# Unusual Vacuum Decay Events in The Early Universe

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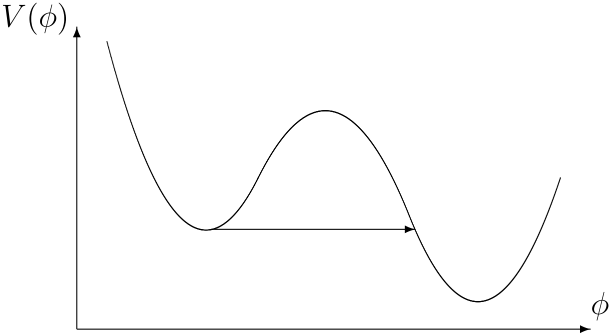
# Motivation

- The existence and consequences of a landscape or multiverse of possible universes, either in string theory or otherwise, is one of the key questions of theoretical physics today.
- Cosmology has reached a level of precision where it now makes sense to ask whether there exist observational signatures resulting from such scenarios.

# Bubble nucleation



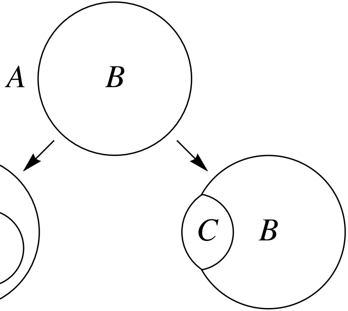

- A potential with two minima may allow decay from the false vacuum.
- A bubble of true vacuum nucleates and grows.
- Bubble wall approaches speed of light.

Coleman (1977), Callan & Coleman (1977), Coleman & de Luccia (1980)

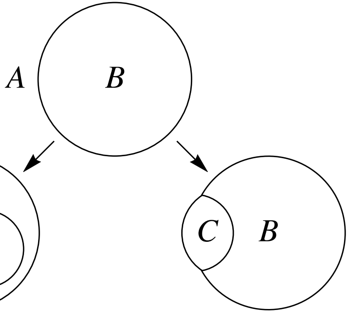
# Decays involving more than two vacua

Usually one only considers two vacua involved in a decay, but what about three vacua?

- Two qualitatively new possibilities:
  - *interior* of bubble undergoes further decay; or
  - *wall* of bubble decays



# Decays involving more than two vacua




Both of the above decays can be described by instantons, but with *two* negative modes

- Coleman (1988) showed that the instantons relevant for decays of empty space have exactly one negative mode of fluctuations.
- Are these instantons physically relevant? How should they be interpreted?

# Instantons and tunnelling rates

Euclidean partition function:

$$Z = \mathrm{e}^{-S_A} = \mathrm{e}^{-V_A \mathrm{Vol}_A}$$

Include $AB$ instantons:

$$Z \to \mathrm{e}^{-V_A \mathrm{Vol}_A} \sum_{n=0}^{\infty} \frac{\left( [\det' S''_{AB}]^{-\frac{1}{2}} \, \mathrm{e}^{-S_{AB}} \mathrm{Vol}_A \right)^n}{n!} = \mathrm{e}^{-(V_A + i\Gamma_{AB})\mathrm{Vol}_A},$$

where

$$\Gamma_{AB} = [-\det' S''_{AB}]^{-\frac{1}{2}} \, \mathrm{e}^{-S_{AB}}$$

Can interpret correction as *decay rate* when $\det' S''_{AB}$ has *one* negative mode.

# Instantons with multiple negative modes

Now imagine that $B$ can decay to $C$:

$$Z \to \mathrm{e}^{-V_A \mathrm{Vol}_A} \sum_{n=0}^{\infty} \frac{\left( i\Gamma_{AB} \sum_m \frac{\left( \left[ \det' S''_{BC} \right]^{-\frac{1}{2}} \mathrm{e}^{-S_{BC}} \mathrm{Vol}_B \right)^m}{m!} \mathrm{Vol}_A \right)^n}{n!}$$

In thin-wall limit $S_{AB} = -(V_A - V_B)\mathrm{Vol}_B + \sigma_{AB}\mathrm{Vol}_{AB}$, so

$$\Gamma_{AB} \to \left[ -\det' S''_{AB} \right]^{-\frac{1}{2}} \mathrm{e}^{(V_A - [V_B + i\Gamma_{BC}])\mathrm{Vol}_B - \sigma_{AB}\mathrm{Vol}_{AB}},$$

where

$$\Gamma_{BC} = \left[ -\det' S''_{BC} \right]^{-\frac{1}{2}} \mathrm{e}^{-S_{BC}}$$

# Instantons with multiple negative modes

Now include barnacles:

$$Z \to \mathrm{e}^{-V_A \mathrm{Vol}_A} \sum_{n=0}^{\infty} \frac{\left( i\Gamma_{AB} \sum_m \frac{\left( \left[\det' \tilde{S}_b''\right]^{-\frac{1}{2}} \mathrm{e}^{-\tilde{S}_b''} \mathrm{Vol}_{AB} \right)^m}{m!} \mathrm{Vol}_A \right)^n}{n!}$$

$$\Gamma_{AB} \to \left[-\det' S_{AB}''\right]^{-\frac{1}{2}} \mathrm{e}^{(V_A - V_B)\mathrm{Vol}_B - [\sigma_{AB} - i\Gamma_b]\mathrm{Vol}_{AB}},$$

