2 + (\mathrm{Im}\, a_l - 1)^2 = R_l^2, \qquad R_l = \sqrt{1 - \sum_{n \in \mathrm{inel.}} S_n \int d\Pi_n |\mathcal{M}(E, 0, l, m \to \{n\})|^2}.$$

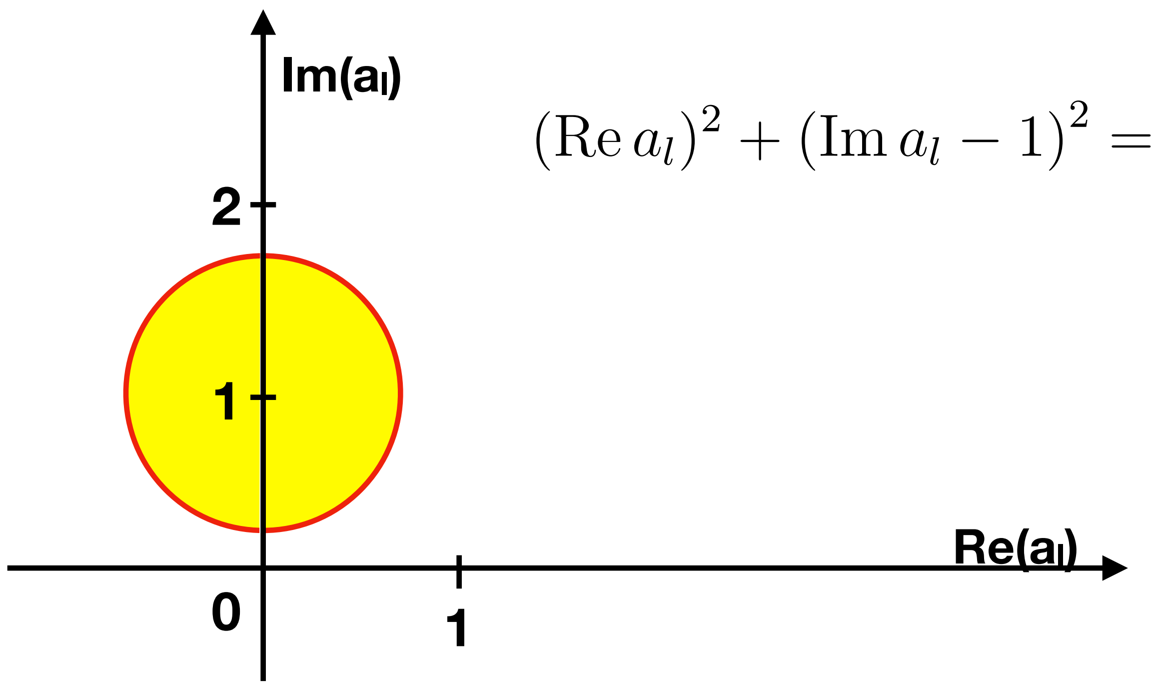
- In a unitary theory, all partial wave amplitudes must lie on the boundary of the Argand circle

- Amplitudes calculated in perturbation theory may violate this condition, which signals that higher order corrections are non-negligible

- This goes under the name of perturbative unitarity violation

- New degrees of freedom must appear around the scale of perturbative unitarity violation, either as a UV completion of the effective theory, or as a strong coupling transition

# Unitarity primer




$$(\mathrm{Re}\, a_l)^2 + (\mathrm{Im}\, a_l - 1)^2 = R_l^2, \qquad R_l = \sqrt{1 - \sum_{n \in \text{inel.}} S_n \int d\Pi_n |\mathcal{M}(E, 0, l, m \to \{n\})|^2}.$$

