Strong CP problem and axion on the lattice

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based on

1506.00370 with Nori Yamada (KEK),

1606.07175 with Nori Yamada, Julien Frison, Shingo Mori, Hideo Matsufuru (KEK)

1611.07150 with Nori Yamada, Julien Frison (KEK)

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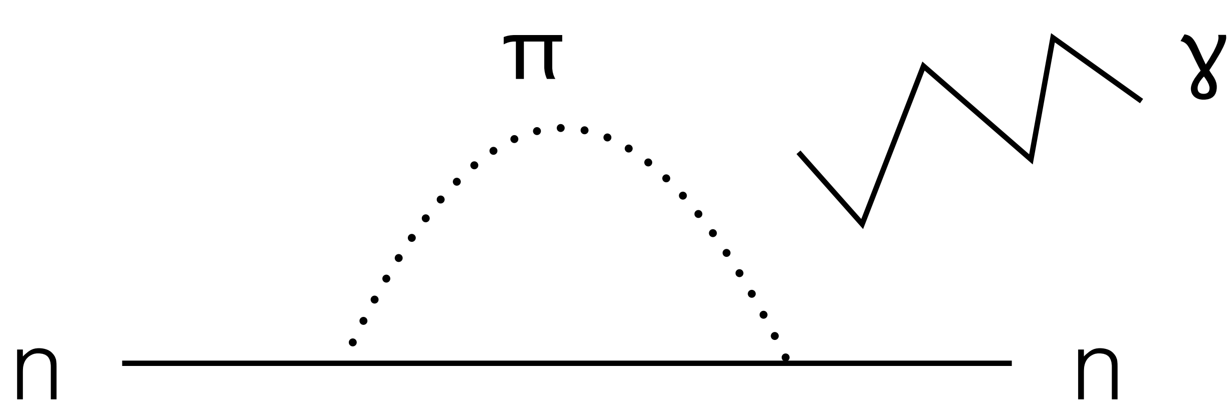
Strong CP problem

$$Z_{\rm QCD} = \int [dA][d\psi][d\bar{\psi}]e^{-S_{\rm QCD}}$$

$$S_{\rm QCD} = \int d^4x \left(\frac{1}{4g^2}F^2 + \frac{i\theta}{32\pi^2}F\tilde{F} + \bar{\psi}(D+m)\psi \right)$$

θ term breaks CP ['t Hooft '76]



$$d_n \sim 10^{-15}\theta e \cdot {\rm cm}$$

$$\theta \lesssim 10^{-10}$$????

[Crewther, Di Vecchia, Veneziano, Witten '79]

Is θ-term really physical?

—> Does the partition function Z depend on θ?

$$\frac{1}{iZ}\frac{dZ}{d\theta}\bigg|_{\theta=0} = \langle\int d^4x \frac{1}{32\pi^2}F\tilde{F}\rangle\bigg|_{\theta=0} = 0$$

(CP)

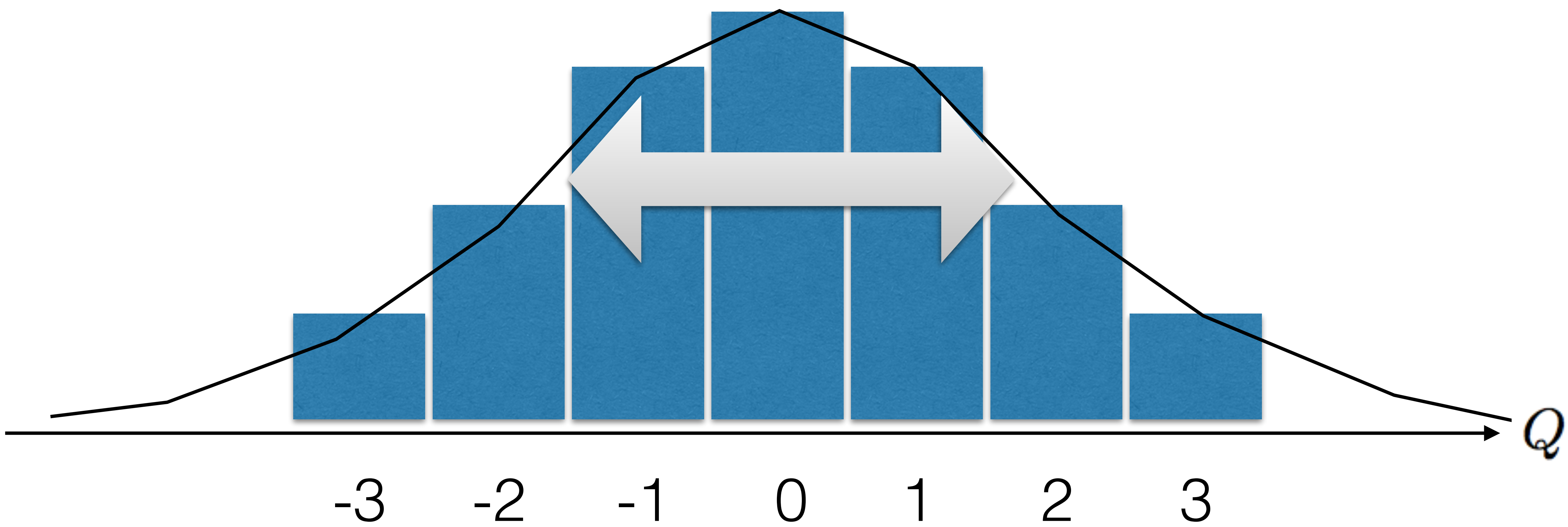
$$= Q$$

(topological charge = integers!)

$$\chi_t = -\frac{1}{V}\frac{1}{Z}\frac{d^2Z}{d\theta^2}\bigg|_{\theta=0} = \frac{\langle Q^2\rangle}{V}$$

(topological susceptibility)

χ_t



$$\langle Q^2 \rangle = \chi_t V$$

χ_t measures how often instantons appear in the path integral.

If χ_t is nonzero, θ is physical.

χ_t and m_u

$$Z_{\rm QCD} = \int [dA][d\psi][d\bar{\psi}]e^{-S_{\rm QCD}} = \int [dA][d\psi][d\bar{\psi}]e^{-S'_{\rm QCD}}$$

$$S_{\rm QCD} = \int d^4x \left(\frac{1}{4g^2}F^2 + \frac{i\theta}{32\pi^2}F\tilde{F} + \bar{\psi}(D+m)\psi \right)$$

$$S'_{\rm QCD} = \int d^4x \left(\frac{1}{4g^2}F^2 + \bar{\psi}(D+me^{-i\gamma_5\theta})\psi \right)$$

$$\chi_t = -\frac{1}{V}\frac{1}{Z}\frac{d^2Z}{d\theta^2}\bigg|_{\theta=0} = -m_u\langle\bar{u}u\rangle + O(m_u^2/m_\pi^2)$$

If m_u is non zero, θ is physical.

If m_u=0, physics does **not** depend on θ.
—> no strong CP problem

$m_u=0$?

[PDG]

LIGHT QUARKS (u, d, s)

OMITTED FROM SUMMARY TABLE

u-QUARK MASS

The *u*-, *d*-, and *s*-quark masses are estimates of so-called "current-quark masses," in a mass- independent subtraction scheme such as $\overline{\text{MS}}$. The ratios m_u/m_d and m_s/m_d are extracted from pion and kaon masses using chiral symmetry. The estimates of *d* and *u* masses are not without controversy and remain under active investigation. Within the literature there are even suggestions that the *u* quark could be essentially massless. The *s*-quark mass is estimated from SU(3) splittings in hadron masses.