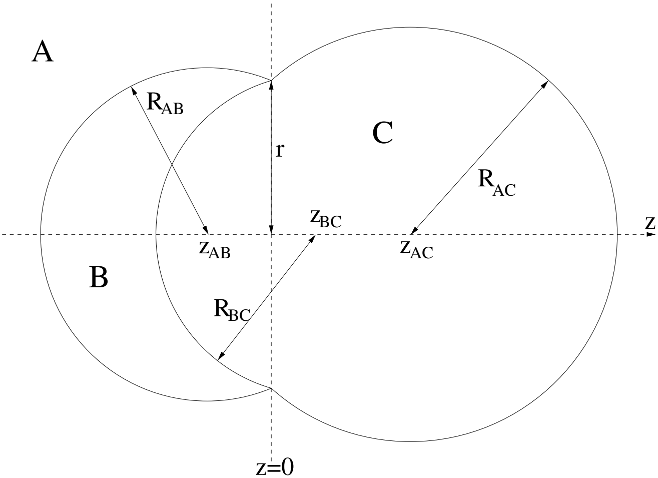
where

$$\Gamma_b = \left[-\det' \tilde{S}_b''\right]^{-\frac{1}{2}} \mathrm{e}^{-\tilde{S}_b''},$$

and $\tilde{S}_b'' \equiv S_b - S_{AB}$ is the difference in Euclidean action between an $AB$ bubble dressed with a barnacle and the $AB$ bubble alone.

# Barnacles in flat space

Balasubramanian, Czech, Larjo, & Levi (2011)

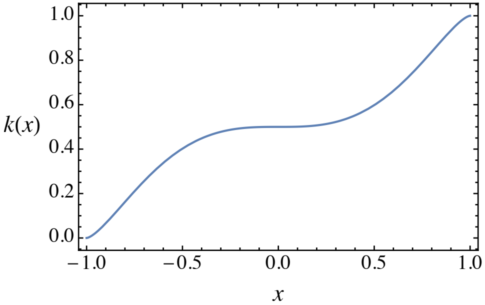



The bubble radii take their usual thin wall values: $R_X = \frac{3\sigma_X}{\Delta V_X}$
and the $z$'s are constrained to satisfy $z_X^2 + r^2 = R_X^2$

# Barnacles in flat space

Balasubramanian, Czech, Larjo, & Levi (2011)

$$S_b = -\sum_{i\in\{A,B,C\}} (V_A - V_i)\mathrm{Vol}_i + \sum_{X\in\{AB,AC,BC\}} \sigma_X \mathrm{Vol}_X$$
$$= S_{AB}\, k\left(-\frac{z_{AB}}{R_{AB}}\right) + S_{AC}\, k\left(\frac{z_{AC}}{R_{AC}}\right) + S_{BC}\, k\left(-\frac{z_{BC}}{R_{BC}}\right),$$

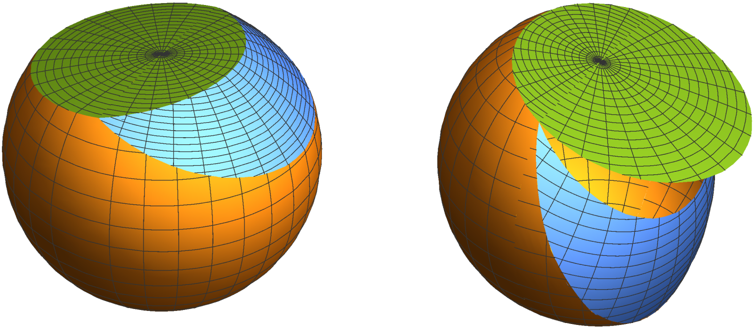


# Barnacles and gravity

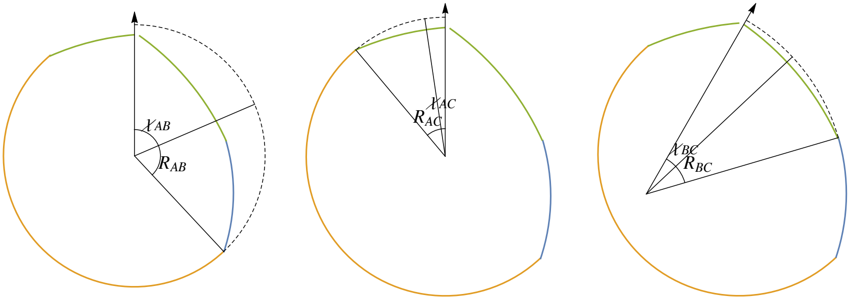
$$
\begin{aligned}
S_b = &\sum_i \left[ \int_{\mathrm{Vol}_i} \mathrm{d}^4x \, \sqrt{|g|} \left( V_i - \frac{1}{2\kappa}\mathcal{R} \right) - \frac{1}{\kappa} \int_{\partial \mathrm{Vol}_i} \mathrm{d}^3y \, \sqrt{|h|} \, \mathcal{K} \right] \\
&+ \sum_X \int_{(\partial \mathrm{Vol})_X} \mathrm{d}^3y \, \sqrt{|h|} \, \sigma_X \\
&+ \int_J \mathrm{d}^2z \sqrt{|\tilde{h}|} \left( \mu - \frac{1}{\kappa}(\pi + \Delta) \right) \\
&- \left( -\frac{24\pi^2}{\kappa^2 V_A} \right)
\end{aligned}
$$

Euclidean de Sitter is a four sphere, so this becomes an exercise in gluing together spheres...