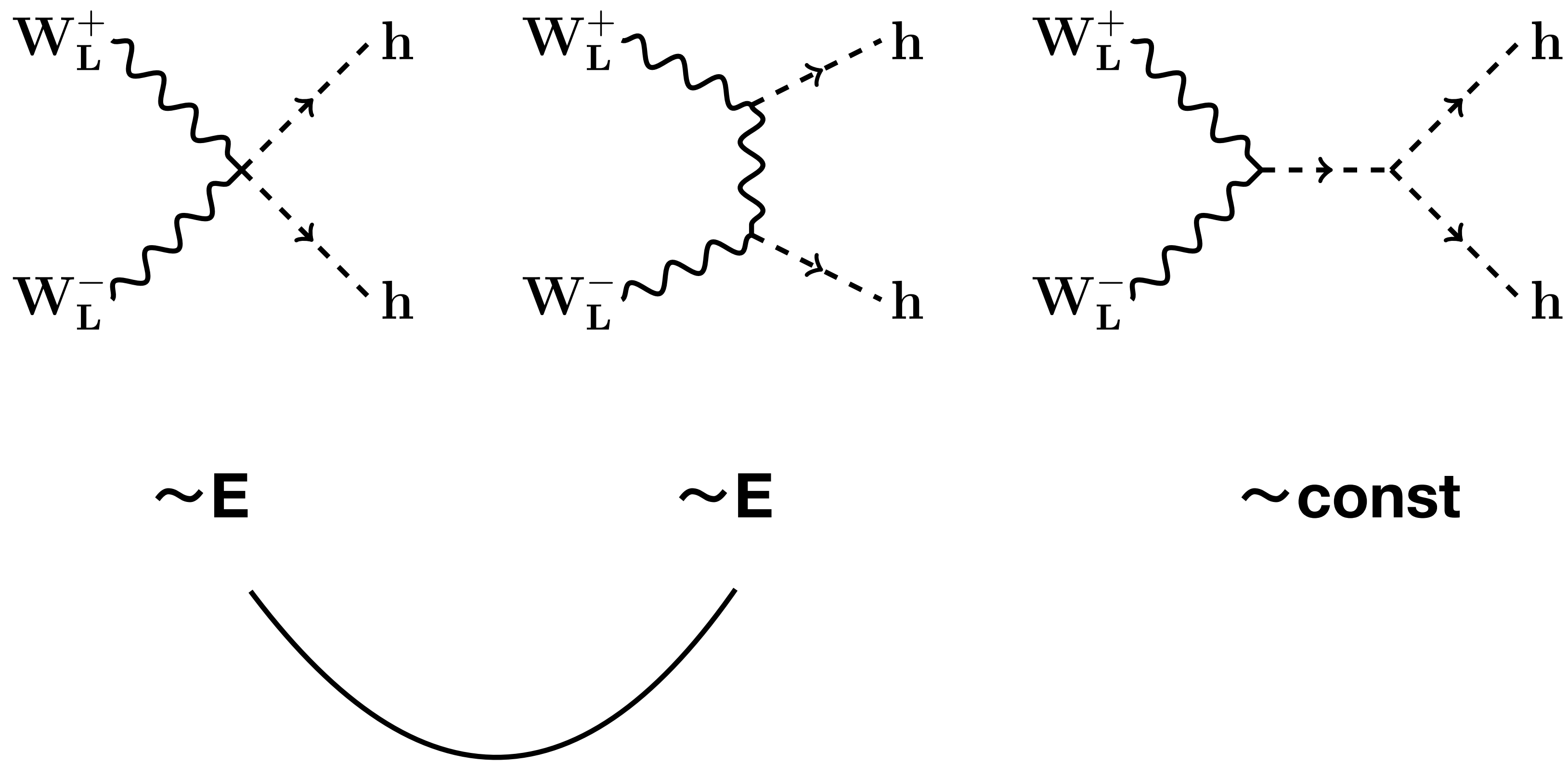
**Scale $\Lambda_u$ where perturbative predictions are no longer reliable**

$$(\mathrm{Re}\, a_l)^2 + \sum_{n \in \text{inel.}} S_n \int d\Pi_n |\mathcal{M}(\sqrt{s}, 0, l, m \to \{n\})|^2 \Big|_{\sqrt{s} = \Lambda_\mathrm{u}} = 1.$$

**Estimated scale $\Lambda_*$ where new degrees of freedom must appear**

$$(\mathrm{Re}\, a_l)^2 + \sum_{n \in \text{inel.}} S_n \int d\Pi_n |\mathcal{M}(\sqrt{s}, 0, l, m \to \{n\})|^2 \Big|_{\sqrt{s} = \Lambda_*} \sim \pi^2.$$

# Tadpole model: elastic channels




**Tadpole model modifies significantly the Higgs boson self-couplings, but not its couplings to W bosons. In 2-to-2 scattering at tree level only the latter are important for unitarity**

**Thus, non-unitary behavior is not visible at the level of 2-to-2 amplitudes**

# Tadpole model: inelastic channels

**In effective theory, unitarity constraints can be used to place bounds on scale $\Lambda_*$ where new degrees of freedom must appear**

$$(\mathrm{Re}\, a_l)^2 + \sum_{n \in \mathrm{inel.}} S_n \int d\Pi_n |\mathcal{M}(\sqrt{s}, 0, l, m \to \{n\})|^2 \Big|_{\sqrt{s}=\Lambda_*} \sim \pi^2.$$

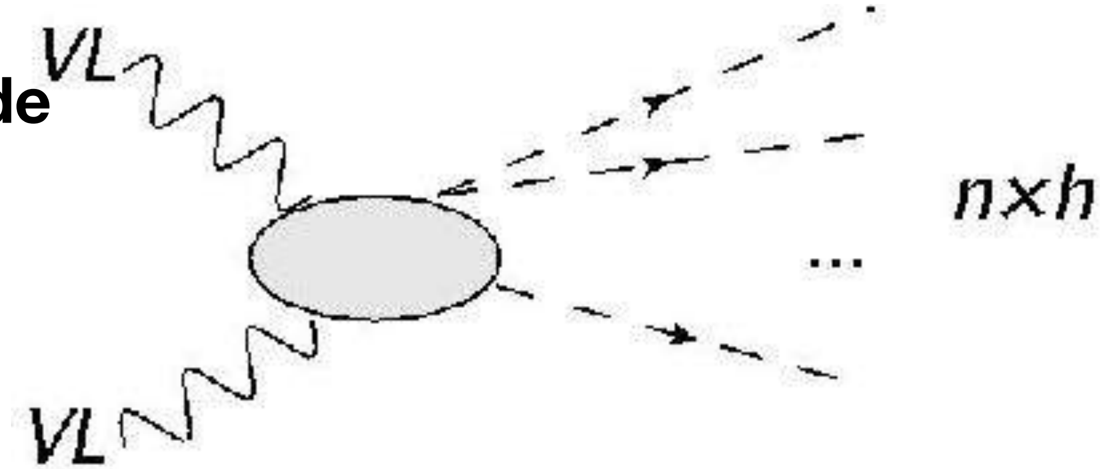
- **2-to-2 amplitude will hit unitarity bounds whenever some partial waves grow with energy**
- **2-to-n amplitude will hit unitarity bounds whenever some partial waves decays at large energies more slowly than $1/E^{2n-4}$**
- **That's because n-body phase space grows more quickly with energy for larger n**

$$V_n \equiv \frac{1}{2(4\pi)^{2n-3}} \frac{s^{n-2}}{(n-1)!(n-2)!}.$$

**in massless limit**

# Unitarity bounds in tadpole model

**Perturbative unitarity bound on non-elastic amplitude**




$$\sum_{n=2}^{\infty} S_n \int d\Pi_n |\mathcal{M}_n|^2 \Big|_{\sqrt{s}=\Lambda_*} = \sum_{n=2}^{\infty} \frac{1}{n!} V_n(\Lambda_*)|\mathcal{M}_n|^2 \sim \pi^2,$$

$$|\mathcal{M}([GG]^{l=0}_{I=0} \to h^n)| \equiv |\mathcal{M}_n| = \frac{1}{4\sqrt{\pi}} \frac{\sqrt{3}\, n!\, m_h^2}{v^n}.$$