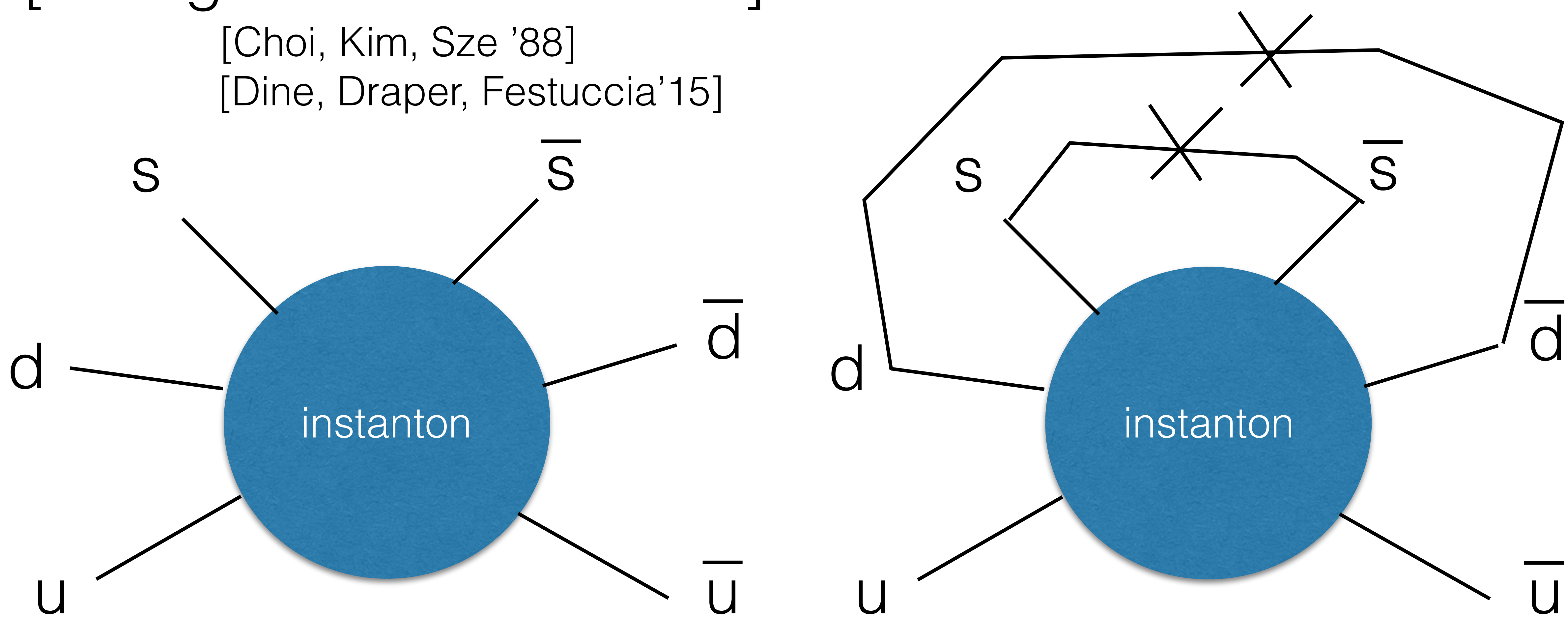
We have normalized the $\overline{\text{MS}}$ masses at a renormalization scale of $\mu = 2$ GeV. Results quoted in the literature at $\mu = 1$ GeV have been rescaled by dividing by 1.35. The values of "Our Evaluation" were determined in part via Figures 1 and 2.

VALUE (MeV)	*DOCUMENT ID*		*TECN*	*COMMENT*
$2.3^{+0.7}_{-0.5}$ OUR EVALUATION	See the ideogram below.			
$2.15 \pm 0.03 \pm 0.10$	[1] DURR	11	LATT	$\overline{\text{MS}}$ scheme
$2.24 \pm 0.10 \pm 0.34$	[2] BLUM	10	LATT	$\overline{\text{MS}}$ scheme

Confusion 1

[Georgi and McArthur ’81]

[Choi, Kim, Sze ’88]
[Dine, Draper, Festuccia’15]



additive shift of $m_u \sim \frac{m_d m_s}{\Lambda_{\rm QCD}} \sim$ MeV

mimic the non-zero mass even if m_u=0?

Confusion 2

[Kaplan and Manohar '86]

$$\begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix} \qquad \begin{pmatrix} m_d m_s & & \\ & m_s m_u & \\ & & m_u m_d \end{pmatrix}$$

these two matrices have the **same** quantum numbers under the chiral symmetry

➡ the chiral Lagrangian cannot distinguish m_u from $m_u + c m_d m_s$

➡ again, mimic nonzero m_u?

Lattice QCD?

Once you break chiral symmetry on the lattice the situation is similar.

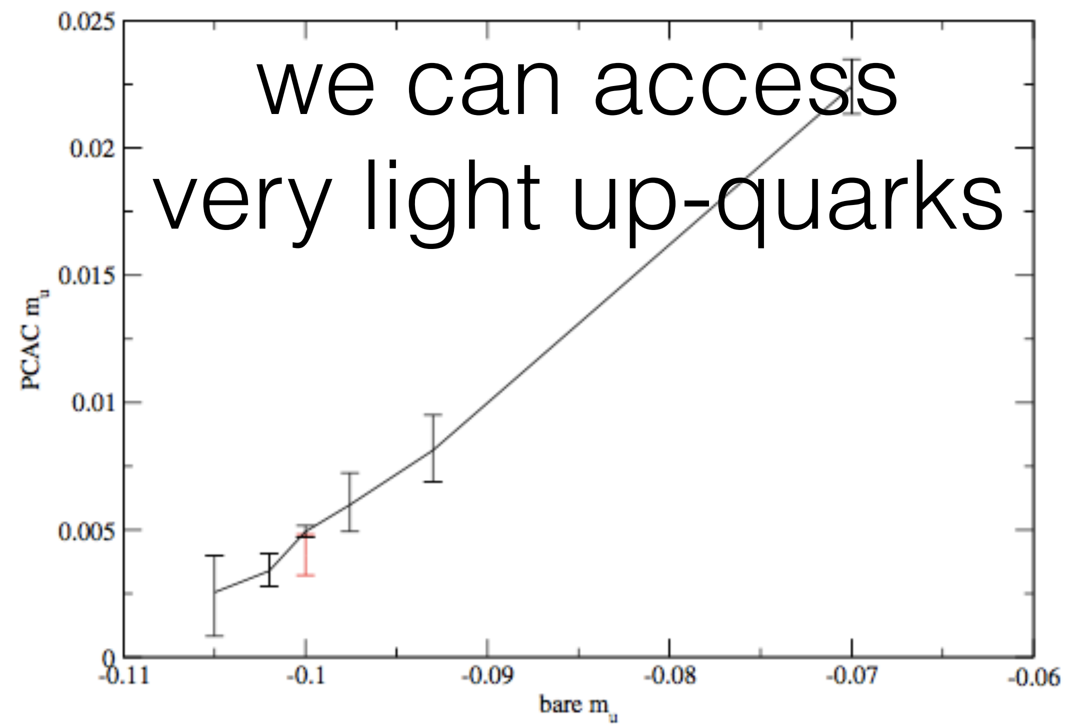
One should define the quark mass so that

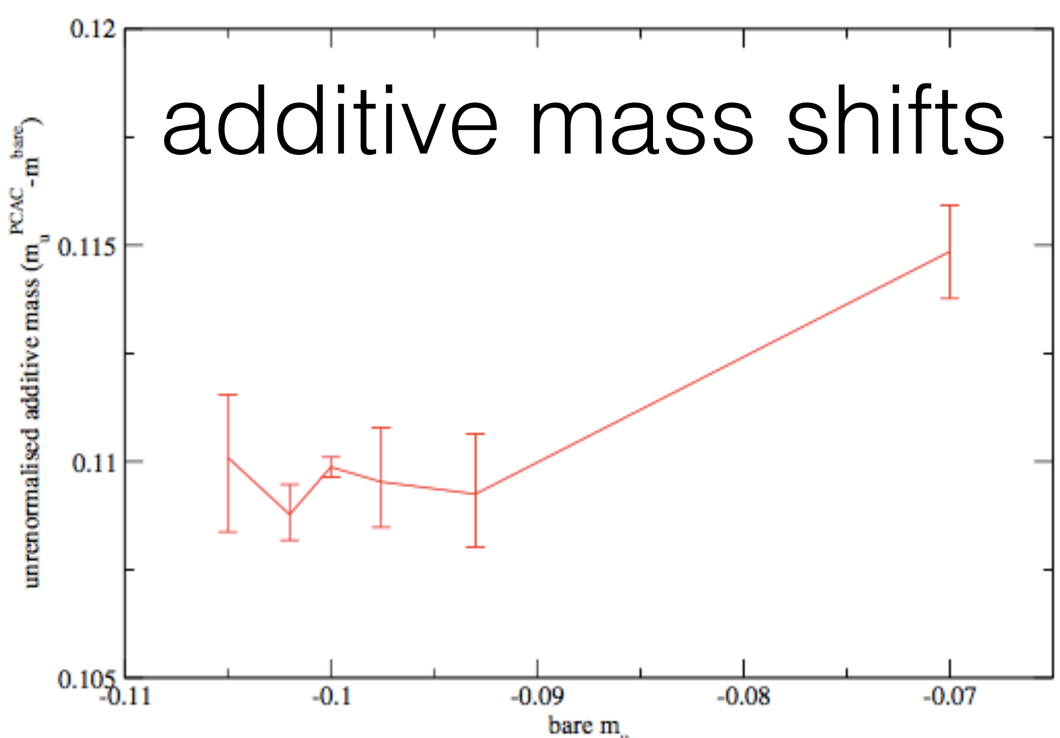
$$\chi_t = -\frac{1}{V}\frac{1}{Z}\frac{d^2 Z}{d\theta^2}\bigg|_{\theta=0} = -m_u \langle \bar{u}u \rangle + O(m_u^2/m_\pi^2)$$