# Barnacles Geometries



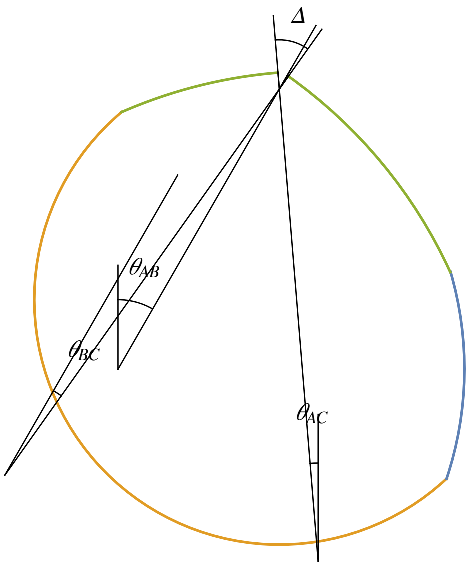
# Barnacles Geometries




The $\chi$'s satisfy a consistency condition (*cf.* $z_X^2 + r^2 = R_X^2$):

$$1 - \frac{\cos^2 R_{AB}}{\cos^2 \chi_{AB}} = 1 - \frac{\cos^2 R_{AC}}{\cos^2 \chi_{AC}} = \frac{V_A}{V_B}\left(1 - \frac{\cos^2 R_{BC}}{\cos^2 \chi_{BC}}\right) = \sin^2 \delta,$$

# Barnacle Geometries




The deficit angle is related the misalignments of the planes which go through a bubble centre and junction point:

$$\Delta = \theta_{AC} + \theta_{AB} + \theta_{BC}$$

# Barnacle Geometries

Junction conditions give

$$\frac{3}{\kappa V_i} \sin^2 R_{ij} = \left( \frac{\kappa V_i}{3} + \left[ \frac{V_i - V_j}{3\sigma} - \frac{\kappa \sigma}{4} \right]^2 \right)^{-1},$$

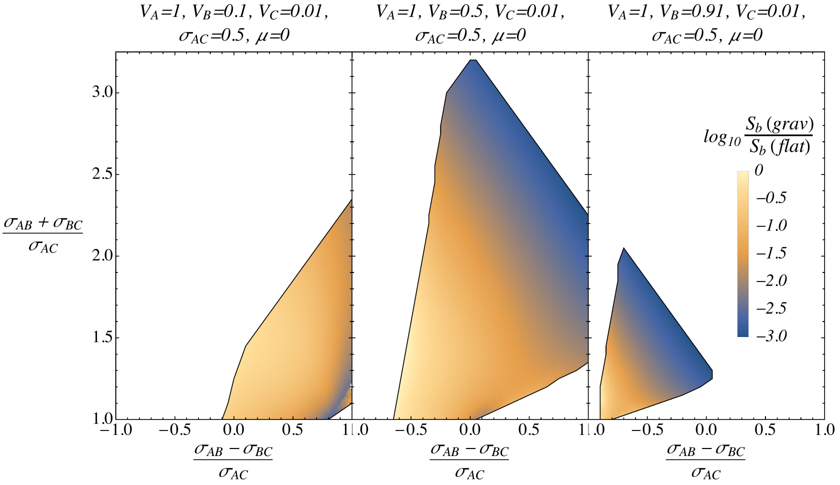
and

$$\Delta = \kappa \mu,$$

which one can (sometimes) solve to determine $\chi_X$, $\theta_X$, *etc.* in terms of $V_i$, $\sigma_X$ and $\mu$.

One can derive simple formulae for the volumes of the bubble segments and walls in terms of $R_X$, $\chi_X$, $\theta_X$, but solving the deficit angle junction condition must be done numerically.