**Sum over n Higgs bosons exponentiates**

$$\begin{aligned}
\pi^2 &\sim \frac{3m_h^4}{16\pi} \sum_{n=2}^{\infty} \frac{1}{n!} \frac{\Lambda_*^{2n-4}}{2(n-1)!(n-2)!(4\pi)^{2n-3}} \frac{(n!)^2}{v^{2n}} \\
&= \frac{3m_h^4}{128\pi^2 v^4} \sum_{n=2}^{\infty} \frac{n\Lambda_*^{2n-4}}{(4\pi v)^{2n-4}} = \frac{3m_h^4}{64\pi^2 v^4} \left(2 + \frac{\Lambda_*^2}{(4\pi v)^2}\right) \exp\left(\frac{\Lambda_*^2}{(4\pi v)^2}\right)
\end{aligned}$$

$$\frac{\Lambda_*}{4\pi v} \sim \log^{1/2}\left(\frac{4\pi v}{m_h}\right)$$

**New physics must enter**
**at scale ≤ few*4 π v**
**to regulate multi-Higgs amplitudes!**

# Unitarity bounds in tadpole model

- Tadpole model loses perturbative unitarity at the scale of order $4 \pi v$, and has to be UV completed around that scale
- This is hardly surprising, given the construction of the model as effective theory of a non-decoupling limit of the two-Higgs doublet model
- However, the same lesson applies to <u>any</u> HEFT theory that is not part of the SMEFT parameter space, even when it is a continuous deformation of the SM!

# Example: cubic Higgs deformation

**Given Lagrangian for Higgs boson h, one can always uplift it to manifestly SU(3)xSU(2)xU(1) invariant form replacing**

$$h \to \sqrt{2H^\dagger H} - v$$

$$\frac{m_h^2}{2}h^2 + \frac{m_h^2}{2v}\left(1+\delta\lambda_3\right)h^3 + \frac{m_h^2}{8v^2}h^4$$

$$\Lambda_3 = \frac{m_h^2}{2v}\delta\lambda_3$$

$$\to m^2 H^\dagger H + \lambda(H^\dagger H)^2 + 3\Lambda_3 v^2 (2H^\dagger H)^{1/2} + \Lambda_3 (2H^\dagger H)^{3/2}$$

$$H = \begin{pmatrix} iG_+ \\ \frac{v+h-iG_z}{\sqrt{2}} \end{pmatrix}$$

$$V \supset \frac{3vm_h^2}{2}\delta\lambda_3((h+v)^2+G^2)^{1/2} + \frac{m_h^2}{2v}\delta\lambda_3((h+v)^2+G^2)^{3/2}.$$

$$G^2 = 2G_+G_- + G_z^2.$$

**Non-analytic terms lead to infinite series of n-point Goldstone and Higgs boson interactions**

$$\mathcal{L} \supset \mathcal{L}_{G^2} + \mathcal{L}_{G^4} + \mathcal{L}_{G^6} + \ldots$$

$$\mathcal{L}_{G^2} = -m_h^2\left(2G_+G_- + G_z^2\right)\left[\frac{h}{2v} + \frac{1+3\delta\lambda_3}{4}\frac{h^2}{v^2} - \frac{3\delta\lambda_3}{4}\frac{h^3}{v^3} + \ldots\right]$$

$$\mathcal{L}_{G^4} = -m_h^2\left(2G_+G_- + G_z^2\right)^2\left(\frac{1}{8v^2} + \frac{3\delta\lambda_3}{8}\frac{h}{v^3} - \frac{15\delta\lambda_3}{16}\frac{h^2}{v^4} + \ldots\right)$$

$$\ldots$$

**Consequence:** in deformed SM with $\delta\lambda_3 \neq 0$,

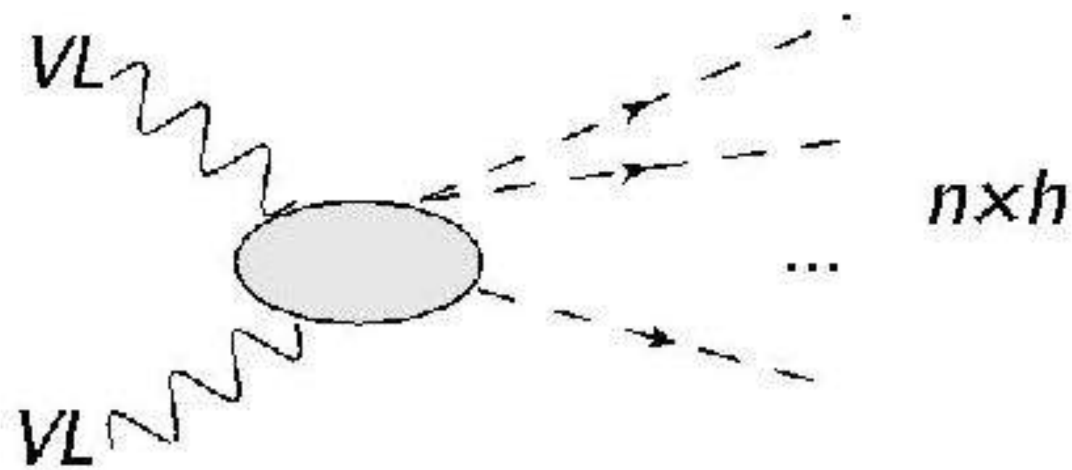
**VV→n × h, VV→ VV + n × h, ...., lose unitarity near scale 4πv**