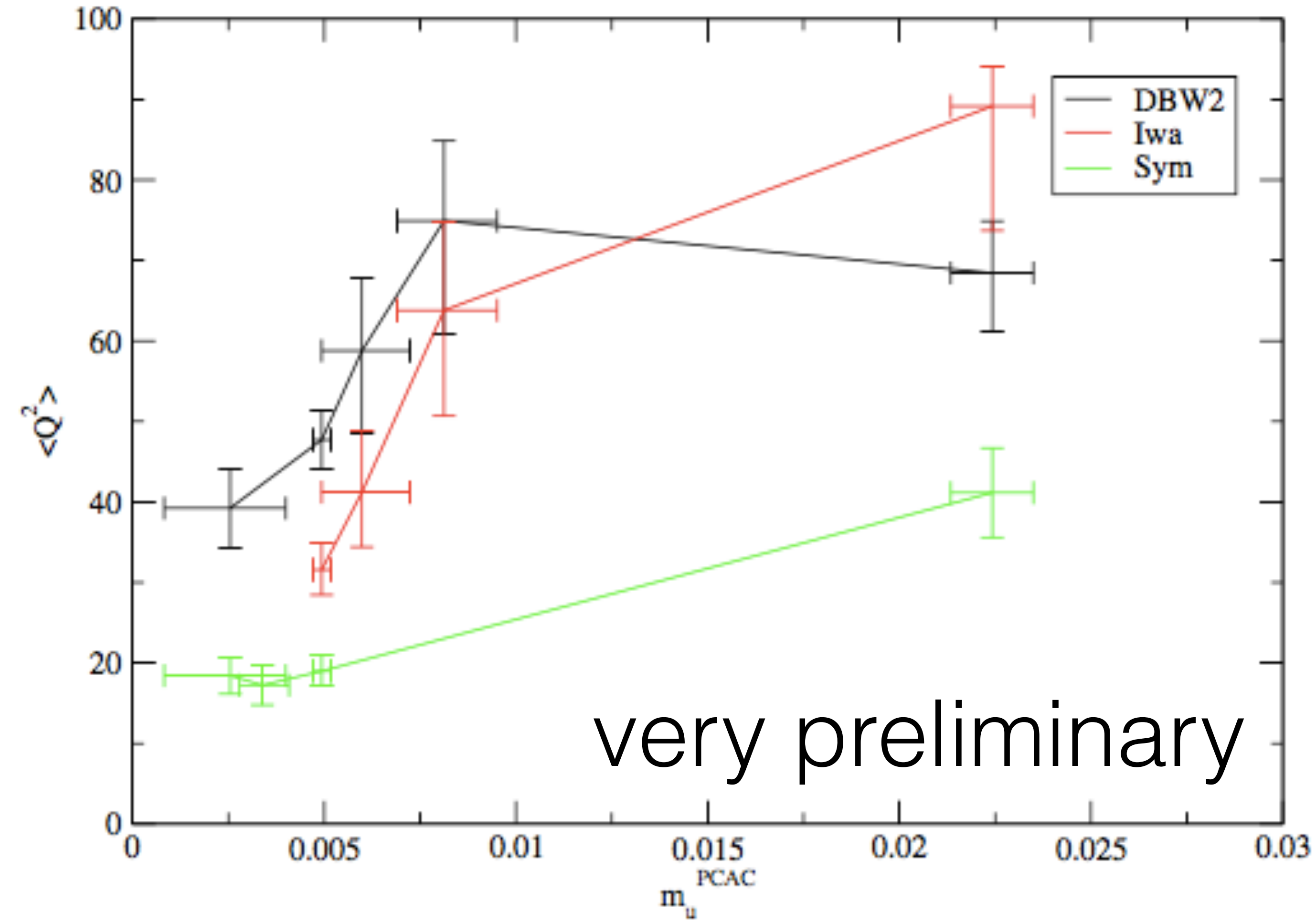
this relation holds in order to establish $m_u \neq 0$ and the strong CP problem is real.

1+2 flavor QCD

(very preliminary yet…) [RK, Yamada, Frison '16]







A large dependence on the definition of Q. We need to study the continuum limit.

Hopefully we can say something soon…

[Peccei, Quinn '77][Weinberg '78][Wilczek '78]

Axion

[Kim '79]
[Shifman, Vainstein, Zakharov '80]
[Zhitnitsky '80]
[Dine, Fischler, Srednicki '81]

OK, maybe m_u is non zero and θ is physical.

Then, why is θ so small?

The axion provides a nice solution.

$$\theta \to \theta + \frac{a(x)}{f_a} \qquad \left(\Delta\mathcal{L} = \frac{ia(x)}{32\pi^2 f_a} F\tilde{F} \right)$$

$$\frac{\chi_t}{f_a^2} = m_a^2$$

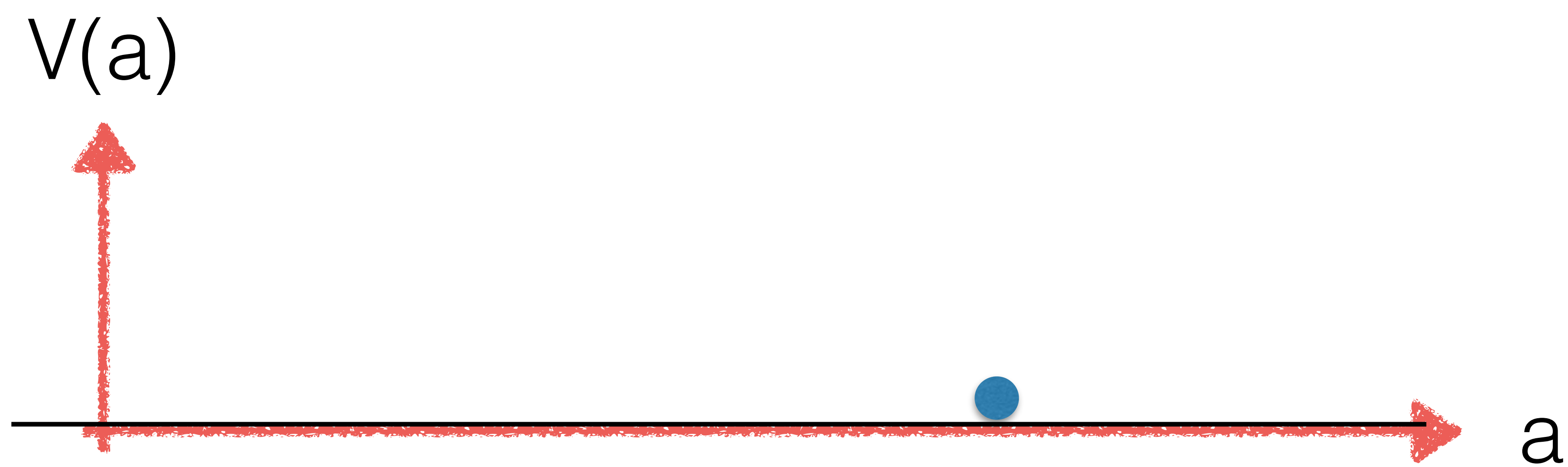
$$\mathcal{L}_{\rm eff} = \frac{\chi_t}{2}\theta^2 + \cdots \quad \Rightarrow \quad \mathcal{L}_{\rm eff} = (\partial_\mu a)^2 + \frac{\chi_t}{2}(\theta + \frac{a}{f_a})^2 + \cdots$$

$$\chi_t > 0 \quad \Rightarrow \quad \theta + \frac{a}{f_a} = 0$$

(dynamically selected)

[Preskill, Wise, Wilczek ’83][Abbott, Sikivie ’83]
[Dine, Fischler ’83]

Axion Dark Matter



$$\ddot{a} + 3H\dot{a} = -V'(a) \sim -m_a^2 a$$

$$\left.\frac{n_a}{T^3}\right|_{\rm now} \sim \frac{m_a(T_*) f_a^2 \theta_{\rm ini}^2}{T_*^3}$$

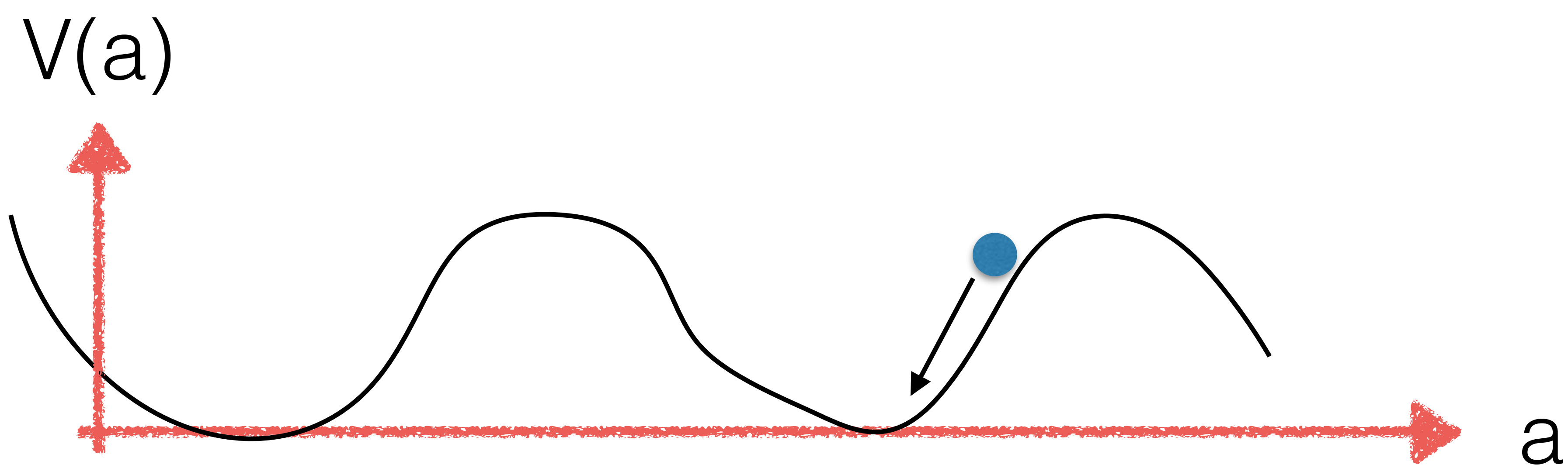
where

$$m_a(T_*) \sim 3H(T_*)$$

temperature dependence of the axion mass
is the essential information to estimate the abundance.