# Barnacle Actions—comparing gravity to flat space



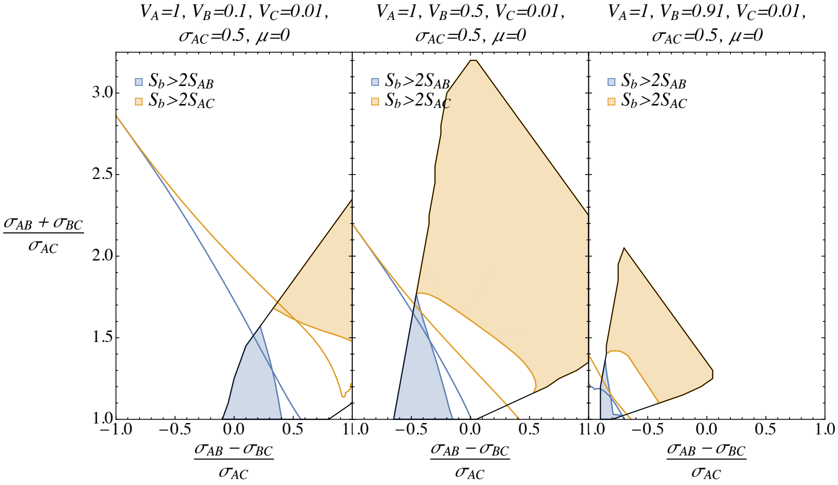
# Barnacle Actions—comparing to other decays

Approximating $\Gamma \sim \mathrm{e}^{-S}$, one can then compare the rate of production of barnacles versus other decay channels.

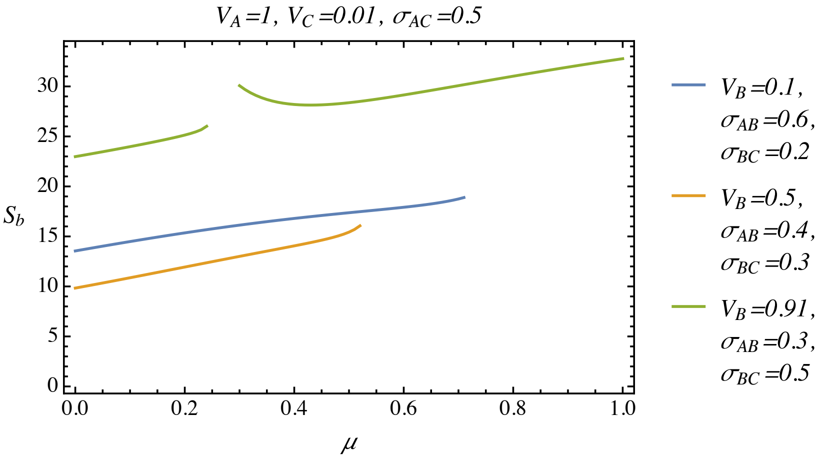
Throughout parameter space (and both with and without gravity), one finds:

- $S_b - S_{AB} < S_{BC}$
  - The wall of a bubble is more likely to decay than its interior
- $S_b - S_{AB} < S_{AC}$ and $S_b - S_{AC} < S_{AB}$
  - It is more likely for a wall of a bubble to decay than the parent vacuum to produce a bubble of the other vaccum