# Multi-Higgs with HEFT-deformed Higgs cubic

**Higgs potential with Goldstones**

$$V \supset \frac{3vm_h^2}{2}\delta\lambda_3((h+v)^2+G^2)^{1/2} + \frac{m_h^2}{2v}\delta\lambda_3((h+v)^2+G^2)^{3/2}.$$

$$G^2 = 2G_+G_- + G_z^2.$$




**Expanding to leading order in $G^2$**

$$V \supset \delta\lambda_3 \frac{3m_h^2 v}{2}\frac{G^2}{h+v} = \delta\lambda_3 \frac{3m_h^2}{2} G^2 \sum_{n=0}^{\infty}\left(\frac{-h}{v}\right)^n.$$

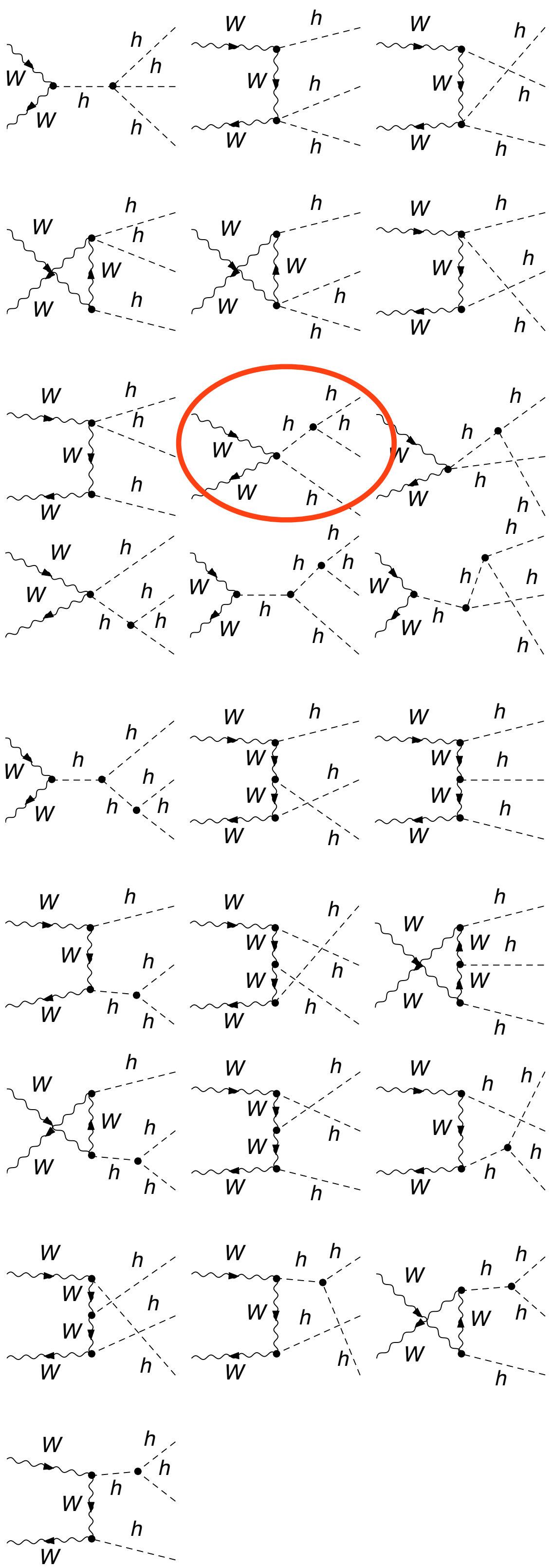
**s-wave isospin-0 amplitude for $GG \to h^n$ is momentum-independent constant proportional to the deformation**

$$|\mathcal{M}([GG]^{l=0}_{I=0} \to h^n)| \equiv |\mathcal{M}_n| = \frac{1}{4\sqrt{\pi}}\delta\lambda_3 \frac{3\sqrt{3}n!m_h^2}{v^n}.$$

**By equivalence theorem, Goldstone scattering amplitudes at large energies are related to those of longitudinal W and Z bosons**

**Amplitudes for multi-Higgs production in W/Z boson fusion are only suppressed by scale v, leading to unitarity loss at some scale above v**

# Multi-Higgs with HEFT-deformed Higgs cubic



**Same calculation can be performed (much more painfully) without resorting to equivalence theorem**

**Consider $V_L V_L \to hhh$ which depends on triple and other Higgs couplings.**
**Diagrams with one triple Higgs vertex contribute**

$$\mathcal{M} \sim \frac{m_W^2}{v^2} \frac{m_h^2}{v} (1 + \delta\lambda_3) \left( \frac{\sqrt{s}}{m_W} \right)^2 \frac{1}{s - m_h^2} + \dots$$