[Preskill, Wise, Wilczek '83][Abbott, Sikivie '83]
[Dine, Fischler '83]

Axion Dark Matter



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where

$$m_a(T_*) \sim 3H(T_*)$$

temperature dependence of the axion mass
is the essential information to estimate the abundance.

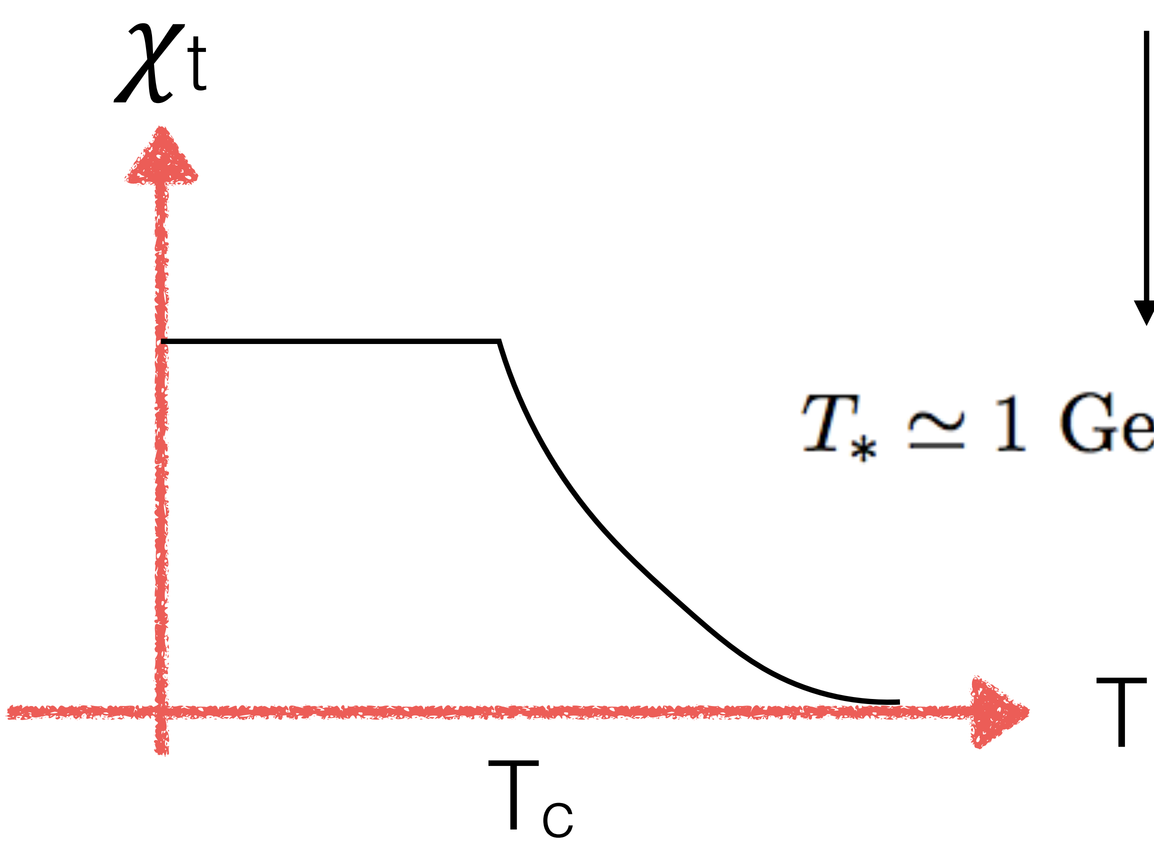
instanton paradigm

The standard way to calculate the temperature dependence of m_a is based on the dilute instanton gas approximation.

[Pisarsky, Yaffe '80]

$$\chi_t(T) = m_a^2(T) f_a^2 \propto m_u m_d m_s \Lambda_{\rm QCD}^b T^{-8} \qquad b = \frac{11}{3}N_c - \frac{2}{3}N_f = 9$$

instanton action $e^{-8\pi^2/g^2}$



$$T_* \simeq 1\ {\rm GeV} \cdot \left(\frac{m_a}{10^{-5}\ {\rm eV}}\right)^{1/6}$$

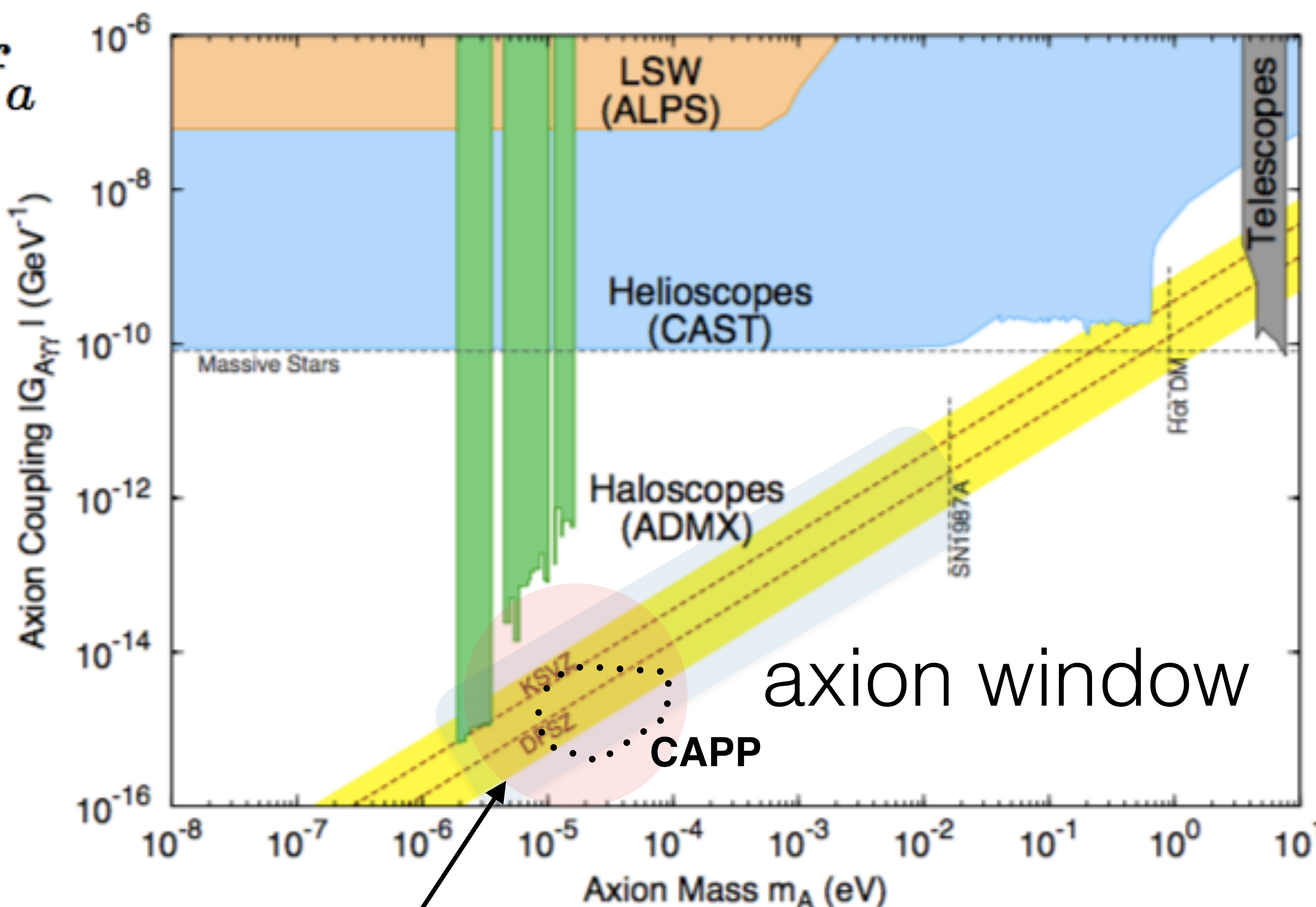
$$\Omega_a \simeq 0.2 \cdot \theta_{\rm ini}^2 \left(\frac{m_a}{10^{-5}\ {\rm eV}}\right)^{-7/6}$$

Axion Dark Matter

$$\Omega_a \simeq 0.2 \cdot \theta_{\rm ini}^2 \left(\frac{m_a}{10^{-5}\ {\rm eV}}\right)^{-7/6}$$

[PDG]

$\sim 1/f_a$



good DM abundance

Is instanton correct?