# Barnacle Actions—comparing to other decays

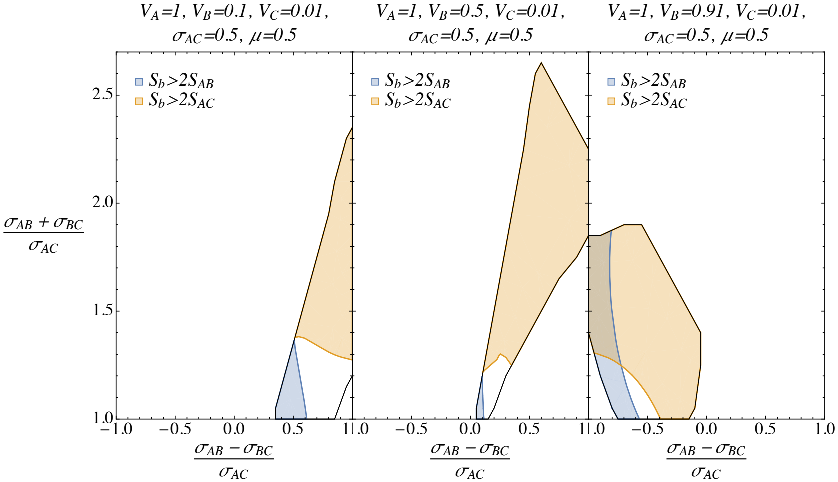


# Barnacle Actions—dependence on $\mu$



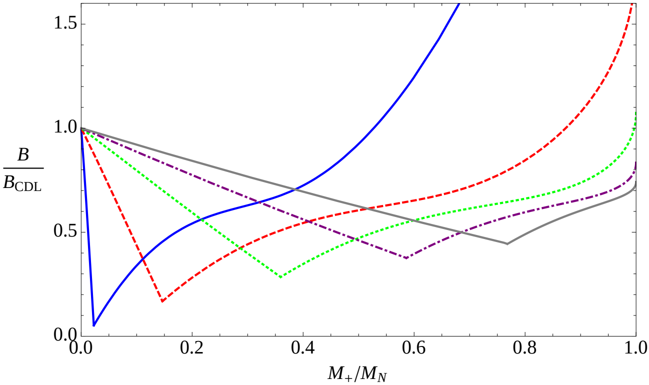

- *n.b.* in flat space $S_b$ is linear in $\mu$

# Barnacle Actions—dependence on $\mu$



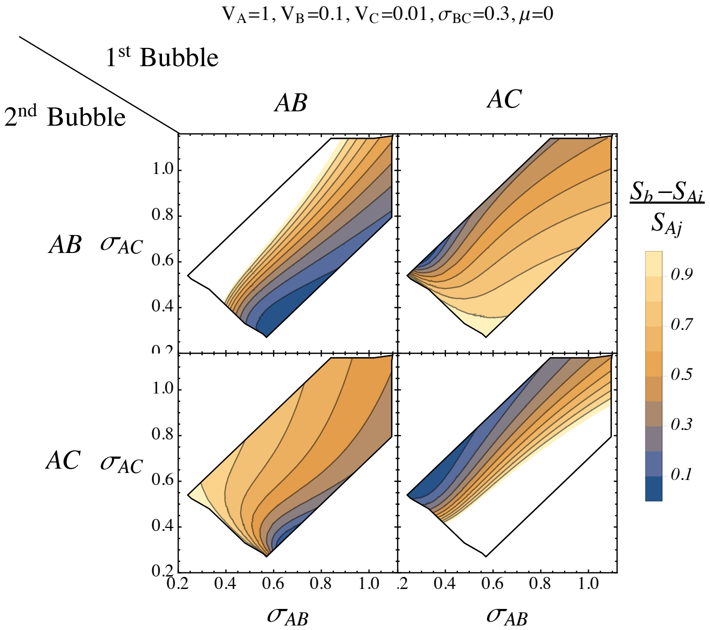
# Black holes as nucleation sites

- In everyday bubble nucleation (*e.g.* Champagne) impurities act as seeds and enhance the rate.
- Gregory, Moss, & Withers (2014) have studied this in the cosmological context, with black holes as the seeds.




Gregory, Moss, & Withers (2014)

# Bubble walls as nucleation sites

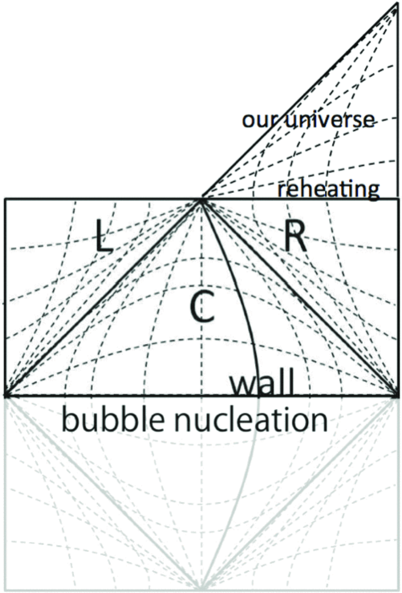


# Summary

- In theories with more than two vacua, sections of the wall of vacuum bubbles can decay.
- The rate of such events is competitive with regular vacuum decays, both inside and outside of the bubble.
- For certain parameters, gravity precludes such events, but when they are possible, gravity somewhat enhances the effect.
- The observational consequences of such events should be investigated.
  - Czech (2011) has pointed out the similarity with bubble collisions.
  - Could also be a source of primordial anisotropy in the power spectrum of perturbations.

# A universe in a bubble

- *Inside* the bubble it is possible to construct an *open* FRW coordinate system. Coleman & de Luccia (1980)
- Bubble wall is infinitely far away.




Sugimura, Yamauchi, & Sasaki (2012)

# Signatures of a previous universe

In general one has:

- $\Omega_{k0} > 0$ if inflation not too long.
- Primordial power spectrum is altered.
- Contribution to tensor modes from bubble wall fluctuations.

Is it possible to determine the nature of the parent vacuum?

# Tunnelling from a smaller number of dimensions

What if the parent vacuum has a *smaller* number of large dimensions than ours?

- More ways to compactify more dimensions, so might expect *more* vacua with *fewer* large dimensions.
- Also possible within the standard model.
- Could tunnelling from these be favoured?
- Some studies have been done into the tunnelling process.
  Blanco-Pillado & Salem (2010), Adamek, Campo, & Niemeyer (2010)

What are the consequences of such a process?

# Tunnelling from a smaller number of dimensions

The nature of the resulting universe depends on how many dimensions decompactify.

If three: $0 + 1 \to 3 + 1$

- Isotropic FRW
- Curvature depends on how the spatial dimensions were compactified.
- Different signatures to usual inflation after false vacuum decay models

# Tunnelling from a smaller number of dimensions

If one or two:

$$ds^2 = -dt^2 + a(t)^2 \left( \frac{dr^2}{1 + \kappa r^2} + r^2 d\phi^2 \right) + b(t)^2 dz^2$$

- Anisotropic
- $1 + 1 \to 3 + 1$: $\kappa$ depends on how the $(r, \phi)$ dimensions were compactified; $b(0) = 0$, $a(0) = a_0$.
- $2 + 1 \to 3 + 1$: as for $4D$ bubbles have $\kappa = -1$; $a(0) = 0$, $b(0) = b_0$.

Will focus on the latter.

# An anisotropic universe

$$ds^2 = -dt^2 + a(t)^2 \left( \frac{dr^2}{1 + \kappa r^2} + r^2 d\phi^2 \right) + b(t)^2 dz^2$$

Two types of anisotropy:

Shear: $H_a = \frac{\dot{a}}{a} \neq H_b = \frac{\dot{b}}{b}$

Curvature: $\Omega_k = \frac{-\kappa}{a^2 H_a^2}$, only in $(r, \phi)$, not $z$.