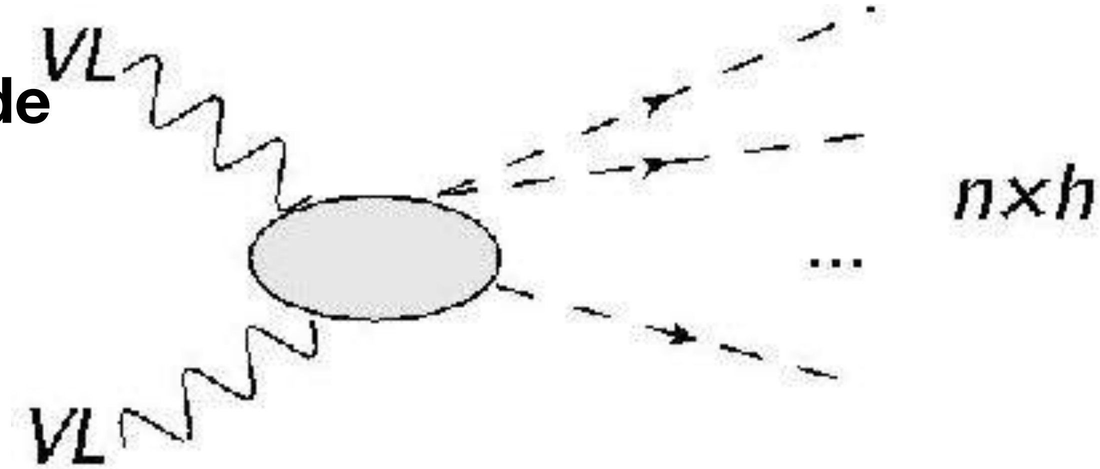
**hhWW vertex** — **Triple Higgs vertex** — **Longitudinal polarization** — **Propagator**

**In SM, various contributions that go like $E^0$ cancel against each other so that full amplitude behaves as 1/E at high energy, consistently with perturbative unitarity**

**However, as soon as $\delta\lambda_3 \neq 0$, cancellation is no longer happening, and then tree level $V_L V_L \to hhh$ cross section explodes at high energies**

# Multi-Higgs with HEFT-deformed Higgs cubic

**Perturbative unitarity bound on non-elastic amplitude**




$$\sum_{n=2}^{\infty} S_n \int d\Pi_n |\mathcal{M}_n|^2 \Big|_{\sqrt{s}=\Lambda_*} = \sum_{n=2}^{\infty} \frac{1}{n!} V_n(\Lambda_*) |\mathcal{M}_n|^2 \sim \pi^2,$$

$$|\mathcal{M}([GG]_{I=0}^{l=0} \to h^n)| \equiv |\mathcal{M}_n| = \frac{1}{4\sqrt{\pi}} \delta\lambda_3 \frac{3\sqrt{3} n! m_h^2}{v^n}.$$

**Sum over n Higgs bosons exponentiates**

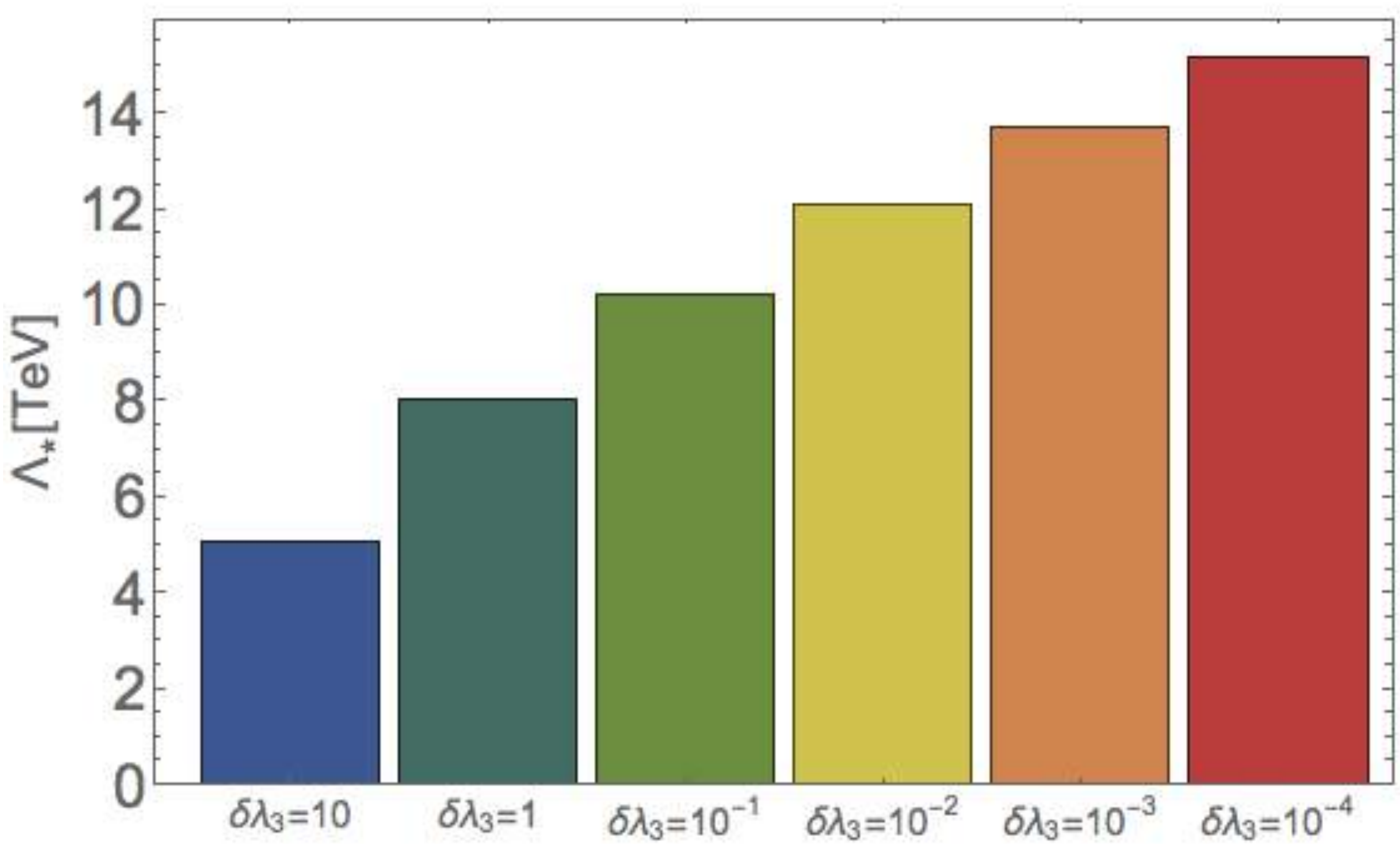
$$\begin{aligned} \pi^2 &\sim \frac{27\delta\lambda_3^2 m_h^4}{16\pi} \sum_{n=2}^{\infty} \frac{1}{n!} \frac{\Lambda_*^{2n-4}}{2(n-1)!(n-2)!(4\pi)^{2n-3}} \frac{(n!)^2}{v^{2n}} \\ &= \frac{27\delta\lambda_3^2 m_h^4}{128\pi^2 v^4} \sum_{n=2}^{\infty} \frac{n\Lambda_*^{2n-4}}{(n-2)!(4\pi v)^{2n-4}} = \frac{27\delta\lambda_3^2 m_h^4}{128\pi^2 v^4} \left(2 + \frac{\Lambda_*^2}{(4\pi v)^2}\right) \exp\left(\frac{\Lambda_*^2}{(4\pi v)^2}\right) \end{aligned}$$