INSTANTONS, THE QUARK MODEL, AND THE 1/N EXPANSION

Edward WITTEN *
Lyman Laboratory of Physics, Harvard University, Cambridge, MA 02138, USA

Received 2 November 1978

An attempt is made to resolve certain discrepancies between instantons, the quark model and the $1/N$ expansion. It is argued that the most attractive resolution of these discrepancies is the possibility that quantum corrections cause the instanton gas to disappear in QCD. A two-dimensional model is described in which it can be seen explicitly that such a disappearance takes place. (This model has been investigated independently by D'Adda, Di Vecchia, and Lüscher.)

Is instanton correct?

TESTING THE INSTANTON METHOD ☆

Ian AFFLECK [1]
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Received 11 February 1980

The consistency of instanton and large-N methods is demonstrated in the CP^{N-1} model. It is argued that similar behaviour should be expected in QCD.

here are two independent zero
ı [see eq. (9)]. The $T \to \infty$ limit
ın effective action is given by
rared terms:

$^2)^{-1}$ as a sum over discrete modes
ip (of width β) onto S^2. The zero-
then normalizable. However, the
ɔportional to M^2, is not; this is why
ı amount of O(M). See ref. [8].
ɜpired by Jevicki's proof that station-
the constraint Det$(-D^2 + \mathrm{i}\lambda) = 0$,
ıl equations of motion [11]. The
that as $T \to \infty$, the equations of mo-
iλ) to be zero.

$\sim -\cos\theta \exp[-Nf(T)]$, for some function $f(T)$. At very low T [$g^2(T) - N$] the one-loop large-N approximation breaks down [$Nf(T) \to 0$], and no simple formula for $F_T(\theta)$ exists. Finally when $T = 0$, $E(\theta)$ is given by ordinary perturbation theory in $1/N$: $E(\theta)/L \sim \theta^2/N$. In conclusion, the classical instanton analysis is correct in precisely the regime where it should be – weak coupling – *but not elsewhere.*

The large-N method has not been developed in QCD; however, it is an attractive conjecture that $E(\theta)/V \sim \theta^2$ [12]. On the other hand the (classical) caloron [13] analysis has been performed giving a finite result [14], [Cohen '96][Aoki, Fukaya, Taniguchi '12]

Is instanton correct?

Based on $\langle \bar{q}q \rangle = O(m_q)$ at high temperatures and the Ward identities, Cohen has argued

$$\chi_t(T) = O(m_q^4) \qquad \text{for } N_f=2$$

whereas the instanton says

$$\chi_t(T) = O(m_q^2) \qquad \text{for } N_f=2$$

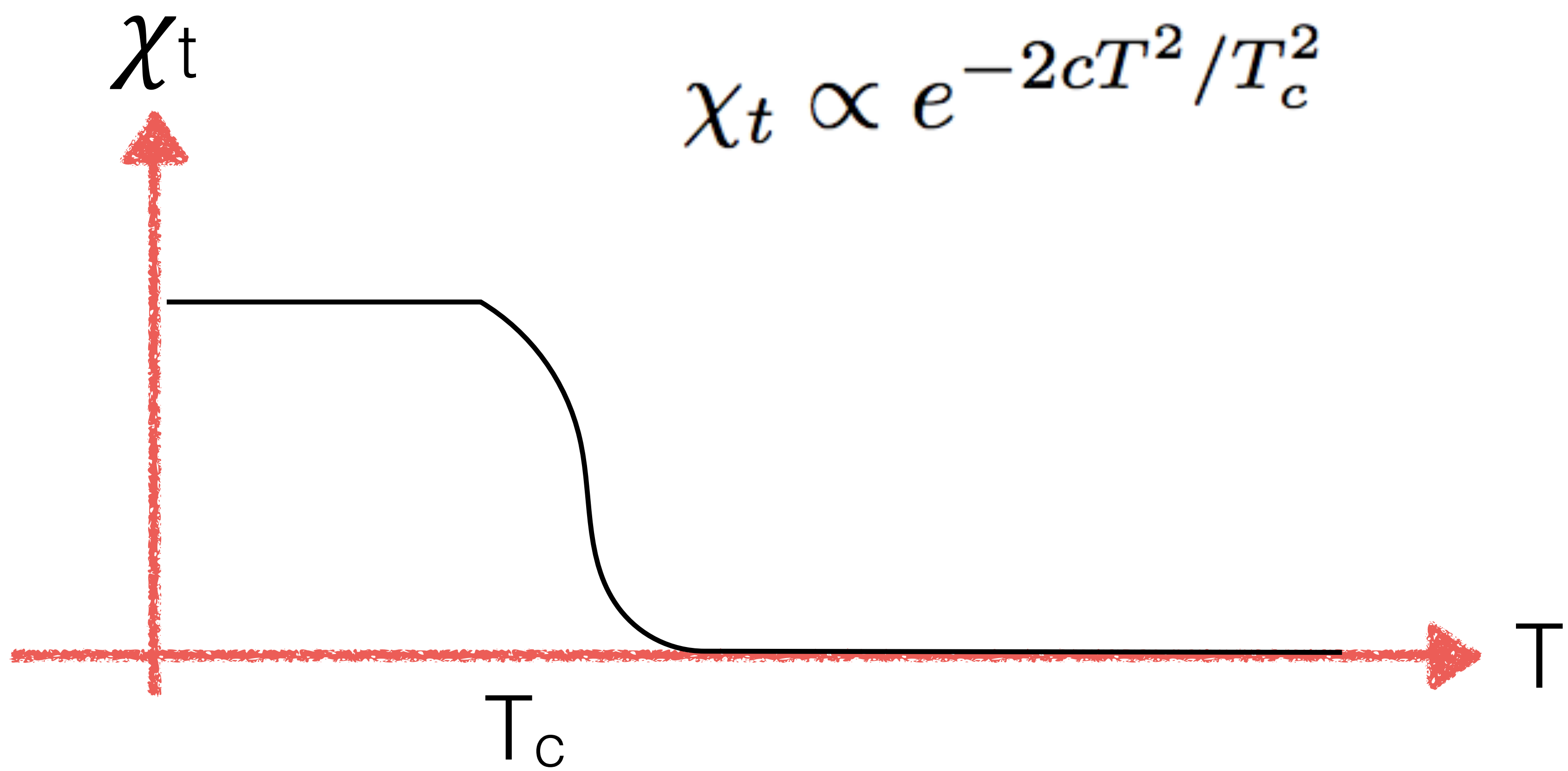
Aoki et al refined the Cohen's analysis and argued

$$\chi_t(T) = 0 \qquad \text{for small but finite } m_q$$

in any case, it is clearly inconsistent with instantons.

if χ_t shuts off very quickly at T_c

the axion suddenly starts to oscillate at $T \sim T_c$



$$\chi_t \propto e^{-2cT^2/T_c^2}$$

$$\Omega_a \sim 0.2\theta_{\rm ini}^2 \left(\frac{m_a}{10^{-5}\ {\rm eV}}\right)^{-1} \times 2.5c \quad (c \gg 1)$$

enhancement due to the non-adiabatic evolution of the potential.

It seems that
the lattice determination of
χ_t is important

χ_t on the lattice

$$\chi_t = \frac{\langle Q^2 \rangle}{V}$$

we just need to measure Q in each configuration.

$$Q = \int d^4x \frac{1}{32\pi^2} F\tilde{F}$$

$$= \mathrm{Tr}\gamma_5 = n_+ - n_- \qquad \gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

There are two ways to measure Q.

Bosonic definition

$$Q = \int d^4x \frac{1}{32\pi^2} F\tilde{F}$$

on the lattice, one would not get integers
due to the ambiguities in the definition of F.

—> The techniques called Cooling or Wilson flow
can make it possible to identify Q.

Fermionic definition

$$Q = \mathrm{Tr}\gamma_5 = n_+ - n_-$$

With a properly defined γ_5, one can get integers.

This method gives unambiguous Q, but the cost of the calculation is high.

Somehow,

three independent calculations appeared recently.