These are related:

$$\frac{H_a - H_b}{H_a} \propto \Omega_k$$

# An anisotropic universe

Relevant in two regimes.

Primordial anisotropy: $\Omega_k = 1$ initially,

then damped away by inflation, until

Late-time anisotropy: $\Omega_k$ grows during the radiation and matter dominated epochs.

Can the former compete with the latter?

# What is the value of $\Omega_k$ today?

- (As we will see) anisotropy leads leads to mixing of CMB modes with $\Delta\ell = 2$
- Monopole feeds into Quadrupole:

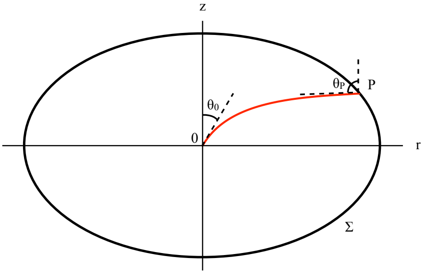
$$T_0 \Omega_{k0} \lesssim \Delta T \implies \Omega_{k0} \lesssim 10^{-4}$$

- Much more constrained than isotropic curvature.

# Late-time anisotropy

Masterfully studied by Graham, Harnik, & Rajendran (2010), who found three effects:

1. Shape of LSS is warped.
2. Reception and emission angles are not the same.
3. Redshift is angle-dependent.




Graham, Harnik, & Rajendran (2010)

# Primordial anisotropy

Power spectrum is no longer isotropic:

$$P(k) \to P(k, \hat{\mathbf{k}} \cdot \hat{\mathbf{z}})$$

Need to deal with two things which change:

1. Cosmological perturbation theory to get $P(k, \hat{\mathbf{k}} \cdot \hat{\mathbf{z}})$.
2. Going from power spectrum to CMB.

# Solving for the mode functions

Approximation: only vacuum energy and curvature driving background.

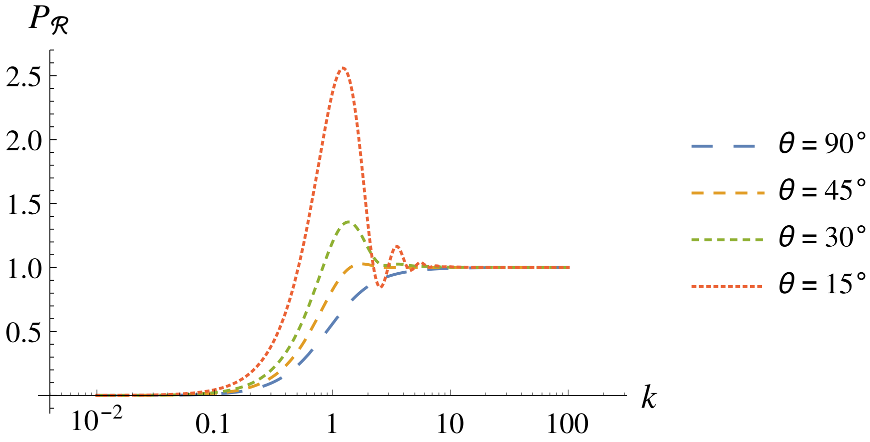
For scalar mode:

$$v'' + \left(k^2(1-\mu^2\tanh^2\eta) - 2\,\mathrm{cosech}^2\,\eta - \frac{1}{4}\,\mathrm{sech}^2\,\eta\right)v = 0,$$

$$k^2 = k_2^2 + k_3^2, \qquad \mu = \hat{\mathbf{k}}\cdot\hat{\mathbf{z}} = \cos\theta$$

$\rightarrow$ solution in terms of Hypergeometric functions.

# Anisotropic primordial power spectrum



# Anisotropic primordial power spectrum

For $k \gg 1$, there are two regimes, depending on the projection of the wavevector onto the old dimensions:

- $k \sin\theta \gtrsim \frac{1}{4} : P \propto 1 + \frac{5}{4} k^{-2} \cos^2\theta$
- $k \sin\theta \lesssim \frac{1}{4} : P \propto \frac{1}{k \sin\theta}$
  - Due to adiabatic vacuum initial conditions.

$P_{+,\times}$ show similar behaviour.

- $h_\times$ can be solved in terms of Heun functions.

# CMB due an anisotropic power spectrum

Usual formulae for $C_\ell$ must be updated, since $P(\mathbf{k}) \neq P(k)$.