$$\frac{\Lambda_*}{4\pi v} \sim 2 \log^{1/2} \left( \frac{4\pi v}{m_h |\delta\lambda_3|^{1/2}} \right)$$

**For any observable cubic Higgs deformations, new physics must enter at scale <=few*4π v to regulate multi-Higgs amplitudes!**

# Multi-Higgs with HEFT-deformed Higgs cubic

**Maximum new physics scale for different $\delta\lambda_3$**

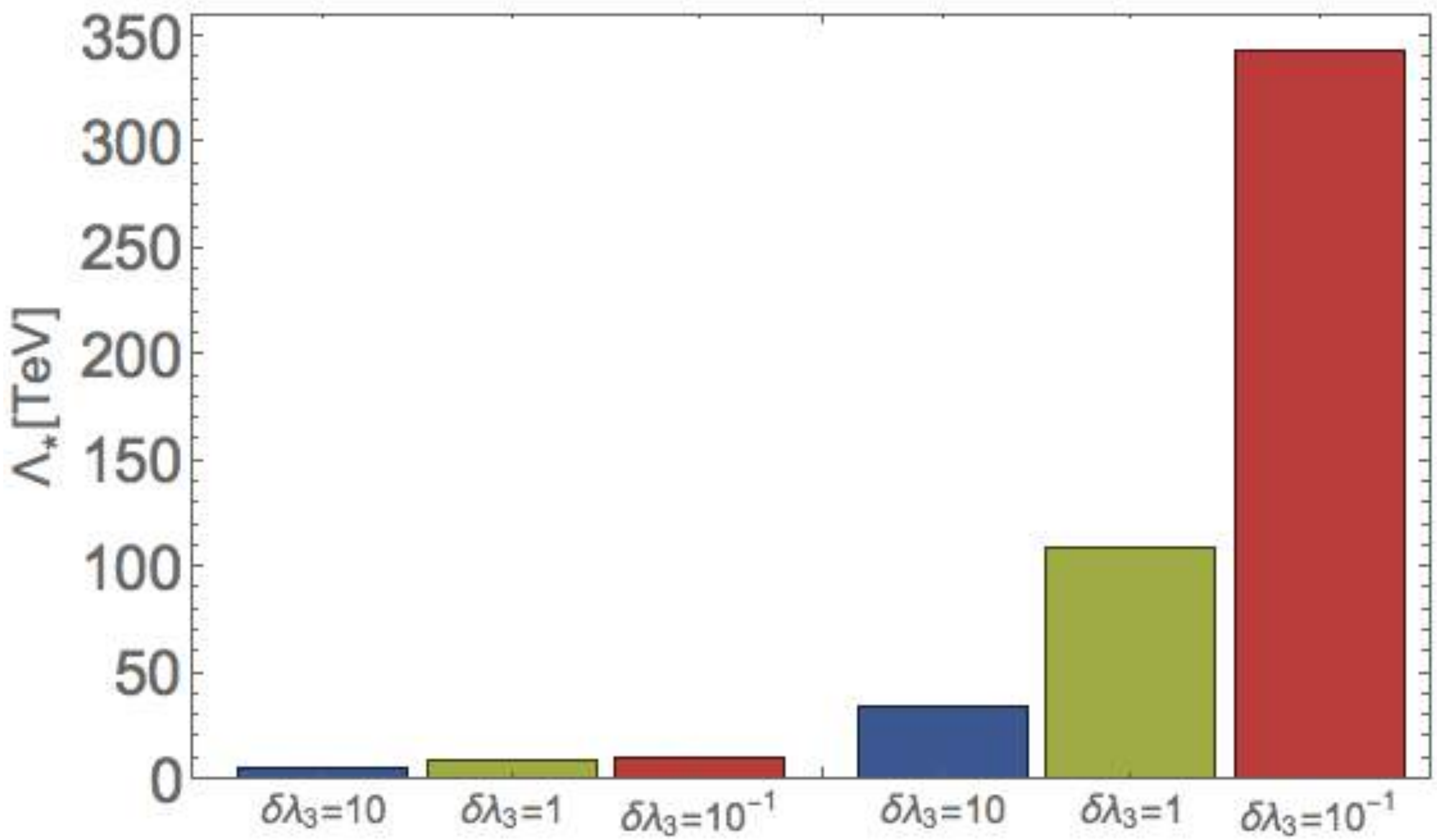



**For observable deformations of Higgs cubic, new physics must enter below ~10 TeV scale**

**Corollary: if we demonstrate no new physics strongly coupled to Higgs below~10 TeV, we practically prove EW symmetry is linearly realized**