(in the SU(3) Yang-Milles theory, **no quarks yet**)

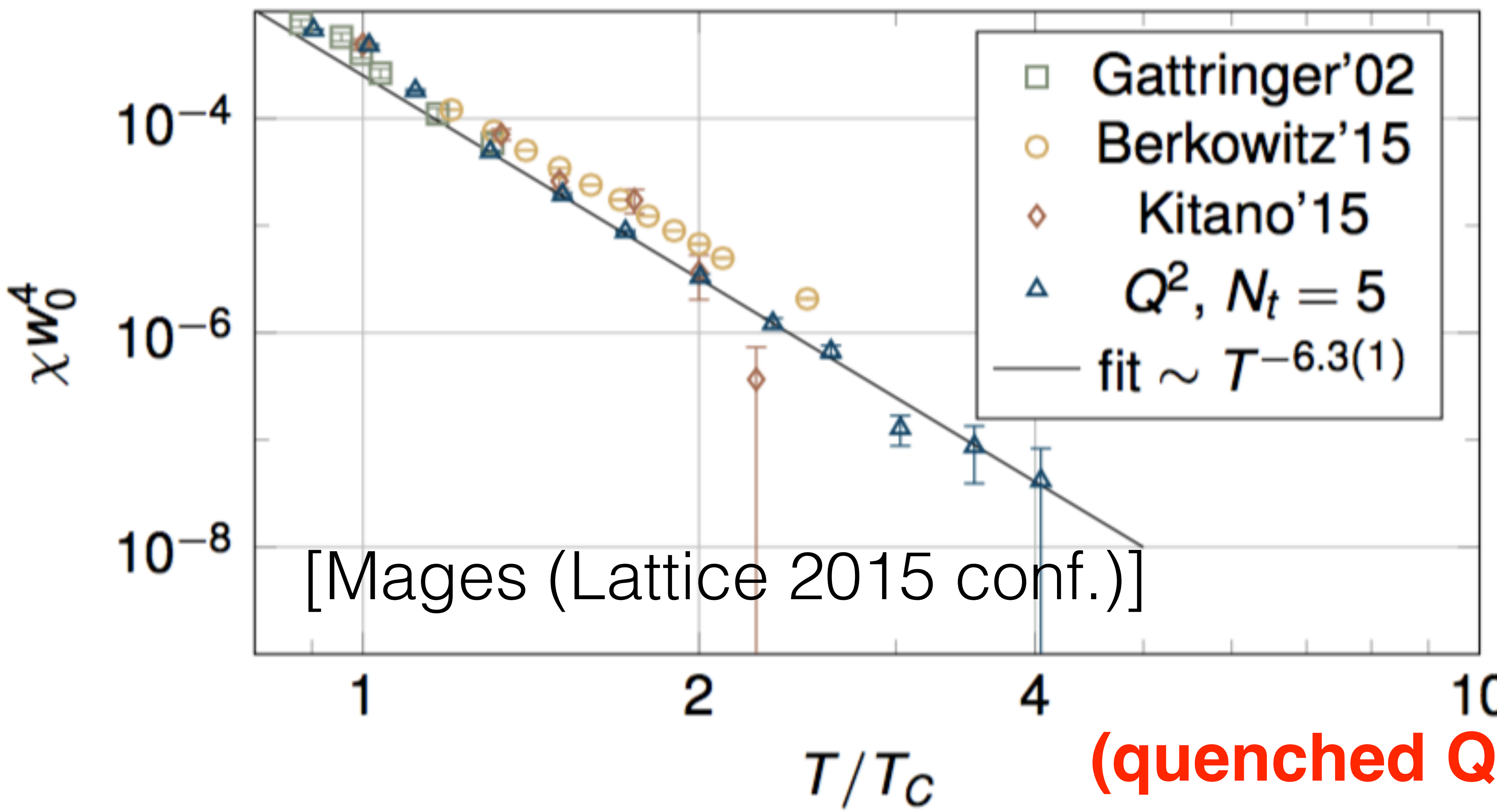
E. Berkowiz, M. Buchoff, E. Rinaldi (LLNL)
Bosonic (cooling)

RK and N. Yamada (KEK) Fermionic (overlap)

S. Mages et al (BMW) Bosonic (Wilson Flow)

All look consistent

(at least qualitatively)



[Mages (Lattice 2015 conf.)]

(quenched QCD!)

We see a clear **power law** even at a very low temperature.

instanton?

The instanton predicts $\chi_t \propto T^{-7}$ for $T \gg T_c$

in SU(3) YM theory

at one-loop level

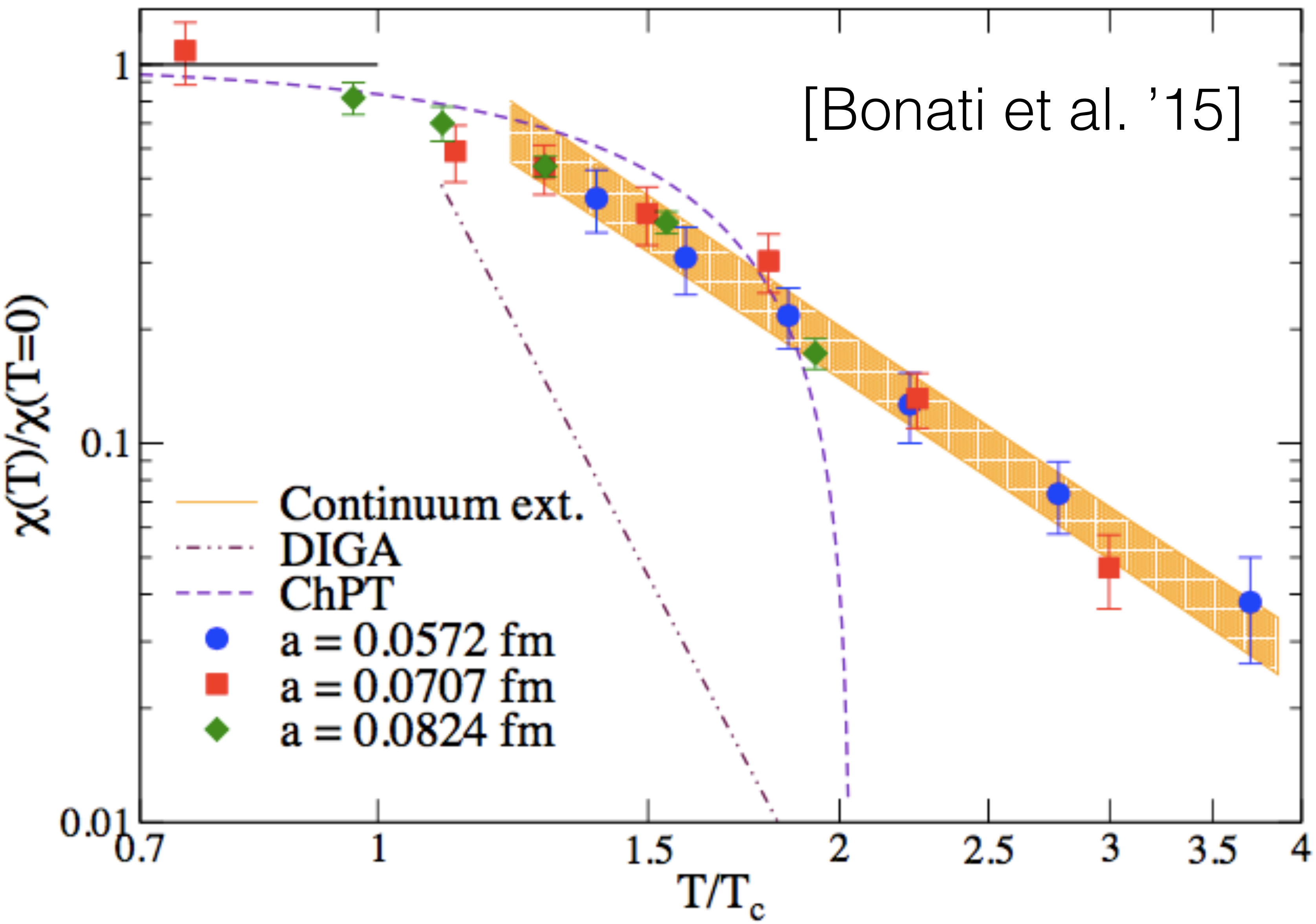
The lattice says

$$\chi_t \propto T^{-6\pm0.?}$$ T ~ 2-4Tc

It seems that the semiclassical instanton picture is qualitatively good in YM theories.

But for the axion study, we need to include quarks.

recent progress



very large deviation from instantons!?