- For scalar modes this is not too difficult:

$$C^{(S)XY}_{ll'mm'} = \frac{\delta_{mm'}}{\pi} \int_0^\infty \frac{dk}{k} \Delta^{(S)X}_l(k) \Delta^{(S)Y}_{l'}(k) \tilde{P}^{(S)}_{ll'm}(k),$$

where

$$\tilde{P}^{(S)}_{ll'm}(k) = f_{ll'm} \int_{-1}^{1} d\mu P^m_l(\mu) P^m_{l'}(\mu) P_{\mathcal{R}}(k,\mu).$$

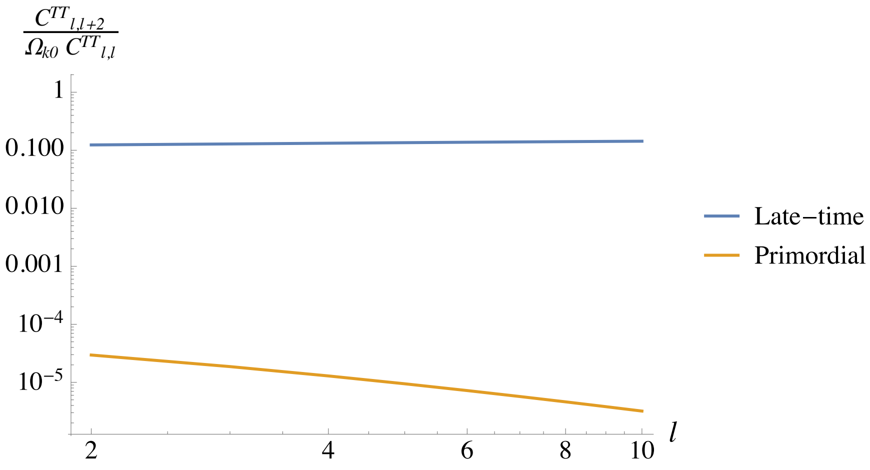
# CMB due an anisotropic power spectrum

$$\tilde{P}^{(S)}_{ll'm}(k) = f_{ll'm} \int_{-1}^{1} d\mu P_l^m(\mu) P_{l'}^m(\mu) P_{\mathcal{R}}(k,\mu).$$

Things to note:

- Reduces to isotropic expression for isotropic $P_{\mathcal{R}}$.
- Parity is not broken, so $C^{(S)}_{ll'mm'} = 0$ for odd $\Delta l$.
- Diagonality in $m$ results from coordinate system aligned with anisotropy direction
  - In general would have to rotate.

# Scalar mode



$\frac{C^{TT}{}_{l,l+2}}{\Omega_{k0}\, C^{TT}{}_{l,l}}$
1
0.100
0.010
0.001
$10^{-4}$
$10^{-5}$
2
4
6
8
10
$l$
Late−time
Primordial

# Scalar mode

Can the primordial effects ever dominate for the scalar modes?

- $\Delta l = 2n$
- Late-time effects $\sim \Omega_{k0}^n$
- Primordial effects $\sim 10^{-3}\Omega_{k0}n^{-4}$
- Signal is very small by this point.

What about the tensor modes?

# CMB due an anisotropic power spectrum

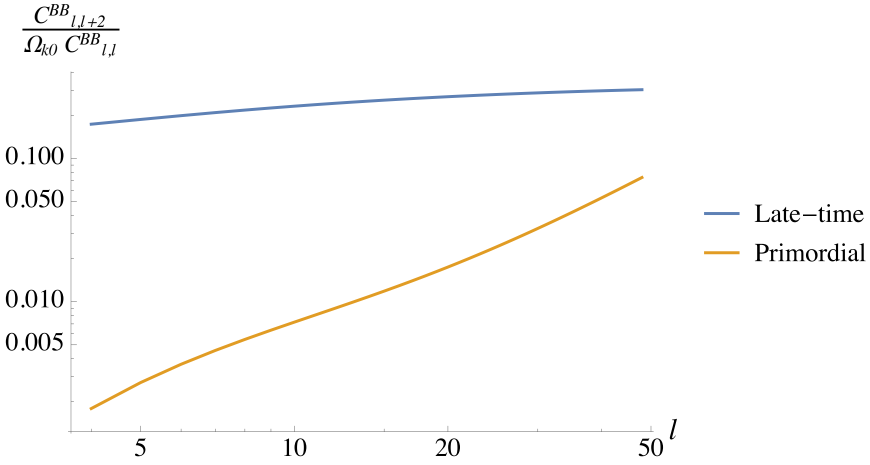
- Tensor modes, being spin-2, are more complicated:

$$C_{ll'mm'}^{(T)XY} = \frac{\delta_{mm'}}{\pi} \int_0^\infty \frac{dk}{k} \Delta_l^{(T)X}(k) \Delta_{l'}^{(T)Y}(k) \tilde{P}_{ll'm}^{(T)}(k),$$