# HEFT with SMEFT

## Maximum new physics scale for different $\delta\lambda_3$

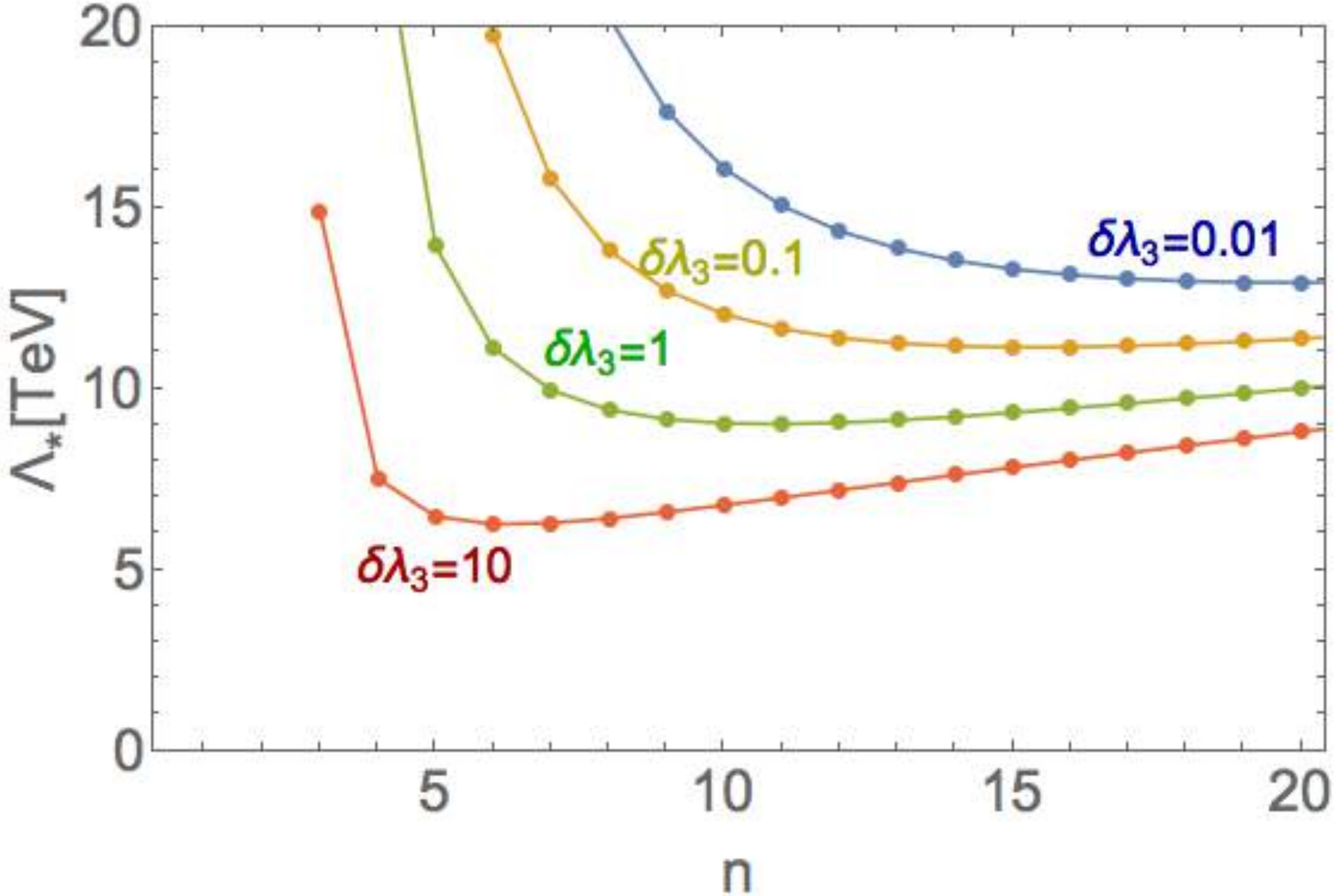



**HEFT** **SMEFT**

**For SMEFT maximum new physics scale increases as $(\delta\lambda_3)^{-1/2}$**

# Multi-Higgs with HEFT-deformed Higgs cubic

**Unitarity bounds separately for each n**




**The smaller $\delta\lambda_3$, the larger multiplicity n which dominates unitarity bounds. But even for tiny $\delta\lambda_3$, dominant n is order 10, so neglecting Higgs masses in phase space is justified a posteriori**

# SMEFT vs HEFT

$$U = \exp(2i\varphi^a T^a / v)$$

$$\mathscr{L}_{\text{HEFT}} = \frac{1}{2} f_h(h) \partial_\mu h \partial_\mu h - V(h) + \frac{v^2}{4} f_1(h) \text{Tr}[\partial_\mu U^\dagger \partial_\mu U] + v^2 f_2(h) \left(\text{Tr}[U^\dagger \partial_\mu U \sigma_3]\right)^2 + \ldots$$

.