(Fukaya seems to get completely different results by using domain wall+overlap reweighting method.)

problem at high temperature and/or with small quark masses

at high temperatures
and/or small quark masses

$$\langle Q^2 \rangle = \chi_t V \ll 1$$

→ We only see Q=0 configurations

→ We cannot calculate $<Q^2>$

Probably we need some method to improve the calculation further.

directly access the exponent

[Frison, RK, Matsufuru, Mori, Yamada '16]

$$\chi_t V(\beta) \simeq \frac{2Z_1(\beta)}{Z_0(\beta)} \qquad \chi_t(\beta) \propto T^k$$

$$\frac{d\ln Z_Q(T)}{d\ln T} = \left(\frac{d\beta}{d\ln T}\frac{\partial}{\partial\beta} + \frac{d\ln \bar{m}_q}{d\ln T}\frac{\partial}{\partial \ln \bar{m}_q}\right)\ln Z_Q(\beta, \bar{m}_q)$$

$$k = \frac{d\log\chi_t}{d\log T} = \frac{d\beta}{d\log T}(\langle S\rangle_{1,\beta} - \langle S\rangle_{0,\beta}) + 4 + N_f\left(1 + \frac{d\log m_q}{d\log a}\right) m_q\left(\langle\bar{q}q\rangle_{1,\beta} - \langle\bar{q}q\rangle_{0,\beta}\right)$$

instanton prediction is "-b+4-Nf"

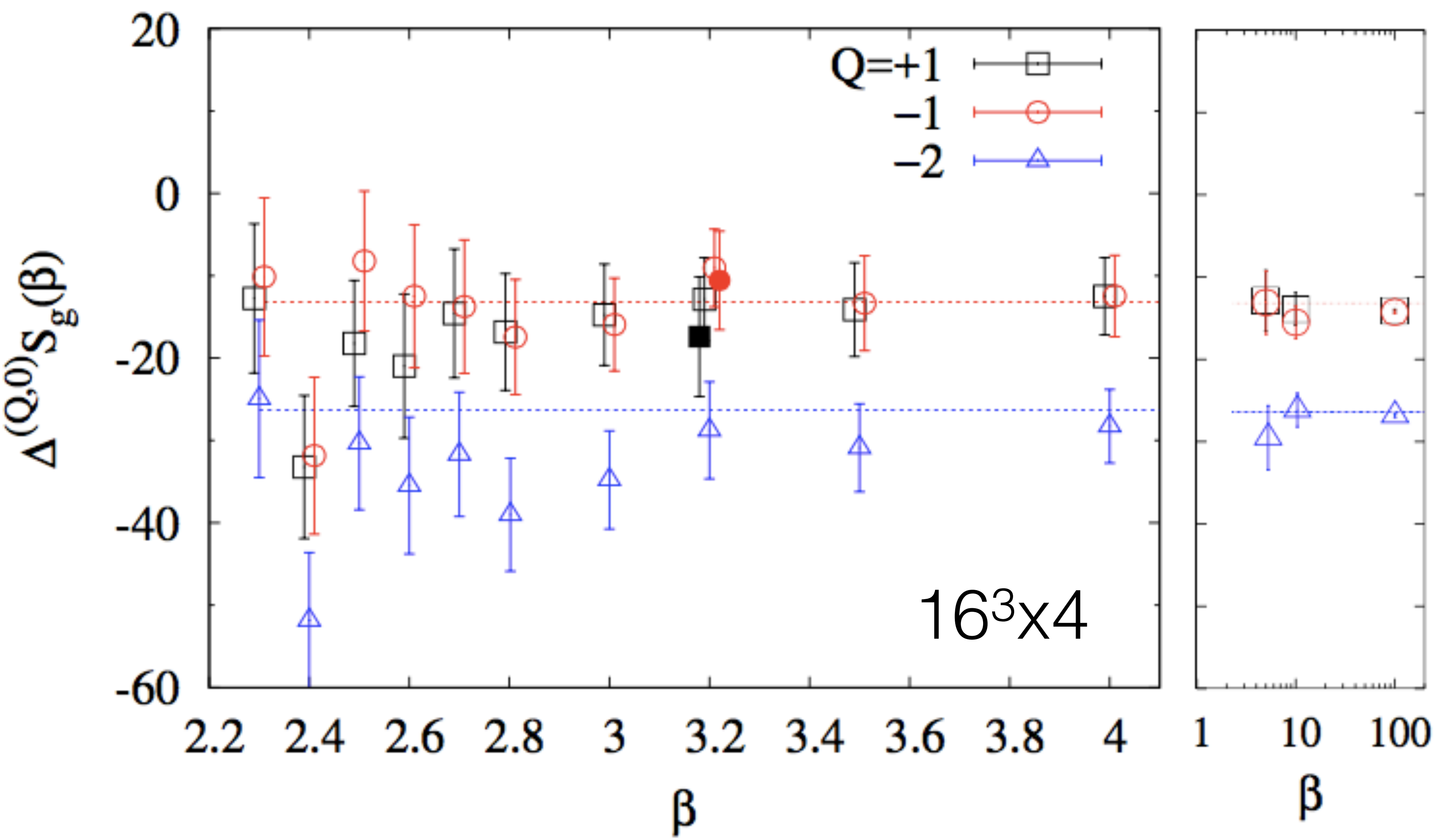
we can measure this by fixing the topology.

[Frison, RK, Matsufuru, Mori, Yamada '16]

Results

$$\Delta^{(Q)}_{S_g} = -\frac{1}{\beta}\left(\langle \hat{S}_g\rangle^{(1)}_{\beta} - \langle \hat{S}_g\rangle^{(0)}_{\beta}\right)$$

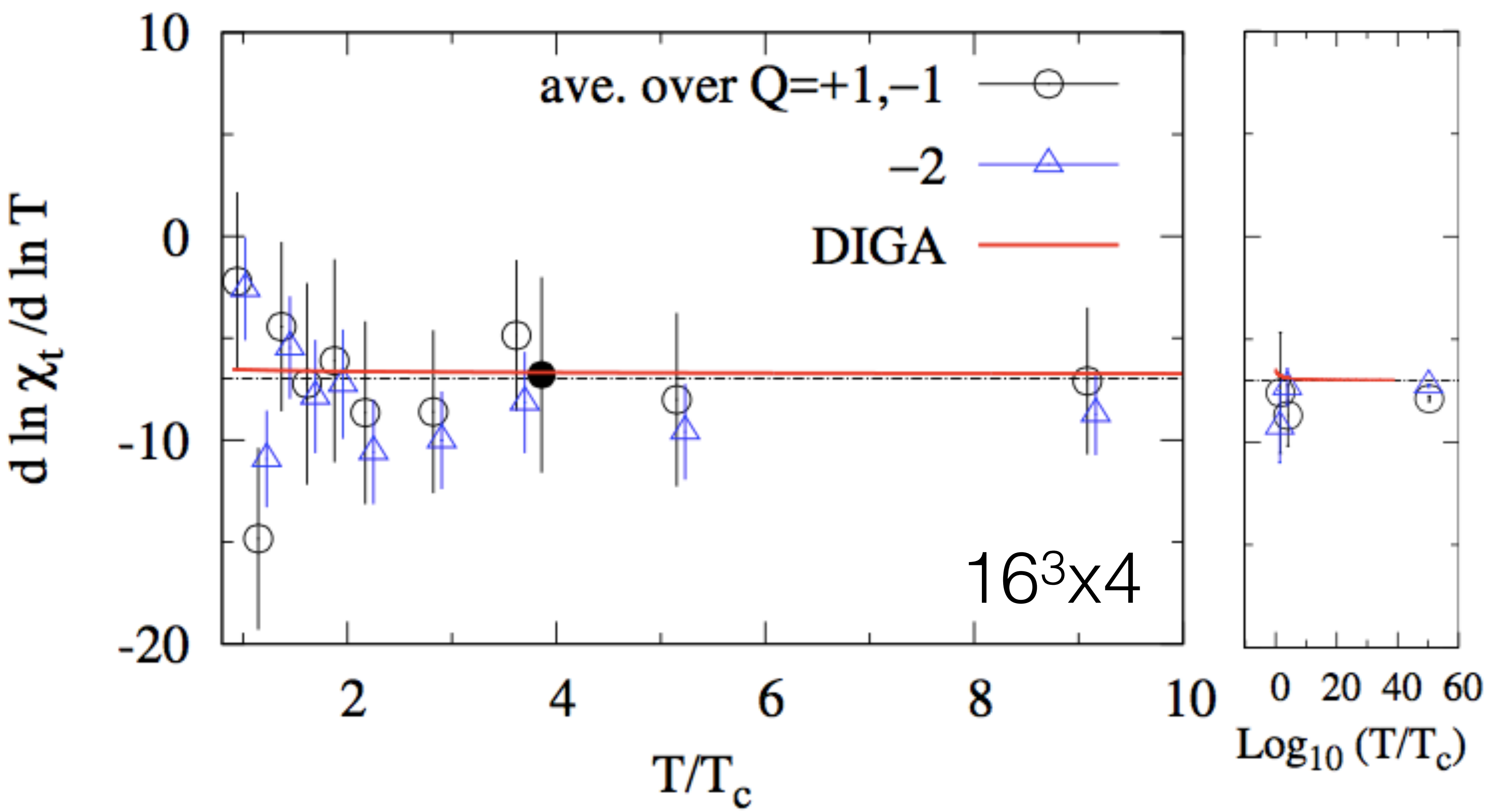
(still quenched…)



[Frison, RK, Matsufuru, Mori, Yamada '16]

results

(still quenched…)