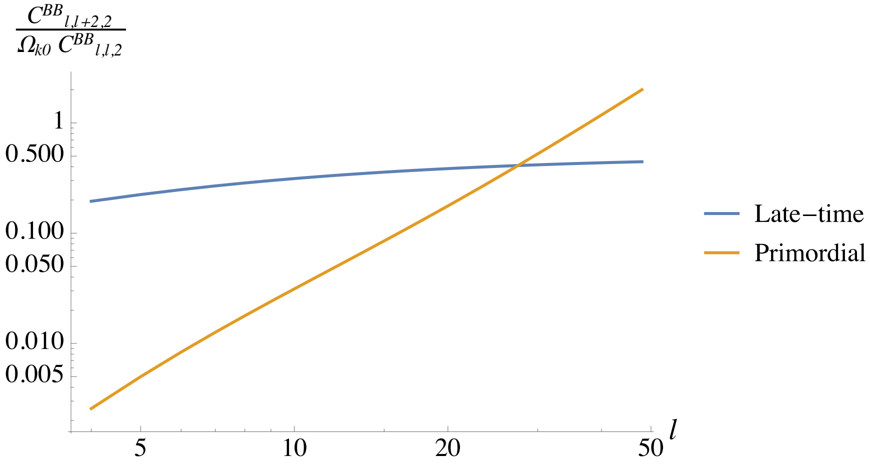
where

$$\tilde{P}_{ll'm}^{(T)}(k) = f_{ll'm} \sum_{i,i'} \beta_{ilm} \beta_{i'l'm} \int_{-1}^{1} d\mu P_{l-i}^{m-2}(\mu) P_{l'-i'}^{m-2}(\mu) \hat{P}^{(T)}(k,\mu).$$

# Tensor modes



# Tensor modes



# $TB$ and $EB$ correlations

$$\tilde{P}^{(T)}_{ll'm}(k) = f_{ll'm} \sum_{i,i'} \beta_{ilm}\beta_{i'l'm} \int_{-1}^{1} d\mu P^{m-2}_{l-i}(\mu) P^{m-2}_{l'-i'}(\mu) \hat{P}^{(T)}(k,\mu),$$

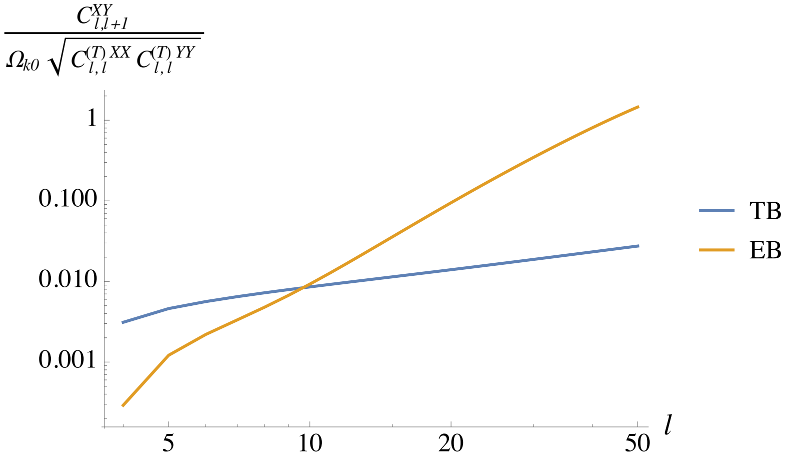
where the following combination of $P_{+,\times}$ sources:

$$\begin{aligned}\hat{P}^{(T)}(k,\mu) =& (1+\sigma_X\sigma_Y(-1)^{\Delta l})(P_+ + P_\times) \\ &+ (-1)^i(\sigma_X + \sigma_Y(-1)^{\Delta l})(P_+ - P_\times).\end{aligned}$$

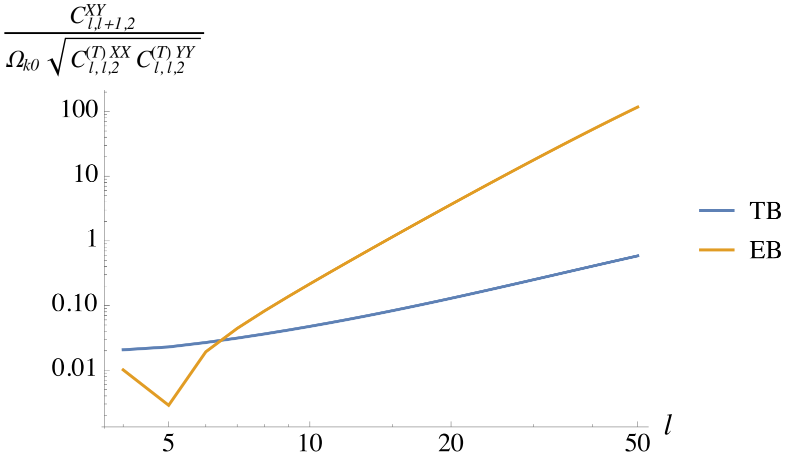
where $\sigma_{T,E} = 1$, $\sigma_B = -1$.

- For even $\Delta l$: correlations as in isotropic case
- For odd $\Delta l$: $TB$, $EB$ correlations are possible!

# $TB$ and $EB$ correlations



# $TB$ and $EB$ correlations



# Summary

- In inflation after false vacuum decay scenarios it is possible to probe some of the features of the parent vacuum.
- Transitions which increase the number of large dimensions are motivated especially from a landscape picture.
- Such transitions lead to an anisotropic universe (in the $2/1+1 \to 3+1$ case).
- Anisotropy makes itself known both at early and late times.
- Whilst primordial anisotropy can be neglected for scalar mode perturbations.
- For the tensor modes it can dominate over the late time effect.
- Such a signal is on the edge of observability.