**One can always re-express non-linear Lagrangian in linear language by replacing:**

$$U \to \frac{(\tilde{H}, H)}{\sqrt{H^\dagger H}}$$

$$h \to \sqrt{2H^\dagger H} - v$$

**After this substitution, Lagrangian has linearly realized electroweak symmetry but, for a generic point in parameter space, it contains terms that are non-analytic (that is, not continuously differentiable) at H=0**

# HEFT vs SMEFT

$$\mathscr{L}_{\mathrm{HEFT}} = \frac{1}{2} f_h(h)\partial_\mu h \partial_\mu h - V(h) + \frac{v^2}{4} f_1(h) \mathrm{Tr}[\partial_\mu U^\dagger \partial_\mu U] + v^2 f_2(h) \left(\mathrm{Tr}[U^\dagger \partial_\mu U \sigma_3]\right)^2 + \ldots$$

**A point in HEFT parameter space is a part of SMEFT if, after the substitution, non-analytic terms cancel (up to equations of motion)**

$$U \to \frac{\left(\tilde{H}, H\right)}{\sqrt{H^\dagger H}}$$

$$h \to \sqrt{2H^\dagger H} - v$$

**For example**

$$V(h) = \frac{m_h^2}{2} h^2 + a_3 v h^3 + a_4 h^4 + \frac{a_5}{v} h^5 + \frac{a_6}{v^2} h^6,$$

**corresponds to analytic potential when**

$$a_4 = \frac{3a_3}{2} - \frac{5m_h^2}{4v^2}, \quad a_5 = \frac{3a_3}{4} - \frac{3m_h^2}{4v^2}, \quad a_6 = \frac{a_3}{8} - \frac{m_h^2}{8v^2}.$$

# HEFT vs SMEFT

$$\mathscr{L}_{\mathrm{HEFT}} = \frac{1}{2} f_h(h)\partial_\mu h \partial_\mu h - V(h) + \frac{v^2}{4} f_1(h)\mathrm{Tr}[\partial_\mu U^\dagger \partial_\mu U] + v^2 f_2(h)\left(\mathrm{Tr}[U^\dagger \partial_\mu U \sigma_3]\right)^2 + \ldots$$

**A point in HEFT parameter space is a part of SMEFT if, after the substitution, non-analytic terms cancel (up to equations of motion)**

$$U \to \frac{(\tilde{H}, H)}{\sqrt{H^\dagger H}}$$

$$h \to \sqrt{2H^\dagger H} - v$$

**More generally, HEFT reduces to dimension-6 SMEFT for**

$$\begin{aligned}
V &= \frac{m_h^2}{2}h^2 + a_3 v h^3 + \left(\frac{3a_3}{2} - \frac{5m_h^2}{4v^2}\right)h^4 + \left(\frac{3a_3}{4} - \frac{3m_h^2}{4v^2}\right)\frac{h^5}{v} + \left(\frac{a_3}{8} - \frac{m_h^2}{8v^2}\right)\frac{h^6}{v^2} \\
f_h &= \frac{2b_0 - b_1}{2} + \frac{b_1}{2}\left(1 + \frac{h}{v}\right)^2, \\
f_1 &= \frac{2b_0 - b_1}{2}\left(1 + \frac{h}{v}\right)^2 + \frac{2 - 2b_0 + b_1}{2}\left(1 + \frac{h}{v}\right)^4, \\
f_2 &= d_0\left(1 + \frac{h}{v}\right)^4 .
\end{aligned}$$

$$\begin{aligned}
\mathcal{L} &= \left(b_0 - \frac{b_1}{2}\right)|D_\mu H|^2 - \frac{3a_3 v^2 - 7m_h^2}{4}H^\dagger H - \left(\frac{5m_h^2}{2v^2} - \frac{3a_3}{2}\right)(H^\dagger H)^2 \\
&- \left(a_3 - \frac{m_h^2}{v^2}\right)\frac{(H^\dagger H)^3}{v^2} + \frac{b_0 - 1}{2}\frac{[\partial_\mu (H^\dagger H)]^2}{v^2} + (2 - 2b_0 + b_1)\frac{H^\dagger H |D_\mu H|^2}{v^2} \\
&+ d_0 \frac{\left(D_\mu H^\dagger H - H^\dagger D_\mu H\right)^2}{v^2} .
\end{aligned}$$

# Perspective on HEFT

- In effective theories, non-analytic terms in Lagrangian appear due to integrating out light degrees of freedom

- More precisely, non-analyticity at $H \to 0$ signals that particle whose mass vanishes as $H \to 0$ has been integrated out (e.g. integrating out 4th chiral generation produces $\mathrm{Log}|H|^2$ in Coleman-Weinberg potential)

- Thus, HEFT is effective theory for UV models containing particles who get their masses from EW symmetry breaking. This clarifies why cutoff cannot be taken parametrically above $4\pi v$.

- In contrast, SMEFT is effective theory for UV models where new particles decouple in the limit $v \to 0$

- For practical purpose, there is no difference between HEFT and SMEFT with $\Lambda$ of order v

# Summary

- HEFT = SMEFT + non-analytic interactions
- Non-analytic term → infinite series of interactions suppressed by $v^n$ → cut-off near $4\pi v$
- Manifested as n>2-body Higgs production violating perturbative unitarity bounds around that scale

# Question for next high-energy collider ?

....

1980 — Sp$\bar{p}$S — UV completion to Fermi theory identified

1990 — LEP — SM-like interactions of fermions confirmed

Tevatron — Top quark mass

2000

2010 — LHC — Higgs boson mass

20XY — *LHC++* — *Linear vs non-linear electroweak symmetry ?*