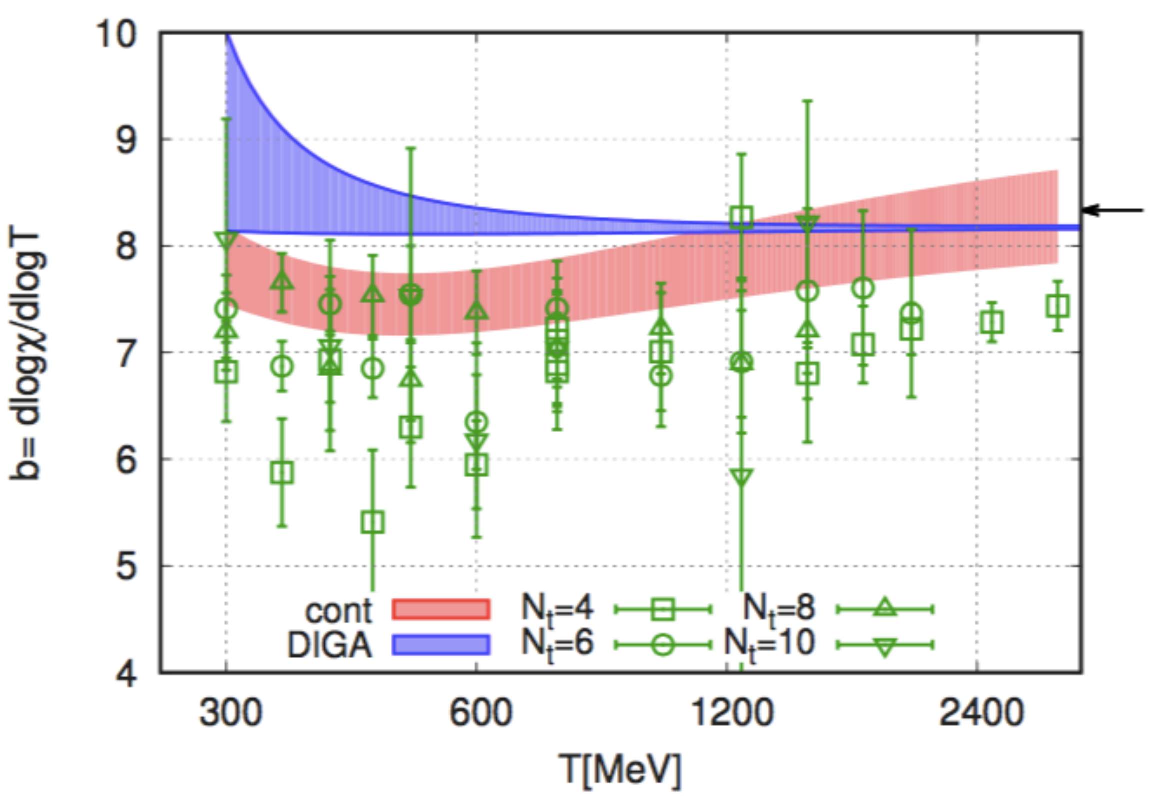


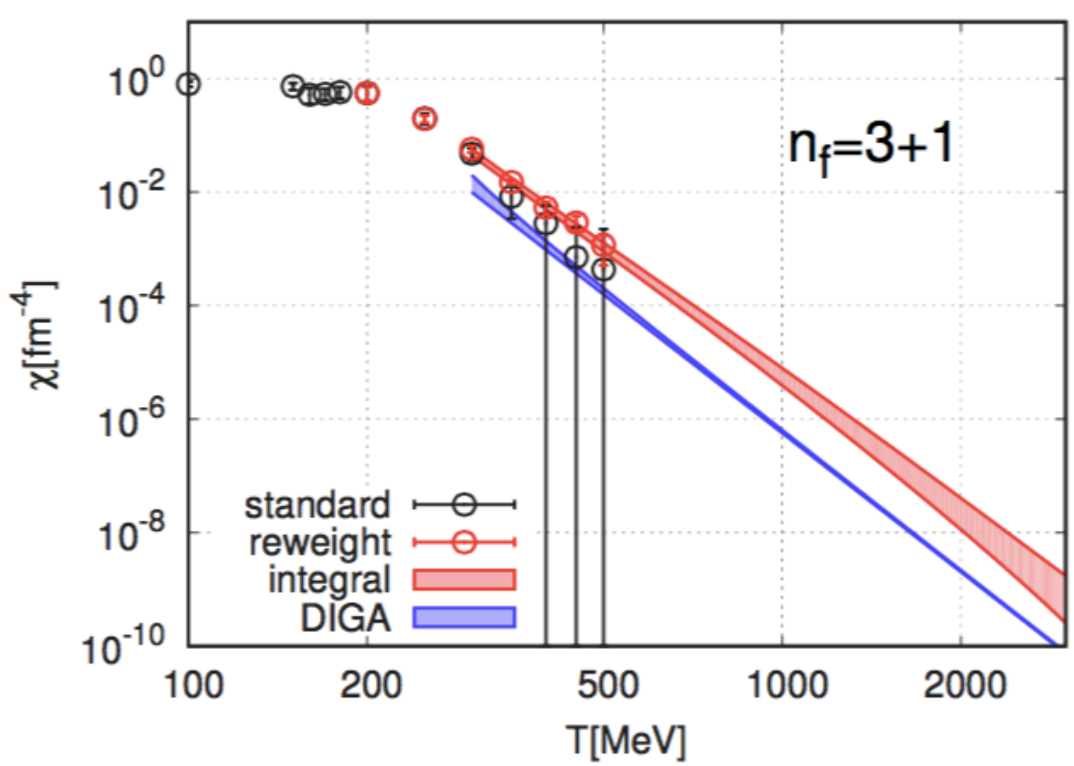
Instanton looks good.

dynamical fermion?

from [1606.07494 Borsanyi et. al.]

$$\langle \bar{\psi}\psi_f \rangle^{\mathrm{rw+zm}}_{Q-0} = \langle \bar{\psi}\psi_f \rangle^{\mathrm{rw}}_{Q-0} + \frac{|Q|}{m_f} - \left\langle \frac{1}{2m_f} \sum_{n=1}^{2|Q|} \frac{4m_f^2}{\lambda_n^2[U] + 4m_f^2} \right\rangle^{\mathrm{rw}}_{Q}$$



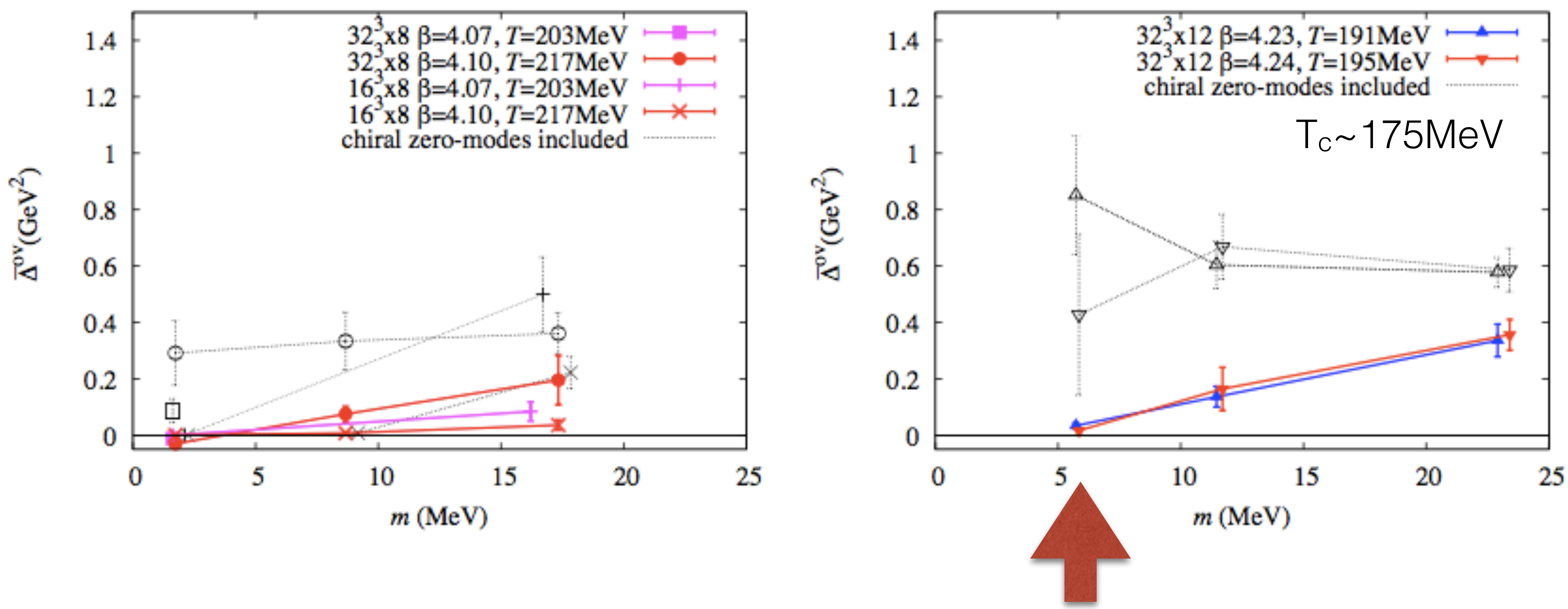


It seems that instanton is good!!
(caveats: finite volume, reweighting+zero mode…)

A more recent results

chiral limit carefully

[Tomiya et al ’16]



$U(1)_A$ violation gets small (zero?) above the phase transition….

Summary

χ_t is a fundamental quantity in QCD which measures the effects of topology.

→ very much related to Strong CP problem

The calculation in YM seems to support the instanton picture, and we probably need more studies with dynamical fermions to make things clearer.